Einstein's Equivalence Principle and Invalidity of Thorne's Theory for LIGO

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The theoretical foundation of LIGO's design is based on the equation of motion derived by Thorne. His formula, motivated by Einstein's theory of measurement, shows that the gravitational wave-induced displacement of a mass with respect to an object is proportional to the distance from the object. On the other hand, based on the observed bending of light and Einstein's equivalence principle, it is concluded that such induced displacement has nothing to do with the distance from another object. It is shown that the derivation of Thorne's formula has invalid assumptions that make it inapplicable to LIGO. This is a good counter example for those who claimed that Einstein's equivalence principle is not important or even irrelevant.

1 Introduction

Since the behavior of binary pulsars has been interpreted successfully as due to gravitational radiation [1, 2], the existence of gravitational waves is generally accepted. Moreover, the Maxwell Newton Approximation,(1) which generates gravitational waves, has been established independent of the Einstein equation [3]. However, a direct observation of the gravitational waves has not been successful because a gravitational wave is very weak in nature [4].

To obtain the required sensitivity of detection for gravitational waves, two gigantic laser interferometer gravitational wave observatories (LIGO) have been built.(2) Currently they represent the hope of detecting the gravitational waves directly. The confidence on these new apparatus is based on the perceived high sensitivity [5] that is designed according to Thorne's equation, which is motivated on Einstein's theory of measurement [6, 7].

Thorne's [8] equation of motion is as follows [9]:

$$m\frac{d^2\delta x^j}{dt^2} = \frac{1}{2}\,m\frac{\partial^2 h^{\mathrm{TT}}_{jk}}{\partial t^2}\,x^k, \tag{1}$$

where δx^k is the displacement of the test particle with mass m from a fixed object, x^k is the Euclidean-like distance (or the particle's Cartesian coordinate position) of the test particle from the fixed object (at the original the space coordinates), and h^{TT}_{jk} is the first order of the dimensionless "gravitational wave field" that induces the displacement. Then the integration of equation (1) gives,

$$\delta x^j = \frac{1}{2}\,h^{\mathrm{TT}}_{jk}x^k. \tag{2}$$

The superscript TT on the gravitational field is to remind us that the field is "transverse and traceless".

On the other hand, according to Einstein's equivalence principle [10], the Euclidean-like structure [11, 12] that determines the distance between two points is independent of gravity, and this is supported the observed bending of light. Thus, the displacement from a fixed object induced by gravitational wave, according the geodesic equation, has nothing to do with the distance between them (see Section 2). In this paper, it will be shown the errors related to eqs. (1) and (2).

2 Problems in the theory of Thorne

Now let us first derive, according the theory of Thorne [8], the induced phase delay in the interferometer. Since the sources of the gravitational waves are very far away, the waves look very nearly planar as they pass through the observer's proper reference frame.(3) If we orient the x, y, z spatial axes, so the propagation in the z direction, then the transversality of the waves and traceless mean that the non-zero components of the wave field are $h^{\mathrm{TT}}_{xx} = -h^{\mathrm{TT}}_{yy}$, $h^{\mathrm{TT}}_{xy} = h^{\mathrm{TT}}_{yx}$, called respectively the $+$ and $\times$-polarization. For a $(+)$-polarization, if the arm length of the interferometer is L, we have

$$\begin{aligned} \delta x(t) &= \frac{1}{2}\,Lh_+(t) \qquad \text{for mass on } x \text{ axis,} \\ \delta y(t) &= -\frac{1}{2}\,Lh_+(t) \quad \text{for mass on } y \text{ axis.} \end{aligned} \tag{3}$$

For a light wavelength λ, if B is the number of bounce back and forth in the arms, the total phase delay is

$$\triangle\phi_{\mathrm{T}} = 8\pi B\,\frac{\delta x}{\lambda} = 4\pi B\,\frac{L}{\lambda}\,h_+\,. \tag{4}$$

Thus, the sensitivity of the interferometer would be increased with longer arms. If Einstein's theory of measurement was valid, then eq. (3) would be an expected result. This explains that eq. (1) was accepted. To show the errors, some detailed analysis is needed.

