Relations Between Physical Constants

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This article discusses the main analytic relationship between physical constants, and applications thereof to cosmology. The mathematical bases herein are group theoretical methods and topological methods. From this it is argued that the Universe was born from an Inversion Explosion of the primordial particle (pre-particle) whose outer radius was that of the classical electron, and inner radius was that of the gravitational radius of the electron. All the mass was concentrated in the space between the radii, and was inverted outside the particle through the pre-particle's surface (the inversion classical radius). This inversion process continues today, determining evolutionary changes in the fundamental physical constants.

As is well known, group theoretical methods, and also topological methods, can be effectively employed in order to interpret physical problems. We know of studies setting up the discrete interior of space-time, and also relationships between atomic quantities and cosmological quantities.

Roberto di Bartini, 1920's (in Italian Air Force uniform)

However, no analytic relationship between fundamental physical quantities has been found. They are determined only by experimental means, because there is no theory that could give a theoretical determination of them.

In this brief article we give the results of our own study, which, employing group theoretical methods and topological methods, gives an analytic relationship between physical constants.

Let us consider a predicative unbounded and hence unique specimen A. Establishing an identity between this specimen A and itself

$$A \equiv A\,, \qquad A\frac{1}{A} = 1\,,$$

is the mapping which transfers images of A in accordance with the pre-image of A.

The specimen A, by definition, can be associated only with itself. For this reason it's inner mapping can, according to Stoilow's theorem, be represented as the superposition of a topological mapping and subsequently by an analytic mapping.

The population of images of A is a point-containing system, whose elements are equivalent points; an n-dimensional affine spread, containing $(n+1)$-elements of the system, transforms into itself in linear manner

$$x'_i = \sum_{k=1}^{n+1} a_{ik} x_k\,.$$

With all a_{ik} real numbers, the unitary transformation

$$\sum_k a^*_{ik} a_{lk} = \sum_k a^*_{ki} a_{kl}\,, \qquad i,k = 1,2,3\ldots,n+1\,,$$

is orthogonal, because $\det a_{ik} = \pm 1$. Hence, this transformation is rotational or, in other words, an inversion twist.

A projective space, containing a population of all images of the object A, can be metrizable. The metric spread R^n (coinciding completely with the projective spread) is closed, according to Hamel's theorem.

A coincidence group of points, drawing elements of the set of images of the object A, is a finite symmetric system, which can be considered as a topological spread mapped into the spherical space R^n. The surface of an $(n+1)$-dimensional sphere, being equivalent to the volume of an n-dimensional torus, is completely and everywhere densely filled by the n-dimensional excellent, closed and finite point-containing system of images of the object A.

The dimension of the spread R^n, which consists only of the set of elements of the system, can be any integer n inside the interval $(1-N)$ to $(N-1)$ where N is the number of entities in the ensemble.

We are going to consider sequences of stochastic transitions between different dimension spreads as stochastic vector

*Brief contents of this paper was presented by Prof. Bruno Pontecorvo to the Proceedings of the Academy of Sciences of the USSR (*Doklady Acad. Sci. USSR*), where it was published in 1965 [19]. Roberto di Bartini (1897–1974), the author, was an Italian mathematician and aircraft engineer who, from 1923, worked in the USSR where he headed an aircraft project bureau. Because di Bartini attached great importance to this article, he signed it with his full name, including his titular prefix and baronial name Oros — from Orosti, the patrimony near Fiume (now Rijeka, located in Croatian territory near the border), although he regularly signed papers as Roberto Bartini. The limited space in the Proceedings did not permit publication of the whole article. For this reason Pontecorvo acquainted di Bartini with Prof. Kyril Stanyukovich, who published this article in his bulletin, in Russian. Pontecorvo and Stanyukovich regarded di Bartini's paper highly. Decades later Stanyukovich suggested that it would be a good idea to publish di Bartini's article in English, because of the great importance of his idea of applying topological methods to cosmology and the results he obtained. (Translated by D. Rabounski and S. J. Crothers.) — Editor's remark.

quantities, i. e. as fields. Then, given a distribution function for frequencies of the stochastic transitions dependent on n, we can find the most probable number of the dimension of the ensemble in the following way.

