On the Rate of Change of Period for Accelerated Motion and Their Implications in Astrophysics

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We have derived in this paper, the relationship that needs to be satisfied when length measurements are expressed in two different units. Interesting relationships emerge when the smaller of the two units chosen is a function of time. We relate these results to the expected periodicities in the observed data when a system of objects are revolving around a common center of mass. We find that these results are highly intriguing and can equally well account for some of the major results in the field of astrophysics.

1 Introduction

In an earlier paper (Rajamohan and Satya Narayanan [1]) we derived the condition that needs to be satisfied for signal from a relatively stationary emitter to meet an observer moving transverse to the line of sight. A receiver moving across the line of sight is equivalent of the receiver accelerating away along the line of sight from the emitter. In this paper, we have derived the period and period derivative for this equivalent situation.

It is well known that signals with uniform period P_e from an emitter will arrive at a receiver, moving with uniform relative velocity V along the line of sight, with a period P given by the equation

$$P = \frac{P_e}{(1 - V/C)}\,,$$

where C is the signal speed. Instead if the receiver or the emitter were to be accelerating with a as the value of acceleration, it is generally assumed that the observed rate of change of period $\dot{P}$ per unit time is governed by the equation (Shklovski [2])

$$\dot{P} = \frac{aP}{C}\,. \tag{1}$$

The above equation does not take into account the relationship between space intervals and time intervals properly. When acceleration is involved, the time interval Δt that corresponds to a given space interval Δx is a function of time. That is, the space interval Δx corresponds to smaller and smaller time interval (along the direction of motion) as the velocity of the accelerating receiver is a function of time.

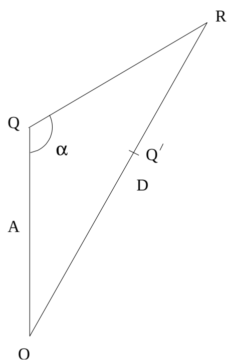

Fig. 1: Schematic representation of the observer and the emitter meeting at a point.

The space-time relationship when properly taken into account leads to an additional term which is enormously larger than that given by equation (1).

2 Relationship between time, length and the unit of length-measurement

Consider the general case when the observer is at a distance A (km) from the emitter moving with uniform velocity V at an angle α to the line of sight as shown in Figure 1. Let the emitter at position O emit signals at regular intervals of P_e seconds.

At time $t = 0$, let a signal start from O when the observer is at Q (at $t = 0$). Let this signal meet the observer at R at time t. Let the initial distance $OQ = A$ at $t = 0$ and the distance $OR = D$ at time t.

From triangle OQR

$$(OR)^2 = (OQ)^2 + (QR)^2 - 2(OQ)(QR)\cos\alpha$$

or

$$D^2 = A^2 + V^2t^2 - 2AV\cos\alpha t = A^2\left[1 + \frac{V^2t^2}{A^2} - \frac{2V\cos\alpha t}{A}\right],$$

$$D = A\left[1 + \frac{V^2t^2}{A^2} - \frac{2V\cos\alpha t}{A}\right]^{\frac{1}{2}} \approx A + \frac{1}{2}\frac{V^2t^2}{A} -$$

$$- V\cos\alpha t - \frac{1}{2}\frac{V^2t^2\cos^2\alpha}{A} = A - V\cos\alpha t + \frac{1}{2}\frac{V^2\sin^2\alpha}{A}\,t^2.$$

Therefore

$$D - A = -V\cos\alpha t + \frac{1}{2}\frac{V^2\sin^2\alpha}{A}\,t^2.$$

We can rewrite $D - A$ as

$$D - A = ut + \frac{1}{2}at^2;$$

u is positive when α is greater than 90° and negative when α is less than 90°. However, $a = V^2\sin^2\alpha/A$ is always positive. If the angle α were to be 0 or 180°, the observer will be moving uniformly along the line of sight and the signals from O will be equally spaced in time. If the observer were to move in a circular orbit around the emitter then too, the period observed would be constant. In all other cases the acceleration due to transverse component that leads to the period derivative will always be positive.

Draw a circle with A as radius. Let it intercept the line

OR at Q'. Therefore $OQ = OQ'$. Let the signal from O reach Q' at time t_e

$$D - A = Q'R = C(t - t_e) = ut + \frac{1}{2}at^2.$$

The signal from O meeting the uniformly moving observer along QR is equivalent to the same signal chasing an observer from Q' to R with initial velocity u and acceleration a

$$\begin{aligned} C(t - t_e) &= ut + \frac{1}{2}at^2 = \\ &= u\big[t_e + (t - t_e)\big] + \frac{1}{2}a\big[t_e + (t - t_e)\big]^2 = \\ &= ut_e + \frac{1}{2}at_e^2 + (u + at_e)(t - t_e) + \frac{1}{2}a(t - t_e)^2. \end{aligned}$$

Let $C(t - t_e) = X$ and $ut_e + \frac{1}{2}at_e^2 = X_e$. The space interval X_e contains N signals where $N = X_e/CP_e$ which will get folded in the space interval $X - X_e$ as the train of signals moving along OR will be spaced at CP_e km.

