The Gravity of Photons and the Necessary Rectification of Einstein Equation

C. Y. Lo

Applied and Pure Research Institute, 17 Newcastle Drive, Nashua, NH 03060, USA

E-mail: c_y_lo@yahoo.com; C_Y_Lo@alum.mit.edu

It is pointed out that Special Relativity together with the principle of causality implies that the gravity of an electromagnetic wave is an accompanying gravitational wave propagating with the same speed. Since a gravitational wave carries energy-momentum, this accompanying wave would make the energy-stress tensor of the light to be different from the electromagnetic energy-stress tensor, and thus can produce a geodesic equation for the photons. Moreover, it is found that the appropriate Einstein equation must additionally have the photonic energy-stress tensor with the antigravity coupling in the source term. This would correct that, in disagreement with the calculations for the bending of light, existing solutions of gravity for an electromagnetic wave, is unbounded. This rectification is confirmed by calculating the gravity of electromagnetic plane-waves. The gravity of an electromagnetic wave is indeed an accompanying gravitational wave. Moreover, these calculations show the first time that Special Relativity and General Relativity are compatible because the physical meaning of coordinates has been clarified. The success of this rectification makes General Relativity standing out further among theories of gravity.

1 Introduction

The physical basis of Special Relativity is constancy of the light speed, which is also the velocity of an electromagnetic wave [1]. On the other hand, the physical basis of quantum mechanics is that light can be considered as consisting of the photons [2]. Currently, it seems, there is no theoretical connection between constancy of light speed and photons, except that both are proposed by Einstein. However, since constancy of the light speed and the notion of photon are two aspects of the same physical phenomenon, from the viewpoint of physics, a theoretical connection of these notions must exist. Moreover, such a connection would be a key to understand the relationship between these two theories.

To this end, General Relativity seems to hold a special position because of the bending of light. The fact that a photon follows the geodesic of a massless particle [3, 4] manifests that there is a connection between the light speed and the photon. This suggests that General Relativity may provide some insight on the existence of the photons. In other words, the existence of the photons, though an observed fact, may be theoretically necessary because the light speed is the maximum.

On the other hand, since electromagnetism is a source for gravity [5], an electromagnetic wave would generate gravity. Thus, it is natural to ask whether its gravity is related to the existence of the photon. In other words, would the existence of the photon be an integral part of the theory of General Relativity? It will be shown here that the answer is affirmative. In fact, this is also a consequence of Special Relativity provided that the theoretical framework of General Relativity is valid.

2 Special Relativity and the accompanying gravity of an electromagnetic wave

In a light ray, the massless light energy is propagating in vacuum with the maximum speed c. Thus, the gravity due to the light energy should be distinct from that generated by massive matter [6–7]. Since a field emitted from an energy density unit means a non-zero velocity relative to that unit, it is instructive to study the velocity addition. According to Special Relativity, the addition of velocities is as follows [1]:

$$u_x = \frac{\sqrt{1-v^2/c^2}}{1+u'_z v/c^2}\, u'_x\,, \quad u_y = \frac{\sqrt{1-v^2/c^2}}{1+u'_z v/c^2}\, u'_y\,, \quad \text{and} \quad u_z = \frac{u'_z + v}{1+u'_z v/c^2}\,, \tag{1}$$

where velocity $\vec{v}$ is in the z-direction, (u'_x, u'_y, u'_z) is a velocity in a system moving with velocity v, c is the light speed, $u_x = dx/dt$, $u_y = dy/dt$, and $u_z = dz/dt$. When $v = c$, independent of (u'_x, u'_y, u'_z) one has

$$u_x = 0\,, \quad u_y = 0\,, \quad \text{and} \quad u_z = c\,. \tag{2}$$

Thus, neither the direction nor the magnitude of the velocity $\vec{v}\ (= \vec{c})$ have been changed.

This implies that nothing can be emitted from a light ray, and therefore no field can be generated outside the light ray. To be more specific, from a light ray, no gravitational field can be generated outside the ray although, accompanying the light ray, a gravitational field g_{ab} ($\neq \eta_{ab}$ the flat metric) is allowed within the ray.

According to the principle of causality [7], this accompanying gravity g_{ab} should be a gravitational wave since an electromagnetic wave is the physical cause. This would put General Relativity into a severe test for theoretical consistency. However, this examination would also have the benefit of knowing that electrodynamics is completely compatible with General Relativity.

