Bootstrap Universe from Self-Referential Noise

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We further deconstruct Heraclitean Quantum Systems giving a model for a universe using pregeometric notions in which the end-game problem is overcome by means of self-referential noise. The model displays self-organisation with the emergence of 3-space and time. The time phenomenon is richer than the present geometric modelling.

1 Heraclitean Quantum Systems

From the beginning of theoretical physics in the 6th and 5th centuries BC there has been competition between two classes of modelling of reality: one class has reality explained in terms of things, and the other has reality explained purely in terms of relationships (information).* While in conventional physics a mix of these which strongly favours the "things" approach is currently and very efficaciously used, here we address the problem of the "ultimate" modelling of reality. This we term the *end-game* problem: at higher levels in the phenomenology of reality one chooses economical and effective models — which usually have to be accompanied by meta-rules for interpretation, but at the lower levels we are confronted by the problem of the source of "things" and their rules or "laws". At one extreme we could have an infinite regress of ever different "things", another is the notion of a Platonic world where mathematical things and their rules reside [1]. In both instances we still have the fundamental problem of why the universe "ticks" — that is, why it is more than a mathematical construct; why is it experienced?

This "end-game" problem is often thought of as the unification of our most successful and deepest, but incompatible, phenomenologies: General Relativity and Quantum Theory. We believe that the failure to find a common underpinning of these models is that it is apparently often thought it would be some amalgamation of the two, and not something vastly different. Another difficulty is that the lesson from these models is often confused; for instance from the success of the geometrical modelling of space and time it is often argued that the universe "*is* a 4-dimensional manifold". However the geometrical modelling of time is actually deficient: it lacks much of the experienced nature of time — for it fails to model both the directionality of time and the phenomenon of the (local) "present moment". Indeed the geometrical model might better be thought of as a "historical model" of time, because in histories the notion of direction and present moment are absent — they must be provided by external meta-rules. General relativity then is about possible histories of the universe, and in this it is both useful and successful. Similarly quantum field theories have fields built upon a possible (historical) spacetime, and subjected to quantisation. But such quantum theories have difficulties with classicalisation and the individuality of events — as in the "measurement problem". At best the theory invokes ensemble measurement postulates as external meta-rules. So our present-day quantum theories are also historical models.

The problem of unifying general relativity and quantum theories then comes down to going beyond *historical* modelling, which in simple terms means finding a better model of time. The historical or *being* model of reality has been with us since Parmenides and Zeno, and is known as the Eleatic model. The *becoming* or *processing* model of reality dates back further to Heraclitus of Ephesus (540 – 480 BC) who argued that common sense is mistaken in thinking that the world consists of stable things; rather the world is in a state of flux. The appearances of "things" depend upon this flux for their continuity and identity. What needs to be explained, Heraclitus argued, is not change, but the appearance of stability.

Although "process" modelling can be traced through to the present time it has always been a speculative notion because it has never been implemented in a mathematical form and subjected to comparison with reality. Various proposals of a *pregeometric* nature have been considered [2, 3, 4]. Here we propose a mathematical *pregeometric process* model of reality — which in [5] was called a *Heraclitean Quantum System* (HQS). There we arrived at a HQS by deconstruction of the functional integral formulation of quantum field theories retaining only those structures which we felt would not be emergent. In this we still started with "things", namely a Grassmann algebra, and ended with the need to de-

*This is the original 1997 version of the paper which introduced the notion that reality has an *information-theoretic* intrinsic randomness. Since this pioneering paper the model of reality known as *Process Physics* has advanced enormously, and has been confirmed in numerous experiments. The book Cahill, R. T. *Process Physics: From Information Theory to Quantum Space and Matter*, Nova Science Pub. NY 2005, reviews subsequent developments. Numerous papers are available at http://www.mountainman.com.au/process_physics/ and http://www.scieng.flinders.edu.au/cpes/people/cahill_r/processphysics.html compose the mathematical structures into possible histories — each corresponding to a different possible decoherent classical sequencing. However at that level of the HQS we cannot expect anything other than the usual historical modelling of time along with its deficiencies. The problem there was that the deconstruction began with ensembled quantum field theory, and we can never recover individuality and actuality from ensembles — that has been the problem with quantum theory since its inception.

