A New Method to Measure the Speed of Gravitation

Dmitri Rabounski
E-mail: rabounski@yahoo.com

According to the standard viewpoint the speed of gravitation is the speed of weak waves of the metrics. This study proposes a new approach, defining the speed as the speed of travelling waves in the field of gravitational inertial force. D'Alembert's equations of the field show that this speed is equal to the velocity of light corrected by gravitational potential. The approach leads to a new experiment to measure the speed of gravitation, which, using "detectors" such as planets and their satellites, is not linked to deviation of geodesic lines and quadrupole mass-detectors with their specific technical problems.

1 Introduction

Herein we use a pseudo-Riemannian space with the signature $(+---)$, where time is real and spatial coordinates are imaginary, because the projection of a four-dimensional impulse on the spatial section of any given observer is positive in this case. We also denote space-time indices in Greek, while spatial indices are Roman. Hence the time term in d'Alembert's operator $\Box = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$ will be positive, while the spatial part (Laplace's operator) will be negative $\Delta = -g^{ik}\nabla_i\nabla_k$.

By applying the d'Alembert operator to a tensor field, we obtain the d'Alembert equations of the field. The non-zero elements are the d'Alembert equations containing the field-inducing sources. The zero elements are the equations without the sources. If there are no sources the field is free, giving a free wave. There is the time term $\frac{1}{a^2}\frac{\partial^2}{\partial t^2}$ containing the linear velocity a of the wave. For this reason, in the case of gravitational fields, the d'Alembert equations give rise to a possibility of calculating the speed of propagation of gravitational attraction (the speed of gravitation). At the same time the result may be different according to the way we define the speed as the velocity of waves of the metric, or something else.

The usual approach sets forth the speed of gravitation as follows [1, 5]. One considers the space-time metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$, composed of a Galilean metric $g^{(0)}_{\alpha\beta}$ (wherein $g^{(0)}_{00} = 1$, $g^{(0)}_{0i} = 0$, $g^{(0)}_{ik} = -\delta_{ik}$) and tiny corrections $\zeta_{\alpha\beta}$ defining a weak gravitational field. Because the $\zeta_{\alpha\beta}$ are tiny, we can raise and lower indices with the Galilean metric tensor $g^{(0)}_{\alpha\beta}$. The quantities $\zeta^{\alpha\beta}$ are defined by the main property of the fundamental metric tensor $g_{\alpha\sigma}g^{\sigma\beta} = \delta^\beta_\alpha$ as follows: $(g^{(0)}_{\alpha\sigma} + \zeta_{\alpha\sigma})\, g^{\sigma\beta} = \delta^\beta_\alpha$. Besides this approach defines $g^{\alpha\beta}$ and $g = \det\|g_{\alpha\beta}\|$ to within higher order terms withheld as $g^{\alpha\beta} = g^{(0)\alpha\beta} - \zeta^{\alpha\beta}$ and $g = g^{(0)}(1+\zeta)$, where $\zeta = \zeta^\sigma_\sigma$. Because $\zeta_{\alpha\beta}$ are tiny we can take Ricci's tensor $R_{\alpha\beta} = R^{\cdots\sigma}_{\alpha\sigma\beta}$ (the Riemann-Christoffel curvature tensor $R_{\alpha\beta\gamma\delta}$ contracted on two indices) to within higher order terms withheld. Then the Ricci tensor for the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$ is

$$R_{\alpha\beta} = \frac{1}{2}\, g^{(0)\mu\nu}\frac{\partial^2\zeta_{\alpha\beta}}{\partial x^\mu \partial x^\nu} = \frac{1}{2}\Box\zeta_{\alpha\beta}\,,$$

which simplifies Einstein's field equations $R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta}R = -\kappa T_{\alpha\beta} + \lambda g_{\alpha\beta}$, where in this case $R = g^{(0)\mu\nu}R_{\mu\nu}$. In the absence of matter and λ-fields ($T_{\alpha\beta} = 0$, $\lambda = 0$), that is, in emptiness, the Einstein equations for the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$ become

$$\Box\zeta^\beta_\alpha = 0\,.$$

Actually, these are the d'Alembert equations of the corrections $\zeta_{\alpha\beta}$ to the metric $g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + \zeta_{\alpha\beta}$ (weak waves of the metric). Taking the flat wave travelling in the direction $x^1 = x$, we see

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\zeta^\beta_\alpha = 0\,,$$

so weak waves of the metric travel at the velocity of light in empty space.