3 The accompanying gravitational wave and the photonic energy-stress tensor

Observations confirm that photons follow a geodesic. One may expect that the light energy-stress tensor $T(L)_{ab}$ would generate the photonic geodesic since the massive tensor $T(m)_{ab}$ generates the geodesic through $\nabla^c T(m)_{cb} = 0$ [5]. This means that $T(L)_{ab}$ is different from the electromagnetic energy-stress tensor $T(E)_{ab}$ since $\nabla^c T(E)_{cb}$ is the Lorentz force [7, 8].

Nevertheless, this can be resolved since a gravitational wave carries an additional energy-stress tensor $T(g)_{ab}$, i. e., one should have

$$T(L)_{ab} = T(E)_{ab} + T(g)_{ab} \tag{3}$$

since there is no other type of energy. Then, one may expect that Eq. (3) allows $\nabla^c T(L)_{cb} = 0$ to generate the necessary geodesic equation for photons.

If the light is emitted and absorbed in terms of photons, physically the photons contain all the energy of the light, i. e., the photonic energy-stress tensor,

$$T(P)_{ab} = T(L)_{ab} \,. \tag{4}$$

One might object on the ground that, in quantum theory, $T(E)_{ab}$ is considered as identical to the photonic energy-stress tensor $T(P)_{ab}$. However, one should note also that gravity is ignored in quantum electrodynamics.

4 The Einstein equation for an electromagnetic wave

Einstein [9] suggested the field equation for the gravity of an electromagnetic wave was

$$G_{ab} = -KT(E)_{ab} \,, \tag{5}$$

where G_{ab} is the Einstein tensor, and K is the coupling constant. However, to generate the photonic geodesic, the source term must include the photonic energy-stress $T(P)_{ab}$. The need of a modified equation is supported by the fact that all existing solutions, in disagreement with light bending calculation, are unbounded [7].

Moreover, if the gravity of an electromagnetic wave is a gravitational wave, validity of Eq. (5) is questionable. It has been known from the binary pulsar experiments, that when radiation is included, the anti-gravity coupling must be included in the Einstein equation [10],

$$G_{ab} = -K\left[\,T(m)_{ab} - t(g)_{ab}\right] , \tag{6}$$

where $T(m)_{ab}$ and $t(g)_{ab}$ are respectively the energy-stress tensors for massive matter and gravity. The need of $t(g)_{ab}$ was first conjectured by Hogarth [12]. The possibility of such an coupling was suggested by Pauli [13]. Moreover, if a space-time singularity is not a reality, the existence of an antigravity coupling is implicitly given by the singularity theorems which assume the coupling constants are of the same sign [14].

There are theories such as the Brans-Dicke's [15] and the Yilmaz's [16] that provide an extra source term in vacuum. However, it is not clear that they can provide the right formula for the gravity of an electromagnetic wave since their connection with the notion of photon was never mentioned. Besides, it is more appropriate to consider a fundamental problem from the basics.

The above analysis suggests that, to obtain an appropriate Einstein equation, one may start from considering the gravitational radiation with Einstein's radiation formula as follows:

(a) For the gravitational wave generated by massive matter, the gravitational energy-stress $t(g)_{ab}$ of Einstein's radiation formula is approximately [11].

$$t(g)_{ab} = \frac{G^{(2)}_{ab}}{K}\,, \quad \text{where} \quad G^{(2)}_{ab} = G_{ab} - G^{(1)}_{ab}\,, \tag{7}$$

where $G^{(1)}_{ab}$ consists of all first order terms of G_{ab}. Moreover, if the gravitational energy is the same as the gravitational wave energy, one has

$$t(g)_{ab} = T(g)_{ab} \,. \tag{8}$$

(b) Since g_{ab} is a wave propagating with the electromagnetic wave, one may have the linear terms, $G^{(1)}_{ab} = 0$ on a time average. This suggests $G_{ab} = KT(g)_{ab}$. Thus, it follows from Eqs. (3) and (4) that

$$G_{ab} = KT(g)_{ab} = -K\left[\,T(E)_{ab} - T(P)_{ab}\right] \tag{9}$$

would be the appropriate Einstein equation. Comparing with Eq. (5), there is an additional term $T(P)_{ab}$.

