

Joint Wave-Particle Properties of the Individual Photon

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Two-slit experiments performed earlier by Tsuchiya et al. and recently by Afshar et al. demonstrate the joint wave-particle properties of the single individual photon, and agree with Einstein's argument against Complementarity. These results cannot be explained by conventional theory in which Maxwell's equations serve as a guiding line and basis. On the other hand a revised quantum electrodynamic theory based on a nonzero electric field divergence in the vacuum yields results which appear to be consistent with the experiments. A model of the individual photon is thus deduced from the theory, in the form of a wave packet behaving as a single entity and having simultaneous wave and particle properties.

1 Introduction

Ever since the earlier epoch of natural science, the wave-particle duality of light has appeared as something of an enigma. In Bohr's principle of Complementarity, this duality has been a cornerstone in the interpretation of quantum mechanics. Thereby the wavelike and particlelike properties are conceived to be complementary, in the sense that they are mutually exclusive, and no experiment can reveal both at once. This formulation of quantum mechanics has been successful in many applications and is widely accepted by physicists, but it is full of apparent paradoxes which made Einstein deeply uncomfortable [1].

During the latest decades additional investigations on the nature of light have been made, among which the two-slit experiments by Tsuchiya et al. [2] and Afshar et al. [3] deserve particular attention. These investigations verify that there is a joint wave-particle duality of the individual photon, thus being in agreement with Einstein's argument against Complementarity.

In this paper part of the results by Tsuchiya et al. and Afshar et al. are reviewed and compared with a revised quantum electrodynamic theory by the author. The latter theory is based on a vacuum state that is not merely an empty space but includes the electromagnetic fluctuations of the zero point energy and a corresponding nonzero electric charge density associated with a nonzero electric field divergence. A short description of the theory is presented, whereas its detailed deductions are given elsewhere [4–7].

2 The two-slit experiments

A photon-counting imaging system has earlier been elaborated by Tsuchiya et al. [2] and incorporates the ability to detect individual photons, spatial resolution, and the capability of real-time imaging and subsequent image analysis. Two parallel slits of size $50\ \mu\text{m} \times 4\ \text{mm}$ at a spacing of $250\ \mu\text{m}$

were arranged to pass light through an interference filter at a wavelength of 253.7 nm. The full size of the obtained image on the monitor screen of the experiment was 11.4 mm at the input plane. Since the purpose of the investigation was to demonstrate the interference property of a single photon itself, the spacing of individual photons was made much longer than their coherence time, so that interference between individual photons could be prevented. For this reason, neutral density filters were used to realize a very low light level, where the counting rates were of the order of 100 per second.

As the measurements started, bright very small dots appeared at random positions on the monitor screen. After 10 seconds had elapsed, a photon-counting image was seen on the screen, containing 10^3 events, but its overall shape was not yet clearly defined. After 10 minutes, however, the total accumulated counts were 6×10^4 , and an interference pattern formed by the dots was clearly detected. The diameter of each dot was of the order of 6×10^{-3} of the screen size, and the fringe distance about 5×10^{-2} of it. The effect of closing one of the double slits was finally observed. Then the interference pattern did not appear, but a diffraction pattern was observed.

As concluded by Tsuchiya et al., these results cannot be explained by mutually exclusive wave and particle descriptions of the photon, but give a clear indication of the wave-particle duality of the single individual photon [2].

These important results appear not to have attracted the wide interest which they ought to deserve. However, as long as 22 years later, Afshar et al. [3] conducted a two-slit experiment based on a different methodology but with a similar outcome and conclusions. In this investigation there was a simultaneous determination of the wave and particle aspects of light in a "welcher-weg" experiment, beyond the limitations set by Bohr's principle of Complementarity. The experiment included a pair of pinholes with diameters of 40 nm and center-to-center separation of $250\ \mu\text{m}$, with light from a

diode laser of the wavelength 638 nm. These parameter values were thus not too far from those of the experiments by Tsuchiya et al. In addition, six thin wires of 127 μm diameter were placed at a distance of 0.55 m from the pinholes, and at the minima of the observed interference pattern. When this pattern was present, the disturbance to the incoming beam by the wire grid was minimal. On the other hand, when the interference pattern was absent, the wire grid obstructed the beam. Also here the investigation was conducted in the low photon flux regime, to preclude loss of which-way information due to the intrinsic indistinguishability of coherent multi-photon systems. When the flux was 3×10^4 photons per second, the average separation between successive photons was estimated to about 10 km. The experiments were performed in four ways, i.e. with both pinholes open in absence of the wire grid, with both pinholes open in presence of the wire grid, and with either pinhole open in presence of the same grid.