This approach leads to an experiment, based on the principle that geodesic lines of two infinitesimally close test-particles will deviate in a field of waves of the metric. A system of two real particles connected by a spring (a quadrupole mass-detector) should react to the waves. Most of these experiments have since 1968 been linked to Weber's detector. The experiments have not been technically decisive until now, because of problems with precision of measurement and other technical problems [3] and some purely theoretical problems [4, 5].

Is the approach given above the best? Really, the resulting d'Alembert equations are derived from that form of the Ricci tensor obtained under the substantial simplifications of higher order terms withheld (i.e. to first order). Eddington [1] wrote that a source of this approximation is a specific reference frame which differs from Galilean reference frames by the tiny corrections $\zeta_{\alpha\beta}$, the origin of which could be very different from gravitation. This argument leads, as Eddington remarked, to a "vicious circle". So the standard approach has inherent drawbacks, as follows:

(1) The approach gives the Ricci tensor and hence the d'Alembert equations of the metric to within higher order terms withheld, so the velocity of waves of the metric calculated from the equations is not an exact theoretical result;

(2) A source of this approximation are the tiny corrections $\zeta_{\alpha\beta}$ to a Galilean metric, the origin of which may be very different: not only gravitation;

(3) Two bodies attract one another because of the transfer of gravitational force. A wave travelling in the field of gravitational force is not the same as a wave of the metric — these are different tensor fields. When a quadrupole mass-detector registers a signal, the detector reacts to a wave of the metric in accordance with this theory. Therefore it is concluded that quadrupole mass detectors would be the means by to discovery of waves of the metric. However, the experiment is only incidental to the measurement of the speed of gravitation.

For these reasons we are lead to consider gravitational waves as waves travelling in the field of gravitational force, which provides two important advantages:

(1) The mathematical apparatus of chronometric invariants (physical observable quantities in the General Theory of Relativity) defines gravitational inertial force F_i without the Riemann-Christoffel curvature tensor [1, 2]. Using this method, we can deduce the exact d'Alembert equations for the force field, giving an exact formula for the velocity of waves of the force;

(2) Experiments to register waves of the force field, using "detectors" such as planets or their satellites, does not involve a quadrupole mass-detector and its specific technical problems.

2 The new approach

The basis here is the mathematical apparatus of chronometric invariants, created by Zelmanov in the 1940's [1, 2]. Its essence is that if an observer accompanies his reference body, his observable quantities (chronometric invariants) are projections of four-dimensional quantities on his time line and the spatial section, made by projecting operators $b^\alpha = \frac{dx^\alpha}{ds}$ and $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$, which fully define his real reference space. Thus, chr.inv.-projections of a world-vector Q^α are $b_\alpha Q^\alpha = \frac{Q_0}{\sqrt{g_{00}}}$ and $h^i_\alpha Q^\alpha = Q^i$, while chr.inv.-projections of a world-tensor of the 2nd rank $Q^{\alpha\beta}$ are $b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}$, $h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}$, $h^i_\alpha h^k_\beta Q^{\alpha\beta} = Q^{ik}$. Physical observable properties of the space are derived from the fact that the chr. inv.-differential operators $\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{^*\partial}{\partial t}$ are non-commutative. They are the chr.inv.-vector of gravitational inertial force F_i, the chr.inv.-tensor of angular velocities of the space rotation A_{ik}, and the chr.inv.-tensor of rates of the space deformations D_{ik}, namely

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right),$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right),$$

$$v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}\,, \qquad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,,$$

$$D_{ik} = \frac{1}{2}\frac{^*\partial h_{ik}}{\partial t}\,, \quad D^{ik} = -\frac{1}{2}\frac{^*\partial h^{ik}}{\partial t}\,, \quad D = D^k_k = \frac{^*\partial \ln\sqrt{h}}{\partial t}\,,$$

where w is gravitational potential, v_i is the linear velocity of the space rotation, $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ is the chr.inv.-metric tensor, and also $h = \det\|h_{ik}\|$, $\sqrt{-g} = \sqrt{h}\,\sqrt{g_{00}}$, $g = \det\|g_{\alpha\beta}\|$. Observable non-uniformity of the space is set up by the chr.inv.-Christoffel symbols $\Delta^i_{jk} = h^{im}\Delta_{jk,m}$, which are built just like Christoffel's usual symbols $\Gamma^\alpha_{\mu\nu} = g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}$, using h_{ik} instead of $g_{\alpha\beta}$.