Let the differential function of distribution of frequencies ν in the spectra of the transitions be given by

$$\varphi(\nu) = \nu^n \exp[-\pi\nu^2]\,.$$

If $n \gg 1$, the mathematical expectation for the frequency of a transition from a state n is equal to

$$m(\nu) = \frac{\int_0^\infty \nu^n \exp[-\pi\nu^2]\,d\nu}{2\int_0^\infty \exp[-\pi\nu^2]\,d\nu} = \frac{\Gamma\left(\frac{n+1}{2}\right)}{2\pi^{\frac{n+1}{2}}}\,.$$

The statistical weight of the time duration for a given state is a quantity inversely proportional to the probability of this state to be changed. For this reason the most probable dimension of the ensemble is that number n under which the function $m(\nu)$ has its minimum.

The inverse function of $m(\nu)$, is

$$\Phi_n = \frac{1}{m(\nu)} = S_{(n+1)} = {}_{\mathrm{T}}V_n\,,$$

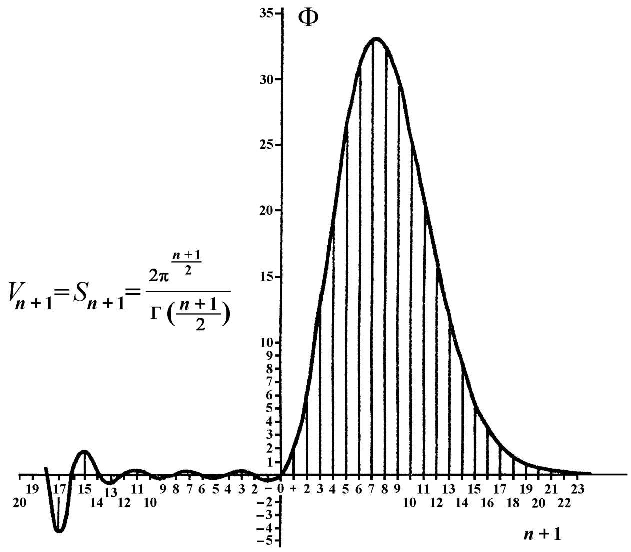
where the function Φ_n is isomorphic to the function of the surface's value $S_{(n+1)}$ of a unit radius hypersphere located in an $(n+1)$-dimensional space (this value is equal to the volume of an n-dimensional hypertorus). This isomorphism is adequate for the ergodic concept, according to which the spatial and time spreads are equivalent aspects of a manifold. So, this isomorphism shows that realization of the object A as a configuration (a form of its real existence) proceeds from the objective probability of the existence of this form.

The positive branch of the function Φ_n is unimodal; for negative values of $(n+1)$ this function becomes sign-alternating (see the figure).

The formation takes its maximum length when $n = \pm 6$, hence the most probable and most unprobable extremal distributions of primary images of the object A are presented in the 6-dimensional closed configuration: the existence of the total specimen A we are considering is 6-dimensional.

Closure of this configuration is expressed by the finitude of the volume of the states, and also the symmetry of distribution inside the volume.

Any even-dimensional space can be considered as the product of two odd-dimensional spreads, which, having the same odd-dimension and the opposite directions, are embedded within each other. Any spherical formation of n dimensions is directed in spaces of $(n+1)$ and higher dimensions. Any odd-dimensional projective space, if immersed in its own dimensions, becomes directed, while any even-dimensional projective space is one-sided. Thus the form of the real existence of the object A we are considering is a $(3+3)$-dimensional complex formation, which is the product of the 3-dimensional spatial-like and 3-dimensional time-like spreads (each of them has its own direction in the $(3+3)$-dimensional complex formation).