(c) Since the Lorentz force $\nabla^c T(E)_{cb} = 0$ and $\nabla^c G_{cb} = 0$, as expected, one has the necessary formula

$$\nabla^c T(P)_{cb} = 0 \tag{10}$$

generate the photonic geodesic equation. However, to verify Eq. (9), one must first show that Eq. (5) cannot be valid for at least one example and then find the photonic energy-stress tensor $T(P)_{ab}$ for Eq. (9).

Alternatively, Eq. (9) can be derived from the principle of causality [7, 8] since the electromagnetic plane-wave as a spatial local idealization has been justified in electrodynamics. In general, without an idealization, to solve the gravity of an electromagnetic wave is very difficult [4].

5 The reduced Einstein equation for plane-waves

Due to the speed of light is the maximum, the influence of an electromagnetic wave to its accompanying gravity is spatially local. Thus, an electromagnetic plane-wave is also a valid modeling for the problem of gravity.

Now, let us consider the electromagnetic potential $A_k(t-z)$ which represents the photons moving in the z-direction. Then, Eq. (5) is reduced to a differential equation of $u\,(=t-z)$ [6] as follows:

$$G'' - g'_{xx}g'_{yy} + (g'_{xy})^2 - G'\,\frac{g'}{2g} = 2GR_{tt} = = 2K\left(F^2_{xt}\,g_{yy} + F^2_{yt}\,g_{xx} - 2F_{xt}F_{yt}\,g_{xy}\right), \tag{11}$$

where

$$G = g_{xx}g_{yy} - g^2_{xy}\,, \qquad g = |\,g_{ab}\,|$$

is the determinant of the metric, $F_{ab} = \partial_a A_b - \partial_b A_a$ is the electromagnetic field tensor, and R_{ab} is the Ricci tensor. The metric elements are connected as follows:

$$g = Gg^2_t\,, \quad \text{where} \quad g_t \equiv g_{tt} + g_{tz}\,. \tag{12}$$

Moreover, the massless of photons implies that

$$g_{tt} + 2g_{tz} + g_{zz} = 0, \quad \text{and} \quad g^{tt} - 2g^{tz} + g^{zz} = 0\,.$$

Note that Eq. (35.31) and Eq. (35.44) in reference [4] and Eq. (2.8) in reference [17] are special cases of Eq. (5). They believed that bounded wave solutions can be obtained [7].

It has been shown that A_t, g_{xt}, g_{yt}, and g_{zt} are allowed to be zero. Although there are four remaining metric elements (g_{xx}, g_{xy}, g_{yy}, and g_{tt}) to be determined, based on Einstein's notion of weak gravity and Eq. (5), it will be shown that there is no physical solution [6]. In other words, in contrast to Einstein's belief [9], the difficulty of his equation is not limited to mathematics.

6 Verification of the rectified Einstein equation

Now, consider an electromagnetic plane-waves of circular polarization, propagating to the z-direction

$$A_x = \frac{1}{\sqrt{2}}\,A_0\cos\omega u\,, \quad \text{and} \quad A_y = \frac{1}{\sqrt{2}}\,A_0\sin\omega u\,, \tag{13}$$

The rotational invariants with respect to the z-axis are constants. These invariants are: G_{tt}, R_{tt}, $T(E)_{tt}$, G, $(g_{xx} + g_{yy})$, g_{tz}, g_{tt}, g, and etc. It follows that [6, 7]

$$\begin{aligned} g_{xx} &= -1 - C + B_\alpha\cos(\omega_1 u + \alpha)\,, \\ g_{yy} &= -1 - C - B_\alpha\cos(\omega_1 u + \alpha)\,, \\ g_{xy} &= \pm\, B_\alpha\sin(\omega_1 u + \alpha)\,, \end{aligned} \tag{14}$$

where C and B_α are small constants, and $\omega_1 = 2\omega$. Thus, metric (14) is a circularly polarized wave with the same direction of polarization as the electromagnetic wave (13). On the other hand, one also has $G = (1+C)^2 - B^2_\alpha \geqslant 0$,

$$G_{tt} = \frac{2\omega^2 B^2_\alpha}{G} \geqslant 0\,, \tag{15}$$

$$T(E)_{tt} = \frac{g_{yy}}{G}\,\omega^2 A^2_0\,(1 + C - B_\alpha\cos\alpha) > 0\,.$$

Thus, it is not possible to satisfy Einstein equation (5) because $T(E)_{tt}$ and G_{tt} have the same sign [6]. Thus, it is necessary to have a photonic energy-stress tensor.

