SPECIAL REPORT

PLANCK, the Satellite: a New Experimental Test of General Relativity

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If the origin of a microwave background (EMB) is the Earth, what would be its density and associated dipole anisotropy measured at different altitudes from the surface of the Earth? The mathematical methods of the General Theory of Relativity are applied herein to answer these questions. The density of the EMB is answered by means of Einstein's equations for the electromagnetic field of the Earth. The dipole anisotropy, which is due to the rapid motion of the source (the Earth) in the weak intergalactic field, is analysed by using the geodesic equations for light-like particles (photons), which are mediators for electromagnetic radiation. It is shown that the EMB decreases with altitude so that the density of its energy at the altitude of the COBE orbit (900km) is 0.68 times less than that at the altitude of a U2 aeroplane (25 km). Furthermore, the density at the 2nd Lagrange point (1.5 million km, the position of the WMAP and PLANCK satellites) should be only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of the EMB doesn't depend on altitude from the surface of the Earth, it should be the same irrespective of the altitude at which measurements are taken. This result is in support to the experimental and observational analysis conducted by P.-M. Robitaille, according to which the 2.7 K microwave background, first observed by Penzias and Wilson, is not of cosmic origin, but of the Earth, and is generated by the oceans. WMAP indicated the same anisotropy of the microwave background at the 2nd Lagrange point that near the Earth. Therefore when PLANCK, which is planned on July, 2008, will manifest the 2.7 K monopole microwave signal deceased at the 2nd Langrange point, it will be a new experimental verification of Einstein's theory.

1 Introduction

Our recent publication [1] was focused on the mathematical proof in support to the claim made by P.-M. Robitaille: according to the experimental and observational analysis conducted by him [3–10], the 2.7 K monopole microwave background, first detected by Penzias and Wilson [2], is not of cosmic origin, but of the Earth, and is generated by oceanic water*. As shown in the framework of Robitaille's concept, the anisotropy of the background, observed on the 3.35 mK dipole component of it†, is due to the rapid motion of the whole field in common with its source, the Earth, in a weak intergalactic field so that the anisotropy of the observed microwave background has a purely relativistic origin [21].‡

If the microwave background is of the earthy origin, the density of the field should obviously decrease with altitude from the surface of the Earth. The ground-bound measurements and those made on board of the COBE satellite, at the altitude 900 km, were processed very near the oceans which aren't point-like sources, so the observations were unable to manifest the change of the field density with altitude. Another case — the 2nd Lagrange point, which is located as far as 1.5 mln km from the Earth, the position of the WMAP satellite and the planned PLANCK satellite.

A problem is that WMAP has only differential instruments on board: such an instrument, having a few channels for incoming photons, registers only the difference between the number of photons in the channels. WMAP therefore targeted measurements of the anisotropy of the field, but was unable to measure the field density. PLANCK, which is planned on July, 2008, is equipped by absolute instruments (with just one channel for incoming photons, an absolute instrument gets the integral density of the monopole and all the multipole components of the field). Hence PLANCK will be able to measure the field density at the 2nd Lagrange point.

We therefore were looking for a theory which would be able to represent the density and anisotropy of the Earth's microwave background as the functions of altitude from the Earth's surface.

In our recent publication [1], we created such a theory with use of the mathematical methods of the General The-

*Robitaille reported the result first in 1999 and 2001 in the short communications [3, 4], then detailed explanation of the problem was given by him in the journal publications [5–8] and also in the reports [9, 10].

†The 3.35 mK dipole component of the background was first observed in 1969 by Conklin [11] in a ground-based observation. Then it was studied by Henry [12], Corey [13], and also Smoot, Gorenstein, and Muller (the latest team organized a stratosphere observation on board of a U2 aeroplane [14]). The history of the discovery and all the observations is given in detail in Lineweaver's paper [15]. The anisotropy of the dipole component was found later, in the COBE space mission then verified by the WMAP space mission [16–20].

‡This conclusion is based on that fact that, according to the General Theory of Relativity, photons exceeded from a source at radial directions should be carried out with the space wherein this source moves so that the spherical distribution of the signals should experience an anisotropy in the direction of the motion of this source in the space [22, 23].

ory of Relativity where the physical characteristics of fields are expressed through the geometrical characteristics of the space itself. We have split our tasks into two particular problems: if a microwave background originates from the Earth, what would be the dependency of its density and relativistic anisotropy with altitude? The first problem was solved via Einstein's equations for the electromagnetic field of the Earth. The second problem was solved using the geodesic equations for light-like particles (photons) which are mediators for electromagnetic radiation.

We have determined, according to our solutions [1], that a microwave background that originates at the Earth decreases with altitude so that the density of the energy of such a background in the COBE orbit (the altitude 900 km) is 0.68 times less than that at the altitude of a U2 aeroplane. The density of the energy of the background at the L2 point is only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of such an earthy microwave background, due to the rapid motion of the Earth relative to the source of a weak intergalactic field which is located in depths of the cosmos, doesn't depend on altitute from the surface of the Earth. Such a dipole will be the same irrespective of the position at which measurements are taken.

