The Neutrosophic Logic View to Schrödinger's Cat Paradox

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This article discusses Neutrosophic Logic interpretation of the Schrodinger's cat paradox. We argue that this paradox involves some degree of indeterminacy (unknown) which Neutrosophic Logic could take into consideration, whereas other methods including Fuzzy Logic could not. For a balanced discussion, other interpretations have also been discussed.

1 Schrödinger equation

As already known, Schrödinger equation is the most used equation to describe non-relativistic quantum systems. Its relativistic version was developed by Klein-Gordon and Dirac, but Schrödinger equation has wide applicability in particular because it resembles classical wave dynamics. For introduction to non-relativistic quantum mechanics, see [1].

Schrödinger equation begins with definition of total energy $E = \vec{p}^{\,2}/2m$. Then, by using a substitution

$$E = i\hbar\frac{\partial}{\partial t}, \qquad P = \frac{\hbar}{i}\nabla, \tag{1}$$

one gets [2]

$$\left[i\hbar\frac{\partial}{\partial t} + \hbar\frac{\bar{\nabla}^2}{2m} - U(x)\right]\psi = 0 \tag{2}$$

or

$$\frac{i\partial}{\partial t}\psi = H\psi\,. \tag{3}$$

While this equation seems quite clear to represent quantum dynamics, the physical meaning of the wavefunction itself is not so clear. Soon thereafter Born came up with hypothesis that the square of the wavefunction has the meaning of chance to find the electron in the region defined by dx (Copenhagen School). While so far his idea was quickly adopted as "standard interpretation", his original "guiding field" interpretation has been dropped after criticism by Heisenberg over its physical meaning [3]. Nonetheless, a definition of "Copenhagen interpretation" is that it gives the wavefunction a role in the actions of something else, namely of certain macroscopic objects, called "measurement apparatus", therefore it could be related to phenomenological formalism [3].

Nonetheless, we should also note here that there are other approaches different from Born hypothesis, including:

- The square of the wavefunction represents a measure of the density of matter in region defined by dx (Determinism school [3, 4, 5]). Schrödinger apparently preferred this argument, albeit his attempt to demonstrate this idea has proven to be unfruitful;
- The square of wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [6];
- The wavefunction in Schrödinger equation represents tendency to make structures;
- The wavemechanics can also be described in terms of topological Aharonov effect, which then it could be related to the notion of topological quantization [7, 8]. Aharonov himself apparently argues in favour of "realistic" meaning of Schrödinger wave equation, whose interpretation perhaps could also be related to Kron's work [9].

So forth we will discuss solution of this paradox.

2 Solution to Schrödinger's cat paradox

2.1 Standard interpretation

It is known that Quantum Mechanics could be regarded more as a "mathematical theory" rather than a physical theory [1, p. 2]. It is wave mechanics allowing a corpuscular duality. Already here one could find problematic difficulties: i. e. while the quantity of wavefunction itself could be computed, the physical meaning of wavefunction itself remains *indefinable* [1]. Furthermore, this notion of wavefunction corresponds to another fundamental indefinable in Euclidean geometry: the point [1, p. 2]. It is always a baffling question for decades, whether the electron could be regarded as wave, a point, or we should introduce a *non-zero* finite entity [4]. Attempts have been made to describe wave equation in such non-zero entity but the question of the physical meaning of wavefunction itself remains mystery.

The standard Copenhagen interpretation advertised by Bohr and colleagues (see DeBroglie, Einstein, Schrödinger who advocated "realistic" interpretation) asserts that it is practically *impossible* to know what really happens in quantum scale. The quantum measurement itself only represents reading in *measurement apparatus*, and therefore it is difficult to separate the object to be measured and the measurement apparatus itself. Bohr's phenomenological viewpoint perhaps could be regarded as pragmatic approach, starting with the request not to attribute a deep meaning to the wave function but immediately go over to statistical likelihood [10]. Consequently, how the process of "wave collapse" could happen remains mystery.

Heisenberg himself once emphasized this viewpoint when asked directly the question: Is there a fundamental level of reality? He replied as follows:

> "This is just the point: I do not know what the words fundamental reality mean. They are taken from our daily life situation where they have a good meaning, but when we use such terms we are usually extrapolating from our daily lives into an area very remote from it, where we cannot expect the words to have a meaning. This is perhaps one of the fundamental difficulties of philosophy: that our thinking hangs in the language. Anyway, we are forced to use the words so far as we can; we try to extend their use to the utmost, and then we get into situations in which they have no meaning" [11].

A modern version of this interpretation suggests that at the time of measurement, the wave collapses instantaneously into certain localized object corresponding to the action of measurement. In other words, the measurement processes define how the wave should define itself. At this point, the wave ceases to become coherent, and the process is known as "decoherence". Decoherence may be thought of as a way of making real for an observer in the large scale world only one possible history of the universe which has a likelihood that it will occur. Each possible history must in addition obey the laws of logic of this large-scale world. The existence of the phenomenon of decoherence is now supported by laboratory experiments [12]. It is worthnoting here, that there are also other versions of decoherence hypothesis, for instance by Tegmark [13] and Vitiello [14].