In a Local frame of free fall, Manasse and Misner [12] claimed that the metric have approximately,

$$-ds^2 = (1 + R_{0l0m}x^l x^m)dt^2 + \left(\frac{4}{3}R_{0ljm}x^l x^m\right)dx^j dt - \\ - \left(\delta_{ij} - \frac{1}{3}R_{iljm}x^l x^m\right)dx^i dx^j - \mathrm{O}(|x^j|^3)dx^\alpha dx^\beta \tag{5}$$

accurate to the second order in small $|x^j|$. The observer in the free fall is located at the origin of the local frame. Eq. (5) is the equation (13.73) in Misner et al. [9]. In the next step (35.12), they claimed to have the equation,

$$\frac{D^2 n^j}{d\tau^2} = -R^j_{0k0}\, n^k = -R_{j0k0}\, n^k, \\ \text{where} \quad n^j = x^j_{\mathrm{B}} - x^j_{\mathrm{A}} = x^j_{\mathrm{B}} \tag{6}$$

since $x^j_{\mathrm{A}} = 0$. In eq. (5), $|x^j|$ is restricted to be small. However, a problem in this derivation is that R_{j0k0} may not be the same at points A and B. Nevertheless, one may argue that $\Gamma^\mu_{\alpha\beta} = 0$ at A, and (6) is reduced to

$$\frac{d^2 x^j_{\mathrm{B}}}{d\tau^2} = -R_{j0k0}\, x^k_{\mathrm{B}} = \frac{1}{2}\frac{\partial h^{\mathrm{TT}}_{jk}}{\partial \tau^2}\, x^k_{\mathrm{B}}\,. \tag{7}$$

If it is applied to the case of LIGO, one must show at least a miles long x^j_{B} could be regarded as very small as (5) requires. From the geodesic equation, clearly it is impossible to justify (7) for any frame of reference.

More important, since LIGO is built on the Earth, its frame of reference is not at free fall when gravitational waves are considered. The radius of the Earth is 6.3×10^3 km, but the expected gravitational wave length is only about 15 km [9]. Thus, the Earth can no longer be considered as a test particle when only the gravity of the Sun is considered. In other words, (5) and (7) are inapplicable to LIGO.

Note that Misner et al. [9] have mistaken Pauli's version(4) as Einstein's equivalence principle [10], it is natural that they made related mistakes. For instance, Thorne [15] incorrectly criticized Einstein's equivalence principle as follows:

"In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

However, Einstein has already explained these problems. For instance, the problem of tidal forces was answered in Einstein's letter of 12 July 1953 to Rehtz [16] as follows:

"The equivalence principle does not assert that every gravitational field (e. g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Clearly, his principle is for a space where physical requirements are sufficiently satisfied.

In fact, Misner et al. [9] do not understand Einstein's equivalence principle and related theorems in Riemannian space [14, 17]. A simple and clear evidence is in their eq. (40.14) [9; p. 1107], and they got a physically incorrect conclusion on the local time of the Earth in the solar system. Moreover, Ohanian and Ruffini [5; p. 198] also ignored the Einstein-Minkowski condition and had the same problems as shown in their eq. (50). However, Liu [18], Straumann [19], Wald [20], and Weinberg [4] did not make the same mistake. Note that Ohanian, Ruffini, and Wheeler have proclaimed that they are non-believers of Einstein's principles [5].

3 Remarks

In the theory of Thorne, there are major errors because his understanding of Einstein's equivalence principle is inadequate. His equation was motivated by Einstein's theory of measurement, and the superficial consistency with such a theory makes many theorists had confidence on his equation. Now, it is clear that such a support from an invalid theory is proven to be useless. Because Misner et al. [9] do not understand Einstein's equivalence principle, they cannot see that Einstein's theory of measurement is not self-consistent [21, 22].

In addition, since LIGO is built on the Earth, the frame is not at free fall. The radius of the Earth is 6.3×10^3 km, but the expected gravitational wave length is only about 15 km [9]. Thus, the Earth cannot be regarded as a test particle for gravitational waves. Moreover, Thorne was not aware that the Einstein equation has no wave solution [1, 2]. Although Misner, Thorne, and Wheeler [9] claimed plane wave solutions exist, their derivation has been found to be invalid [2, 23]. The second problem has been resolved by a modified Einstein equation, and it has the Maxwell-Newton Approximation as the first order equation [1].

In short, the current theory on the detection of gravitational waves for LIGO is incorrect. The root of these problems is due to that they do not understand Einstein's equivalence principle.(5) Consequently, they also failed to see the Euclidean-like structure is necessary(6) in a physical space [12]. This is a very good counter example for those who believed the Einstein's equivalence principle is not important or even irrelevant [2]. The sensitivity of LIGO will be addressed in a separate paper [24].

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Endnotes

(1) The Maxwell-Newton Approximation, whose sources are massive matter, could be identified as a special case of the so-called linearized approximation that has been found to be incompatible with Einstein equation for a dynamic situation [1].

(2) M. Bartusiak [25] has written an interesting book on the great efforts to build LIGO.

(3) Einstein equation has no physically valid wave solution because there is no term in Einstein's equation to accommodate the energy-stress tensor of a gravitational wave that must move with the wave [23]. Thus, a wave solution must come from the modified equation of 1995.

