

Zelmanov's Anthropic Principle and the Infinite Relativity Principle

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Zelmanov's Anthropic Principle, although introduced in the 1940's, has been published only recently: "The Universe has the interior we observe because we observe the Universe in this way. It is impossible to divorce the Universe from the observer. If the observer is changed, then the observed world will present in some other way, so the Universe observed will also be changed. If no observers exist then the observable Universe as well does not exist." Zelmanov's mathematical apparatus of physical observable quantities employs the Principle to the General Theory of Relativity. Using this apparatus he developed the Infinite Relativity Principle: "In homogeneous isotropic cosmological models spatial infinity of the Universe and infinity of its evolution span depend on our choice of the observer's reference frame."

Abraham Zelmanov (1913–1987), a prominent cosmologist, introduced his Anthropic Principle in the 1940's, but it has been published only recently. It is probable that he reached his ideas not only from his pure mathematical studies on the General Theory of Relativity and relativistic cosmology — besides these he had an excellent knowledge of religious considerations on world-genesis and the origin of humanity. We can now only guess at the way in which he came to his idea of the Anthropic Principle. The fact is that for more than 60 years his Anthropic Principle remained known only a close circle of several of his pupils. His book containing his main fundamental studies on the General Theory of Relativity and relativistic cosmology was written in 1944 and had survived only in manuscript until it has been published in 2004 [1].

Zelmanov stated his Anthropic Principle in two versions. The first version sets forth the law of human evolution dependent upon fundamental physical constants:

Humanity exists at the present day and we observe world constants completely because the constants bear their specific numerical values at this time. When the world constants bore other values humanity did not exist. When the constants change to other values humanity will disappear. That is, humanity can exist only with the specific scale of the numerical values of the cosmological constants. Humanity is only an episode in the life of the Universe. At the present time cosmological conditions are such that humanity develops.

In the second form he argues that any observer depends on the Universe he observes in the same way that the Universe depends on him:

The Universe has the interior we observe, because we observe the Universe in this way. It is impossible to divorce the Universe from the observer. The observable Universe depends on the observer and the observer depends on the Universe. If the contemporary

physical conditions in the Universe change then the observer is changed. And vice versa, if the observer is changed then he will observe the world in another way. So the Universe he observes will be also changed. If no observers exist then the observable Universe as well does not exist.

It is probable that by proceeding from his Anthropic Principle, in 1941–1944 Zelmanov solved the well-known problem of physical observable quantities in the General Theory of Relativity [1, 2]. It should be noted, many researchers were working on the theory of observable quantities in the 1940's. For example, Landau and Lifshitz in their famous *The Classical Theory of Fields* [3] introduced observable time and the observable three-dimensional interval similar to those introduced by Zelmanov. But they limited themselves only to this particular case and did not arrive at general mathematical methods to define physical observable quantities in pseudo-Riemannian spaces. It was only Cattaneo, an Italian mathematician, who developed his own approach to the problem, not far removed from Zelmanov's solution. Cattaneo published his results on the theme in 1958 and later [4, 5].

In 1944 Zelmanov completed a complete mathematical apparatus [1, 2] to calculate physical observable quantities in four-dimensional pseudo-Riemannian space, that is the strict solution of that problem. He called the apparatus the *theory of chronometric invariants*. The essence of his theory is that if an observer accompanies his physical reference body, his observable quantities are projections of four-dimensional quantities on his time line and the spatial section — *chronometrically invariant quantities*, made by projecting operators $b^\alpha = \frac{dx^\alpha}{ds}$ and $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$ which fully define his real reference space (here b^α is his velocity with respect to his real references). Thus, the chr.inv.-projections of a world-vector Q^α are $b_\alpha Q^\alpha = \frac{Q_0}{\sqrt{g_{00}}}$ and $h_\alpha^i Q^\alpha = Q^i$, while chr.inv.-projections of a world-tensor of the 2nd rank $Q^{\alpha\beta}$

are $b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}$, $h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q_0^i}{\sqrt{g_{00}}}$, $h_\alpha^i h_\beta^k Q^{\alpha\beta} = Q^{ik}$. Physically observable properties of the space are derived from the fact that chr.inv.-differential operators $\frac{\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{\partial}{\partial t}$ are non-commutative $\frac{\partial}{\partial t} \frac{\partial}{\partial x^i} - \frac{\partial}{\partial x^i} \frac{\partial}{\partial t} = \frac{1}{c^2} F_i \frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x^i} \frac{\partial}{\partial x^k} - \frac{\partial}{\partial x^k} \frac{\partial}{\partial x^i} = \frac{2}{c^2} A_{ik} \frac{\partial}{\partial t}$, and also from the fact that the chr.inv.-metric tensor h_{ik} may not be stationary. The observable characteristics are the chr.inv.-vector of gravitational inertial force F_i , the chr.inv.-tensor of angular velocities of the space rotation A_{ik} , and the chr.inv.-tensor of rates of the space deformations D_{ik} , namely

