

Completing Einstein's Proof of $E = mc^2$

C. Y. Lo

Applied and Pure Research Institute, 17 Newcastle Drive, Nashua, NH 03060, USA

E-mail: c_y_lo@yahoo.com; C_Y_Lo@alum.mit.edu

It is shown that Einstein's proof for $E = mc^2$ is actually incomplete and therefore is not yet valid. A crucial step is his implicit assumption of treating the light as a bundle of massless particles. However, the energy-stress tensor of massless particles is incompatible with an electromagnetic energy-stress tensor. Thus, it is necessary to show that the total energy of a light ray includes also non-electromagnetic energy. It turns out, the existence of intrinsic difference between the photonic and the electromagnetic energy tensors is independent of the coupling of gravity. Nevertheless, their difference is the energy-stress tensor of the gravitational wave component that is accompanying the electromagnetic wave component. Concurrently, it is concluded that Einstein's formula $E = mc^2$ necessarily implies that the photons include non-electromagnetic energy and that the Einstein equation of 1915 must be rectified.

1 Introduction

In physics, the most famous formula is probably $E = mc^2$ [1]. However, it is also this formula that many⁽¹⁾ do not understand properly [2, 3]. Einstein has made clear that this formula must be understood in terms of energy conservation [4]. In other words, there is energy related to a mass, but there may not be an equivalent mass for any type of energy [2]. As shown by the Reissner-Nordstrom metric [5, 6], the gravity generated by mass and that by the electromagnetic energy are different because an electromagnetic energy stress tensor is traceless. Thus, the relationship between mass and energy would be far more complicated than as commonly believed.

In Einstein's 1905 derivation,⁽²⁾ he believed [7] that the corresponding was between mass and any type of energy although he dealt with only the light, which may include more than just electromagnetic energy. Moreover, although his desired generality has not been attained, his belief was very strong. On this, Stachel [7] wrote,

“Einstein returned to the relation between inertial mass and energy in 1906 and in 1907 giving more general arguments for their complete equivalence, but he did not achieve the complete generality to which he inspired. In his 1909 Salzburg talk, Einstein strongly emphasized that inertial mass is a property of all form of energy, and therefore electromagnetic radiation must have mass. This conclusion strengthened Einstein's belief in the hypothesis that light quanta manifest particle-like properties.”

Apparently, the publications of the papers of Reissner [6] and Nordstrom [5] have changed the view of Einstein as shown in his 1946 article [4].

Perhaps, a root of misunderstanding $E = mc^2$ is related to the fact that the derivation of this formula [8] has not been fully understood. In Einstein's derivation, a crucial step is his

implicit assumption of treating light as a bundle of massless particles. However, because gravity has been ignored in Einstein's derivation, it was not clear that an electromagnetic energy-stress tensor is compatible with the energy-stress tensor of massless particles.

Such an issue is valid since the divergence of an electromagnetic energy-stress tensor $\nabla_c T(E)^{cb}$ (where ∇_c is a covariant derivative) generates only the Lorentz force, whereas the divergence of a massive energy-stress tensor $\nabla_c T(m)^{cb}$ would generate the geodesic equation [9].

Thus, the energy-stress of photons $T(L)_{ab}$ would be

$$T(L)_{ab} = T(E)_{ab} + T(N)_{ab} \quad (1)$$

or

$$T(N)_{ab} = T(L)_{ab} - T(E)_{ab}$$

where $T(E)_{ab}$ and $T(N)_{ab}$ are respectively the electromagnetic energy-stress tensor and a non-electromagnetic energy-stress tensor. Besides, being intrinsically traceless, $T(E)_{cb}$ would not be compatible with Einstein's formula $\Delta E = \Delta mc^2$. Based on the fact that the electromagnetic energy is dominating experimentally, it is natural to assume as shown later that $T(N)_{ab}$ is in fact the gravitational energy-stress tensor $T(g)_{ab}$.