Therefore

$$t - t_e = \frac{X}{C} = \frac{X_e}{C} + \frac{u + at_e}{C}(t - t_e) + \frac{1}{2}\frac{a}{C}(t - t_e)^2.$$

Hence the average observed period in the time interval $(t - t_e)$ is

$$\bar{P} = \frac{(t - t_e)}{N} = \frac{(t - t_e)CP_e}{X_e} = \frac{X}{X_e}P_e\,,$$

$$\bar{P} = \frac{X}{X_e}P_e = P_e + \frac{u(t - t_e)CP_e}{CX_e} + \frac{at_e(t - t_e)CP_e}{CX_e} + \frac{\frac{1}{2}a(t - t_e)^2CP_e}{CX_e}\,,$$

$$\bar{P} = P_e + \frac{u}{C}\frac{X}{X_e}P_e + \frac{at_e}{C}\frac{X}{X_e}P_e + \frac{1}{2}\frac{a(t - t_e)}{C}\frac{X}{X_e}P_e\,.$$

For N signals in the time interval $(t - t_e)$, we can write

$$(t - t_e) = P_iN + \frac{1}{2}\dot{P}P_iN^2,$$

where P_i is the initial period. Hence

$$\bar{P} = \frac{t - t_e}{N} = P_i + \frac{1}{2}\dot{P}P_iN\,.$$

Comparing this with

$$\bar{P} = P_e + \frac{u}{C}\frac{X}{X_e}P_e + \frac{at_e}{C}\frac{X}{X_e}P_e\left[1 + \frac{1}{2}\frac{t - t_e}{t_e}\right]$$

we derive

$$\bar{P} = P_i + \frac{at_e}{C}\frac{X}{X_e}P_e\left[1 + \frac{1}{2}\frac{(t - t_e)}{t_e}\right]$$

as $P_i = P_e/(1 - u/C)$. Hence $\frac{1}{2}\dot{P}N \approx \frac{at_e}{C}\left[1 + \frac{1}{2}\frac{(t - t_e)}{t_e}\right]$ or

$$\dot{P} = \frac{2at_e}{CN}\left[1 + \frac{1}{2}\frac{t - t_e}{t_e}\right] = \frac{2at_e(CP_e)}{CX_e}\left[1 + \frac{1}{2}\frac{t - t_e}{t_e}\right].$$

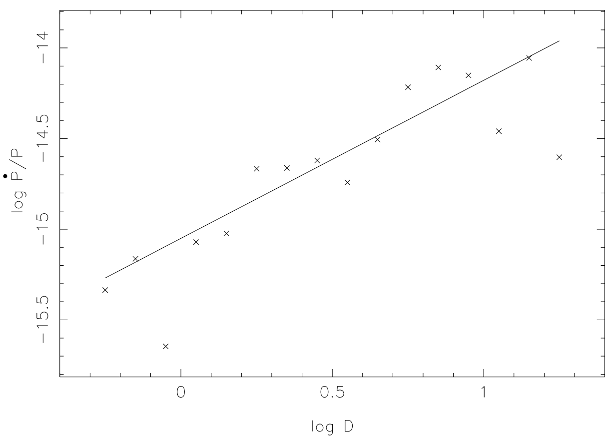

Fig. 2: $\log \dot{P}/P$ plotted as a function of $\log$ D.

As $|X_e| = |u|t_e + \frac{1}{2}at_e^2$,

$$\dot{P} \approx \frac{2a}{|u|} + \frac{aP_e}{C}\,.$$

The second term on the right hand side of the above equation is the Shklovski's [2] solution which is u/C times smaller than the first term

$$\dot{P} = \frac{2at_e}{X_e}P_e\Big(1 + \frac{u}{C}\Big) = \frac{2at_e}{|u|}\Big(1 + \frac{u}{C}\Big) \approx \frac{2P_e}{t_e}\Big(1 + \frac{u}{C}\Big)\,.$$

The acceleration a due to transverse component of velocity is always positive and hence $\dot{P}$ will be positive even when the observer is moving toward the emitter at an angle α less than 90°.

3 The period derivatives of pulsars

If V_τ is the relative transverse velocity between the Sun and the Pulsar, then the relative acceleration is V_τ^2/d. As $\sqrt{2}d/V_\tau = t$ is the relative time of free fall over $\frac{\pi}{2}$ radians, we can write $\dot{P} = 2P_e/t = \frac{1}{2}V_\tau/d = \frac{\pi}{2}V_\tau/d$. This is of the order of the average observed period derivate of pulsars. If we assume that an inverse square law is applicable the average observed period derivatives of pulsars must increase as a function of distance from the Sun.

