

# Funnel's Fluctuations in Dyonic Case: Intersecting D1–D3 Branes

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The fluctuations of funnel solutions of intersecting D1 and D3 branes are quite explicitly discussed by treating different modes and different directions of the fluctuation at the presence of world volume electric field. The boundary conditions are found to be Neumann boundary conditions.

## 1 Introduction

D-branes described by Non-abelian Born-Infeld (BI) action [1] have many fascinating features. Among these there is the possibility for D-branes to morph into other D-branes of different dimensions by exciting some of the scalar fields [2, 3]. It's known in the literature that there are many different but physically equivalent descriptions of how a D1-brane may end on a D3-brane. From the point of view of the D3 brane the configuration is described by a monopole on its world volume. From the point of view of the D1-brane the configuration is described by the D1-brane opening up into a D3-brane where the extra three dimensions form a fuzzy two-sphere whose radius diverges at the origin of the D3-brane. These different view points are the stringy realization of the Nahm transformation [4, 5]. Also the dynamics of the both bion spike [2, 6] and the fuzzy funnel [5, 7, 8] were studied by considering linearized fluctuations around the static solutions.

The present work is devoted to study the fluctuations of funnel solutions in the presence of a world-volume electric field. By discussing the solutions and the potentials for this particular case we end by the system D1⊥D3 branes gets a special property because of the presence of electric field; the system is divided to two regions corresponding to small and large electric field. Consequently, the system has Neumann boundary conditions and the end of open string can move freely on the brane which is agree with its dual discussed in [9] considering Born-Infeld action dealing with the fluctuation of the bion skipe in D3⊥D1-case.

The paper is organized as follows: In section 2, we start by a brief review on D1⊥D3 branes in dyonic case by using the non-Born-Infeld action. Then, we discuss the fluctuations of the fuzzy funnel in section 3 for zero and high modes. We give the solutions of the linearized equations of motion of the fluctuations for both cases the overall transverse and the relative one. We also discuss the solutions and the potential depending on the presence of electric field which is leading to Neumann boundary conditions as special property of the system. Then the waves on the brane cause the fuzzy funnel to freely oscillate.

## 2 D1⊥D3 branes with electric field swished on

In this section, we review in brief the funnel solutions for D1⊥D3 branes from D3 and D1 branes points of view. First, using abelian BI action for the world-volume gauge field and one excited transverse scalar in dyonic case, we give the funnel solution. It was showed in [10] that the BI action, when taken as the fundamental action, can be used to build a configuration with a semi-infinite fundamental string ending on a D3-brane [11]. The dyonic system is given by using D-string world-volume theory and the fundamental strings introduced by adding a  $U(1)$  electric field. Thus the system is described by the following action

$$\begin{aligned} S &= \int dt L = \\ &= -T_3 \int d^4\sigma \sqrt{-\det(\eta_{ab} + \lambda^2 \partial_a \phi^i \partial_b \phi^i + \lambda F_{ab})} = \\ &= -T_3 \int d^4\sigma \left[ 1 + \lambda^2 \left( |\nabla\phi|^2 + B^2 + E^2 \right) + \right. \\ &\quad \left. + \lambda^4 \left( (B \cdot \nabla\phi)^2 + (E \cdot B)^2 + |E \wedge \nabla\phi|^2 \right) \right]^{\frac{1}{2}} \end{aligned} \quad (1)$$

in which  $F_{ab}$  is the field strength and the electric field is denoted as  $F_{09} = EI_{ab}$ , ( $I_{ab}$  is  $N \times N$  matrix).  $\sigma^a$  ( $a = 0, \dots, 3$ ) denote the world volume coordinates while  $\phi^i$  ( $i = 4, \dots, 9$ ) are the scalars describing transverse fluctuations of the brane and  $\lambda = 2\pi\ell_s^2$  with  $\ell_s$  is the string length. In our case we excite just one scalar so  $\phi^i = \phi^9 \equiv \phi$ . Following the same process used in the reference [10] by considering static gauge, we look for the lowest energy of the system. Accordingly to (1) the energy of dyonic system is given as

$$\begin{aligned} \Xi &= T_3 \int d^3\sigma \left[ \lambda^2 |\nabla\phi + \vec{B} + \vec{E}|^2 + (1 - \lambda^2 \nabla\phi \cdot \vec{B})^2 - \right. \\ &\quad \left. - 2\lambda^2 \vec{E} \cdot (\vec{B} + \nabla\phi) + \lambda^4 \left( (\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla\phi|^2 \right) \right]^{\frac{1}{2}}, \end{aligned} \quad (2)$$

then if we require  $\nabla\phi + \vec{B} + \vec{E} = 0$ ,  $\Xi$  reduces to  $\Xi_0 \geq 0$  and we find

$$\begin{aligned} \Xi_0 &= T_3 \int d^3\sigma \left[ (1 - \lambda^2 (\nabla\phi) \cdot \vec{B})^2 + 2\lambda^2 \vec{E} \cdot \vec{E} + \right. \\ &\quad \left. + \lambda^4 \left( (\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla\phi|^2 \right) \right]^{\frac{1}{2}} \end{aligned} \quad (3)$$

as minimum energy. By using the Bianchi identity  $\nabla \cdot B = 0$  and the fact that the gauge field is static, the funnel solution is then

$$\phi = \frac{N_m + N_e}{2r}, \quad (4)$$

with  $N_m$  is magnetic charge and  $N_e$  electric charge.

