Upper Limit in the Periodic Table of Elements

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The method of rectangular hyperbolas is developed for the first time, by which a means for estimating the upper bound of the Periodic Table is established in calculating that its last element has an atom mass of 411.663243 and an atomic number (the nuclear charge) of 155. The formulating law is given.

1 Introduction. The mathematical basis

The periodic dependence of the properties of the elements on their atomic mass, as discovered by D. I. Mendeleev in 1869, predicted the existence of new elements in appropriate locations in the Periodic Table.

Progress in synthesis and in the study of the properties of the far transuranium elements has increased interest in the question of the upper limits of the Periodic Table. G. T. Seaborg, J. L. Bloom and V. I. Goldanskii emphasized that the charge of the atomic nucleus and the position occupied by the element "define unambiguously the structure of electron jackets of its atoms and characterize the whole set of its chemical properties". They suggested the existence of nuclei containing 114, 126 and 164 protons, 184, and 258 neutrons and the Table arrangement of the relevant elements [1, 2].

The objective of this study is to determine the possible number of chemical elements, along with atomic masses and atomic numbers up to the final entry in the Periodic Table.

The calculations were performed on the basis of IUPAC [3] table data for all known elements. The basic principle resides in the idea that the proportion of the defined element Y in any chemical compound of molecular mass X should be related to its single gram-atom. In this case, if K is the atomic mass, the equation $Y = \frac{K}{X}$ would represent a rectangular hyperbola in the first quadrant ($K > 0$). Its asymptotes conform to the axis coordinates, and semi-axis $a = b = \sqrt{2\,|K|}$. The peak of the curve should occur on the virtual axis inclined at an angle of 45° to the positive direction of the abscissa axis. The necessary conditions associated with this chemical conception are: $Y \leqslant 1$ and $K \leqslant X$.

The foregoing equation differs only in the atomic mass for each element of the Periodic Table and allows calculation of the proportion of the element in any compound. Accuracy plotting the curve and the associated straight line in the logarithmic coordinates depends on the size of the steps in the denominator values, which can be entirely random but must be on the relevant hyperbola in terms of X. Consequently, it can be computed without difficulty by prescribing any value of the numerator and denominator. In Table 1a are given both known oxygen containing compounds and random data on X arranged in the order of increasing molecular mass. Fig. 1 depicts the hyperbola (the value of the approximation certainty $R^2 = 1$), calculated for 1 gram-atom of oxygen.

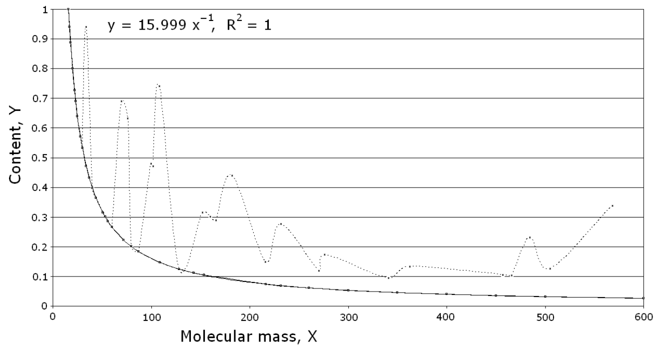

Fig. 1: Oxygen content versus the molecular mass of compounds on estimation to 1 gram-atom (hyperbola $y = k/x$) and the total amount of O (maxima, leaders). The molecular mass in the table is given according to its increase.

Estimation of the unobserved content in the chemical compound as determined by the formula is expressed on the plot by the polygonal line (Table 1b, Fig. 1). It is obvious from the Fig. 2a that the hyperbolic function of the elemental proportion in chemical compounds plotted against molecular mass, by the example of the second Group, is true ($R^2 = 1$). In the logarithmic coordinates (Fig. 2b) it is represented as the straight lines arranged in the fourth quadrant (to the right of hydrogen) all with slope 1. With the view to expansion of the basis of the arguments, this example is given for the first Group including "roentgenium" No. 111, a more recently identified element, and the predicted No. 119 and No. 155. The real axis is shown here, on which the peaks of all hyperbolas of the Periodic Table are arranged (see below).