In the meantime, the "standard" Copenhagen interpretation emphasizes the role of observer where the "decoherence viewpoint" may not. The problem becomes more adverse because the axioms of standard statistical theory themselves are not fixed forever [15, 16]. And here is perhaps the source of numerous debates concerning the interpretation and philosophical questions implied by Quantum Mechanics. From this viewpoint, Neutrosophic Logic offers a new viewpoint to problems where indeterminacy exists. We will discuss this subsequently. For a sense of balance, we also discuss a number of alternative interpretations. Nonetheless this article will not discuss all existing interpretations of the quantum wavefunction in the literature.

2.2 Schrödinger's cat paradox

To make the viewpoint on this paradox a bit clearer, let us reformulate the paradox in its original form.

According to Uncertainty Principle, any measurement of a system must disturb the system under investigation, with a resulting lack of precision in the measurement. Soon after reading Einstein-Podolsky-Rosen's paper discussing incompleteness of Quantum Mechanics, Schrödinger in 1935 came up with a series of papers in which he used the "cat paradox" to give an illustration of the problem of viewing these particles in a "thought experiment" [15, 17]:

> "One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The wave-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared into equal parts."

In principle, Schrödinger's thought experiment asks whether the cat is dead or alive after an hour. The most logical solution would be to wait an hour, open the box, and see if the cat is still alive. However once you open the box to determine the state of the cat you have viewed and hence disturbed the system and introduced a level of uncertainty into the results. The answer, in quantum mechanical terms, is that before you open the box the cat is in a state of being half-dead and half-alive.

Of course, at this point one could ask whether it is possible to find out the state of the cat without having to disturb its wavefunction via action of "observation".

If the meaning of word "observation" here is defined by *to open the box and see the cat*, and then it seems that we could argue whether it is possible to propose another equally possible experiment where we introduce a pair of twin cats, instead of only one. A cat is put in the box while another cat is located in a separate distance, let say 1 meter from the box. If the state of the cat inside the box altered because of poison reaction, it is likely that we could also observe its effect to its twin, perhaps something like "sixth sense" test (perhaps via monitoring frequency of the twin cat's brain).

This plausible experiment could be viewed as an alternative "thought experiment" of well-known Bell-Aspect-type experiment. One could also consider an entangled pair of photons instead of twin cats to conduct this "modified" cat paradox. Of course, for this case then one would get a bit complicated problem because now he/she should consider two probable state: the decaying atom and the photon pair.

We could also say that using this alternative configuration, we know exact information about the Cat outside, while indeterminate information about the Cat inside. However, because both Cats are entangled (twin) we are sure of all the properties of the Cat inside "knows" the state of the Cat outside the box, via a kind of "spooky action at distance" reason (in Einstein's own word)*.

Therefore, for experimental purpose, perhaps it would be useful to simplify the problem by using "modified" Aspect-type experiment [16]. Here it is proposed to consider a decaying atom of Cesium which emits two correlated photons, whose polarization is then measured by Alice (A) on the left and by Bob (B) on the right (see Fig. 1). To include the probable state as in the original cat paradox, we will use a switch instead of Alice A. If a photon comes to this switch, then it will turn on a coffee-maker machine, therefore the observer will get a cup of coffee†. Another switch and coffee-maker set also replace Bob position (see Fig. 2). Then we encapsulate the whole system of decaying atom, switch, and coffee-maker at A, while keeping the system at B side open. Now we can be sure, that by the time the decaying atom of Cesium emits photon to B side and triggers the switch at this side which then turns on the coffee-maker, it is "likely" that we could also observe the same cup of coffee at A side, even if we do not open the box.

We use term "likely" here because now we encounter a "quasi-deterministic" state where there is also small chance that the photon is shifted different from -0.0116, which is indeed what the Aspect, Dalibard and Roger experiment demonstrated in 1982 using a system of two correlated photons [16]. At this "shifted" phase, it could be that the switch will not turn on the coffee-maker at all, so when an observer opens the box at A side he will not get a cup of coffee.

If this hypothetical experiment could be verified in real world, then it would result in some wonderful implications, like prediction of ensembles of multi-particles system, — or a colony of cats.

Another version of this cat paradox is known as GHZ paradox: "The Greenberger-Horne-Zeilinger paradox exhibits some of the most surprising aspects of multiparticle entanglement" [18]. But we limit our discussion here on the original cat paradox.

2.3 Hidden-variable hypothesis

It would be incomplete to discuss quantum paradoxes, in particular Schrödinger's cat paradox, without mentioning hidden-variable hypothesis. There are various versions of this argument, but it could be summarised as an assertion that there is "something else" which should be included in the Quantum Mechanical equations in order to explain thoroughly all quantum phenomena. Sometimes this assertion can be formulated in question form [19]: Can Quantum Mechanics be considered complete? Interestingly, however, the meaning of "complete" itself remains quite abstract (fuzzy).