One of the main concepts in dimension theory and combinatorial topology is nerve. Using this term, we come to the statement that any compact metric space of n dimensions can be mapped homeomorphicly into a subset located in a Euclidean space of $(2n+1)$ dimensions. And conversely, any compact metric space of $(2n+1)$ dimensions can be mapped homeomorphicly way into a subset of n dimensions. There is a unique correspondence between the mapping $7 \to 3$ and the mapping $3 \to 7$, which consists of the geometrical realization of the abstract complex A.

The geometry of the aforementioned manifolds is determined by their own metrics, which, being set up inside them, determines the quadratic interval

$$\Delta s^2 = \Phi_n^2 \sum_{ik}^{n} g_{ik}\,\Delta x^i \Delta x^k, \qquad i,k = 1,2,\ldots,n\,,$$

which depends not only on the function g_{ik} of coordinates i and k, but also on the function of the number of independent parameters Φ_n.

The total length of a manifold is finite and constant, hence the sum of the lengths of all formations, realized in the manifold, is a quantity invariant with respect to orthogonal transformations. Invariance of the total length of the formation is expressed by the quadratic form

$$N_i r_i^2 = N_k r_k^2\,,$$

where N is the number of entities, r is the radial equivalent of the formation. From here we see, the ratio of the radii is

$$\frac{R\rho}{r^2} = 1\,,$$

where R is the largest radius; ρ is the smallest radius, realised in the area of the transformation; r is the radius of spherical inversion of the formation (this is the calibre of the area). The transformation areas are included in each other, the inversion twist inside them is cascaded

$$\sqrt{\frac{Rr}{2\pi}} = R_{\text{e}}\,, \qquad \sqrt{R\rho} = r\,, \qquad \sqrt{\frac{r\rho}{2\pi}} = \rho_{\text{e}}\,.$$

Negative-dimensional configurations are inversion images, corresponding to anti-states of the system. They have mirror symmetry if $n = l(2m-1)$ and direct symmetry if $n = 2(2m)$, where $m = 1, 2, 3$. Odd-dimensional configurations have no anti-states. The volume of the anti-states is

$$V_{(-n)} = 4\,\frac{-1}{V_n}\,.$$

Equations of physics take a simple form if we use the LT kinematic system of units, whose units are two aspects l and t of the radius through which areas of the space R^n undergo inversion: l is the element of the spatial-like spread of the subspace L, and t is the element of time-like spread of the subspace T. Introducing homogeneous coordinates permits reduction of projective geometry theorems to algebraic equivalents, and geometrical relations to kinematic relations.

The kinematic equivalent of the formation corresponds the following model.

An elementary $(3+3)$-dimensional image of the object A can be considered as a wave or a rotating oscillator, which, in turn, becomes the sink and source, produced by the singularity of the transformation. There in the oscillator polarization of the background components occurs — the transformation $L \to T$ or $T \to L$, depending on the direction of the oscillator, which makes branching L and T spreads. The transmutation $L \leftrightarrow T$ corresponds the shift of the field vector at $\pi/2$ in its parallel transfer along closed arcs of radii R and r in the affine coherence space R^n.

The effective abundance of the pole is

$$e = \frac{1}{2}\,\frac{1}{4\pi}\int_s E ds\,.$$

A charge is an elementary oscillator, making a field around itself and inside itself. There in the field a vector's length depends only on the distance r_i or $1/r_i$ from the centre of the peculiarity. The inner field is the inversion map of the outer field; the mutual correspondence between the outer spatial-like and the inner time-like spreads leads to torsion of the field.

The product of the space of the spherical surface and the strength in the surface is independent of r_i; this value depends only on properties of the charge q

$$4\pi q = S\dot{V} = 4\pi r^2\,\frac{d^2 l}{dt^2}\,.$$

Because the charge manifests in the spread R^n only as the strength of its field, and both parts of the equations are equivalent, we can use the right side of the equation instead of the left one.

