A New Derivation of Biquaternion Schrödinger Equation and Plausible Implications

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In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we discuss some possible interpretation of this imaginary part of the solution of biquaternionic KGE (BQKGE); thereafter we offer a new derivation of biquaternion Schrödinger equation using this method. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

There were some attempts in literature to generalise Schrödinger equation using quaternion and biquaternion numbers. Because quaternion number use in Quantum Mechanics has often been described [1, 2, 3, 4], we only mention in this paper the use of biquaternion number. Sapogin [5] was the first to introduce biquaternion to extend Schrödinger equation, while Kravchenko [4] use biquaternion number to describe neat link between Schrödinger equation and Riccati equation.

In the present article we discuss a new derivation of biquaternion Schrödinger equation using a method used in the preceding paper. Because the previous method has been used for Klein-Gordon equation [1], now it seems natural to extend it to Schrödinger equation. This biquaternion effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [6]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Some interpretations of preceding result of biquaternionic KGE

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows

$$\left[\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)+i\left(\frac{\partial^2}{\partial t^2}-\nabla^2\right)\right]\varphi(x,t)=-m^2\varphi(x,t). \quad (1)$$

Or this equation can be rewritten as

$$\left(\diamondsuit\bar{\diamondsuit}+m^2\right)\varphi(x,t)=0 \quad (2)$$

provided we use this definition

$$\diamondsuit=\nabla^q+i\nabla^q=\left(-i\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\right)+ +i\left(-i\frac{\partial}{\partial T}+e_1\frac{\partial}{\partial X}+e_2\frac{\partial}{\partial Y}+e_3\frac{\partial}{\partial Z}\right) \quad (3)$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1=i$, $e_2=j$, $e_3=k$)

$$i^2=j^2=k^2=-1\,,\quad ij=-ji=k\,,\qquad jk=-kj=i\,,\quad ki=-ik=j\,, \quad (4)$$

and quaternion *Nabla operator* is defined as [7]

$$\nabla^q=-i\frac{\partial}{\partial t}+e_1\frac{\partial}{\partial x}+e_2\frac{\partial}{\partial y}+e_3\frac{\partial}{\partial z}\,. \quad (5)$$

Note that equation (3) and (5) included partial time-differentiation.

It is worth nothing here that equation (2) yields solution containing imaginary part, which differs appreciably from known solution of KGE:

$$y(x,t)=\left(\frac{1}{4}-\frac{i}{4}\right)m^2t^2+\text{constant}. \quad (6)$$

Some possible alternative interpretations of this *imaginary part* of the solution of biquaternionic KGE (BQKGE) are:

(a) The imaginary part implies that there is exponential term of the wave solution, which is quite similar to the Ginzburg-Landau extension of London phenomenology [8]

$$\psi(r)=|\psi(r)|\,e^{i\varphi(r)}\,, \quad (7)$$

because (6) can be rewritten (approximately) as:

$$y(x,t)=\frac{e^i}{4}\,m^2t^2; \quad (8)$$

(b) The aforementioned exponential term of the solution (8) can be interpreted as signature of vortices solution. Interestingly Navier-Stokes equation which implies vorticity equation can also be rewritten in terms of Yukawa equation [3];

(c) The imaginary part implies that there is spiral wave, which suggests spiralling motion of meson or other particles. Interestingly it has been argued that one can explain electron phenomena by assuming spiralling electrons [9]. Alternatively this spiralling wave may already be known in the form of Bierkeland flow. For meson observation, this could be interpreted as another form of meson, which may be called “supersymmetric-meson” [1];

(d) The imaginary part of solution of BQKGE also implies that it consists of standard solution of KGE [1], and its alteration because of imaginary differential operator. That would mean the resulting wave is composed of two complementary waves;

(e) Considering some recent proposals suggesting that neutrino can have *imaginary mass* [10], the aforementioned imaginary part of solution of BQKGE can also imply that the (supersymmetric-) meson may be composed of neutrino(s). This new proposition may require new thinking both on the nature of neutrino and also supersymmetric-meson [11].

While some of these propositions remain to be seen, in deriving the preceding BQKGE we follow Dirac’s phrase that “*One can generalize his physics by generalizing his mathematics*”. More specifically, we focus on using a “theorem” from this principle, i.e.: “*One can generalize his mathematics by generalizing his (differential) operator*”.

3 Extended biquaternion Schrödinger equation

One can expect to use the same method described above to generalize the standard Schrödinger equation [12]

$$\left[-\frac{\hbar^2}{2m}\,\Delta u + V(x)\right] u = Eu\,, \tag{9}$$

or, in simplified form, [12, p.11]:

$$(-\Delta + w_k) f_k = 0\,, \qquad k = 0, 1, 2, 3. \tag{10}$$

In order to generalize equation (9) to biquaternion version (BQSE), we use first quaternion Nabla operator (5), and by noticing that $\Delta \equiv \nabla\nabla$, we get

$$-\frac{\hbar^2}{2m}\left(\nabla^q \bar{\nabla}^q + \frac{\partial^2}{\partial t^2}\right) u + \big(V(x) - E\big)\, u = 0\,. \tag{11}$$

Note that we shall introduce the second term in order to ‘neutralize’ the partial time-differentiation of $\nabla^q \bar{\nabla}^q$ operator.