2 Using the theorem of Lagrange

It is clear from the Fig. 2a that with the rise of the atomic mass the curvature of the hyperbola decreases (the radius of curvature increases), and the possibility to define its peak, for example, by means of graphical differentiation, becomes a problem due to errors of both subjective and objective character (instrument, vision and so on). Therefore, to estimate the curve peak of the hyperbola the mathematical method of the theorem of Lagrange was used [4].

K	X	$Y = \frac{K}{X}$	$\ln X$	$\ln Y$	Compound		Compound	X	$Y = n\,\frac{K}{X}$
15.9994	15.999	1	2.77255	0	O		O	15.9994	1
15.9994	17.007	0.9408	2.83363	−0.0611	$\frac{1}{2}H_2O_2$		H_2O	18.015	0.88811546
15.9994	18.015	0.8881	2.8912	−0.1187	H_2O		BeO	25.01	0.63972011
15.9994	20	0.8	2.99573	−0.2232	—		CO	28.01	0.57120314
15.9994	22	0.7272	3.09104	−0.3185	—		NO	30.006	0.53320669
15.9994	23.206	0.6895	3.14441	−0.3719	$\frac{1}{3}B_2O_3$		H_2O_2	34.01	0.94089974
15.9994	25.01	0.6397	3.21928	−0.4467	BeO		MgO	40.304	0.39698293
15.9994	28.01	0.5712	3.33256	−0.56	CO		N_2O	44.012	0.36353722
15.9994	30.006	0.5332	3.4014	−0.6288	NO		CaO	56.077	0.28532197
15.9994	33.987	0.4708	3.52598	−0.7534	$\frac{1}{3}Al_2O_3$		COS	60.075	0.26633375
15.9994	37	0.4324	3.61092	−0.8384	—		B_2O_3	69.618	0.68947686
15.9994	40.304	0.397	3.69645	−0.9239	MgO		N_2O_3	76.01	0.63149586
15.9994	44.012	0.3635	3.78446	−1.0119	N_2O		CuO	79.545	0.20114401
15.9994	50.663	0.3158	3.9252	−1.1526	$\frac{1}{3}Cr_2O_3$		Cl_2O	86.905	0.18410908
15.9994	53.229	0.3006	3.9746	−1.2021	$\frac{1}{3}Fe_2O_3$		CrO_3	99.993	0.4800336
15.9994	56.077	0.2853	4.02673	−1.2542	CaO		Al_2O_3	101.96	0.47077285
15.9994	60.075	0.2663	4.09559	−1.323	COS		N_2O_5	108.008	0.74068588
15.9994	71.844	0.2227	4.2745	−1.5019	FeO		CdO	128.41	0.12460089
15.9994	79.545	0.2011	4.37632	−1.6038	CuO		Cr_2O_3	151.99	0.31581025
15.9994	86.905	0.1841	4.46482	−1.6923	Cl_2O		Fe_2O_3	159.687	0.30058803
15.9994	108.6	0.1473	4.6877	−1.9151	$\frac{1}{3}La_2O_3$		Co_2O_3	165.86	0.2894007
15.9994	128.41	0.1246	4.85523	−2.0827	CdO		V_2O_5	181.88	0.43985045
15.9994	143.09	0.1118	4.96348	−2.1909	Cu_2O		WO_2	215.84	0.14825797
15.9994	153.33	0.1043	5.03257	−2.26	BaO		Fe_3O_4	231.53	0.27642206
15.9994	216.59	0.0739	5.37801	−2.6055	HgO		UO_2	270.027	0.11850667
15.9994	231.74	0.069	5.44562	−2.6731	Ag_2O		Ag_2CO_3	275.75	0.174064
15.9994	260	0.0615	5.56068	−2.7881	—		UO_2Cl_2	340.94	0.0938546
15.9994	300	0.0533	5.70378	−2.9312	—		Gd_2O_3	362.5	0.132409
15.9994	350	0.0457	5.85793	−3.0854	—		Tl_2O_3	456.764	0.10508709
15.9994	400	0.04	5.99146	−3.2189	—		Bi_2O_3	465.96	0.103009
15.9994	450	0.0356	6.10925	−3.3367	—		Re_2O_7	484.4	0.231205
15.9994	500	0.032	6.21461	−3.4421	—		Tl_2SO_4	504.8	0.1267781
15.9994	600	0.0267	6.39693	−3.6244	—		$Ce_2(SO_4)_3$	568.43	0.33776