The field vector takes its ultimate value

$$c = \frac{l}{t} = \sqrt{\frac{S\dot{V}}{4\pi r_i}} = 1$$

in the surface of the inversion sphere with the radius r. The ultimate value of the field strength lt^{-2} takes a place in the same surface; $\nu = t^{-1}$ is the fundamental frequency of the oscillator. The effective (half) product of the sphere surface space and the oscillation acceleration equals the value of the pulsating charge, hence

$$4\pi q = \frac{1}{2}\,4\pi\nu r_i^2\,\frac{l}{t} = 2\pi r_i c^2.$$

In LT kinematic system of units the dimension of a charge (both gravitational and electric) is

$$\dim m = \dim e = L^3 T^{-2}.$$

In the kinematic system LT, exponents in structural formulae of dimensions of all physical quantities, including electromagnetic quantities, are integers.

Denoting the fundamental ratio l/t as C, in the kinematic system LT we obtain the generalized structural formula for physical quantities

$$D^{\Sigma n} = c^{\gamma} T^{n-\gamma},$$

where $D^{\Sigma n}$ is the dimensional volume of a given physical quantity, Σn is the sum of exponents in the formula of dimensions (see above), T is the radical of dimensions, n and γ are integers.

Thus we calculate dimensions of physical quantities in the kinematic LT system of units (see Table 1).

Physical constants are expressed by some relations in the geometry of the ensemble, reduced to kinematic structures. The kinematic structures are aspects of the probability and configuration realization of the abstract complex A. The most stable form of a kinematic state corresponds to the most probable form of the stochastic existence of the formation.

The value of any physical constant can be obtained in the following way.

The maximum value of the probability of the state we are considering is the same as the volume of a 6-dimensional torus,

$$V_6 = \frac{16\pi^3}{15}\,r^3 = 33.0733588\,r^6.$$

The extreme numerical values — the maximum of the positive branch and the minimum of the negative branches of the function Φ_n are collected in Table 2.

Table 1

Parameter	Σn	Quantity $D^{\Sigma n}$, taken under γ equal to:							
		5	4	3	2	1	0	-1	-2
		C^5T^{n-5}	C^4T^{n-4}	C^3T^{n-3}	C^2T^{n-2}	C^1T^{n-1}	C^0T^{n-0}	$C^{-1}T^{n+1}$	$C^{-2}T^{n+2}$
Surface power				L^3T^{-5}					
Pressure					L^2T^{-4}				
Current density	-2					L^1T^{-3}			
Mass density, angular acceleration							L^0T^{-2}		
Volume charge density								$L^{-1}T^{-1}$	
Electromagnetic field strength					L^2T^{-3}				
Magnetic displacement, acceleration	-1					L^1T^{-2}			
Frequency							L^0T^{-1}		
Power		L^5T^{-5}							
Force			L^4T^{-4}						
Current, loss mass				L^3T^{-3}					
Potential difference	0				L^2T^{-2}				
Velocity						L^1T^{-1}			
Dimensionless constants							L^0T^0		
Conductivity								$L^{-1}T^1$	
Magnetic permittivity									$L^{-2}T^2$
Force momentum, energy		L^5T^{-4}							
Motion quantity, impulse			L^4T^{-3}						
Mass, quantity of magnetism or electricity	$+1$			L^3T^{-2}					
Two-dimensional abundance					L^2T^{-1}				
Length, capacity, self-induction						L^1T^0			
Period, duration							L^0T^1		
Angular momentum, action		L^5T^{-3}							
Magnetic momentum			L^4T^{-2}						
Loss volume	$+2$			L^3T^{-1}					
Surface					L^2T^0				
						L^1T^1			
							L^0T^2		
Moment of inertia		L^5T^{-2}							
			L^4T^{-1}						
Volume of space	$+3$			L^3T^0					
Volume of time							L^0T^3		