$$F_i = \frac{1}{\sqrt{g_{00}}} \left(\frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t} \right), \quad \sqrt{g_{00}} = 1 - \frac{w}{c^2}$$

$$A_{ik} = \frac{1}{2} \left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k} \right) + \frac{1}{2c^2} (F_i v_k - F_k v_i),$$

$$D_{ik} = \frac{1}{2} \frac{\partial h_{ik}}{\partial t}, \quad D^{ik} = -\frac{1}{2} \frac{\partial h^{ik}}{\partial t}, \quad D_k^k = \frac{\partial \ln \sqrt{h}}{\partial t},$$

where w is gravitational potential, $v_i = -c \frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ is the chr.inv.-metric tensor, and also $h = \det \|h_{ik}\|$, $h_{g00} = -g$, $g = \det \|g_{\alpha\beta}\|$. Observable inhomogeneity of the space is set up by the chr.inv.-Christoffel symbols $\Delta_{jk}^i = h^{im} \Delta_{jk,m}$, which are built just like Christoffel's usual symbols $\Gamma_{\mu\nu}^\alpha = g^{\alpha\sigma} \Gamma_{\mu\nu,\sigma}$ using h_{ik} instead of $g_{\alpha\beta}$.

A four-dimensional generalization of the main chr.inv.-quantities F_i , A_{ik} , and D_{ik} (by Zelmanov, the 1960's [10]) is: $F_\alpha = -2c^2 b^\beta a_{\beta\alpha}$, $A_{\alpha\beta} = ch_\alpha^\mu h_\beta^\nu a_{\mu\nu}$, $D_{\alpha\beta} = ch_\alpha^\mu h_\beta^\nu d_{\mu\nu}$, where $a_{\alpha\beta} = \frac{1}{2} (\nabla_\alpha b_\beta - \nabla_\beta b_\alpha)$, $d_{\alpha\beta} = \frac{1}{2} (\nabla_\alpha b_\beta + \nabla_\beta b_\alpha)$.

In this way, for any equations obtained using general covariant methods, we can calculate their physically observable projections on the time line and the spatial section of any particular reference body and formulate the projections in terms of their real physically observable properties, from which we obtain equations containing only quantities measurable in practice.

Zelmanov deduced chr.inv.-formulae for the space curvature [1]. He followed that procedure by which the Riemann-Christoffel tensor was built: proceeding from the non-commutativity of the second derivatives of an arbitrary vector ${}^* \nabla_i {}^* \nabla_k Q_l - {}^* \nabla_k {}^* \nabla_i Q_l = \frac{2A_{ik}}{c^2} \frac{\partial Q_l}{\partial t} + H_{lki}^{...j} Q_j$, he obtained the chr.inv.-tensor $H_{lki}^{...j} = \frac{\partial \Delta_{il}^j}{\partial x^k} - \frac{\partial \Delta_{kl}^j}{\partial x^i} + \Delta_{il}^m \Delta_{km}^j - \Delta_{kl}^m \Delta_{im}^j$, which is similar to Schouten's tensor from the theory of non-holonomic manifolds. The tensor $H_{lki}^{...j}$ differs algebraically from the Riemann-Christoffel tensor because of the presence of the space rotation A_{ik} in the formula.

Nevertheless its generalization gives the chr.inv.-tensor

$$C_{lkij} = \frac{1}{4} (H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}),$$

which possesses all the algebraic properties of the Riemann-Christoffel tensor in this three-dimensional space and, at the same time, the property of chronometric invariance. Therefore Zelmanov called C_{iklj} the *chr.inv.-curvature tensor* as the tensor of the observable curvature of the observer's spatial section. Its contraction step-by-step

$$C_{kj} = C_{kij}^{...i} = h^{im} C_{kimj}, \quad C = C_j^j = h^{lj} C_{lj}$$

gives the chr.inv.-scalar C , which is the *observable three-dimensional curvature* of this space.

Chr.inv.-projections of the Riemann-Christoffel tensor are [1]: $X^{ik} = -c^2 \frac{R_{0..0}^{i..k}}{g_{00}}$, $Y^{ijk} = -c \frac{R_{0...}^{i..jk}}{\sqrt{g_{00}}}$, $Z^{ijkl} = c^2 R^{ijkl}$.