Table 1: Content of oxygen Y in compounds X per gram-atom (Table 1a) left and summarized O (Table 1b) on the right.

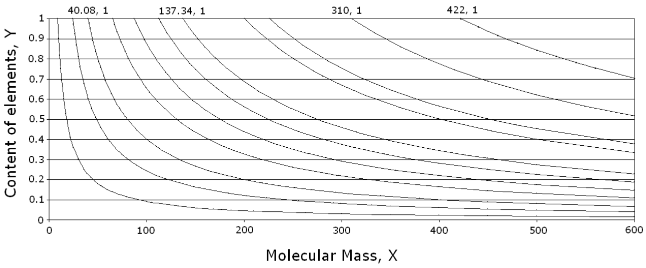

Fig. 2a: Element proportion in chemical compounds against molecular mass ($y = k/x$) on the example of the 2nd Group of the Periodic Table, plus No.126 and No.164.

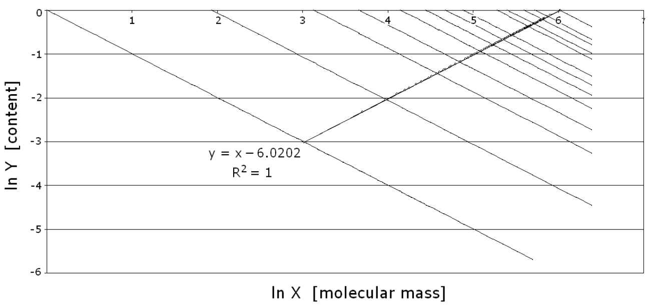

Fig. 2b: Element content versus the molecular mass in chemical compounds of the 1st Group and No.111, calculated No.119, No.155; + virtual axis.

For example, the coordinates of the peak for beryllium are as follows: $X = 60.9097$, $Y = 0.14796$, the normal equation is $Y = 0.0024292\,X$. Taking into consideration that the semiaxis of the rectangular hyperbola $a = b = \sqrt{2\,|K|}$, the coordinates of the point $X_0 = Y_0 = \sqrt{K}$.

Let us examine this fact in relation to elements with the following atomic masses (K): beryllium Be (9.0122), random Z (20), chromium Cr (51.9961), mercury Hg (200.59), No. 126 (310), random ZZ (380), No. 164 (422), random ZZZ (484). In this case $X_0 = Y_0 = \sqrt{K}$, and correspondingly, 3.00203, 4.472136, 7.210825, 14.16298, 17.606817, 19.493589, 20.54264, 22.

The obtained values are the coordinates of the rectangular hyperbola peaks ($X_0 = Y_0$), arranged along the virtual axis, the equation of which is $Y = X$ (because $\tan\alpha = 1$).

3 The point of crossing and the scaling coefficient

Our attention is focused on the point of crossing of the virtual axis with the line $Y = 1$ in Fig. 3 when the atomic mass and the molecular mass are equal, i.e. $K = X$. It is possible only in the case when the origin of the hyperbola and its peak coincide in the point with the maximum content Y according to the equation $Y = \frac{K}{X}$.