Here we carry the deconstruction one step further by exploiting the fact that functional integrals can be thought of as arising as ensemble averages of Wiener processes. These are normally associated with Brownian-type motions in which random processes are used in modelling many-body dynamical systems. We argue that random processes are a fundamental and necessary aspect of reality — that they arise in the resolution presented here to the end-game problem of modelling reality. In sect. 2 we argue that this "noise" arises as a necessary feature of the self-referential nature of the universe. In sect. 3 we discuss the nature of the self-organised space and time phenomena that arise, and argue that the time modelling is richer and more "realistic" than the geometrical model. In sect. 4 we show how the ensemble averaging of possible universe behaviour is expressible as a functional integral.

2 Self-Referential Noise

Our proposed solution to the end-game problem is to avoid the notion of things and their rules; rather to use a boot-strapped self-referential system. Put simply, this models the universe as a self-organising and self-referential information system — "information" denoting relationships as distinct from "things". In such a system there is no bottom level and we must consider the system as having an iterative character and attempt to pick up the structure by some mathematical modelling.

Chaitin [6] developed some insights into the nature of complex self-referential information systems: combining Shannon's information theory and Turing's computability theory resulted in the development of Algorithmic Information Theory (AIT). This shows that number systems contain randomness and unpredictability, and extends Gödel's discovery, which resulted from self-referenc-ing problems, of the incompleteness of such systems (see [7] for various discussions of the *physics of information*; here we are considering *information as physics*).

Hence if we are to model the universe as a closed system, and thus self-referential, then the mathematical model must necessarily contain randomness. Here we consider one very simple such model and proceed to show that it produces a dynamical 3-space and a theory for time that is richer than the historical/geometrical model.

We model the self-referencing by means of an iterative map

$$\begin{aligned} &B_{ij} \to B_{ij} - (B + B^{-1})_{ij}\,\eta + w_{ij}, \\ &i, j = 1, 2, \ldots, M \to \infty. \end{aligned} \tag{1}$$

We think of B_{ij} as relational information shared by two monads i and j. The monads concept was introduced by Leibniz, who espoused the *relational* mode of thinking in response to and in contrast to Newton's *absolute* space and time. Leibniz's ideas were very much in the *process* mould of thinking: in this the monad's *view* of available information and the commonality of this information is intended to lead to the emergence of space. The monad i acquires its meaning entirely by means of the information $B_{i1}, B_{i2}, \ldots$, where $B_{ij} = -B_{ji}$ to avoid self-information, and real number valued. The map in (1) has the form of a Wiener process, and the $w_{ij} = -w_{ji}$ are independent random variables for each ij and for each iteration, and with variance 2η for later convenience. The w_{ij} model the self-referential noise. The beginning of a universe is modelled by starting the iterative map with $B_{ij} \approx 0$, representing the absence of information or order. Clearly due to the B^{-1} term iterations will rapidly move the B_{ij} away from such starting conditions.

The non-noise part of the map involves B and B^{-1}. Without the non-linear inverse term the map would produce independent and trivial random walks for each B_{ij} — the inverse introduces a linking of all information. We have chosen B^{-1} because of its indirect connection with quantum field theory (see sec. 4) and because of its self-organising property. It is the conjunction of the noise and non-noise terms which leads to the emergence of self-organisation: without the noise the map converges (and this determines the signs in formula 1), in a deterministic manner to a degenerate condensate type structure, discussed in [5], corresponding to a pairing of linear combinations of monads. Hence the map models a non-local and noisy information system from which we extract spatial and time-like behaviour, but we expect residual non-local and random processes characteristic of quantum phenomena including EPR/Aspect type effects. While the map already models some time-like behaviour, it is in the nature of a bootstrap system that we start with *process*. In this system the noise corresponds to the Heraclitean flux which he also called the "cosmic fire", and from which the emergence of stable structures should be understood. To Heraclitus the flame represented one of the earliest examples of the interplay of order and disorder. The contingency and self-ordering of the process clearly suggested a model for reality.