2 A field equation for the accompanying gravitational wave

Physics requires also that the energy-stress tensor for photons $T(L)_{ab}$ is: (1) traceless, (2) $T(L)_{ab} \approx T(E)_{ab}$ and $[T(L)_{tt} - T(E)_{tt}] > 0$ on the average, and (3) related to a gravitational wave, i. e. satisfying

$$R_{ab} - \frac{1}{2} g_{ab} R = K T(g)_{ab} = -K [T(E)_{ab} - T(L)_{ab}], \quad (2)$$

where R_{ab} is the Ricci tensor, and $R = g^{mn} R_{mn}$. Eq. (2) dif-

fers from Einstein equation with an additional term $T(L)_{ab}$ having a coupling of different sign. However, Eq. (2) is similar to the modified Einstein equation,

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = -K [T(m)_{ab} - T(g)_{ab}], \quad (3)$$

which is necessitated by the Hulse-Taylor experiment [10, 11]. $T(g)_{ab}$ is non-zero since a gravitational wave carries energy. From Eq. (2), we have $\nabla_c T(L)^{cb} = 0$ since there $\nabla_c T(E)^{cb} = 0$ and $\nabla_c G^{cb} \equiv 0$.

Related to Eq. (2), a crucial question is whether the Einstein equation with only the electromagnetic wave energy-stress tensor as the source is valid. It has been found that such an equation cannot produce a physically valid solution [12]. Historically, it is due to that the Einstein equation does have a physical plane-wave solution that the need of a photonic energy-stress tensor is recognized (see also Sect. 3). One may object that the general form of gravitational energy-stress tensor is not yet known although its approximation for the weak gravity with the massive source is known to be equivalent to Einstein's pseudo-tensor for the gravitational energy-stress [10]. However, for this case, the related gravitational energy-stress tensor is defined by formula (1).

Now the remaining question is whether (2) would produce a gravitational wave. However, we should address first whether an electromagnetic wave has an accompanying gravitational wave. The answer is affirmative because the electromagnetic energy is propagating with the allowed maximum speed in Special Relativity.⁽³⁾ Thus, the gravity due to the light energy should be distinct from that generated by massive matter [13].

Since a field emitted from an energy density unit means a non-zero velocity relative to that unit, it is instructive to study the velocity addition. According to Special Relativity, the addition of velocities is as follows:

$$u_x = \frac{\sqrt{1 - v^2/c^2}}{1 + u'_z v/c^2} u'_x, \quad u_y = \frac{\sqrt{1 - v^2/c^2}}{1 + u'_z v/c^2} u'_y, \quad (4)$$

$$\text{and } u_z = \frac{u'_z + v}{1 + u'_z v/c^2},$$

where velocity \vec{v} is in the z -direction, (u'_x, u'_y, u'_z) is a velocity w. r. t. a system moving with velocity v , c is the light speed, $u_x = dx/dt$, $u_y = dy/dt$, and $u_z = dz/dt$. When $v = c$, independent of (u'_x, u'_y, u'_z) one has

$$u_x = 0, \quad u_y = 0, \quad \text{and } u_z = c. \quad (5)$$

Thus, neither the direction nor the magnitude of the velocity \vec{v} ($= \vec{c}$) have been changed.

This implies that nothing can be emitted from a light ray, and therefore no field can be generated outside the light ray. To be more specific, from a light ray, no gravitational field can be generated outside the ray although, accompanying the

light ray, a gravitational field $g_{ab} (\neq \eta_{ab}$ the flat metric) is allowed within the ray.

According to the principle of causality [13], this accompanying gravity g_{ab} should be a gravitational wave since an electromagnetic wave is the physical cause. This would put General Relativity into a severe test for theoretical consistency. But, this examination would also have the benefit of knowing whether Einstein's implicit assumption in his proof for $E = mc^2$ is valid.