The atomic mass of this element was calculated with application of the scaling coefficient and the value of the slope of the virtual axis (the most precise mean is 0.00242917): $\tan\alpha = \frac{y}{x} = 0.00242917$, from which $x = \frac{y}{\tan\alpha}$. Due to

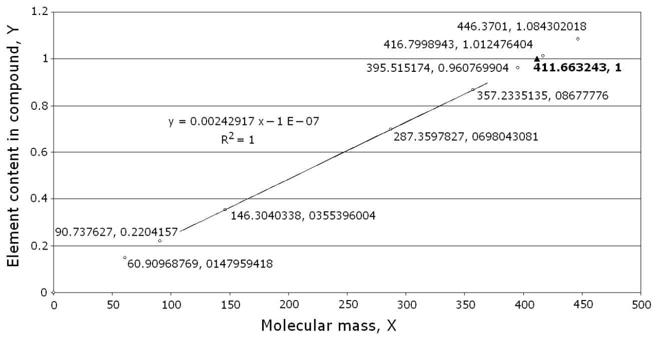

Fig. 3: The virtual axis of the hyperbolas $y = k/x$ with application of the scaling coefficient.

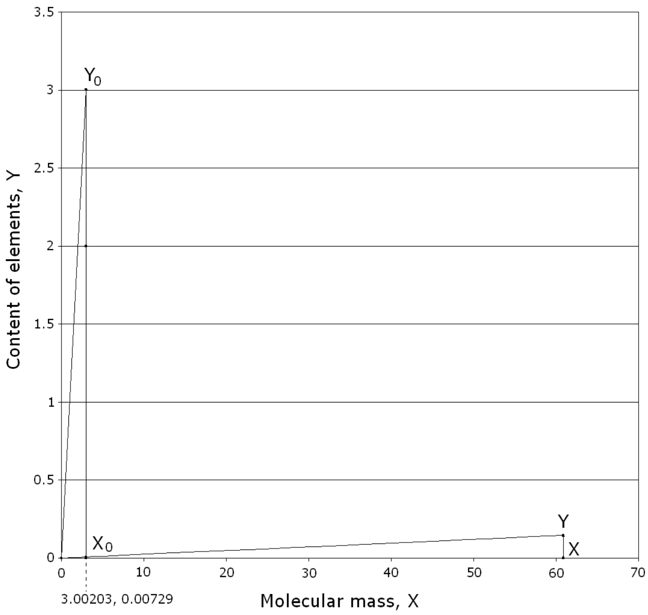

Fig. 4: Inversely proportional dependency in coordinates at calculation of the scaling coefficient.

the fact that at this point $k = x$ we have: $\frac{y}{\tan\alpha} = \frac{1}{\tan\alpha} = 411.663243$. This value is equal to the square of the scaling coefficient too: $20.2895^2 = 411.6638$, $\Delta = 0.0006$.

The coefficient was calculated from matching of the coordinates of the peak hyperbola for beryllium: $X_0 = Y_0 = \sqrt{K}$ and $X = 60.9097$, $Y = 0.14796$. Using this data to construct two triangles (Fig. 4), one easily sees an inversely proportional relationship: $\frac{X}{X_0} = \frac{Y_0}{Y}$, whence $\frac{X}{X_0} = \frac{60.9097}{3.00203} = 20.2895041$ and $\frac{Y_0}{Y} = \frac{3.00203}{0.14976} = 20.28947013$, $\Delta = 0.000034$.

The calculated value $M = 20.2895$ is the scaling coefficient. With its help the scale of system coordinates can be reorganised.

Now if one rectangular hyperbola peak is known, $X_0 = Y_0 = \sqrt{K}$, then the new coordinates will be: $X = X_0\,M$ or $X = M\sqrt{K}$, $Y = \frac{\sqrt{K}}{M}$. Furthermore, $\tan\alpha_0 = \frac{Y_0}{X_0} = 1$, so $\tan\alpha = \frac{Y}{X} = \frac{1}{M^2}$. At the same time at $Y = 1$ and $K = X$, $X = \frac{Y}{\tan\alpha}$ or $K = \frac{Y}{\tan\alpha} = \frac{1}{\tan\alpha} = M^2$.