Solving Einstein's equations with this mathematical apparatus, Zelmanov obtained the total system of all cosmological models (senarios of the Universe's evolution) which could be possible as derived from the equations [1, 6]. In particular, he had arrived at the possibility that infinitude may be relative. Later, in the 1950's, he enunciated the *Infinite Relativity Principle*:

In homogeneous isotropic cosmological models spatial infinity of the Universe depends on our choice of that reference frame from which we observe the Universe (the observer's reference frame). If the three-dimensional space of the Universe, being observed in one reference frame, is infinite, it may be finite in another reference frame. The same is just as well true for the time during which the Universe evolves.

In other words, using purely mathematical methods of the General Theory of Relativity, Zelmanov showed that any observer forms his world-picture from a comparison between his observation results and some standards he has in his laboratory — the standards of different objects and their physical properties. So the "world" we see as a result of our observations depends directly on that set of physical standards we have, so the "visible world" depends directly on our considerations about some objects and phenomena.

The mathematical apparatus of physical observable quantities and those results it gave in relativistic cosmology were the first results of Zelmanov's application of his Anthropic Principle to the General Theory of Relativity. To obtain the results with general covariant methods (standard in the General Theory of Relativity), where observation results do not depend on the observer's reference properties, would be impossible.

Now, according to the wishes of those who knew Zelmanov closely, I would like to say a few good words in memory of him.

Abraham Leonidovich Zelmanov was born in May 15, 1913 in Poltava Gubernya of the Russian Empire. His father

was a Judaic religious scientist, a specialist in comments on Torah and Kabbalah. In 1937 Zelmanov completed his education at the Mechanical Mathematical Department of Moscow University. After 1937 he was a research-student at the Sternberg Astronomical Institute in Moscow, where he presented his dissertation in 1944. In 1953 he was arrested for “cosmopolitanism” within the framework of Stalin’s campaign against Jews, however as soon as Stalin had died Zelmanov was set free after some months of in gaol. For several decades Zelmanov and his paralyzed parents lived in a room in a shared flat with neighbours. He took everyday care of his parents, so they lived into old age. Only in the 1970’s did he obtain a personal municipal flat. He was married three times. Zelmanov worked on the academic staff of the Sternberg Astronomical Institute all life, until his death on the winter’s day, the 2nd of February, 1987.



Abraham Zelmanov, 1940's

He was very thin in physique, like an Indian yogi, rather shorter than average, and a very delicate man. From his appearance it was possible to think that his life and thoughts were rather ordinary or uninteresting. However, in acquaintance with him and his scientific discussions in friendly company one formed another opinion about him. Those were discussions with a great scientist and humanist who reasoned in a very unorthodox way. Some-

times we thought that we were not speaking with a contemporary scientist of the 20th century, but some famous philosopher from Classical Greece or the Middle Ages. So the themes of those discussions are eternal – the interior of the Universe, what is the place of a human in the Universe, what are space and time.

Zelmanov liked to remark that he preferred to make mathematical “instruments” more than to use them in practice. Perhaps thereby his main goal in science was the mathematical apparatus of physical observable quantities in the General Theory of Relativity known as the *theory of chronometric invariants* [1, 2]. In developing the apparatus he also created other mathematical methods, namely – *kinematic invariants* [9] and *monad formalism* [10]. Being very demanding of himself, Zelmanov published less than a dozen scientific articles during his life (see References), so every publication is a concentrate of his fundamental scientific ideas.

Most of his time was spent in scientific work, but he sometimes gave lectures on the General Theory of Relativity and relativistic cosmology as a science for the geometrical structure of the Universe. Stephen Hawking, a young scientist in the 1960’s, attended Zelmanov’s seminars on cosmology at the Sternberg Astronomical Institute in Moscow. Zelmanov presented him as a “promising young cosmologist”. Hawking read a brief report at one of those seminars.

Because Zelmanov made scientific creation the main goal of his life writing articles was a waste of time to him. However he never regreted time spent on long discussions in friendly company, where he set forth his philosophic concepts on the geometrical structure of the Universe and the ways of human evolution. In those discussions he formulated his famous Anthropic Principle and the Infinite Relativity Principle.

Now everyone may read it. I hope that Zelmanov’s classical works on the General Theory of Relativity and cosmology, in particular his Anthropic Principle and the Infinite Relativity Principle known to a very close circle of his pupils, will become more widely known and appreciated.

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