Let us consider the energy-stress tensor $T(L)_{ab}$ for photons. If a geodesic equation must be produced, for a monochromatic wave with frequency ω , the form of a photonic energy tensor should be similar to that of massive matter. Observationally, there is very little interaction, if any, among photons of the same ray. Theoretically, since photons travel in the velocity of light, there should not be any interaction (other than collision) among them. Therefore, the photons can be treated as a bundle of massless particles just as Einstein [8] did.

Thus, the photonic energy tensor of a wave of frequency ω should be dust-like and traceless as follows:

$$T^{ab}(L) = \rho P^a P^b, \quad (6)$$

where ρ is a scalar and is a function of $u (= ct - z)$. In the units $c = \hbar = 1$, $P^t = \omega$. The geodesic equation, $P^c \nabla_c P^b = 0$, is implied by $\nabla_c T(L)^{cb} = 0$ and also $\nabla_c (\rho P^c) = 0$. Since $\nabla_c (\rho P^c) = [\rho g^{bc} g'_{bc} + \rho'] (P^t - P^z) = 0$, formula (6) does produce a geodesic equation if Eq. (2) is satisfied.

3 The reduced Einstein equation for an electromagnetic plane wave

Let us consider a ray of uniform electromagnetic waves (i. e. a laser beam) propagating in the z -direction. Within the ray, one can assume that the wave amplitude is independent of x and y . Thus, the electromagnetic potentials are plane-waves, and in the unit that light speed $c = 1$,

$$A_k(x, y, z, t) = A_k(t - z), \quad \text{where } k = x, y, z, t. \quad (7)$$

Due to the principle of causality, the metric g_{ab} is functions of $u (= t - z)$, i. e.,

$$g_{ab}(x, y, z, t) = g_{ab}(u), \quad \text{where } a, b = x, y, z, t. \quad (8)$$

Since, for this case, the coordinates for Special Relativity are also valid for General Relativity [14–16], such a consideration is valid. Let P^k be the momentum of a photon. If a photon is massless, one obtains the conditions,

$$P^z = P^t, \quad P^x = P^y = 0, \quad \text{and } P^m g_{mk} = P_k = 0, \quad (9)$$

for $k = x, y$, and $v (= t + z)$. Eq. (9a) is equivalent to

$$\begin{aligned} g_{xt} + g_{xz} &= 0, & g_{yt} + g_{yz} &= 0, \\ \text{and } g_{tt} + 2g_{tz} + g_{zz} &= 0, \end{aligned} \quad (10)$$

or

$$\begin{aligned} g^{xt} - g^{xz} &= 0, & g^{yt} - g^{yz} &= 0, \\ \text{and } g^{tt} - 2g^{zt} + g^{zz} &= 0. \end{aligned} \quad (11)$$

The transverse of an electromagnetic wave implies

$$\begin{aligned} P^m A_m &= 0, \\ \text{or equivalently } A_z + A_t &= 0. \end{aligned} \quad (12)$$

Eqs. (7) to (9) imply that not only the geodesic equation, the Lorentz gauge, but also Maxwell's equation are satisfied. Moreover, the Lorentz gauge becomes equivalent to a covariant expression.

For an electromagnetic wave being the source, Einstein [17] believed the field equation is $G_{ab} = -KT(E)_{ab}$, where $T(E)_{ab} = -g^{mn} F_{ma} F_{nb} + \frac{1}{4} g_{ab} F^{mn} F_{mn}$, while $F_{ab} = \partial_a A_b - \partial_b A_a$ is the field tensor. Since the trace of the energy-stress tensor is zero, $R = 0$. It follows that

$$R_{tt} = -R_{tz} = R_{zz}, \quad (13)$$

because $F^{mn} F_{mn} = 0$ due to Eq. (9). The other components are zero [12]. Then,

$$\begin{aligned} R_{tt} &\equiv -\frac{\partial \Gamma_{tt}^m}{\partial x^m} + \frac{\Gamma_{mt}^m}{\partial t} - \Gamma_{mn}^m \Gamma_{tt}^n + \Gamma_{nt}^m \Gamma_{mt}^n = \\ &= -KT(E)_{tt} = Kg^{mn} F_{mt} F_{nt}. \end{aligned} \quad (14)$$