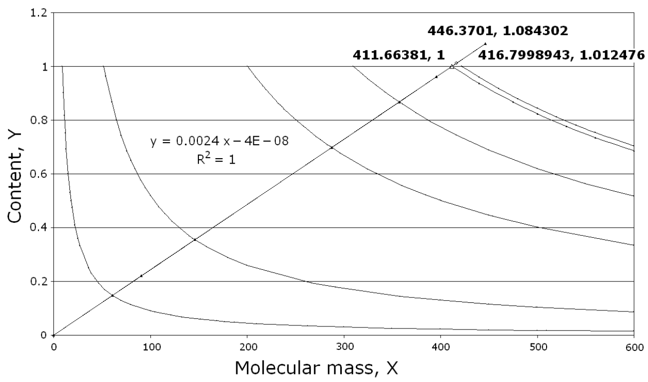

Fig. 5: Element content versus the compound's molecular mass and the hyperbola virtual axes of type $y = k/x$ for the entire Periodical Table. Additionally No.126, No.164 and that rated on (ZZZZ) are introduced.

The results obtained are plotted in Fig. 5 in comparison with the hyperbolas of such elements as Be, Cr, Hg and the hypothetical No.126 (atomic mass = 310), No.164 (atomic mass = 422), ZZZZ (atomic mass = 411.66). It is obvious that it is practically impossible to choose and calculate precisely the curve peak for an atomic mass exceeding the value 250 without the application of the mathematical method adduced herein.

The rated element ZZZZ is the last in the Periodic Table because the hyperbola No.164 after it crosses the virtual axis at the point which coordinates are: $X_0 = Y_0 = \sqrt{422} = 20.5426386$.

After scaling we have $X = 20.2895 \times 20.5426386 = 416.8$ and $Y = 20.5426386/20.2895 = 1.0125$, but this makes no sense because Y cannot exceed the value 1. In addition, the hypothetical atomic mass 422 occurred higher than the molecular mass 416.8, i.e. $X < K$, but that is absurd. Similarly, it is obvious from Fig. 2b how the virtual axis (the equation $Y = X - 6.0202$ where $Y = \ln y$, $X = \ln x$) crossing all the logarithmic straight lines at the points corresponding to the hyperbola peaks, takes the value $\ln x = 6.0202$ at $\ln y = 0$, or after taking logarithms, $X = 411.66$, $Y = 1$.

4 The atomic (ordinal) number

To determine important characteristics of the atomic number some variants of graphical functions of the atomic mass versus the nucleus of all the elements were studied, including No.126. One of them is exponential, with the equation $Y = 1.6091\,e^{1.0992x}$ (where y is the atomic mass, x is ln No) at $R^2 = 0.9967$. After taking the logarithm of the both sides and inserting the atomic mass of 411.66 we have No.155. The calculations also demonstrated that the ordinal No.126 should have the atomic mass 327.2 but not 310.

Finally, the following atomic masses were obtained: No.116 — 298.7, No.118 — 304.4, No.119 — 307.2, No.120 — 310, No.126 — 327.3, No.155 — 411.66.

5 The new law

Based on the foregoing, the heretofore unknown Hyperbolic law of the Periodic Table of Elements is established.

This law is due to the fact that the element content Y when estimated in relation to 1 gram-atom, in any chemical combination with molecular mass X, may be described by the adduced equations for the positive branches of the rectangular hyperbolas of the type $Y = \frac{K}{X}$ (where $Y \leqslant 1$, $K \leqslant X$), arranged in the order of increasing nuclear charge, and having the common virtual axis with their peaks tending to the state $Y = 1$ or $K = X$ as they become further removed from the origin of coordinates, reaching a maximum atomic mass designating the last element.

References

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