(4) Pauli's [26] version of the principle of equivalence was commonly but mistakenly regarded as Einstein's principle, although Einstein strongly objected to this version as a misinterpretation [15]. In fact, Misner, Thorne, and Wheeler [9; p. 386] falsely claimed that Einstein's equivalence principle is as follows:

"In any and every local Lorentz frame, anywhere and anytime in the universe, all the (Nongravitational) laws of physics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to infinitestimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame any other local Lorentz frame in the same or any other region."

However, this is only an alternative version of Pauli's because the Einstein-Minkowski condition,(7) which requires that the local space in a free fall must have a local Lorentz frame, is missing.

(5) There are other surprises. In spite of Einstein's clarification, many theorists, including the editors of Nature, Physical Review, and Science, still do not fully understand special relativity, in particular $E = mc^2$ [27–30].

(6) An existence of the Euclidean-like structure (that Einstein [6] called as "in the sense of Euclidean geometry") is necessary for a physical space [11, 12]. The Euclidean-like structure is operationally defined in terms of spatial measurements essentially the same as Einstein defined the frame of reference for special relativity [31]. Since the attached measuring instruments and the coordinates being measured are under the influence of the same gravity, a Euclidean-like structure emerges from such measurements as if gravity did not exist.

(7) For the Einstein-Minkowski condition, Einstein [10] addressed only the metrics without a crossing space-time element. This creates a false impression that the Einstein-Minkowski condition is trivial.

References

1. Lo C. Y. *Astrophys. J.*, 1995, v.455, 421.
2. Lo C. Y. *Phys. Essays*, 2000, v. 13(4), 527.
3. Lo C. Y. *Phys. Essays*, 1999, v. 12(3), 508.
4. Weinberg S. Gravitation and cosmology. John Wiley Inc., New York, 1972.
5. Ohanian H. C. & Ruffini R. Gravitation and spacetime. Norton, New York, 1994.
6. Einstein A., Lorentz H. A., Minkowski H., & Weyl H. The principle of Relativity. Dover, New York, 1923.
7. Landau L. D. & Lifshitz E. M. The classical theory of fields. Pergamon Press, New York, 1975.
8. Thorne K. S. Gravitational radiation. In *300 Years of Gravitation*. Eds. S. W. Hawking & W. Israel, Camb. Univ. Press, New York, 1987.
9. Misner C. W., Thorne K. S., & Wheeler J. A. Gravitation. Freeman, San Francisco, 1973.
10. Einstein A. The meaning of Relativity (1921). Princeton Univ. Press, 1954.
11. Lo C. Y. *Phys. Essays*, 2002, v. 15(3), 303.
12. Lo C. Y. *Chinese J. of Phys.*, 2003, v. 41(4), 233.
13. Lo C. Y. *Phys. Essays*, 2003, v. 16(1), 84–100.
14. Manasse F. K. & Misner C. W., *J. Math. Phys.*, 1963, v. 4, 735–745.
15. Thorne K. S. Black holes & time warps. Norton, New York, 1994.
16. Norton J. What was Einstein's principle of equivalence? *Einstein's Studies*, v. 1: Einstein and the history of General Relativity, Eds. D. Howard and J. Stachel (Birkhäuser).
17. Synge J. L. Relativity: The General Theory. North-Holland, Amsterdam, 1971.
18. Liu Liao. General Relativity. High Education Press, Shanghai, China), 1987, p. 42.
19. Straumann N. General Relativity and relativistic astrophysics. Springer, New York, 1984.
20. Wald R. M. General Relativity. The Univ.of Chicago Press, Chicago, 1984.
21. Lo C. Y. *Phys. Essays*, 2005, v. 18(4).
22. Lo C. Y. *Progress in Physics*, 2006, v. 1, 10.
23. Lo C. Y. *Phys. Essays*, 1997, v. 10(3), 424.
24. Lo C. Y. The detection of gravitational wave with the laser interferometer and Einstein's theoretical errors on measurements. *In preparation*.
25. Bartusiak M. Einstein's unfinished symphony. Berkley Books, New York, 2000.
26. Pauli W. Theory of Relativity. Pergamon Press, London, 1958, p. 163.
27. Lo C. Y. *Astrophys. J.*, 1997, 477, 700–704.
28. Lo C. Y. *Phys. Essays*, 1997, v. 10 (4), 540–545.
29. Lo C. Y. Remarks on interpretations of the Eötvös experiment and misunderstandings of $E = mc^2$. *In preparation*.
30. Lo C. Y. *Progress in Physics*, 2006, v. 1, 46.
31. Einstein A. The problem of space, ether, and the field in physics. In *Ideas and Opinions*, Dover, 1982.