To get biquaternion form of equation (11) we can use our definition in equation (3) rather than (5), so we get

$$-\frac{\hbar^2}{2m}\left(\diamond\bar{\diamond} + \frac{\partial^2}{\partial t^2} - i\,\frac{\partial^2}{\partial T^2}\right) u + \big(V(x) - E\big)\, u = 0\,. \tag{12}$$

This is an alternative version of *biquaternionic* Schrödinger equation, compared to Sapogin’s [5] or Kravchenko’s [4] method. We also note here that the route to *quaternionize* Schrödinger equation here is rather different from what is described by Horwitz [13, p. 6]

$$\tilde{H}\,\psi = \psi\, e_1\, E\,, \tag{13}$$

or

$$\tilde{H}\,\psi\, q = \psi\, q\,\left(q^{-1}\, e_1\, q\right)\, E\,, \tag{14}$$

where the quaternion number q, can be expressed as follows (see [13, p. 6] and [4])

$$q = q_0 + \sum_{i=1}^{3} q_i\, e_i\,. \tag{15}$$

Nonetheless, further observation is of course recommended in order to refute or verify this proposition (12).

4 Numerical solution of biquaternion Schrödinger equation

It can be shown that numerical solution (using Maxima [14]) of biquaternionic extension of Schrödinger equation yields different result compared to the standard Schrödinger equation, as follows. For clarity, all solutions were computed in 1-D only.

For standard Schrödinger equation [12], one can rewrite equation (9) as follows:

(a) For $V(x) > E$:

$$-\frac{\hbar^2}{2m}\,\Delta u + a \cdot u = 0\,; \tag{16}$$

(b) For $V(x) < E$:

$$-\frac{\hbar^2}{2m}\,\Delta u - a \cdot u = 0\,. \tag{17}$$

Numerical solution of equation (16) and (17) is given (by assuming $\hbar = 1$ and $m = 1/2$ for convenience)

(%i44) -’diff (y, x, 2) + a*y;
(%o44) $a \cdot y - \frac{d^2}{d^2 x} y$

(a) For $V(x) > E$:

(%i46) ode2 (%o44, y, x);
(%o46) $y = k_1 \cdot \exp(\sqrt{a \cdot x}) + k_2 \cdot \exp(-\sqrt{ax})$

(b) For $V(x) < E$:

(%i45) ode2 (%o44, y, x);
(%o45) $y = k_1 \cdot \sinh(\sqrt{a \cdot x}) + k_2 \cdot \cosh(\sqrt{a \cdot x})$

In the meantime, numerical solution of equation (12), is given (by assuming $\hbar = 1$ and $m = 1/2$ for convenience)

(a) For $V(x) > E$:

(%i38) (%i+1)*’diff (y, x, 2) + a*y;
(%o38) $(i+1)\,\frac{d^2}{d^2 x} y + a \cdot y$
(%i39) ode2 (%o38, y, x);
(%o39) $y = k_1 \cdot \sin(\sqrt{\frac{a}{i+1}} \cdot x) + k_2 \cdot \cos(\sqrt{\frac{a}{i+1}} \cdot x)$

(b) For $V(x) < E$:

(%i40) (%i+1)*’diff (y, x, 2) - a*y;
(%o40) $(i+1)\,\frac{d^2}{d^2 x} y - a \cdot y$
(%i41) ode2 (%o40, y, x);
(%o41) $y = k_1 \cdot \sin(\sqrt{-\frac{a}{i+1}} \cdot x) + k_2 \cdot \cos(\sqrt{-\frac{a}{i+1}} \cdot x)$

Therefore, we conclude that numerical solution of biquaternionic extension of Schrödinger equation yields different result compared to the solution of standard Schrödinger equation. Nonetheless, we recommend further observation in order to refute or verify this proposition/numerical solution of biquaternion extension of spatial-differential operator of Schrödinger equation.

As side remark, it is interesting to note here that if we *introduce imaginary number* in equation (16) and equation (17), the numerical solutions will be quite different compared to solution of equation (16) and (17), as follows

$$-\frac{i\hbar^2}{2m}\,\Delta u + au = 0\,, \tag{18}$$

where $V(x) > E$, or

$$-\frac{i\hbar^2}{2m}\,\Delta u - au = 0\,, \tag{19}$$

where $V(x) < E$.

Numerical solution of equation (18) and (19) is given (by assuming $\hbar = 1$ and $m = 1/2$ for convenience)

(a) For $V(x) > E$:

(%i47) -%i*'diff (y, x, 2) + a*y;

(%o47) $a \cdot y - i\frac{d^2}{d^2x}y$

(%i48) ode2 (%o47, y, x);

(%o48) $y = k_1 \cdot \sin(\sqrt{ia \cdot x}) + k_2 \cdot \cos(\sqrt{ia \cdot x})$

(b) For $V(x) < E$:

(%i50) -%i*'diff (y, x, 2) - a*y;

(%o50) $-a \cdot y - i\frac{d^2}{d^2x}y$

(%i51) ode2 (%o50, y, x);

(%o51) $y = k_1 \cdot \sin(-\sqrt{ia \cdot x}) + k_2 \cdot \cos(-\sqrt{ia \cdot x})$

It shall be clear therefore that using different sign for differential operator yields quite different results.

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