The four-dimensional generalization of the chr.inv.-quantities F_i, A_{ik}, and D_{ik} had been obtained by Zelmanov [8] as $F_\alpha = -2c^2 b^\beta a_{\beta\alpha}$, $A_{\alpha\beta} = ch^\mu_\alpha h^\nu_\beta a_{\mu\nu}$, $D_{\alpha\beta} = ch^\mu_\alpha h^\nu_\beta d_{\mu\nu}$, where $a_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha b_\beta - \nabla_\beta b_\alpha\right)$, $d_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha b_\beta + \nabla_\beta b_\alpha\right)$.

Following the study [9], we consider a field of the gravitational inertial force $F_\alpha = -2c^2 b^\beta a_{\beta\alpha}$, the chr.inv.-spatial projection of which is F^i, so that $F_i = h_{ik}F^k$. The d'Alembert equations of the vector field $F^\alpha = -2c^2 b^\beta a_{\beta\cdot}^{\cdot\alpha}$ in the absence of sources are

$$\Box F^\alpha = 0\,.$$

Their chr.inv.-projections (referred to as the chr.inv.-d'Alembert equations) can be deduced as follows

$$b_\sigma\, g^{\alpha\beta}\,\nabla_\alpha\nabla_\beta F^\sigma = 0\,, \qquad h^i_\sigma\, g^{\alpha\beta}\,\nabla_\alpha\nabla_\beta F^\sigma = 0\,.$$

After some algebra we obtain the chr.inv.-d'Alembert equations for the field of the gravitational inertial force $F^\alpha = -2c^2 b^\beta a_{\beta\cdot}^{\cdot\alpha}$ in their final form. They are

$$\frac{1}{c^2}\frac{^*\partial}{\partial t}\left(F_k F^k\right) + \frac{1}{c^2}F_i\frac{^*\partial F^i}{\partial t} + D^k_m\frac{^*\partial F^m}{\partial x^k} + h^{ik}\frac{^*\partial}{\partial x^i}\left[(D_{kn} + A_{kn})F^n\right] - \frac{2}{c^2}A_{ik}F^iF^k + \frac{1}{c^2}F_mF^m D + \Delta^m_{kn}D^k_m F^n - h^{ik}\Delta^m_{ik}\left(D_{mn} + A_{mn}\right)F^n = 0\,,$$

$$\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2} - h^{km}\frac{{}^*\partial^2 F^i}{\partial x^k \partial x^m} + \frac{1}{c^2}\left(D_k^i + A_{k\cdot}^{\cdot i}\right)\frac{{}^*\partial F^k}{\partial t} + \frac{1}{c^2}\frac{{}^*\partial}{\partial t}\left[\left(D_k^i + A_{k\cdot}^{\cdot i}\right)F^k\right] + \frac{1}{c^2}D\frac{{}^*\partial F^i}{\partial t} + \frac{1}{c^2}F^k\frac{{}^*\partial F^i}{\partial x^k} + \frac{1}{c^2}\left(D_n^i + A_{n\cdot}^{\cdot i}\right)F^n D + \frac{1}{c^4}F_k F^k F^i + \frac{1}{c^2}\Delta_{km}^i F^k F^m - h^{km}\left\{\frac{{}^*\partial}{\partial x^k}\left(\Delta_{mn}^i F^n\right) + \left(\Delta_{kn}^i \Delta_{mp}^n - \Delta_{km}^n \Delta_{np}^i\right)F^p + \Delta_{kn}^i \frac{{}^*\partial F^n}{\partial x^m} - \Delta_{km}^n \frac{{}^*\partial F^i}{\partial x^n}\right\} = 0\,.$$