Figure 2, is a plot of $\log \dot{P}/P$ versus $\log$ D of all pulsars in the period range 0.1 to 3 seconds and in the distance range $\log \mathrm{D} = -0.3$ to $+1.3$. The data is taken from Taylor et al. [3]. Table 1 gives the values of $\log \dot{P}/P$ averaged in different distance bins. N is the number of pulsars in each bin. Leaving the two points that are slightly further away from the mean relationship, the best fit straight line $Y = mX + k$ gives a slope of 0.872 and the constant as -15.0506. The constant k gives the value of $\sqrt{2}V_\tau/d$ at an average distance of the Sun. In short we expect that this should more or less correspond with the accepted values for the Sun's motion around the galactic center. Taking $V_\odot = 210$ km×s^{-1} and $d_\odot = 8$ kpc , we get $\sqrt{2}V_\odot/d_\odot = 1.24{\times}10^{-15}$ and the value from k is $1.12{\times}10^{-15}$.

log D	log $\dot{P}/P$	N
−0.25	−15.3353	10
−0.15	−15.1632	17
−0.05	−15.6460	12
0.05	−15.0711	16
0.15	−15.0231	27
0.25	−14.6667	38
0.35	−14.6620	46
0.45	−14.6205	51
0.55	−14.7416	78
0.65	−14.5052	66
0.75	−14.2172	49
0.85	−14.1072	51
0.95	−14.1509	30
1.05	−14.4594	12
1.15	−14.0552	10
1.25	−14.6030	3

Table 1: log $\dot{P}/P$ as a function of log D and the number of pulsars in each bin.

This is given more as an illustration of the application of this effect. The real (true) spin down rates of the large majority of pulsars, may be much lower than the canonical value of 3×10^{-15}. Hence the average observed period derivatives of pulsars is due to the differential galactic rotation effect. This result is fully in conformity with the observed relationship between transverse motion and $\dot{P}$ by Anderson and Lyne [4] and Cordes [5] and that the correlation found by them cannot be accounted for purely by selection effects alone (Stollman and Van den Heuvel [6]).

4 Bending of light

As the photon angle accelerates in the gravitational field of the Sun, the angle $\Delta\phi$ at which the light from the limb of the Sun would be seen to meet the observer is the instantaneous value of the second derivative of α with respect to time at the distance of the earth. This is given by

$$\Delta\phi = \frac{\pi}{2}\,\frac{d^2\alpha}{dt^2}\,t\,(1s) = \frac{\pi}{2}\,\frac{2CV_\tau}{d^2}\,(1s)\,\frac{d}{C} = \frac{\pi V_\tau\,(1s)}{d}\,,$$

where $\frac{\pi}{2}$ is introduced as a scale factor to relate the free-fall height to the actual arc length that an object traverses in a gravitational field; V_τ is the relative transverse velocity and d is the distance between the Sun and the Earth. This will result in an observed bending of light as

$$\Delta\phi = \frac{\pi V_\tau\,(1s)}{d} = \frac{407\,\pi}{1.5\times10^8 \text{ radians}} = 1.76 \text{ arc sec.}$$

5 Precession of Mercury's orbit

The arrival time acceleration when not taken into account will appear as though the orbit is precessing. A good example is the precesion of Mercury's orbit. Treating Mercury as a rotating object with a period equal to its synodic period $P_s = 115.88$ days,

$$\Delta\omega = \frac{\pi V_\tau}{d} = \frac{3.14\times18.1}{0.917\times10^8} = 61.98\times10^{-8} \text{ rad},$$

which is the change per synodic period. Hence,

$$\frac{\Delta\omega}{P_s} = \frac{61.98\times10^{-8}}{115.88\times86400} = \\ = 6.19\times10^{-14} \text{ rad}\times\text{s}^{-1} = 40 \text{ arc sec/century.}$$

6 Binary pulsars

In the case of a binary pulsar, the relative transverse motion of the common centre of mass of the binary system and the Sun will lead to a secular increase in the period. Over and above this effect, the acceleration of the pulsar in the gravitational field of its companion will lead to further periodic deceleration in the arrival times. In analogy with Mercury, we can therefore expect a similar phenomenon in the case of binary pulsars. That is, the orbit might appear to precess if the arrival time delays caused by the pulsar acceleration in the gravitational field of the companion is not taken into account. The apparent precesion per pulse period P_e will be (Rajamohan and Satya Narayanan [1])

$$\Delta\omega = \frac{\pi}{4}\,\frac{V^2}{a^2}\,P_e^2\,.$$

Approximating the orbit to be circular and expressing the above equation in terms of well determined quantities,

$$\Delta\omega \approx \pi^3 P_e^2/P_b^2\,,$$

P_b is the orbital period and a is the semi-major axis of the orbit. Introducing appropriate values for PSR1913+16, we find

$$\Delta\omega \approx 1.386\times10^{-10} \text{ rad/pulse} \approx 4.24^\circ \text{ yr}^{-1},$$

which is in very good agreement with the observed value of 4.2261° yr^{-1} by Taylor and Weisberg [7]. For PSR1534+12 we find

$$\Delta\omega \approx 0.337\times10^{-10} \text{ rad/pulse} \approx 1.61^\circ \text{ yr}^{-1},$$

while the observed value is 1.756° yr^{-1} (Taylor et al. [8]).

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