After some lengthy algebra [12], Eq. (14) is simplified to a differential equation of u as follows:

$$\begin{aligned} G'' - g'_{xx} g'_{yy} + (g'_{xy})^2 - G'(g'/2g) &= 2GR_{tt} = \\ = 2K(F_{xt}^2 g_{yy} + F_{yt}^2 g_{xx} - 2F_{xt} F_{yt} g_{xy}), \end{aligned} \quad (15)$$

where

$$G \equiv g_{xx} g_{yy} - g_{xy}^2, \quad \text{and } g = |g_{ab}|,$$

the determinant of the metric. The metric elements are connected by the following relation:

$$-g = Gg_t^2, \quad \text{where } g_t = g_{tt} + g_{tz}. \quad (16)$$

Note that Eqs. (35.31) and (35.44) in reference [18] and Eq. (2.8) in reference [19] are special cases of Eq. (15). But, their solutions are unbounded [17]. However, compatibility with Einstein's notion of weak gravity is required by the light bending calculation and is implied by the equivalence principle [20].

Equations (9)–(16) allow A_t , g_{xt} , g_{yt} , and g_{zt} to be set to zero. In any case, these assigned values have little effect in subsequent calculations. For the remaining metric elements (g_{xx} , g_{xy} , g_{yy} , and g_{tt}), however, Eq. (15) is sufficient to show that there is no physical solution. In other words, in contrast to Einstein's belief [17], the difficulty of this equation is not limited to mathematics.

4 Verification of the rectified Einstein equation

Now, consider an electromagnetic plane-wave of circular polarization, propagating to the z -direction

$$A_x = \frac{1}{\sqrt{2}} A_0 \cos \omega u, \quad \text{and } A_y = \frac{1}{\sqrt{2}} A_0 \sin \omega u, \quad (17)$$

where A_0 is a constant. The rotational invariants with respect to the z -axis are constants. These invariants are: G_{tt} , R_{tt} , $T(E)_{tt}$, G , $(g_{xx} + g_{yy})$, g_{tz} , g_{tt} , g , and etc. It follows that [12–13]

$$\begin{aligned} g_{xx} &= -1 - C + B_\alpha \cos(\omega_1 u + \alpha), \\ g_{yy} &= -1 - C - B_\alpha \cos(\omega_1 u + \alpha), \\ g_{xy} &= \pm B_\alpha \sin(\omega_1 u + \alpha), \end{aligned} \quad (18)$$

where C and B_α are small constants, and $\omega_1 = 2\omega$. Thus, metric (18) is a circularly polarized wave with the same direction of polarization as the electromagnetic wave (17). On the other hand, one also has

$$\begin{aligned} G_{tt} &= 2\omega^2 B_\alpha^2 / G \geq 0, \quad \text{and} \\ T(E)_{tt} &= \frac{1}{2G} \omega^2 A_0^2 (1 + C - B_\alpha \cos \alpha) > 0, \end{aligned} \quad (19)$$

where $G = (1 + C)^2 - B_\alpha^2 > 0$. Thus, it is not possible to satisfy Einstein's equation because $T(E)_{tt}$ and G_{tt} have the same sign. Therefore, it is necessary to have a photonic energy-stress tensor.