Now we consider the dual description of the  $D1 \perp D3$  from D1 branes point of view. To get D3-branes from D-strings, we use the non-abelian BI action

$$S = -T_1 \int d^2\sigma \times \text{Str} \left[ -\det(\eta_{ab} + \lambda^2 \partial_a \phi^i Q_{ij}^{-1} \partial_b \phi^j) \det Q^{ij} \right]^{\frac{1}{2}} \quad (5)$$

where  $Q_{ij} = \delta_{ij} + i\lambda [\phi_i, \phi_j]$ . Expanding this action to leading order in  $\lambda$  yields the usual non-abelian scalar action

$$S \cong -T_1 \int d^2\sigma \times \left[ N + \lambda^2 \text{tr} \left( \partial_a \phi^i + \frac{1}{2} [\phi_i, \phi_j] [\phi_j, \phi_i] \right) + \dots \right]^{\frac{1}{2}}.$$

The solutions of the equation of motion of the scalar fields  $\phi_i$ ,  $i = 1, 2, 3$  represent the D-string expanding into a D3-brane analogous to the bion solution of the D3-brane theory [2, 3]. The solutions are

$$\phi_i = \pm \frac{\alpha_i}{2\sigma}, \quad [\alpha_i, \alpha_j] = 2i \epsilon^{ijk} \alpha_k,$$

with the corresponding geometry is a long funnel where the cross-section at fixed  $\sigma$  has the topology of a fuzzy two-sphere.

The dyonic case is taken by considering  $(N, N_f)$ -strings. We have  $N$  D-strings and  $N_f$  fundamental strings [5]. The theory is described by the action

$$S = -T_1 \int d^2\sigma \times \text{Str} \left[ -\det(\eta_{ab} + \lambda^2 \partial_a \phi^i Q_{ij}^{-1} \partial_b \phi^j + \lambda E I_{ab}) \det Q^{ij} \right]^{\frac{1}{2}} \quad (6)$$

in which we replaced the field strength  $F_{ab}$  by  $E I_{ab}$  ( $I_{ab}$  is  $N \times N$ -matrix) meaning that the fundamental string is introduced by adding a  $U(1)$  electric field  $E$ .

The action can be rewritten as

$$S = -T_1 \int d^2\sigma \text{Str} \left[ -\det \begin{pmatrix} \eta_{ab} + \lambda E I_{ab} & \lambda \partial_a \phi^j \\ -\lambda \partial_b \phi^i & Q^{ij} \end{pmatrix} \right]^{\frac{1}{2}}, \quad (7)$$

then the bound states of D-strings and fundamental strings are made simply by introducing a background  $U(1)$  electric field on D-strings, corresponding to fundamental strings dissolved on the world-sheet. By computing the determinant, the action becomes

$$S = -T_1 \int d^2\sigma \times \text{Str} \left[ (1 - \lambda^2 E^2 + \alpha_i \alpha_i \hat{R}^2)(1 + 4\lambda^2 \alpha_j \alpha_j \hat{R}^4) \right]^{\frac{1}{2}}, \quad (8)$$

where the following ansatz were inserted

$$\phi_i = \hat{R} \alpha_i. \quad (9)$$

Hence, we get the funnel solution for dyonic string by solving the equation of variation of  $\hat{R}$ , as follows

$$\phi_i = \frac{\alpha_i}{2\sigma \sqrt{1 - \lambda^2 E^2}}. \quad (10)$$

### 3 Fluctuations of dyonic funnel solutions

In this section, we treat the dynamics of the funnel solutions. We solve the linearized equations of motion for small and time-dependent fluctuations of the transverse scalar around the exact background in dyonic case.

We deal with the fluctuations of the funnel (10) discussed in the previous section. By plugging into the full  $(N - N_f)$  string action (6, 7) the ‘‘overall transverse’’  $\delta\phi^m(\sigma, t) = f^m(\sigma, t) I_N$ ,  $m = 4, \dots, 8$  which is the simplest type of fluctuation with  $I_N$  the identity matrix, together with the funnel solution, we get

$$S = -T_1 \int d^2\sigma \text{Str} \left[ (1 + \lambda E) \left( 1 + \frac{\lambda^2 \alpha^i \alpha^i}{4\sigma^4} \right) \times \left( \left( 1 + \frac{\lambda^2 \alpha^i \alpha^i}{4\sigma^4} \right) (1 + (\lambda E - 1) \lambda^2 (\partial_t \delta\phi^m)^2) + \lambda^2 (\partial_\sigma \delta\phi^m)^2 \right) \right]^{\frac{1}{2}} \approx -NT_1 \int d^2\sigma H \left[ (1 + \lambda E) - (1 - \lambda^2 E^2) \frac{\lambda^2}{2} (f^m)^2 + \frac{(1 + \lambda E)\lambda^2}{2H} (\partial_\sigma f^m)^2 + \dots \right] \quad (11)$$

where

$$H = 1 + \frac{\lambda^2 C}{4\sigma^4}$$

and  $C = \text{tr} \alpha^i \alpha^i$ . For the irreducible  $N \times N$  representation we have  $C = N^2 - 1$ . In the last line we have only kept the terms quadratic in the fluctuations as this is sufficient to determine the linearized equations of motion

$$\left( (1 - \lambda E) \left( 1 + \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) \partial_t^2 - \partial_\sigma^2 \right) f^m = 0. \quad (12)$$