3 Emergent Space and Time

Here we show that the HQS iterative map naturally results in dynamical 3-dimensional spatial structures. Under the mapping the noise term will produce rare large value B_{ij}.

Because the order term is generally much smaller, for small η, than the disorder term these values will persist under the mapping through more iterations than smaller valued B_{ij}. Hence the larger B_{ij} correspond to some temporary background structure which we now identify.

Consider this relational information from the point of view of one monad, call it monad i. Monad i is connected via these large B_{ij} to a number of other monads, and the whole set forms a tree-graph relationship. This is because the large links are very improbable, and a tree-graph relationship is much more probable than a similar graph with additional links. The simplest distance measure for any two nodes within a graph is the smallest number of links connecting them. Let $D_1, D_2, \ldots, D_L$ be the number of nodes of distance $1, 2, \ldots, L$ from node i (define $D_0 = 1$ for convenience), where L is the largest distance from i in a particular tree-graph, and let N be the total number of nodes in the tree. Then $\sum_{k=0}^{L} D_k = N$. See fig.1 for an example.

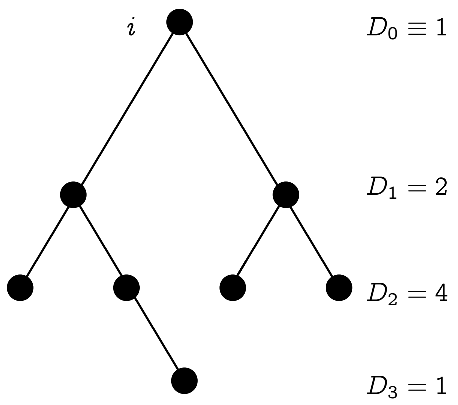

Fig. 1: An $N = 8$, $L = 3$ tree, with indicated distance distributions from monad i.

Now consider the number of different N-node trees, with the same distance distribution $\{D_k\}$, to which i can belong. By counting the different linkage patterns, together with permutations of the monads we obtain

$$\mathcal{N}(D, N) = \frac{(M-1)! D_1^{D_2} D_2^{D_3} \ldots D_{L-1}^{D_L}}{(M-N-2)! D_1! D_2! \ldots D_L!}, \qquad (2)$$

here $D_k^{D_{k+1}}$ is the number of different possible linkage patterns between levels k and $k+1$, and $(M-1)!/(M-N-2)!$ is the number of different possible choices for the monads, with i fixed. The denominator accounts for those permutations which have already been accounted for by the $D_k^{D_{k+1}}$ factors. Nagels [8] analysed $\mathcal{N}(D, N)$, and the results imply that the most likely tree-graph structure to which a monad can belong has the distance distribution

$$D_k \approx \frac{L^2 \ln L}{2\pi^2} \sin^2\left(\frac{\pi k}{L}\right) \quad k = 1, 2, \ldots, L. \qquad (3)$$

for a given arbitrary L value. The remarkable property of this most probable distribution is that the $\sin^2$ indicates that the tree-graph is embeddable in a 3-dimensional hypersphere, S^3. Most importantly, monad i "sees" its surroundings as being 3-dimensional, since $D_k \sim k^2$ for small $\pi k/L$. We call these 3-spaces *gebits* (geometrical bits). We note that the $\ln L$ factor indicates that larger gebits have a larger number density of points.

Now the monads for which the B_{ij} are large thus form disconnected gebits. These gebits however are in turn linked by smaller and more transient B_{kl}, and so on, until at some low level the remaining B_{mn} are noise only; that is they will not survive an iteration. Under iterations of the map this spatial network undergoes growth and decay at all levels, but with the higher levels (larger $\{B_{ij}\}$ gebits) showing most persistence. By a similarity transformation we can arrange the gebits into block diagonal matrices $b_1, b_2, \ldots$, within B, and embedded amongst the smaller and more common noise entries. Now each gebit matrix has $\det(b) = 0$, since a tree-graph connectivity matrix is degenerate. Hence under the mapping the B^{-1} order term has an interesting dynamical effect upon the gebits since, in the absence of the noise, B^{-1} would be singular. The outcome from the iterations is that the gebits are seen to compete and to undergo mutations, for example by adding extra monads to the gebit. Numerical studies reveal gebits competing and "consuming" noise, in a Darwinian process.