Given that a geodesic equation must be produced, for a monochromatic wave, the form of a photonic energy tensor should be similar to that of massive matter. Observationally, there is very little interaction, if any, among photons of the same ray. Theoretically, since photons travel in the velocity of light, there should not be any interaction among them.

Therefore, the photonic energy tensor should be dust-like with the momentum of the photon P_a as follows:

$$T_{ab}(P) = \rho\,P_a\,P_b\,, \tag{16}$$

where ρ is a scalar and is a function of u. In the units $c = \hbar = 1$, $P_t = \omega$. It has been obtained [6] that

$$\rho(u) = -A_m\,g^{mn}A_n \geqslant 0\,. \tag{17}$$

Here, $\rho(u)$ is related to gravity through g^{mn}. Since light intensity is proportional to the square of the wave amplitude, ρ which is Lorentz gauge invariant, can be considered as the density function of photons. Then

$$T_{ab} = -T(g)_{ab} = T(E)_{ab} - T(P)_{ab} = = T(E)_{ab} + A_m\,g^{mn}A_n\,P_a\,P_b\,. \tag{18}$$

Thus, $T_{ab}(P)$ has been derived completely from the electromagnetic wave A_k and g_{ab}.

Physically, such a tensor should be unique. It remains to see whether all the severe physical requirements can be satisfied. In particular, validity of the light bending calculation requires compatibility with the notion of weak gravity [3]. Also, the photonic energy tensor of Misner et al. [4], is an approximation of the time average of $T_{ab}(P)$.

As expected, this tensor $T_{ab}(P)$ enables a gravity solution for wave (13). According to Eq. (8),

$$T_{tt} = -\frac{1}{G}\,\omega^2 A^2_0 B_\alpha\cos\alpha \leqslant 0\,, \tag{19}$$

since $B_\alpha = \frac{K}{2}\,A^2_0\cos\alpha$. the energy density of the photonic energy tensor is indeed larger than that of the electromagnetic wave. $T(g)_{tt}$ is of order K. Note that, pure electromagnetic waves can exist since $\cos\alpha = 0$ is also possible. To confirm the general validity of (16), consider a wave linearly polarized in the x-direction,

$$A_x = A_0 \cos\omega(t - z)\,. \tag{20}$$

Then, one has

$$T_{tt} = \frac{g_{yy}}{G}\,\omega^2 A_0^2 \cos 2\omega(t - z)\,, \tag{21}$$

since the gravitational component is not an independent wave, $T(g)_{tt}$ is allowed to be negative. Eq. (21) implies that its polarization has to be different.

It turns out that the solution is a linearly polarized gravitational wave and that the time-average of $T(g)_{tt}$ is positive of order K [7]. From the viewpoint of physics, for an x-directional polarization, gravitational components related to the y-direction, remains the same. In other words,

$$g_{xy} = 0, \quad \text{and} \quad g_{yy} = -1\,. \tag{22}$$

It follows that the general solution of wave (20) is:

$$\begin{aligned} -g_{xx} &= 1 + C_1 - \frac{K}{2}\,A_0^2 \cos 2\omega(t - z)\,, \\ \text{and} \quad g_{tt} &= -g_{zz} = \sqrt{\frac{g}{g_{xx}}}\,, \end{aligned} \tag{23}$$

where C_1 is a constant. Note that he frequency ratio is the same as that of a circular polarization, but there is no phase difference to control the amplitude of the gravitational wave.

For a polarization in the diagonal direction of the $x - y$ plane, the solution is:

$$g_{xx} = g_{yy} = -1 - \frac{C_1}{2} + \frac{K}{4}\,A_0^2 \cos 2\omega(t - z)\,, \tag{24}$$

$$g_{xy} = -\frac{C_1}{2} + \frac{K}{4}\,A_0^2 \cos 2\omega(t - z)\,, \tag{25}$$

$$g_{tt} = -g_{zz} = \sqrt{\frac{-g}{1 - 2g_{xy}}}\,. \tag{26}$$

Note that for a perpendicular polarization, the metric element g_{xy} changes sign. The time averages of their T_{tt} are also negative as required. If $g = -1$, relativistic causality requires $C_1 \geqslant KA_0^2/2$.