In the overall case, all the points of the fuzzy funnel move or fluctuate in the same direction of the dyonic string by an equal distance  $\delta x^m$ . First, the funnel solution is  $\phi^i = \frac{1}{2\sqrt{1 - \lambda^2 E^2}} \frac{\alpha^i}{\sigma}$  and the fluctuation  $f^m$  waves in the direction of  $x^m$ ;  $f^m(\sigma, t) = \Phi(\sigma) e^{-i\omega t} \delta x^m$ . (13)

With this ansatz the equation of motion is

$$\left( (1 - \lambda E) H \omega^2 + \partial_\sigma^2 \right) \Phi(\sigma) = 0, \quad (14)$$

then the problem is reduced to finding the solution of a single scalar equation.

Thus, we remark that the equation (14) is an analog one-dimensional Schrödinger equation and it can be rewritten as

$$\left(-\partial_\sigma^2 + V(\sigma)\right) \Phi(\sigma) = w^2(1 - \lambda E) \Phi(\sigma), \quad (15)$$

with

$$V(\sigma) = w^2(\lambda E - 1) \lambda^2 \frac{N^2 - 1}{4\sigma^4}.$$

We notice that, if the electric field dominates  $E \gg 1$ , the potential goes to  $w^2 \lambda^3 E \frac{N^2}{4\sigma^4}$  for large  $N$  and if  $E \ll 1$  we find  $V = -w^2 \lambda^2 \frac{N^2}{4\sigma^4}$ . This can be seen as two separated systems depending on electric field so we have Neumann boundary condition separating the system into two regions  $E \gg 1$  and  $E \ll 1$ .

Now, let's find the solution of a single scalar equation (14). First, the equation (14) can be rewritten as follows

$$\left(\frac{1}{w^2(1 - \lambda E)} \partial_\sigma^2 + 1 + \frac{\lambda^2 N^2}{4\sigma^4}\right) \Phi(\sigma) = 0, \quad (16)$$

for large  $N$ . If we suggest  $\tilde{\sigma} = w\sqrt{1 - \lambda E} \sigma$  the latter equation becomes

$$\left(\partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right) \Phi(\tilde{\sigma}) = 0, \quad (17)$$

with the potential is

$$V(\tilde{\sigma}) = \frac{\kappa^2}{\tilde{\sigma}^4}, \quad (18)$$

and  $\kappa = \frac{\lambda N w^2}{2}(1 - \lambda E)$ . This equation is a Schrödinger equation for an attractive singular potential  $\propto \tilde{\sigma}^{-4}$  and depends on the single coupling parameter  $\kappa$  with constant positive Schrödinger energy. The solution is then known by making the following coordinate change

$$\chi(\tilde{\sigma}) = \int_{\sqrt{\kappa}}^{\tilde{\sigma}} dy \sqrt{1 + \frac{\kappa^2}{y^4}}, \quad (19)$$

and

$$\Phi = \left(1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right)^{-\frac{1}{4}} \tilde{\Phi}. \quad (20)$$

Thus, the equation (17) becomes

$$\left(-\partial_\chi^2 + V(\chi)\right) \tilde{\Phi} = 0, \quad (21)$$

with

$$V(\chi) = \frac{5\kappa^2}{\left(\tilde{\sigma}^2 + \frac{\kappa^2}{\tilde{\sigma}^2}\right)^3}. \quad (22)$$

Then, the fluctuation is found to be

$$\Phi = \left(1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right)^{-\frac{1}{4}} e^{\pm i\chi(\tilde{\sigma})}. \quad (23)$$

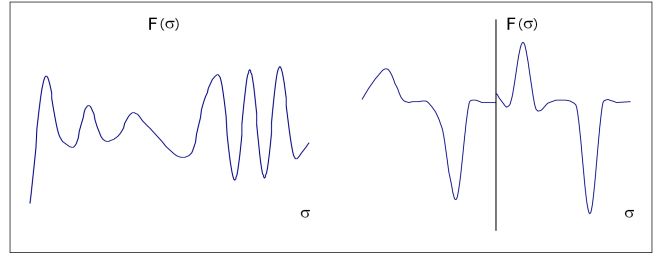


Fig. 1: Left hand curve represents the overall fluctuation wave in zero mode and low electric field. Right hand curve shows the scattering of the overall fluctuation wave in zero mode and high electric field. This latter caused a discontinuity of the wave which means Neumann boundary condition.