Hence in combination the order and disorder terms synthesise an evolving dynamical 3-space with hierarchical structures, possibly even being fractal. This emergent 3-space is entirely relational; it does not arise within any *a priori* geometrical background structure. By construction it is the most robust structure, — however other softer emergent modes of behaviour will be seen as attached to or embedded in this flickering 3-space. The possible fractal character could be exploited by taking a higher level view: identifying each gebit $\rightarrow I$ as a higher level monad, with appropriate informational connections $\mathcal{B}_{IJ}$, we could obtain a higher level iterative map of the form (1), with new order/disorder terms. This would serve to emphasise the notion that in self-referential systems there are no "things", but rather a complex network of iterative relations.

In the model the iterations of the map have the appearance of a cosmic time. However the analysis to reveal the internal experiential time phenomenon is non-trivial, and one would certainly hope to recover the local nature of experiential time as confirmed by special and general relativity experiments. However it is important to notice that the modelling of the time phenomenon here is much richer than that of the historical/geometric model. First the map is clearly uni-directional (there is an "arrow of time") as there is no way to even define an inverse mapping because of the role of the noise term, and this is very unlike the conventional differential equations of traditional physics. In the analysis of the gebits we noted that they show strong persistence, and in that sense the mapping shows a natural partial-memory phenomenon, but the far "future" detailed structure of even this spatial network is completely unknowable without performing the iterations. Furthermore the sequencing of the spatial and other structures is individualistic in that a rerun of the model will always produce a different outcome. Most important of all is that we also obtain a modelling of the "present moment" effect, for the outcome of the next iteration is contingent on the noise. So the system shows overall a sense of a recordable past, an unknowable future and a contingent present moment.

The HQS process model is expected to be capable of a better modelling of our experienced reality, and the key to this is the noisy processing the model requires. As well we need the "internal view", rather than the "external view" of conventional modelling in physics. Nevertheless we would expect that the internally recordable history could be indexed by the usual real-number/geometrical time coordinate.

This new self-referential process modelling requires a new mode of analysis since one cannot use externally imposed meta-rules or interpretations, rather, the internal experiential phenomena and the characterisation of the simpler ones by emergent "laws" of physics must be carefully determined. There has indeed been an ongoing study of how (unspecified) closed self-referential noisy information systems acquire self-knowledge and how the emergent hierarchical structures can "recognise" the same "individuals" [9]. These *Combinatoric Hierarchy* (CH) studies use the fact that only recursive constructions are possible in Heraclitean/Leibnizian systems. We believe that our HQS process model may provide an explicit representation for the CH studies.

4 Possible-Histories Ensemble

While the actual history of the noisy map can only be found in a particular "run", we can nevertheless show that averages over an ensemble of possible histories can be determined, and these have the form of functional integrals. The notion of an ensemble average for any function f of the B, at iteration $c = 1, 2, 3, \ldots$, is expressed by

$$< f[B] >_c = \int \mathcal{D}B f[B]\, \Phi_c[B]\,, \tag{4}$$

where $\Phi_c[B]$ is the ensemble distribution. By the usual construction for Wiener processes we obtain the Fokker-Planck equation

$$\Phi_{c+1}[B] = \Phi_c[B] - \\ -\sum_{ij} \eta \left\{ \frac{\partial}{\partial B_{ij}} \big[(B+B^{-1})_{ij}\, \Phi_c[B]\big] - \frac{\partial^2}{\partial B_{ij}^2} \Phi_c[B] \right\}. \tag{5}$$