7 Compatibility between Special Relativity and General Relativity

We implicitly use the same coordinate system whether the calculation is done in terms of Special Relativity or General Relativity. However, according to Einstein's "covariance principle" [1], coordinates have no physical meaning whereas the coordinates in Special Relativity have very clear meaning [18]. Thus, all the above calculations could have no meaning. Recently, it has been proven that a physical coordinate system for General Relativity necessarily has a frame of reference(1) with the Euclidean-like structure [19–21]. Moreover, the time coordinate will be the same as in Special Relativity if the metric is asymptotically flat.

Many theorists, including Einstein, overlooked that the metric of a Riemannian space actually is compatible with the space coordinates with the Euclidean-like structure. Let us illustrate this with the Schwarzschild solution in quasi-Minkowskian coordinates [11],

$$\begin{aligned} -ds^2 = -\left(1 - \frac{2M\kappa}{r}\right)c^2 dt^2 + \left(1 - \frac{2M\kappa}{r}\right)^{-1} dr^2 + \\ + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \end{aligned} \tag{27}$$

where $(r,\,\theta,\,\varphi)$ transforms to $(x,\,y,\,z)$ by,

$$\begin{aligned} &x = r\sin\theta\ \cos\varphi\,, \quad y = r\sin\theta\ \sin\varphi\,, \\ &\text{and} \quad z = r\cos\theta\,. \end{aligned} \tag{28}$$

Coordinate transformation (28) tells that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure represents this fact, but avoids confusion with the notion of a Euclidean subspace, determined by the metric. Metric (27) and the Euclidean-like structure (28) are complementary to each other in the Riemannian space. Then, a light speed ($ds^2 = 0$) is defined in terms of dx/dt, dy/dt, and dz/dt [1]. This is necessary though insufficient for a physical space [19–21].

Einstein's oversight made his theory inconsistent, and thus rejected by Whitehead [22] for being not a theory in physics. For instance, his theory of measurement is incorrect because it is modeled after(2) measurements for a Riemannian space embedded in a higher dimensional space [19–21]. In General Relativity, the local distance ($\sqrt{-ds^2}$, where $dt = 0$) represents the space contraction, which is measured in a free fall local space [1, 3]. Thus, this is a dynamic measurement since the measuring instrument is in a free fall state.

Einstein's error is that he overlooked the free fall state, and thus has mistaken this dynamic local measurement as a static measurement. Moreover, having different states at different points, this makes such a measurement for an extended object not executable.

The Euclidean-like structure determines the distance between two points in a frame of reference, and the observed light bending supports this physical meaning. This is why the interpretation of Hubble's law as a consequence of receding velocity(3) is invalid [23]. Because the measurement theory of Einstein is invalid, the miles long arms of the laser interferometer in LIGO would not change their length under the influence of gravitational waves [24]. In other words, LIGO would inadvertently further confirm that Einstein's theory of measurement is invalid.

It has been solved that the coordinate system of General Relativity and that of Special Relativity are actually the same for this problem. We must show also that the plane waves would satisfy the Maxwell equation in General Relativity, see [11; p. 125],

$$\frac{\partial}{\partial x^a}\sqrt{g}\,F^{ab} = -\sqrt{g}\,J^b, \tag{29}$$

$$\frac{\partial}{\partial x^a}F^{bc} + \frac{\partial}{\partial x^b}F^{ca} + \frac{\partial}{\partial x^c}F^{ab} = 0\,. \tag{30}$$

Since equation (30) is the same as in Special Relativity, it remains to show that (29) is satisfied for $J^a = 0$. To show this, we can use the facts that g^{ab} and F^{ab} are function of u, and that $g^{tt} + g^{zz} = 0$. It follows that

$$\begin{aligned}\frac{\partial}{\partial x^a}\sqrt{g}\,F^{ab} &= \frac{\partial\sqrt{g}}{\partial t}(F^{tb} - F^{zb}) = \\ &= \frac{\partial\sqrt{g}}{\partial t}\,g^{tt}\,(\partial_t A_c + \partial_z A_c)\,g^{cb} = 0\,.\end{aligned} \tag{31}$$

We thus complete the compatibility proof.

8 Conclusions and Discussions

A crucial argument for this case is that both Special Relativity and General Relativity use the same coordinate system. This is impossible, according to Einstein's theory of measurement. A major problem of Einstein's theory is that the physical meaning of coordinates is not only ambiguous, but also confusing(4) since the physical meaning of the coordinates depends on the metric. Moreover, Einstein's equivalence principle actually contradicts the so-called "covariance principle". P. Morrison of MIT [21, 25] remarked that the "covariance principle" is physically invalid because it disrupts the necessary physical continuity from Special Relativity to General Relativity.