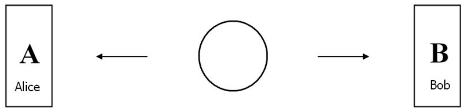

Figure 1: Aspect-type experiment

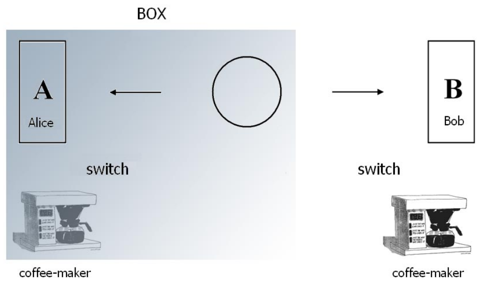

Figure 2: Aspect-type experiment in box

An interpretation of this cat paradox suggests that the problem arises because we mix up the macroscopic systems (observer's wavefunction and apparatus' wavefunction) from microscopic system to be observed. In order to clarify this, it is proposed that "...the measurement apparatus should be described by a classical model in our approach, and the physical system eventually by a quantum model" [20].

2.4 Hydrodynamic viewpoint and diffusion interpretation

In attempt to clarify the meaning of wave collapse and decoherence phenomenon, one could consider the process from (dissipative) hydrodynamic viewpoint [21]. Historically, the hydrodynamic/diffusion viewpoint of Quantum Mechanics has been considered by some physicists since the early years of wave mechanics. Already in 1933, Fuerth showed that Schrödinger equation could be written as a diffusion equation with an imaginary diffusion coefficient [1]

$$D_{qm} = \frac{i\hbar}{2m}\,. \tag{4}$$

But the notion of imaginary diffusion is quite difficult to comprehend. Alternatively, one could consider a classical Markov process of diffusion type to consider wave mechanics equation. Consider a continuity equation

$$\frac{\partial \rho}{\partial t} = -\nabla\left(\rho v\right), \tag{5}$$

where $v = v_0 = D\nabla \ln\rho$ (see [1]), which is a Fokker-Planck equation. Then the expectation value for the energy of particle can be written as [1]

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†The "coffee-maker" analogue came to mind after a quote: "A mathematician is a device for turning coffee into theorems" — Alfréd Rényi, a Hungarian mathematician, 1921–1970. (As quoted by Christopher J. Mark.)

$$<E> = \int \left(\frac{mv^2}{2} + \frac{D^2 m}{2} D \ln \rho^2 + eV \right) \rho d^3 x \,. \quad (6)$$

Alternatively, it could be shown that there is exact mapping between Schrödinger equation and viscous dissipative Navier-Stokes equations [6], where the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [6]. This Navier-Stokes interpretation differs appreciably from more standard Euler-Madelung fluid interpretation of Schrödinger equation [1], because in Euler method the fluid is described only in its inviscid limit.

2.5 How neutrosophy could offer solution to Schrödinger's paradox

In this regard, Neutrosophic Logic as recently discussed by one of these authors [22, 23, 24] could offer an interesting application in the context of Schrödinger's cat paradox. It could explain how the "mixed" state could be. It could be shown, that Neutrosophic probability is useful to those events, which involve some degree of indeterminacy (unknown) and more criteria of evaluation — as quantum physics. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events [25]. This new viewpoint for quantum phenomena is required because it is known that Quantum Mechanics is governed by uncertainty, but the meaning of "uncertainty" itself remains uncertain [16].

For example the Schrödinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a problem of "alternative worlds theory well represented by the neutrosophic set theory.

In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function ψ, which describes the superposition of possible states, may be simulated by a neutrosophic function, i. e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Now let's return to our cat paradox [25]. Let's consider a Neutrosophic set of a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between $^{-}0$ and 1^{+}. For example: x (0.5, 0.2, 0.3) belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is undecidable); or y (0, 0, 1) belongs to A (which normally means y is not for sure in A); or z (0, 1, 0) belongs to A (which means one does know absolutely nothing about z's affiliation with A). More general, x $\{$(0.2–0.3), (0.40–0.45)$\cup$[0.50–0.51], (0.2, 0.24, 0.28)$\}$ belongs to the set A, which mean:

- Owning a likelihood in between 20–30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- Owning a probability of 20% or 24% or 28% x is not in A;
- The indeterminacy related to the appurtenance of x to A is in between 40–45% or between 50–51% (limits included);
- The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $n\_\sup = 30\% + 51\% + 28\% > 100\%$ in this case.

To summarize our proposition [25], given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i. e. we don't know if it is dead or alive), then herein the Neutrosophic Logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic Logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We could expect that someday this proposition based on Neusotrophic Logic could be transformed into a useful guide for experimental verification of quantum paradox [15, 10].

Above results will be expanded into details in our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

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