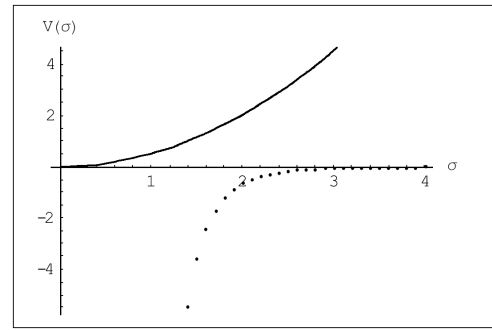


Fig. 2: The up line shows the potential in zero mode of the overall funnel's fluctuations at the absence of electric field  $E$  and the dots represent the potential in the same mode at the presence of  $E$ . The presence of  $E$  is changing the potential totally to the opposite.

This fluctuation has the following limits; at large  $\sigma$ ,  $\Phi \sim e^{\pm i\chi(\tilde{\sigma})}$  and if  $\sigma$  is small  $\Phi = \frac{\sqrt{\kappa}}{\tilde{\sigma}} e^{\pm i\chi(\tilde{\sigma})}$ . These are the asymptotic wave function in the regions  $\chi \rightarrow \pm\infty$ , while around  $\chi \sim 0$ ; i.e.  $\tilde{\sigma} \sim \sqrt{\kappa}$ ,  $f^m \sim 2^{-\frac{1}{4}} e^{-i\omega t} \delta x^m$  (Fig. 1).

The potential (22) in large and small limits of electric field becomes (Fig. 2):

- $E \gg 1$ ,  $V(\chi) \sim \frac{-5\lambda N^2}{E\sigma^6}$ ;
- $E \ll 1$ ,  $V(\chi) \sim \frac{5\lambda^2 N^2 w^2}{4(w^2 \sigma^2 + \frac{\lambda^2 N^2 w^2}{4\sigma^2})}$ .

At the presence of electric field we remark that around  $\sigma \sim 0$  there is a symmetric potential which goes to zero very fast and more fast as electric field is large  $\sim \frac{-1}{E\sigma^2}$ . As discussed above, again we get the separated systems in different regions depending on the values of electric field. Also if we have a look at the fluctuation (23) we find that  $f^m$  in the case of  $E \gg 1$  is different from the one in  $E \ll 1$  case and as shown in the Fig. 1 the presence of electric field causes a discontinuity of the fluctuation wave which means free boundary condition. Contrarily, at the absence of electric field the fluctuation wave is continue. Then, this is seen as Neumann boundary condition from non-Born-Infeld dynamics separating the system into two regions  $E \gg 1$  and  $E \ll 1$  which is agree with its dual discussed in [9].

The fluctuations discussed above could be called the zero mode  $\ell = 0$  and for high modes  $\ell \geq 0$ , the fluctuations are

$$\delta\phi^m(\sigma, t) = \sum_{\ell=0}^{N-1} \psi_{i_1 \dots i_\ell}^m \alpha^{i_1} \dots \alpha^{i_\ell}$$

with  $\psi_{i_1 \dots i_\ell}^m$  are completely symmetric and traceless in the lower indices.

The action describing this system is

$$\begin{aligned} S \approx & -NT_1 \int d^2\sigma \left[ (1 + \lambda E)H - (1 - \lambda^2 E^2) \times \right. \\ & \times H \frac{\lambda^2}{2} (\partial_t \delta\phi^m)^2 + \frac{(1 + \lambda E)\lambda^2}{2H} (\partial_\sigma \delta\phi^m)^2 - \\ & \left. - (1 - \lambda^2 E^2) \frac{\lambda^2}{2} [\phi^i, \delta\phi^m]^2 - \frac{\lambda^4}{12} [\partial_\sigma \phi^i, \partial_t \delta\phi^m]^2 + \dots \right] \end{aligned} \quad (24)$$

Now the linearized equations of motion are

$$\begin{aligned} & \left[ (1 + \lambda E)H \partial_t^2 - \partial_\sigma^2 \right] \delta\phi^m + (1 - \lambda^2 E^2) \times \\ & \times [\phi^i, [\phi^i, \delta\phi^m]] - \frac{\lambda^2}{6} [\partial_\sigma \phi^i, [\partial_\sigma \phi^i, \partial_t^2 \delta\phi^m]] = 0. \end{aligned} \quad (25)$$

Since the background solution is  $\phi^i \propto \alpha^i$  and we have  $[\alpha^i, \alpha^j] = 2i\epsilon_{ijk}\alpha^k$ , we get

$$\begin{aligned} [\alpha^i, [\alpha^i, \delta\phi^m]] &= \sum_{\ell < N} \psi_{i_1 \dots i_\ell}^m [\alpha^i, [\alpha^i, \alpha^{i_1} \dots \alpha^{i_\ell}]] \\ &= \sum_{\ell < N} 4\ell(\ell + 1) \psi_{i_1 \dots i_\ell}^m \alpha^{i_1} \dots \alpha^{i_\ell} \end{aligned} \quad (26)$$

To obtain a specific spherical harmonic on 2-sphere, we have

$$\begin{aligned} [\phi^i, [\phi^i, \delta\phi_\ell^m]] &= \frac{\ell(\ell + 1)}{\sigma^2} \delta\phi_\ell^m, \\ [\partial_\sigma \phi^i, [\partial_\sigma \phi^i, \partial_t^2 \delta\phi_\ell^m]] &= \frac{\ell(\ell + 1)}{\sigma^4} \partial_t^2 \delta\phi_\ell^m. \end{aligned} \quad (27)$$