If the photons are massless particles, the photonic energy-stress tensor (6) has a density function [12],

$$\rho(u) = -A_m g^{mn} A_n \geq 0 \quad (20)$$

which is a scalar function of $u (= t - z)$. Since light intensity is proportional to the square of the wave amplitude, which is Lorentz gauge invariant, $\rho(u)$ can be considered as the density function of photons. Then

$$\begin{aligned} T_{ab} &= -T(g)_{ab} = T(E)_{ab} - T(L)_{ab} = \\ &= T(E)_{ab} + A_m g^{mn} A_n P_a P_b. \end{aligned} \quad (21)$$

Note that since $\rho(u)$ is a positive non-zero scalar consisting of A_k and/or fields such that, on the average, $T(L)_{ab}$ is approximately $T(E)_{ab}$ and Eq. (2) would have physical solutions, $\rho = -A_m g^{mn} A_n$ is the only choice.

As expected, tensor $T(L)_{ab}$ enables a valid solution for wave (17). According to Eq. (2) and formula (21),

$$T_{tt} = -\frac{1}{G} \omega^2 A_0^2 B_\alpha \cos \alpha < 0, \quad (22)$$

since $B_\alpha = (K/2) A_0^2 \cos \alpha$. Thus, $T(g)_{tt} = -T_{tt}$ is of order K . It will be shown that $\cos \alpha = 1$.

To confirm the general validity of (2) further, consider a wave linearly polarized in the x -direction,

$$A_x = A_0 \cos \omega(t - z). \quad (23)$$

Then,

$$\begin{aligned} T(E)_{tt} &= -\frac{g_{yy}}{2G} \omega^2 A_0^2 [1 - \cos 2\omega(t - z)] \quad \text{and} \\ T_{tt} &= \frac{g_{yy}}{2G} \omega^2 A_0^2 \cos 2\omega(t - z). \end{aligned} \quad (24)$$

Note that independent of the coupling K , T_{tt} is non-zero. Since the gravitational component is not an independent wave, $T(g)_{tt} (= -T_{tt})$ is allowed to be negative or positive [13]. Eq.(19) implies $(g_{xx} + g_{yy})'$ to be of first order [13], and thus its polarization has to be different.

It turns out that the solution is a linearly polarized gravitational wave and that, as expected, the time-average of $T(g)_{tt}$ is positive of order K [13]. From the viewpoint of physics, for an x -directional polarization, gravitational components related to the y -direction, remains the same. In other words,

$$g_{xy} = 0 \quad \text{and} \quad g_{yy} = -1. \quad (25)$$

It follows [10, 11] that $G = -g_{xx}$ and the general solution for wave (18) is:

$$-g_{xx} = 1 + C_1 - (K/2)A_0^2 \cos[2\omega(t - z)], \quad (26)$$

$$\text{and } g_{tt} = -g_{zz} = \sqrt{g/g_{xx}},$$

where C_1 is a constant and g is the determinant of the metric. The frequency ratio is the same as that of a circular polarization. However, there is no phase difference as α in (18). According to the principle of causality, α has a value, and to be consistent with (26) $\alpha = 0$.

However, if $T(L)_{ab}$ were absent, one would have,

$$\begin{aligned} -g_{xx} &= 1 + C_1 - (K/4)A_0^2 (2\omega^2(t - z)^2 + \\ &+ \cos[2\omega(t - z)]) + C_2(t - z), \end{aligned} \quad (27)$$

where C_1 and C_2 are constants. But solution (27) is invalid in physics since $(t - z)^2$ grows very large as time goes by. This would "represent" the effects if Special Relativity were invalid, and the wave energy were equivalent to mass. This illustrates that Einstein's notion of weak gravity, which is the theoretical basis for his calculation on the bending of light, may not be compatible with the Einstein equation with an inadequate source term.

5 Conclusions and discussions

A photonic energy-stress tensor has been obtained to satisfy the demanding physical requirements. The energy and momentum of a photon are proportional to its frequency

although, as a classical theory, their relationship with the Planck constant h is not yet clear. Just as expected from Special Relativity, indeed, the gravity of an electromagnetic wave is an accompanying gravitational wave propagating with the same speed.⁽⁴⁾ Concurrently, for this case, the need of modifying the Einstein equation is accomplished. Then, clearly the gravity due to the light is negligible in calculating the light bending [8].