Table 2

$n+1$	+7.256946404	−4.99128410
S_{n+1}	+33.161194485	−0.1209542108

The ratio between the ultimate values of the function S_{n+1} is

$$\bar{E} = \frac{\left|+S_{(n+1)_{max}}\right|}{\left|-S_{(n+1)_{min}}\right|} = 274.163208\, r^{12}.$$

On the other hand, a finite length of a spherical layer of R^n, homogeneously and everywhere densely filled by doublets of the elementary formations A, is equivalent to a vortical torus, concentric with the spherical layer. The mirror image of the layer is another concentric homogeneous double layer, which, in turn, is equivalent to a vortical torus coaxial with the first one. Such formations were studied by Lewis and Larmore for the $(3+1)$-dimensional case.

Conditions of stationary vortical motion are realized if

$$V \times \operatorname{rot} V = \operatorname{grad}\varphi\,, \qquad 2v\,ds = d\Gamma\,,$$

where φ is the potential of the circulation, Γ is the main kinematic invariant of the field. A vortical motion is stable only if the current lines coincide with the trajectory of the vortex core. For a $(3+1)$-dimensional vortical torus we have

$$V_x = \frac{\Gamma}{2\pi D}\left[\ln\frac{4D}{r} - \frac{1}{4}\right],$$

where r is the radius of the circulation, D is the torus diameter.

The velocity at the centre of the formation is

$$V_{\odot} = \frac{u\pi D}{2r}\,.$$

The condition $V_x = V_{\odot}$, in the case we are considering, is true if $n = 7$

$$\ln\frac{4D}{r} = (2\pi + 0.25014803)\,\frac{2n+1}{2n} = \\ = 2\pi + 0.25014803 + \frac{n}{2n+1} = 7\,,$$

$$\frac{D}{r} = \bar{E} = \frac{1}{4}\,e^{7} = 274.15836\,.$$

In the field of a vortical torus, with Bohr radius of the charge, $r = 0.999\,9028$, the quantity π takes the numerical value $\pi^* = 0.999\,9514\,\pi$. So $E = \frac{1}{4}e^{6.9996968} = 274.074996$. In the LT kinematic system of units, and introducing the relation $B = V_6 E/\pi = 2885.3453$, we express values of all constants by prime relations between E and B

$$K = \delta\,\widetilde{E}^{\alpha}\,\widetilde{B}^{\beta},$$

where δ is equal to a quantized turn, α and β are integers.

Table 3 gives numerical values of physical constants, obtained analytically and experimentally. The appendix gives experimental determinations in units of the CGS system (cm, gramme, sec), because they are conventional quantities, not physical constants.

The fact that the theoretically and experimentally obtained values of physical constants coincide permits us to suppose that all metric properties of the considered total and unique specimen A can be identified as properties of our observed World, so the World is identical to the unique "particle" A. In another paper it will be shown that a $(3+3)$-dimensional structure of space-time can be proven in an experimental way, and also that this 6-dimensional model is free of logical difficulties derived from the $(3+1)$-dimensional concept of the space-time background*.

In the system of units we are using here the gravitational constant is

$$\kappa = \frac{1}{4\pi}\left[\frac{l^0}{t^0}\right].$$

If we convert its dimensions back to the CGS system, so that $G = \left[\frac{l^3}{mt^2}\right]$, appropriate numerical values of the physical quantities will be determined in another form (Column 5 in Table 3). Reduced physical quantities are given in Column 8. Column 9 gives evolutionary changes of the physical quantities with time according to the theory, developed by Stanyukovich [17]†.

The gravitational "constant", according to his theory, increases proportionally to the space radius (and also the world-time) and the number of elementary entities, according to Dirac [18], increases proportional to the square of the space radius (and the square of world-time as well). Therefore we obtain $N = T_m^2 \simeq B^{24}$, hence $B \approx T_m^{\frac{1}{12}}$.