Now, a photonic energy-stress tensor has been obtained as physics requires. The energy and momentum of a photon is proportional to its frequency although, as expected from a classical theory, their relationship with the Planck constant $\hbar$ is not yet clear; and the photonic energy-stress tensor is a source term in the Einstein equation. As predicted by Special Relativity, the gravity of an electromagnetic wave is an accompanying gravitational wave propagating with the same speed. Moreover, the gravity of light is proven to be compatible with the notion of weak gravity.

In the literature [4, 26–29], however, solutions of Eq. (5) are unbounded.(5) Thus, they are incompatible with the approximate validity of electrodynamics and violate physical principles including the equivalence principle and the principle of causality [7, 30]. (The existence of local Minkowski spaces is only a necessary condition(6) for Einstein's equivalence principle [31].) Naturally, one may question whether the gravity due to the light is negligible. Now, the claim that the bending of light experiment confirms General Relativity, is no longer inflated.

In addition, the calculation answers a long-standing question on the propagation of gravity in General Relativity. Since an electromagnetic wave has an accompanying gravitational wave, gravity should propagate in the same speed as electromagnetism. It is interesting to note that Rabounski [32] reached the same conclusion on the propagation of gravity with a completely different method, which is independent of the Einstein equation.

One might argue that since $E = mc^2$ and the gravitational effect of the wave energy density should be outside a light ray. However, this is a misinterpretation [33, 34]. One should not, as Tolman [35] did, ignore Special Relativity and the fact that the light energy density is propagating with the maximum velocity possible. There are intrinsically different characteristics in such an energy form according to Special Relativity. This calculation confirms a comment of Einstein [23] that $E = mc^2$ must be understood in the contact of energy conservation.

To illustrate this, consider the case of a linear polarization, for which Eq. (5) still has a solution [6]

$$-g_{xx} = 1 - \frac{K}{4}\,A_0^2\left[2\omega^2(t-z)^2 + \cos 2\omega(t-z)\right]. \tag{32}$$

However, solution (32) is invalid since $(t - z)^2$ grows very large as time goes by. This would "represent" the effects that the wave energy were equivalent to mass. This illustrates also that Einstein's notion of weak gravity may not be compatible with an inadequate source.

The theoretical consistency between Special Relativity and General Relativity is further established. This is a very strong confirming evidence for General Relativity beyond the requirements of the equivalence principle. Moreover, this rectification makes General Relativity standing out among all theories of gravity. Moreover, since light has a gravitational wave component, it would be questionable to quantize gravity independently as in the current approach.

Acknowledgments

The author gratefully acknowledges stimulating discussions with J. E. Hogarth, and P. Morrison. Special thanks are to D. Rabounski for valuable comments and useful suggestions. This work is supported in part by the Chan Foundation, Hong Kong.

Endnotes

(1) In a Riemannian geometry, a frame of reference may not exist since the coordinates can be arbitrary. However, for a physical space, a frame of reference with the Euclidean-like structure must exist because of physical requirements [19–21]. Note that the Euclidean-like structure is independent of the metric.

(2) In the initial development of Riemannian geometry, the metric was identified formally with the notion of distance in analogy as the case of the Euclidean space. Such geometry is often illustrated with the surface of a sphere, a subspace embedded in a flat space [4, 36]. Then, the distance is determined by the flat space and can be measured with a static method. For a general case, however, the issue of measurement was not addressed before Einstein's theory.

(3) Einstein's theory of measurements is not supported by observation, which requires [21, 37] that the light speed must be defined in terms of the Euclidean-like structure as in Einstein's own papers [1, 3].

(4) If the "covariance principle" was valid, it has been shown that the "event of horizon" for a black hole could be just any arbitrary constant [38].

(5) In fact, all existing solutions involving waves are unbounded because the term to accommodate gravitational wave energy-stress is missing. It is interesting that Einstein and Rosen are the first to discover the non-existence of wave solutions [39]. However, their arguments that led to their correct conclusion was incorrect. Robertson as a referee of Physical Review pointed out that the singularities mentioned are actually removable [39]. However, there are other reasons for a wave solution to be invalid. It has been found that a wave solution necessarily violates Einstein's equivalence principle and the principle of causality [10, 19].