In this derivation, it is crucial that the spatial coordinates are proven the same in Special and General Relativity [14–16] because the space coordinates must have the Euclidean-like structure.⁽⁵⁾ For this case, even the time coordinate is the same, and the plane wave satisfied the Maxwell equation in terms of both Special and General Relativity [16]. Thus, Special Relativity and General Relativity are consistent with each other. Einstein's proof is clearly incomplete since the energy-stress tensor of photons is different from that of electromagnetism.

A particle such as the photon has no inertial mass since it is subjected to only absorption and emission, but not acceleration and deceleration. Based on Special Relativity, it has been shown that the electromagnetic energy is distinct from the energy of a rest mass.⁽⁶⁾ Interestingly, *it is precisely because of this non-equivalence of mass and energy that photonic energy-stress tensor (6) is valid, and the formula $E = mc^2$ can be proven.*

One might argue that experiment shows the notion of massless photons is valid, and thus believed the equivalence of mass and electromagnetic energy. However, while the addition of two massless particles may end up with a rest mass, the energy-stress tensor of electromagnetism cannot represent a rest mass since such a tensor is traceless. Thus, the formula⁽⁷⁾ $\Delta E = \Delta mc^2$ necessarily implies that $T(L)_{ab}$ must include non-electromagnetic energy. Note that $[T(L)_{tt} - T(E)_{tt}]$ being non-zero, is independent of the gravitational coupling constant K . This makes it clear that the photonic energy tensor is intrinsically different from the electromagnetic energy tensor.

Although the formula $E = mc^2$ has been verified in numerous situations [1, 18], its direct physical meaning related to gravity was not understood;⁽⁸⁾ and thus this formula is often misinterpreted, in conflict with General Relativity [2, 9], as any type of energy being equivalent to a mass [3]. A related natural question is how to measure the gravitational component of a light ray. However, in view of the difficulties encountered in measuring pure gravitational waves, the quantitative measurement of such a gravitational component is probably very difficult with our present level of technology although its qualitative existence is proven by the formula $E = mc^2$.

Both quantum theory and relativity are based on the phenomena of light. The gravity of photons finally shows that there is a link between them. It is gravity that makes the notion of photons compatible with electromagnetic waves.

Clearly, gravity is no longer just a macroscopic phenomena, but also a microscopic phenomena of crucial importance to the formula $E = mc^2$. In Einstein's proof, it has not been shown whether his implicit assumption is compatible with electromagnetism. This crucial problem is resolved with the gravity of an electromagnetic wave. Einstein probably would smile heartily since his formula confirms the link that relates gravity to quantum theory.

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Endnotes

- (1) They include, but not limited to, Fock [21], Hawking [22], Misner, Thorne, & Wheeler [18], Tolman [23], and Will [3].
- (2) In 1907 Plank [24] criticized the Einstein argument, and presented his own argument to show that the transfer of heat is associated with a similarly related transfer of inertial mass [7].
- (3) In this paper, the convention of the metric signature for Special Relativity is $(1, -1, -1, -1)$.
- (4) Some arguments, which were presented differently in the literature [13], are included in this paper for the convenience of the readers. For instance, now the value of α in (18) is obtained.
- (5) Einstein called this structure as "in the sense of Euclidean geometry" [8], but failed to understand its physical meaning in terms of measurements [15, 25]. Weinberg [26] has showed, however, that in a curved space the coordinates can be straight.
- (6) However, there are theorists such as Tolman [23], who incorrectly saw no difference in terms of gravity between mass and the energy in a light ray.
- (7) Einstein's formula $\Delta E = \Delta mc^2$ is proven for radiating energy. Thus, it is applicable to the atomic bomb.
- (8) Bodanis [1] gives a good account of how the formula $E = mc^2$ is applied. However, like many others, he also misinterpreted the formula as general equivalence between any type of energy and mass.

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