Calling upon the formulae for chr.inv.-derivatives, we transform the first term in the chr.inv.-d'Alembert vector equations into the form

$$\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2} = \frac{1}{c^2 g_{00}}\frac{\partial^2 F^i}{\partial t^2} + \frac{1}{c^4\sqrt{g_{00}}}\frac{{}^*\partial \mathrm{w}}{\partial t}\frac{{}^*\partial F^i}{\partial t}\,,$$

so waves of gravitational inertial force travel at a velocity u^k, the square of which is $u_k u^k = c^2 g_{00}$ and the modulus

$$u = \sqrt{u_k u^k} = c\left(1 - \frac{\mathrm{w}}{c^2}\right).$$

Because waves of the field of gravitational inertial force transfer gravitational interaction, this wave speed is the speed of gravitation as well. The speed depends on the scalar potential w of the field itself, which leads us to the following conclusions:

(1) In a weak gravitational field, the potential w of which is negligible but its gradient F_i is non-zero, the speed of gravitation equals the velocity of light;

(2) According to this formula, the speed of gravitation will be less than the velocity of light near bulky bodies like stars or planets, where gravitational potential is perceptible. On the Earth's surface slowing gravitation will be slower than light by 21 cm/sec. Gravitation near the Sun will be about 6.3×10^4 cm/sec slower than light;

(3) Under gravitational collapse ($\mathrm{w} = c^2$) the speed of gravitation becomes zero.

Let us turn now from theory to experiment. An idea as to how to measure the speed of gravitation as the speed to transfer of the attracting force between space bodies had been proposed by the mathematician Dombrowski [10] in conversation with me more than a decade ago. But in the absence of theory the idea had not developed to experiment in that time. Now we have an exact formula for the speed of waves travelling in the field of gravitational inertial force, so we can propose an experiment to measure the speed (a Weber detector reacts to weak waves of the metric, so it does not apply to this experiment).

The Moon attracts the Earth's surface, causing the flow "hump" in the ocean surface that follows the moving Moon, producing ebbs and flows. An analogous "hump" follows the Sun: its magnitude is more less. A satellite in an Earth orbit has the same ebb and flow oscillations — its orbit rises and falls a little, following the Moon and the Sun as well. A satellite in space experiences no friction, contrary of the viscous waters of the oceans. A satellite is a perfect system, which reacts instantly to the flow. If the speed of gravitation is limited, the moment of the satellite's maximum flow rise should be later than the lunar/solar upper transit by the amount of time taken by waves of the gravitational force field to travel from the Moon/Sun to the satellite.

The Earth's gravitational field is not absolutely symmetric, because of the imperfect form of the terrestrial globe. A real satellite reacts to the field defects during its orbital flight around the Earth — the height of its orbit oscillates in decimetres, giving rise to substantial noise in the experiment. For this reason a geostationary satellite would be best. Such a satellite, having an equatorial orbit, requires an angular velocity the same as that of the Earth. As a result, the height of a geostationary satellite above the Earth does not depend on non-uniformities of the Earth's gravitational field. The height could be measured with high precision by a laser range-finder, almost without interruption, providing a possibility of registering the moment of the maximum flow rise of the satellite, perfectly.

In accordance with our formula the speed of gravitation near the Earth is 21 cm/sec less than the velocity of light. In this case the maximum of the lunar flow wave in a satellite orbit will be about 1 sec later than the lunar upper culmination. The lateness of the flow wave of the Sun will be about 500 sec after the upper transit of the Sun. The question is how precisely could the moment of the maximum flow rise of a satellite in its orbit be determined, because the real maximum can be "fuzzy" in time.