From the measured data the which-way information and the visibility of an interference pattern could then be determined within the same experimental setup. The which-way information thus indicates through which pinhole the particlelike photon has passed. At the same time the interference indicates that the same wavelike photon must have *sampled* both pinholes so that an interference pattern could be formed. These derived properties of the individual photon refer back to the same space-time event, i.e. to the moment when the single photon passed the plane of the pinholes.

Consequently, also these experimental results force us to agree with Einstein's argument against Complementarity [3].

3 Shortcomings of conventional theory

In conventional quantum electrodynamics (QED), Maxwell's equations have served as a guiding line and basis when there is a vacuum state with a vanishing electric charge density and a zero electric field divergence [8]. According to Schiff [8] and Heitler [9] the Poynting vector then defines the momentum of the pure radiation field, expressed by sets of quantized plane waves. As pointed out by Feynman [10], there are nevertheless unsolved problems which lead to difficulties with Maxwell's equations that are not removed by and not directly associated with quantum mechanics. Consequently, QED will also become subject to the shortcomings of the conventional field theory.

To be more specific in connection with a theoretical model of the individual photon, we start here with the following general physical requirements to be fulfilled:

- The model should have the form of a wave or a wave packet of preserved and limited geometrical shape, propagating with undamped motion in a defined direction of three-space. This leads to an analysis in a cylindrical frame (r, φ, z) with z in the direction of propagation;

- The obtained general solutions for the field quantities should extend all over space, and no artificial boundaries would have to be introduced in the vacuum;
- The integrated total field energy should remain finite;
- The solutions should result in an angular momentum (spin) of the photon as a propagating boson particle.

Maxwell's equations in the vacuum state yield solutions for any field quantity Q having the normal mode form

$$Q = \hat{Q}(r) \exp[i(-\omega t + \bar{m}\varphi + kz)] \quad (1)$$

in cylindrical geometry where ω is the frequency and k and \bar{m} are the wave numbers with respect to the z and φ directions. We further introduce

$$K_0^2 = \left(\frac{\omega}{c}\right)^2 - k^2. \quad (2)$$

When $K_0^2 > 0$ the phase velocity becomes larger and the group velocity smaller than the velocity c of light. The general solution then has field components in terms of Bessel functions $Z_{\bar{m}}(K_0 r)$ of the first and second kind, where the r -dependence of every component is of the form $Z_{\bar{m}}/r$ or $Z_{\bar{m}+1}$ [11]. Application to a photon model then leads to the following results:

- Already the purely axisymmetric case $\bar{m} = 0$ results in a Poynting vector which yields zero spin;
- The spin also vanishes when $K_0 = 0$ and the phase and group velocities both are equal to c ;
- There is no clearly defined spatial limitation of the solutions;
- With no material boundaries such as walls, the total integrated field energy becomes divergent.

Consequently, conventional theory based on Maxwell's equations in the vacuum state does not lead to a physically relevant model for the individual photon.

4 Photon physics in revised quantum electrodynamics

An extended electromagnetic theory applied to the vacuum state and aiming beyond Maxwell's equations serves as a guiding line and basis of the present theoretical approach [4–7]. In four-dimensional representation the theory has the following form

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) A_\mu = \mu_0 J_\mu, \quad \mu = 1, 2, 3, 4, \quad (3)$$

where A_μ are the electromagnetic potentials. As deduced from the requirement of Lorentz invariance, the four-current density of the right-hand member of equation (3) becomes

$$J_\mu = (\mathbf{j}, ic\bar{\rho}) = \varepsilon_0(\text{div } \mathbf{E})(\mathbf{C}, ic), \quad \mathbf{C}^2 = c^2 \quad (4)$$

with c as the velocity of light, \mathbf{E} denoting the electric field strength, and SI units being adopted. Further $\mathbf{B} = \text{curl } \mathbf{A}$ is

the magnetic field strength derived from the three-space magnetic vector potential \mathbf{A} . In equation (4) the velocity vector \mathbf{C} has the modulus c . Maxwell's equations in the vacuum are recovered when $\text{div } \mathbf{E} = 0$, whereas $\text{div } \mathbf{E} \neq 0$ leads to a space-charge current density (4) in the vacuum. The corresponding three-space part $\mathbf{j} = \epsilon_0(\text{div } \mathbf{E})\mathbf{C}$ appears in addition to the displacement current.