Then for each mode the equations of motion are

$$\begin{aligned} & \left[ \left( (1 + \lambda E) \left( 1 + \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) - \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) \partial_t^2 - \right. \\ & \left. - \partial_\sigma^2 + (1 - \lambda^2 E^2) \frac{\ell(\ell + 1)}{\sigma^2} \right] \delta\phi_\ell^m = 0. \end{aligned} \quad (28)$$

The solution of the equation of motion can be found by taking the following proposal. Let's consider  $\phi_\ell^m = f_\ell^m(\sigma) e^{-i\omega t} \delta x^m$  in direction  $m$  with  $f_\ell^m(\sigma)$  is some function of  $\sigma$  for each mode  $\ell$ .

The last equation can be rewritten as

$$\left[ -\partial_\sigma^2 + V(\sigma) \right] f_\ell^m(\sigma) = \omega^2 (1 + \lambda E) f_\ell^m(\sigma), \quad (29)$$

with

$$\begin{aligned} V(\sigma) = & -\omega^2 \left( (1 + \lambda E) \frac{\lambda^2 N^2}{4\sigma^4} - \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) + \\ & + (1 - \lambda^2 E^2) \frac{\ell(\ell + 1)}{\sigma^2}. \end{aligned}$$

Let's write the equation (29) in the following form

$$\begin{aligned} & \left[ \omega^2 \left( (1 + \lambda E)H - \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) - \right. \\ & \left. - (1 - \lambda^2 E^2) \frac{\ell(\ell + 1)}{\sigma^2} + \partial_\sigma^2 \right] f_\ell^m(\sigma) = 0. \end{aligned} \quad (30)$$

and again as

$$\begin{aligned} & \left[ 1 + \frac{1}{\sigma^4} \left( \lambda^2 \frac{N^2 - 1}{4} - \frac{\lambda^2 \ell(\ell + 1)}{6(1 + \lambda E)} \right) - \right. \\ & \left. - (1 - \lambda E) \frac{\ell(\ell + 1)}{\omega^2 \sigma^2} + \frac{1}{\omega^2 (1 + \lambda E)} \partial_\sigma^2 \right] f_\ell^m(\sigma) = 0. \end{aligned} \quad (31)$$

We define new coordinate  $\tilde{\sigma} = \omega \sqrt{1 + \lambda E} \sigma$  and the latter equation becomes

$$\left[ \partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4} + \frac{\eta}{\tilde{\sigma}^2} \right] f_\ell^m(\sigma) = 0, \quad (32)$$

where

$$\begin{aligned} \kappa^2 &= \omega^2 (1 + \lambda E) \left( \lambda^2 \frac{N^2 - 1}{4} - \frac{\lambda^2 \ell(\ell + 1)}{6(1 + \lambda E)} \right)^{\frac{1}{2}}, \\ \eta &= -(1 - \lambda^2 E^2) \ell(\ell + 1) \end{aligned}$$

such that

$$N > \sqrt{\frac{2\ell(\ell + 1)}{3(1 + \lambda E)}} + 1.$$

For simplicity we choose small  $\sigma$ , then the equation (32) is reduced to

$$\left[ \partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4} \right] f_\ell^m(\sigma) = 0, \quad (33)$$

as we did in zero mode, we get the solution by using the steps (19–22) with new  $\kappa$ . Since we considered small  $\sigma$  we get

$$V(\chi) = \frac{5\tilde{\sigma}^6}{\kappa^4},$$

then

$$f_\ell^m = \frac{\tilde{\sigma}}{\sqrt{\kappa}} e^{\pm i\chi(\tilde{\sigma})}. \quad (34)$$

This fluctuation has two different values at large  $E$  and small  $E$  (Fig. 3) and a closer look at the potential at large and fixed  $N$  in large and small limits of electric field leads to

- $E \gg 1$ ,  $V(\chi) \sim \frac{20\omega^2 E \sigma^6}{\lambda N^2}$ ;
- $E \ll 1$ ,  $V(\chi) \sim \frac{5\omega^2 \sigma^6}{\lambda^2 \left( \frac{N^2}{4} - \frac{\ell(\ell + 1)}{6} \right)}$ .

The potential in the first case is going fast to infinity than the one in the second case because of the electric field if  $\sigma \ll 1$  (Fig. 4).