(6) Many theorists do not understand Einstein's equivalence principle because they failed in understanding the Einstein-Minkowski condition that the local space of a particle under gravity must be locally Minkowskian [1, 3]. This condition is crucial to obtain the time dilation and space contractions [21].

References

1. Einstein A., Lorentz H. A., Minkowski H., and Weyl H. The Principle of Relativity. Dover, New York, 1923.
2. Einstein A. *Annalen der Physik*, 1905, Bd. 17, 132–148.
3. Einstein A. The Meaning of Relativity (1921). Princeton Univ. Press, 1954.
4. Misner C. W., Thorne K. S., and Wheeler J. A. Gravitation. Freeman, San Francisco, 1973.
5. Lo C. Y. *Phys. Essays*, 1997, v.10(4), 540–545.
6. Lo C. Y. *Proc. Sixth Marcel Grossmann Meeting On Gen. Relativity, Kyoto 1991*, ed. H. Sato, and T. Nakamura, World Sci., Singapore, 1992, 1496.
7. Lo C. Y. *Phys. Essays*, 1997, v.10(3),424–436.
8. Lo C. Y. *Phys. Essays*, 1999, v.12(2), 226–241.
9. Einstein A. Physics and Reality (1936) in Ideas and Opinions. Crown, New York, 1954, p. 311.
10. Lo C. Y. *Astrophys. J.*, 1995, v.455, 421–428.
11. Weinberg S. Gravitation and Cosmology. John Wiley Inc., New York, 1972.
12. Hogarth J. E., Ph. D. Thesis, 1953, Dept. of Math. Royal Holloway College, University of London, p. 6.
13. Pauli W. Theory of Relativity. Pergamon Press, London, 1958, p. 163.
14. Hawking S. W. and Penrose R., *Proc. Roy. Soc. London A*, 1970, v. 314, 529–548.
15. Brans C. and Dicke R. H. *Phys. Rev.*, 1961, v. 124, 925.
16. Yilmaz H. *Hadronic J.*, 1979, v. 2, 997–1020.
17. Bondi H., Pirani F. A. E., and Robinson I. *Proc. R. Soc. London A*, 1959, v. 251, 519–533.
18. Einstein A. The Problem of Space, Ether, and the Field in Physics (1934) in Ideas and Opinions Crown, New York, 1954.
19. Lo C. Y. *Phys. Essays*, 2002, v. 15(3), 303–321.
20. Lo C. Y. *Chinese J. of Phys.*, 2003, v. 41(4), 1–11.
21. Lo C. Y. *Phys. Essays*, 2005, v. 18(4).
22. Whitehead A. N. The Principle of Relativity. Cambridge Univ. Press, Cambridge, 1962.
23. Lo C. Y. *Progress in Phys.*, 2006, v. 1, 10–13.
24. Lo C. Y. The Detection of Gravitational Wave with the Laser Interferometer and Einstein's Theoretical Errors on Measurements. (*In preparation*).
25. Lo C. Y. *Phys. Essays*, 2005, v. 18(1).
26. Peres A. *Phys. Rev.*, 1960, v. 118, 1105.
27. Penrose R. *Rev. Mod. Phys.*, 1965, v. 37(1), 215–220.
28. Bonnor W. B. *Commun. Math. Phys.*, 1969, v. 13, 173.
29. Kramer D., Stephani H., Herlt E., and MacCallum M. *Exact Solution of Einstein's Field Equations*, ed. E. Schmutzer, Cambridge Univ. Press, Cambridge, 1980.
30. Lo C. Y. *Phys. Essays*, 1994, v. 7(4), 453–458.
31. Lo C. Y. *Phys. Essays*, 1998, v. 11(2), 274–272.
32. Rabounski D. *Progress in Physics*, 2005, v. 1, 3–6.
33. Einstein A. $E = Mc^2$ (1946) Ideas and Opinions. Crown, New York, 1954, p. 337
34. Lo C. Y. *Astrophys. J.*, 1997, v.477, 700–704.
35. Tolman R. C. Relativity, Thermodynamics, and Cosmology. Dover, New York, 1987, p. 273.
36. Dirac P. A. M. General Theory of Relativity. John Wiley, New York, 1975.
37. Lo C. Y. *Phys. Essays*, 2003, v. 16(1), 84–100.
38. Lo C. Y. *Chin. Phys.*, 2004, v. 13(2), 159.
39. Kennefick D. Einstein versus the Physical Review. *Physics Today*, September 2005.