The revised basic field equations of dynamic states in a three-dimensional representation are now given by the wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2\right) \mathbf{E} + \left(c^2 \nabla + \mathbf{C} \frac{\partial}{\partial t}\right) (\text{div } \mathbf{E}) = 0 \quad (5)$$

for the electric field, and the equation

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (6)$$

of electromagnetic induction. The characteristic features of the field equations (3)–(6) are as follows:

- The theory is based on the pure radiation field in the vacuum state, including contributions from a nonzero electric charge density;
- The associated nonzero electric field divergence introduces an additional degree of freedom, leading to new solutions and new physical phenomena. This also becomes important in situations where this divergence appears to be small;
- The theory is both Lorentz and gauge invariant;
- The velocity of light is no longer a scalar c but a vector \mathbf{C} with the modulus c .

To become complete, the theory has to be quantized. In absence as well as in presence of source terms, such as the right-hand member of equation (3), the quantized field equations are generally equivalent to the original field equations in which all field quantities are replaced by their expectation values, as shown by Heitler [9]. As a first step and a simplification, the general solutions of the field equations will therefore first be determined, and relevant quantum conditions will afterwards be imposed on these solutions. This is justified by the expectation values due to Heitler. The present theory may therefore not be too far from the truth, in the sense that it represents the most probable states in a first approximation to a rigorous quantum-theoretical deduction.

4.1 Application to a model of the individual photon

The theory of equations (3)–(6) is now applied to the model of an individual photon in the axisymmetric case where $\partial/\partial\varphi = 0$ in a cylindrical frame (r, φ, z) with z along the direction of propagation. Screw-shaped modes where $\partial/\partial\varphi \neq 0$ end in several respects up with similar results, but become more involved and have been described elsewhere [6, 7].

The velocity vector of equation (4) is in this axisymmetric case given by

$$\mathbf{C} = c(0, \cos \alpha, \sin \alpha) \quad (7)$$

where α is a constant angle, and $\cos \alpha$ and $\sin \alpha$ could in principle have either sign but are here limited to positive values for the sake of simplicity. The form (7) can be shown to imply that the electromagnetic energy has one part which propagates in the z -direction, and another part which circulates in the φ -direction around the axis of symmetry and becomes associated with the spin [6, 7]. Normal modes of the form (1) with $\tilde{m} = 0$ then result in general solutions for the components of \mathbf{E} and \mathbf{B} , being given in terms of differential expressions of a generating function

$$F = G_0 R(\rho) \exp[i(-\omega t + kz)]. \quad (8)$$

(Here G_0 is an amplitude factor, $\rho = r/r_0$, and r_0 represents a characteristic radius of the geometrical configuration in question.) The corresponding dispersion relation becomes

$$\omega = kv, \quad v = c(\sin \alpha) \quad (9)$$

thus resulting in axial phase and group velocities, both being equal to $v < c$. Not to get into conflict with the experiments by Michelson and Morley, the condition $0 < \cos \alpha \ll 1$ has to be imposed on the parameter $\cos \alpha$. As an example, $\cos \alpha \leq 10^{-4}$ would make the velocity v differ from c by less than the eight decimal in the value of c . As a consequence of the dispersion relation (9) with $v < c$ and of the detailed deductions, the total integrated field energy mc^2 further becomes equivalent to a total mass m and a rest mass

$$m_0 = m \sqrt{1 - (v/c)^2} = m(\cos \alpha). \quad (10)$$

This rest mass is associated with the angular momentum which only becomes nonzero for a nonzero electric field divergence. When $\text{div } \mathbf{E}$, $\cos \alpha$, and m_0 vanish, we are thus back to the conventional case of Section 3 with its spinless and physically irrelevant basis for a photon model. Even if the electric field divergence at a first glance appears to be a small quantity, it thus has a profound effect on the physics of an individual photon model.