For simplicity, in the quasi-stationary regime, we find

$$\Phi[B] \sim \exp\left(-S[B]\right), \tag{6}$$

where the action is

$$S[B] = \sum_{i>j} B_{ij}^2 - \mathrm{TrLn}(B)\,. \tag{7}$$

Then the ensemble average is

$$\frac{1}{Z} \int \mathcal{D}B f[B] \exp\left(-S[B]\right), \tag{8}$$

where Z ensures the correct normalisation for the averages. The connection between (1) and (7) is given by

$$(B^{-1})_{ij} = \frac{\partial}{\partial B_{ji}} \mathrm{TrLn}(B) = \frac{\partial}{\partial B_{ji}} \ln \prod_{\alpha} \lambda_\alpha[B]\,. \tag{9}$$

which probes the sensitivity of the invariant ensemble information to changes in B_{ji}, where the information is in the eigenvalues $\lambda_\alpha[B]$ of B. A further transformation is possible [5]:

$$< f[B] > = \frac{1}{Z} \int \mathcal{D}\overline{m}\,\mathcal{D}m\,\mathcal{D}B\, f[B] \times \\ \times \exp\left[-\sum_{i>j} B_{ij}^2 + \sum_{i,j} B_{ij}(\overline{m}_i m_j - \overline{m}_j m_i)\right] = \\ = \frac{1}{Z} f\left[\frac{\partial}{\partial J}\right] \int \mathcal{D}\overline{m}\,\mathcal{D}m \exp\left[-\sum_{i>j} \overline{m}_i m_j \overline{m}_j m_i + \right. \\ \left. + \textstyle\sum_{ij} J_{ij}(\overline{m}_i m_j - \overline{m}_j m_i\right]. \tag{10}$$

This expresses the ensemble average in terms of an anticommuting Grassmannian algebraic computation [5]. This suggests how the noisy information map may lead to fermionic modes. While functional integrals of the above forms are common in quantum field theory, it is significant that in forming the ensemble average we have lost the contingency or present-moment effect. This always happens — ensemble averages do not tell us about individuals — and then the meta-rules and "interpretations" must be supplied in order to generate some notion of what an individual might have been doing.

The Wiener iterative map can be thought of as a resolution of the functional integrals into different possible histories. However this does not imply the notion that in some sense *all* these histories must be realised, rather only *one* is required. Indeed the basic idea of the process modelling is that of individuality. Not unexpectedly we note that the modelling in (1) must be done from within that *one* closed system.

In conventional quantum theory it has been discovered that the individuality of the measurement process — the "click" of the detector — can be modelled by adding a noise term to the Schrödinger equation [10]. Then by performing an ensemble average over many individual runs of this modified Schrödinger equation one can derive the ensemble measurement postulate — namely $<A> = (\psi, A\psi)$ for the "expectation value of the operator A". This individualising of the ensemble average has been shown to also relate to the decoherence functional formalism [11]. There are a number of other proposals considering noise in spacetime modelling [12, 13].

5 Conclusion

We have addressed here the unique end-game problem which arises when we attempt to model and comprehend the universe as a closed system. The outcome is the suggestion that the peculiarities of this end-game problem are directly relevant to our everyday experience of time and space; particularly the phenomena of the contingent present moment and the three-dimensionality of space. This analysis is based upon the basic insight that a closed self-referential system is necessarily noisy. This follows from Algorithmic Information Theory. To explore the implications we have considered a simple *pregeometric non-linear noisy iterative map*. In this way we construct a process bootstrap system with minimal structure. The analysis shows that the first self-organised structure to arise is a dynamical 3-space formed from competing pieces of 3-geometry — the gebits. The analysis of experiential time is more difficult, but it will clearly be a contingent and process phenomenon which is more complex than the current geometric/historic modelling of time. To extract emergent properties of self-referential systems requires that an internal view be considered, and this itself must be a recursive process. We suggest that the non-local self-referential noise has been a major missing component of our modelling of reality. Two particular applications are an understanding of why quantum detectors "click" and of the physics of consciousness [1], since both clearly have an essential involvement with the modelling of the present-moment effect, and cannot be understood using the geometric/historic modelling of time.

We thank Susan Gunner and Khristos Nizamis for useful comments. Research supported by an ARC Small Grant from Flinders University.

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