3 Effect of the curvature

If a space is homogeneous ($\Delta_{km}^i = 0$) and it is free of rotation and deformation ($A_{ik} = 0$, $D_{ik} = 0$), then the chr.inv.-d'Alembert equations for the field of gravitational inertial force take the form

$$\frac{1}{c^2}\frac{{}^*\partial}{\partial t}\left(F_k F^k\right) + \frac{1}{c^2}F_i\frac{{}^*\partial F^i}{\partial t} = 0\,,$$

$$\frac{1}{c^2}\frac{{}^*\partial^2 F^i}{\partial t^2} - h^{km}\frac{{}^*\partial^2 F^i}{\partial x^k \partial x^m} + \frac{1}{c^2}F^k\frac{{}^*\partial F^i}{\partial x^k} + \frac{1}{c^4}F_k F^k F^i = 0\,,$$

so waves of gravitational inertial force are permitted even in this very simple case.

Are waves of the metric possible in this case or not?

As it is known, waves of the metric are linked to the space-time curvature derived from the Riemann-Christoffel curvature tensor. If the first derivatives of the metric (the space deformations) are zero, then its second derivatives (the curvature) are zero too. Therefore waves of the metric have no place in a non-deforming space, while waves of gravitational inertial force are possible there.

In connection with this fact, following the study [9], another question arises. By how much does the curvature affect waves of gravitational inertial force?

To answer the question let us recall that Zelmanov, following the same procedure by which the Riemann-Christoffel tensor was introduced, after considering non-commutativity of the chr.inv.-second derivatives of a vector ${}^*\nabla_i{}^*\nabla_k Q_l - {}^*\nabla_k{}^*\nabla_i Q_l = \frac{2A_{ik}}{c^2}\frac{{}^*\partial Q_l}{\partial t} + H_{lki}^{\cdot\cdot\cdot j} Q_j$, had obtained the chr. inv.-tensor $H_{lki}^{\cdot\cdot\cdot j}$ like Schouten's tensor [11]. Its generalization gives the chr.inv.-curvature tensor $C_{lkij} = \frac{1}{4}\left(H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}\right)$, which has all the properties of the Riemann-Christoffel tensor in the observer's spatial section. So the chr.inv.-spatial projection $Z^{iklj} = -c^2 R^{iklj}$ of the Riemann-Christoffel tensor $R_{\alpha\beta\gamma\delta}$, after contraction twice by h_{ik}, is $Z = h^{il} Z_{il} = D_{ik} D^{ik} - D^2 - A_{ik} A^{ik} - c^2 C$, where $C = C_j^j = h^{lj} C_{lj}$ and $C_{kj} = C_{kij\cdot}^{\cdot\cdot\cdot i} = h^{im} C_{kimj}$ [1].

At the same time, as Synge's well-known book [12] shows, in a space of constant four-dimensional curvature, $K = \text{const}$, we have $R_{\alpha\beta\gamma\delta} = K\left(g_{\alpha\gamma}\, g_{\beta\delta} - g_{\alpha\delta}\, g_{\beta\gamma}\right)$, $R_{\alpha\beta} = -3K g_{\alpha\beta}$, $R = -12K$. With these formulae as a basis, after calculation of the chr.inv.-spatial projection of the Riemann-Christoffel tensor, we deduce that in a constant curvature space $Z = 6c^2 K$. Equating this to the same quantity in an arbitrary curvature space, we obtain a correlation between the four-dimensional curvature K and the observable three-dimensional curvature in the constant curvature space

$$6c^2 K = D_{ik} D^{ik} - D^2 - A_{ik} A^{ik} - c^2 C\,.$$

If the four-dimensional curvature is zero ($K = 0$), and the space does no deformations ($D_{ik} = 0$ — its metric is stationary, $h_{ik} = \text{const}$), then no waves of the metric are possible. In such a space the observable three-dimensional curvature is

$$C = -\frac{1}{c^2} A_{ik} A^{ik},$$

which is non-zero ($C \neq 0$), only if the space rotates ($A_{ik} \neq 0$). If aside of these factors, the space does not rotate, then its observable curvature also becomes zero; $C = 0$. Even in this case the chr.inv.- d'Alembert equations show the presence of waves of gravitational inertial force.

What does this imply? As a matter of fact, gravitational attraction is an everyday reality in our world, so waves of gravitational inertial force transferring the attraction shall be incontrovertible. Therefore we adduce the alternatives:

(1) Waves of gravitational inertial force depend on a curvature of space — then the real space-time is not a space of constant curvature, or,

(2) Waves of gravitational inertial force do not depend on the curvature.

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