From the obtained general solutions it has further been shown that the total integrated charge and magnetic moment vanish, whereas the total integrated mass m and angular momentum s remain nonzero.

From the solutions of the normal wave modes, a wave packet has to be formed. In accordance with experimental experience, such a packet should have a narrow line width. Its spectrum of wave numbers k should then be piled up around a main wave number k_0 and a corresponding wavelength $\lambda_0 = 2\pi/k_0$. The effective axial length $2z_0$ of the packet is then much larger than λ_0 .

To close the system, two relevant quantum conditions have further to be imposed. The first concerns the total integrated field energy, in the sense that $mc^2 = h\nu_0$ according

to Einstein and Planck, where the frequency $\nu_0 \cong c/\lambda_0$, for $\cos \alpha \ll 1$. The second condition is imposed on the total integrated angular momentum which should become equal to $s = h/2\pi$ for the photon to behave as a boson particle.

From combination with the wave packet solutions, the imposed quantum conditions result in expressions for an effective transverse diameter $2\hat{r}$ of the wave packet. In respect to the radial part R of the generating function (8), there are two alternatives which are both given by

$$2\hat{r} = \frac{\varepsilon \lambda_0}{\pi (\cos \alpha)} \tag{11}$$

and become specified as follows:

- When $\varepsilon = 1$ expression (11) stands for a part $R(\rho)$ which is convergent at the origin $\rho = 0$. This results in an effective photon diameter being only moderately small, but still becoming large as compared to atomic dimensions;
- When $\varepsilon \ll 1$ there are solutions for a part $R(\rho)$ which is divergent at $\rho = 0$. Then finite field quantities can still be obtained within a whole range of small ε , in the limit of a shrinking characteristic radius $r_0 = c_0 \varepsilon$ where c_0 is a positive constant having the dimension of length. This alternative results in an effective photon diameter which can become very small, such as to realize a state of “needle radiation” first proposed by Einstein. Then the diameter (11) can become comparable to atomic dimensions.

It is thus seen that the requirements on a photon model can be fulfilled by the present revised theory. Its wave packet solutions have joint wave-particle properties. In some respects this appears to be similar to the earlier wave-particle duality outlined by de Broglie, where there is a “pilot wave” propagating along the axis, on which wave a “particle-like” part is “surfing”. However, such a subdivision is not necessary in the present case where the wave packet behaves as one single entity, having wave and particle properties at the same time.

Attention is finally called to a comparison between the definition of the momentum of the pure radiation field in terms of the Poynting vector on one hand, and that given by the expression $\mathbf{p} = -i\hbar\nabla$ in the deduction of the Schrödinger equation for a particle with mass on the other [5]. For normal modes the axial component of \mathbf{p} becomes $p_z = \hbar k$ as expected. However, in the transverse direction of a photon model being spatially limited and having a finite effective diameter (11), there would arise a nonzero transverse momentum p_r as well, but this appears to be physically unacceptable for a photon model.

4.2 The present photon model and its relation to two-slit experiments

The limits of the effective photon diameter (11) can be estimated by assuming an upper limit of $2\hat{r}$ when $\varepsilon = 1$ and

$\cos \alpha = 10^{-4}$, and a lower limit of $2\hat{r}$ when $\varepsilon = \cos \alpha$. Then the effective diameter would be in the range of the values $\lambda_0/\pi \leq 2\hat{r} \leq 10^4 \lambda_0/\pi$, but the lower limit could even be lower when $\varepsilon < \cos \alpha$ for strongly pronounced needle radiation. From this first order estimate, and from the features of the theory, the following points should be noticed:

- The diameter of the dot-shaped marks on the monitor screen of the experiment by Tsuchiya et al. is of the order of 6×10^{-3} of the screen size, i.e. about 10^{-4} m. With the wave length $\lambda_0 = 253.7$ nm, the effective photon diameter would then be in the range of the values $7 \times 10^{-4} \geq 2\hat{r} \geq 7 \times 10^{-8}$ m. This range covers the observed size of the dots;
- The width of the parallel slits in the experiments by Tsuchiya et al. is 5×10^{-5} m and their separation distance is 25×10^{-5} m. The corresponding pinhole diameters and their center-to-center separation in the experiments by Afshar et al. are 4×10^{-5} m and 25×10^{-5} m, respectively, and the wavelength is $\lambda_0 = 638$ nm. In the latter experiments the effective diameter is estimated to be in the range $2 \times 10^{-7} \leq 2\hat{r} \leq 2 \times 10^{-3}$ m. In both experiments the estimated ranges of $2\hat{r}$ are thus seen to cover the slit widths and separation distances;
- A large variation of a small $\cos \alpha$ has only a limited effect on the phase and group velocities of equation (9). Also a considerable variation of a small ε does not influence the general deductions of the theory [4, 6, 7] even if it ends up with a substantial change of the diameter (11). This leads to the somewhat speculative question whether the state of the compound parameter $\varepsilon/\cos \alpha$ could adopt different values during the propagation of the wave packet. This could then be related to “photon oscillations” as proposed for a model with a nonzero rest mass, in analogy with neutrino oscillations [4, 7];
- As compared to the slit widths and the separation distances, the obtained ranges of $2\hat{r}$ become consistent with the statement by Afshar et al. that the same wave-like photon can *sample* both pinholes to form an interference pattern;
- Interference between cylindrical waves should take place in a similar way as between plane waves. In particular, this becomes obvious at the minima of the interference pattern where full cancellation takes place;
- Due to the requirement of a narrow line width, the wave packet length $2z_0$ by far exceeds the wave length λ_0 and the effective diameter $2\hat{r}$. Therefore the packet forms a very long and narrow wave train;
- Causality raises the question how the photon can “know” to form the interference pattern on the monitor screen already when it passes the slits. An answer may be provided by the front part of the elongated packet

which may serve as a “precursor”, thereby also representing the quantum mechanical wave nature of the packet. Alternatively, there may exist a counterpart to the precursor phenomenon earlier discussed by Stratton [11] for conventional electromagnetic waves.

5 Conclusions

The two-slit experiments by Tsuchiya et al. and by Afshar et al. demonstrate the joint wave-particle properties of the individual photon, and agree with Einstein’s argument against Complementarity. These experiments cannot be explained by conventional theory. The present revised theory appears on the other hand to become consistent with the experiments.

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References

1. Merali Z. Free will — you only think you have it. *New Scientist*, 6 May 2006, p. 8–9.
2. Tsuchiya T., Inuzuka E., Kurono T., Hosoda M. Photon-counting imaging and its applications. *Advances in Electronics and Electron Physics*, 1985, v. 64A, 21–31.
3. Afshar S. S., Flores E., McDonald K. F., Knoesel E. Paradox in wave-particle duality. *Foundations of Physics*, 2007, v. 37, 295–305.
4. Lehnert B. Photon physics of revised electromagnetics. *Progress in Physics*, 2006, v. 2, 78–85.
5. Lehnert B. Momentum of the pure radiation field. *Progress in Physics*, 2007, v. 1, 27–30.
6. Lehnert B. Revised quantum electrodynamics with fundamental applications. In: Proceedings of 2007 ICTP Summer College on Plasma Physics (Edited by P. K. Shukla, L. Stenflo, and B. Eliasson), World Scientific Publishers, Singapore, 2008.
7. Lehnert B. A revised electromagnetic theory with fundamental applications. Swedish Physics Archive (Edited by D. Rabounski), The National Library of Sweden, Stockholm, 2007; and Bogoljubov Institute for Theoretical Physics (Edited by A. Zagorodny), Kiev, 2007.
8. Schiff L. I. Quantum Mechanics. McGraw-Hill Book Comp. Inc., New York, 1949, Chs. XIV, IV, and II.
9. Heitler W. The quantum theory of radiation. Third edition. Clarendon Press, Oxford, 1954, Ch. II and Appendix.
10. Feynman R. P. Lectures on physics: mainly eletromagnetism and matter. Addison-Wesley, Reading, Massachusetts, 1964.
11. Stratton J. A. Electromagnetic theory. McGraw-Hill Book Comp. Inc., New York and London, 1941, Ch. VI, Sec. 5.18.