Because $T_m = t_0\omega_0 \simeq 10^{40}$, where $t_0 \simeq 10^{17}$ sec is the space age of our Universe and $\omega_0 = \frac{c}{\rho} = 10^{23}\,\text{sec}^{-1}$ is the frequency of elementary interactions, we obtain $B \simeq 10^{\frac{10}{3}} = 10^{\frac{1}{3}} \times 1000$.

In this case we obtain $m \sim e^2 \sim \hbar \sim T_m^{-2} \sim B^{-24}$, which is in good agreement with the evolution concept developed by Stanyukovich.

Appendix

Here is a determination of the quantity 1 cm in the CGS system of units. The analytic value of Rydberg constant is

*Roberto di Bartini died before he prepared the second paper. He died sitting at his desk, looking at papers with drawings of vortical tori and draft formulae. According to Professor Stanyukovich, Bartini was not in the habit of keeping many drafts, so unfortunately, we do not know anything about the experimental statement that he planned to provide as the proof to his concept of the $(3+3)$-dimensional space-time background. — D. R.

†Stanyukovich's theory is given in Part II of his book [17]. Here T_{0m} is the world-time moment when a particle (electron, nucleon, etc.) was born, T_m is the world-time moment when we observe the particle. — D. R.

Table 3

Parameter	Notation	Structural formula	$K=\delta E^{\alpha}B^{\beta}$	Analytically obtained numerical values			Observed numerical values in CGS-system	Structural formula in CGS	Dependence on time
				LT-system of units		CGS-system			
Sommerfeld constant	$1/\alpha$	$1/2E$	$2^{-1}\pi^{0}E^{0}B^{0}$	1.370375×10^{2}	$l^{0}t^{0}$	1.370375×10^{2}	$1.370374\times10^{2}\text{cm}^{0}\text{gm}^{0}\text{sec}^{0}$	$\frac{1}{2}E$	const
Gravitational constant	κ	$1/4\pi F^{*}$	$2^{-2}\pi^{-1}E^{0}B^{0}$	7.986889×10^{-2}	$l^{0}t^{0}$	6.670024×10^{-8}	$6.670\times10^{-8}\text{cm}^{3}\text{gm}^{-1}\text{sec}^{-1}$	κ	$\kappa\,\frac{T_m}{T_{0m}}$
Fundamental velocity	c	l/t	$2^{0}\pi^{0}E^{0}B^{0}$	1.000000×10^{0}	$l^{1}t^{-1}$	2.997930×10^{10}	$2.997930\times10^{10}\text{cm}^{1}\text{gm}^{0}\text{sec}^{-1}$	C	const
Mass basic ratio	n/m	$2B/\pi$	$2^{1}\pi^{-1}E^{0}B^{1}$	1.836867×10^{3}	$l^{0}t^{0}$	1.836867×10^{3}	$1.83630\times10^{3}\text{cm}^{0}\text{gm}^{0}\text{sec}^{0}$	$\frac{n}{m}$	$\frac{n}{m}\left(\frac{T_m}{T_{0m}}\right)^{\frac{1}{12}}$
Charge basic ratio	e/m	B^{6}	$2^{0}\pi^{0}E^{0}B^{6}$	5.770146×10^{20}	$l^{0}t^{0}$	5.273048×10^{17}	$5.273058\times10^{17}\text{cm}^{\frac{2}{3}}\text{gm}^{-2}\text{sec}^{\frac{1}{2}}$	$\frac{e}{\sqrt{\kappa m}}$	$\frac{e}{\sqrt{\kappa m}}\left(\frac{T_m}{T_{0m}}\right)^{\frac{1}{2}}$
Gravitational radius of electron	ρ	$r/2\pi B^{12}$	$2^{-1}\pi^{-1}E^{0}B^{-12}$	4.7802045×10^{-43}	$l^{1}t^{0}$	1.346990×10^{-55}	$1.348\times10^{-55}\text{cm}^{1}\text{gm}^{0}\text{sec}^{0}$	S	const
Electric radius of electron	ρ_{e}	$r/2\pi B^{6}$	$2^{-1}\pi^{-1}E^{0}B^{-6}$	2.753248×10^{-21}	$l^{1}t^{0}$	7.772329×10^{-35}	—	S_{e}	$S_{\text{e}}\left(\frac{T_{0m}}{T_m}\right)^{\frac{1}{2}}$