For large  $\sigma$  the equation of motion (30) of the fluctuation becomes

$$\left[ -\partial_\sigma^2 + \tilde{V}(\sigma) \right] f_\ell^m(\sigma) = \omega^2 (1 + \lambda E) f_\ell^m(\sigma), \quad (35)$$

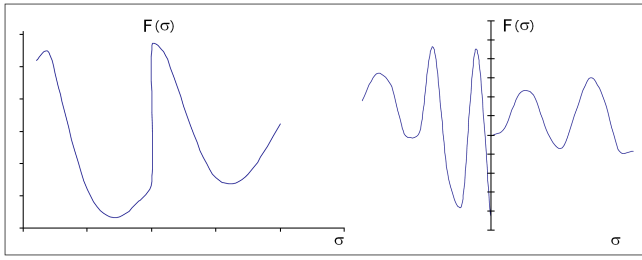


Fig. 3: The left figure shows the continuity of the fluctuation wave in high mode of the overall fluctuation at the absence of electric field E. The right figure shows the discontinuity of the wave at the presence of E in high mode meaning free boundary condition.

with  $\tilde{V}(\sigma) = \frac{(1-\lambda^2 E^2)\ell(\ell+1)}{\sigma^2}$  and  $f_\ell^m$  is now a Sturm-Liouville eigenvalue problem (Fig. 3). We found that the fluctuation has discontinuity at the presence of electric field meaning free boundary condition. Also we remark that the potential has different values in the different regions of electric field  $E \gg 1$  and  $E \ll 1$  and this time for large  $\sigma$ . In this side, the potential drops with opposite sign from one case to other and as shown in (Fig. 4). The presence of E is changing the potential totally to the opposite in both cases zero and high modes.

Consequently, by discussing explicitly the fluctuations and the potential of intersecting D1-D3 branes in D1-brane world volume theory we found that the system has Neumann boundary conditions and the end of the string can move freely on the brane for both zero and high modes of the overall transverse fluctuations case.

### 3.1 Relative Transverse Fluctuations

Now if we consider the “relative transverse”  $\delta\phi^i(\sigma, t) = f^i(\sigma, t)I_N$ ,  $i = 1, 2, 3$  the action is

$$S = -T_1 \int d^2\sigma \times \text{Str} \left[ -\det \begin{pmatrix} \eta_{ab} + \lambda E I_{ab} & \lambda \partial_a(\phi^j + \delta\phi^j) \\ -\lambda \partial_b(\phi^i + \delta\phi^i) & Q_*^{ij} \end{pmatrix} \right]^{\frac{1}{2}}, \quad (36)$$

with  $Q_*^{ij} = Q^{ij} + i\lambda([\phi_i, \delta\phi_j] + [\delta\phi_i, \phi_j] + [\delta\phi_i, \delta\phi_j])$ . As before we keep only the terms quadratic in the fluctuations and the action becomes

$$S \approx -NT_1 \int d^2\sigma \left[ (1 - \lambda^2 E^2) H - (1 - \lambda E) \frac{\lambda^2}{2} (f^i)^2 + \frac{(1 + \lambda E)\lambda^2}{2H} (\partial_\sigma f^i)^2 + \dots \right]. \quad (37)$$

Then the equations of motion of the fluctuations are

$$\left( -\partial_\sigma^2 - w^2 \frac{1 - \lambda E}{1 + \lambda E} \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) f^i = w^2 \frac{1 - \lambda E}{1 + \lambda E} f^i. \quad (38)$$

If we write  $f^i = \Phi^i(\sigma) e^{-i\omega t} \delta x^i$  in the direction of  $x^i$ , the potential will be

$$V(\sigma) = -\frac{1 - \lambda E}{1 + \lambda E} \lambda^2 \frac{N^2 - 1}{4\sigma^4} w^2.$$

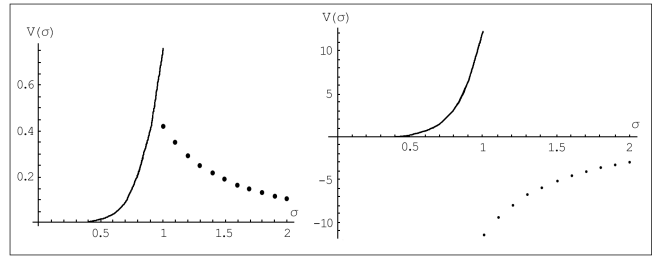


Fig. 4: The line represents the potential for small  $\sigma$  and dots for large  $\sigma$  in both figures. In high mode of overall fluctuations at the absence of electric field E, the left figure shows high potential at some stage of  $\sigma$  where the two curves meet. The right figure shows a critical case. The curves represent the potentials at the presence of E for small and large  $\sigma$ . As a remark, there is no intersecting point for these two potentials! At some stage of  $\sigma$  there is a singularity.

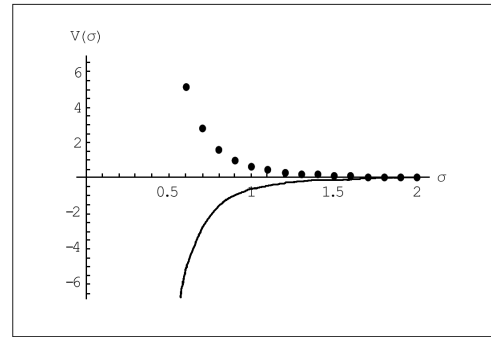


Fig. 5: The line shows the potential in zero mode of the relative funnel’s fluctuations at the absence of electric field E and the dots represent the potential in the same mode at the presence of E. The presence of E is changing the potential totally to the opposite.

Let’s discuss the cases of electric field:

- $E \ll 1$ ,  $V(\sigma) \sim -\lambda^2 \frac{N^2 - 1}{4\sigma^4} w^2$ ;
- $E \gg 1$ ,  $V(\sigma) \sim \lambda^2 \frac{N^2 - 1}{4\sigma^4} w^2$ .