Classical radius of inversion	r	$\sqrt{R\rho}$	$2^{0}\pi^{0}E^{0}B^{0}$	1.000000×10^{0}	$l^{1}t^{0}$	2.817850×10^{-13}	$2.817850\times10^{-13}\text{cm}^{1}\text{gm}^{0}\text{sec}^{0}$	r	const
Space radius	R	$2\pi B^{12}r$	$2^{1}\pi^{1}E^{0}B^{12}$	2.091961×10^{42}	$l^{1}t^{0}$	5.894831×10^{29}	$10^{29}>10^{28}\text{cm}^{1}\text{gm}^{0}\text{sec}^{0}$	R	$R\,\frac{T_m}{T_{0m}}$
Electron mass	m	$2\pi\rho c^{2}$	$2^{0}\pi^{0}E^{0}B^{-12}$	3.003491×10^{-42}	$l^{3}t^{-2}$	9.108300×10^{-28}	$9.1083\times10^{-28}\text{cm}^{0}\text{gm}^{1}\text{sec}^{0}$	κm	$\kappa m\,\frac{T_{0m}}{T_m}$
Nucleon mass	n	$2rc^{2}/\pi B^{11}$	$2^{1}\pi^{-1}E^{0}B^{-11}$	5.517016×10^{-39}	$l^{3}t^{-2}$	1.673074×10^{-24}	$1.67239\times10^{-24}\text{cm}^{0}\text{gm}^{1}\text{sec}^{0}$	κn	$\kappa n\left(\frac{T_{0m}}{T_m}\right)^{\frac{11}{12}}$
Electron charge	e	$2\pi\rho_{\text{e}}c^{2}$	$2^{0}\pi^{0}E^{0}B^{-6}$	1.733058×10^{-21}	$l^{3}t^{-2}$	4.802850×10^{-10}	$4.80286\times10^{-10}\text{cm}^{\frac{3}{2}}\text{gm}^{\frac{1}{2}}\text{sec}^{-1}$	$\sqrt{\kappa e}$	$\sqrt{\kappa e}\left(\frac{T_{0m}}{T_m}\right)^{\frac{1}{2}}$
Space mass	M	$2\pi Rc^{2}$	$2^{2}\pi^{2}E^{0}B^{12}$	1.314417×10^{43}	$l^{3}t^{-2}$	3.986064×10^{57}	$10^{57}>10^{56}\text{cm}^{0}\text{gm}^{1}\text{sec}^{0}$	κM	$\kappa M\,\frac{T_{0m}}{T_m}$
Space period	T	$2\pi B^{12}t$	$2^{1}\pi^{1}E^{0}B^{12}$	2.091961×10^{42}	$l^{0}t^{1}$	1.966300×10^{19}	$10^{19}>10^{17}\text{cm}^{0}\text{gm}^{0}\text{sec}^{1}$	T	$T\,\frac{T_{0m}}{T_m}$
Space density	γ_{k}	$M/2\pi^{2}R^{3}$	$2^{-2}\pi^{-3}E^{0}B^{-24}$	7.273495×10^{-86}	$l^{0}t^{-2}$	9.858261×10^{-34}	$\sim10^{-31}\text{cm}^{-3}\text{gm}^{1}\text{sec}^{0}$	$\kappa\gamma_{\text{k}}$	$\kappa\gamma_{\text{k}}\left(\frac{T_{0m}}{T_m}\right)^{2}$
Space action	H	$Mc2\pi R$	$2^{4}\pi^{4}E^{0}B^{24}$	1.727694×10^{86}	$l^{5}t^{-3}$	4.426057×10^{98}	— $\text{cm}^{2}\text{gm}^{1}\text{sec}^{-1}$	H	const
Number of actual entities	N	R/ρ	$2^{2}\pi^{2}E^{0}B^{24}$	4.376299×10^{84}	$l^{0}t^{0}$	4.376299×10^{84}	$>10^{82}\text{cm}^{0}\text{gm}^{0}\text{sec}^{0}$	N	$N\,\frac{T_m^{2}}{T_{0m}^{2}}$
Number of primary interactions	A	NT	$2^{3}\pi^{3}E^{0}B^{36}$	9.155046×10^{126}	$l^{0}t^{0}$	9.155046×10^{126}	— $\text{cm}^{0}\text{gm}^{0}\text{sec}^{0}$	NT	$NM\left(\frac{T_m}{T_{0m}}\right)^{3}$
Planck constant	$\hbar$	$mc\pi Er$	$2^{0}\pi^{1}E^{1}B^{-12}$	2.586100×10^{-39}	$l^{5}t^{-3}$	6.625152×10^{-27}	$6.62517\times10^{-27}\text{cm}^{2}\text{gm}^{1}\text{sec}^{-1}$	$\kappa\hbar$	$\kappa\hbar\frac{T_{0m}}{T_m}$
Bohr magneton	μ_{b}	$Er^{2}c^{2}/4B^{6}$	$2^{-2}\pi^{0}E^{1}B^{-6}$	1.187469×10^{-19}	$l^{4}t^{-2}$	9.273128×10^{-21}	$9.2734\times10^{-21}\text{cm}^{\frac{5}{2}}\text{gm}^{\frac{1}{2}}\text{sec}^{-1}$	$\sqrt{\kappa\mu}$	$\sqrt{\kappa\mu}\left(\frac{T_{0m}}{T_m}\right)^{\frac{1}{2}}$
Compton frequency	ν_{c}	$c/2\pi Er$	$2^{-1}\pi^{-1}E^{-1}B^{0}$	5.806987×10^{-4}	$l^{0}t^{-1}$	6.178094×10^{19}	$6.1781\times10^{19}\text{cm}^{0}\text{gm}^{0}\text{sec}^{-1}$	$\sqrt{c}$	const