Also in the relative case, this is Neumann boundary condition (Fig. 5) which can be also shown by finding the solution of (38) for which we follow the same way as above by making a coordinate change suggested by WKB. This case is seen as a zero mode of what is following so we will treat this in general case by using this coordinate change for high modes.

Now let’s give the equation of motion of relative transverse fluctuations of high  $\ell$  modes with  $(N - N_f)$  strings intersecting D3-branes. The fluctuation is given by

$$\delta\phi^i(\sigma, t) = \sum_{\ell=1}^{N-1} \psi_{i_1 \dots i_\ell}^i \alpha^{i_1} \dots \alpha^{i_\ell}$$

with  $\psi_{i_1 \dots i_\ell}^i$  are completely symmetric and traceless in the lower indices.

The action describing this system is

$$S \approx -NT_1 \int d^2\sigma \left[ (1 - \lambda^2 E^2) H - (1 - \lambda E) H \frac{\lambda^2}{2} (\partial_t \delta\phi^i)^2 + \frac{(1 + \lambda E)\lambda^2}{2H} (\partial_\sigma \delta\phi^i)^2 - (1 - \lambda E) \frac{\lambda^2}{2} [\phi^i, \delta\phi^i]^2 - \frac{\lambda^4}{12} [\partial_\sigma \phi^i, \partial_t \delta\phi^i]^2 + \dots \right]. \quad (39)$$

The equation of motion for relative transverse fluctuations in high mode is as follows

$$\left[ \frac{1 - \lambda E}{1 + \lambda E} H \partial_t^2 - \partial_\sigma^2 \right] \delta\phi^i + (1 - \lambda E) [\phi^i, [\phi^i, \delta\phi^i]] - \frac{\lambda^2}{6} [\partial_\sigma \phi^i, [\partial_\sigma \phi^i, \partial_t^2 \delta\phi^i]] = 0. \quad (40)$$

By the same way as done for overall transverse fluctuations the equation of motion for each mode is

$$\left[ -\partial_\sigma^2 + \left( \frac{1 - \lambda E}{1 + \lambda E} \left( 1 + \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) - \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) \partial_t^2 + (1 - \lambda E) \frac{\ell(\ell + 1)}{\sigma^2} \right] \delta\phi_\ell^i = 0. \quad (41)$$

We take  $\delta\phi_\ell^i = f_\ell^i e^{-i\omega t} \delta x^i$ , then the equation (41) becomes

$$\left[ -\partial_\sigma^2 - \left( \frac{1 - \lambda E}{1 + \lambda E} \left( 1 + \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) - \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) \omega^2 + (1 - \lambda E) \frac{\ell(\ell + 1)}{\sigma^2} \right] f_\ell^i = 0. \quad (42)$$

To solve the equation we choose for simplicity the boundaries of  $\sigma$ ; For small  $\sigma$ , the equation is reduced to

$$\left[ -\partial_\sigma^2 - \left( \frac{1 - \lambda E}{1 + \lambda E} \left( 1 + \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) - \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) \omega^2 \right] f_\ell^i = 0, \quad (43)$$

which can be rewritten as follows

$$\left[ -\frac{1 + \lambda E}{1 - \lambda E} \partial_\sigma^2 - \left( \left( 1 + \lambda^2 \frac{N^2 - 1}{4\sigma^4} \right) - \frac{1 + \lambda E}{1 - \lambda E} \frac{\lambda^2 \ell(\ell + 1)}{6\sigma^4} \right) \omega^2 \right] f_\ell^i = 0. \quad (44)$$

We change the coordinate to  $\tilde{\sigma} = \sqrt{\frac{1 - \lambda E}{1 + \lambda E}} \omega \sigma$  and the equation (44) becomes

$$\left[ \partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4} \right] f_\ell^i(\tilde{\sigma}) = 0, \quad (45)$$

with

$$\kappa^2 = w^4 \lambda^2 \frac{3(1 - \lambda E)^2 (N^2 - 1) - 2(1 - \lambda^2 E^2) \ell(\ell + 1)}{12(1 + \lambda E)^2}.$$

Then we follow the suggestions of WKB by making a coordinate change;

$$\beta(\tilde{\sigma}) = \int_{\sqrt{\kappa}}^{\tilde{\sigma}} dy \sqrt{1 + \frac{\kappa^2}{y^4}}, \quad (46)$$

and

$$f_\ell^i(\tilde{\sigma}) = \left( 1 + \frac{\kappa^2}{\tilde{\sigma}^4} \right)^{-\frac{1}{4}} \tilde{f}_\ell^i(\tilde{\sigma}). \quad (47)$$

Thus, the equation (45) becomes

$$\left( -\partial_\beta^2 + V(\beta) \right) \tilde{f}^i = 0, \quad (48)$$

with

$$V(\beta) = \frac{5\kappa^2}{(\tilde{\sigma}^2 + \frac{\kappa^2}{\tilde{\sigma}^2})^3}. \quad (49)$$

Then

$$f_\ell^i = \left( 1 + \frac{\kappa^2}{\tilde{\sigma}^4} \right)^{-\frac{1}{4}} e^{\pm i\beta(\tilde{\sigma})}. \quad (50)$$