$^{*}F=E/(E-1)=1.003662$

$[R_\infty] = (1/4\pi E^3) l^{-1} = 3.0922328\times10^{-8} l^{-1}$, the experimentally obtained value of the constant is $(R_\infty) = 109737.311\pm$ $\pm 0.012 \text{cm}^{-1}$. Hence 1 cm is determined in the CGS system as $(R_\infty)/[R_\infty] = 3.5488041\times10^{12} l$.

Here is a determination of the quantity 1 sec in the CGS system of units. The analytic value of the fundamental velocity is $[c] = l/t = 1$, the experimentally obtained value of the velocity of light in vacuum is $(c) = 2.997930\pm$ $\pm\, 0.0000080\times10^{-10} \text{cm}\times\text{sec}^{-1}$. Hence 1 sec is determined in the CGS system as $(c)/l\,[c] = 1.0639066\times10^{23} t$.

Here is a determination of the quantity 1 gramme in the CGS system of units. The analytic value of the ratio e/mc is $[e/mc] = \widetilde{B}^6 = 5.7701460\times10^{20} l^{-1} t$. This quantity, measured in experiments, is $(e/mc) = 1.758897\pm0.000032\times10^7$ $(\text{cm}\times\text{gm}^{-1})^{\frac{1}{2}}$. Hence 1 gramme is determined in the CGS system as $\dfrac{(e/mc)^2}{l[e/mc]^2} = 3.297532510\times10^{-15} l^3 t^{-2}$, so CGS' one gramme is 1 gm (CGS) $= 8.351217\times10^{-7} \text{cm}^3 \text{sec}^{-2}$ (CS).

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