The discussion is similar to the overall case; so the obtained fluctuation has the following limits; at large  $\sigma$ ,  $f_\ell^i \sim e^{\pm i\beta(\tilde{\sigma})}$  and if  $\sigma$  is small  $f_\ell^i = \frac{\sqrt{\kappa}}{\tilde{\sigma}} e^{\pm i\beta(\tilde{\sigma})}$ . These are the asymptotic wave function in the regions  $\beta \rightarrow \pm\infty$ , while around  $\beta \sim 0$ ; i.e.  $\tilde{\sigma} \sim \sqrt{\kappa}$ ,  $f_\ell^i \sim 2^{-\frac{1}{4}}$ .

Then let's have a look at the potential in various limits of electric field:

- $E \sim \frac{1}{\lambda}$ ,  $V(\beta) \sim 0$ ;
- $E \gg 1$ ,  $\kappa^2 \equiv \kappa_+^2 \sim w^4 \lambda^2 \frac{3(N^2 - 1) + 2\ell(\ell + 1)}{12}$ , then  $\sigma \sim 0 \Rightarrow V(\beta) \sim \frac{5\tilde{\sigma}^6}{\kappa_+^4}$ ;
- $E \ll 1$ ,  $\kappa^2 \equiv \kappa_-^2 \sim w^4 \lambda^2 \frac{3(N^2 - 1) - 2\ell(\ell + 1)}{12}$ ; for this case we get  $\sigma \sim 0 \Rightarrow V(\beta) \sim \frac{5\tilde{\sigma}^6}{\kappa_-^4}$ ;

this means that we have a Neumann boundary condition with relative fluctuations at small  $\sigma$  (Fig. 6).

Now, if  $\sigma$  is too large the equation of motion (42) becomes

$$\left[ -\partial_\sigma^2 + (1 - \lambda E) \frac{\ell(\ell + 1)}{\sigma^2} \right] f_\ell^i = \frac{1 - \lambda E}{1 + \lambda E} \omega^2 f_\ell^i. \quad (51)$$

We see, the associated potential  $V(\sigma) = (1 - \lambda E) \frac{\ell(\ell + 1)}{\sigma^2}$  goes to  $-\epsilon$  in the case of  $E \gg 1$  and to  $+\epsilon$  if  $E \ll 1$  since  $\sigma$  is too large with  $\epsilon \sim 0$ , (Fig. 6). We get the same remark as before by dealing with the fluctuations for small and large  $\sigma$  (50) and solving (51) respectively, at the presence of electric field that we have two separated regions depending on the electric field (Fig. 7).

We discussed quite explicitly through this section the fluctuation of the funnel solution of D1  $\perp$  D3 branes by treating different modes and different directions of the fluctuation. We found that the system got an important property because of the presence of electric field; the system has Neumann boundary condition.

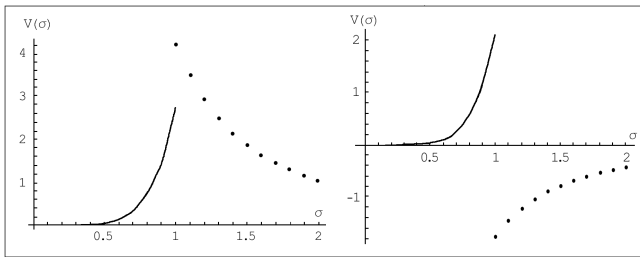


Fig. 6: As we saw in high mode of overall fluctuations, also for relative case we get high potential at some stage of  $\sigma$  where the two curves meet representing potentials for small and large  $\sigma$  at the absence of electric field  $E$  in the left figure. Right figure shows again a singularity this time in relative case because of the presence of  $E$ .

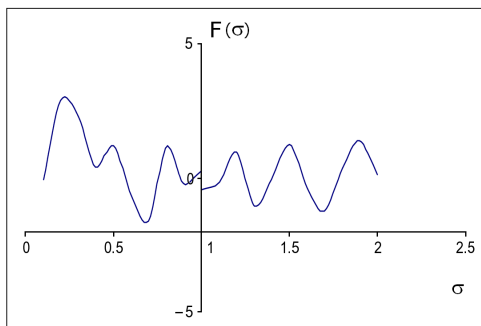


Fig. 7: The presence of electric field  $E$  causes a discontinuity of the wave in high mode of relative case meaning free boundary condition.

#### 4 Conclusion

We have investigated the intersecting D1-D3 branes through a consideration of the presence of electric field. We have treated the fluctuations of the funnel solutions and we have discussed explicitly the potentials in both systems. We found a specific feature of the presence of electric field. When the electric field is going up and down the potential of the system is changing and the fluctuations of funnel solutions as well which cause the division of the system to two regions. Consequently, the end point of the dyonic strings move on the brane which means we have Neumann boundary condition.

The present study is in flat background and there is another interesting investigation is concerning the perturbations propagating on a dyonic string in the supergravity background [12, 5] of an orthogonal 3-brane. Then we can deal with this important case and see if we will get the same boundary conditions by treating the dyonic fluctuations.

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