In principle, the first problem — how the density of an earthy-origin microwave background decreases with altitude — may be resolved by the methods of classical physics. But this is possible only in a particular case where the space is free of rotation. In real, the Earth experiences daily rotation. We therefore should take into account that fact that the rotation makes the observer's local space non-holonomic: in such a space the time lines are non-orthogonal to the spatial section, so the Riemannian curvature of the space is non-zero. A satellite's motion around the Earth should be also taken into account for the local space of an observer which is located on board of the satellite. Therefore in concern of a real experiment, in both cases of ground-based and satellite-based observations, the first problem can be resolved only in the framework of the General Theory of Relativity.

The second problem can never been resolved in the framework of classical physics due to the purely relativistic origin of the field anisotropy we are considering.

WMAP registered the same parameters of the microwave background anisotropy that the registered by COBE near the Earth. This is according to our theory.

Therefore when PLANCK will manifest the 2.7 K monopole microwave signal deceased at the 2nd Langrange point, with the same anisotropy of the background that the measured near the Earth (according to WMAP which is as well located at the 2nd Langange point), this will be a new experimental verification of the General Theory of Relativity.

A drawback of our theory was only that complicate way in which it was initially constructed. As a result, our recently published calculation [1] is hard to reproduce by the others who have no mathematical skills in the very specific areas of General Relativity, which are known to only a close circle of the specialists who are no many in the world. We therefore were requested for many additional explanations by those readers who tried to repeat the calculation.

Due to that discussion, we found another way to give representation of our result with much unused stuff removed. We also gave an additional explanation to those parts of our calculation, which were asked by the readers. As a result a new representation of our calculation, with the same result, became as simple as easy to peroduce by everyone who is free in tensor algebra. This representation is given here.

2 The local space metric of a satellite-bound observer

A result of real measurement processed by an observer depends on the properties of his local space. These properties are completely determined by the metric of this space. We therefore are looking for the metric of the local space of an observer, who is located on board of a satellite moved in the Earth's gravitational field.

As one regularly does in construction for a metric, we take a simplest metric which is close to the case we are considering, then modify the metric by introduction of those additional factors which are working in our particular case.

Here is how we do it.

As was proven in the 1940's by Abraham Zelmanov, on the basis of the theory of hon-holonomic manifolds [24] constructed in the 1930's by Schouten then applied by Zelmanov to the four-dimensional pseudo-Riemannian space of General Relativity, the non-holonomity of such a space (i.e. the non-orthogonality of the time lines to the spatial section, that is expressed as $g_{0i} \neq 0$ in the fundamental metric tensor $g_{\alpha\beta}$) is manifest as the three-dimensional rotation of this space. Moreover, Zelmanov proven that any non-holonomic space has nonzero Riemannian curvature (nonzero Riemann-Christoffel tensor) due to $g_{0i} \neq 0$. All these was first reported in 1944 by him in his dissertation thesis [25], then also in the latter publications [26–28].

In practice this means that the physical space of the Earth, the planet, is non-holonomic and curved due to the daily rotation of it. This is in addition to that fact that the Earth's space is curved due to the gravitational field of the Earth, described in an approximation by Schwarzschild metric of a centrally symmetric gravitational field, created by a spherical mass in emptiness. The space metric of a satellite-bound observer should also take into account that fact that the satellite moves along its orbit in the Earth's space around the terrestrial globe (the central mass that produces the field). In addition to it the Earth, in common with the satellite and the observer located in it, rapidly moves in the physical space of the Universe associated to the weak intergalactic microwave field. This fact should also be taken into account in the metric.

First, we consider a simplest non-holonomic space —

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 + \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdt\, dr + \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdt\, d\varphi + \frac{2v}{c}\, cdt\, dz - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2 \qquad (7)$$

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}\right) c^2 dt^2 + \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdt\, dr + \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdt\, d\varphi + \frac{2v}{c}\, cdt\, dz - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 + \frac{2v\mathrm{v}\,(\cos\varphi + \sin\varphi)}{c^2}\, dr\, dz - r^2 d\varphi^2 + \frac{2r\mathrm{v}\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c^2}\, d\varphi\, dz - \left(1 - \frac{2v\mathrm{v}}{c^2}\right) dz^2 \qquad (8)$$

a space wherein all $g_{0i} \neq 0$, and they have the same numerical values. According to Zelmanov [25–28], such a space experiences rotation around all three axes with the same linear velocity $v = v_1 = v_2 = v_3$, where $v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}$. As obvious, the metric of such a non-holonomic space is

$$ds^2 = c^2 dt^2 + \frac{2v}{c}\, cdt\,(dx + dy + dz) - dx^2 - dy^2 - dz^2. \quad (1)$$

For easy taking the Earth's field into account, we change to the cylindrical coordinates r, φ, z, where the r-axis is directed from the centre of gravity of the Earth along its radius. The corresponding transformations of the coordinates are $x = r\cos\varphi$, $y = r\sin\varphi$, $z = z$ so that the metric (1) represented in the new coordinates is

$$ds^2 = c^2 dt^2 + \frac{2v}{c}\,(\cos\varphi + \sin\varphi)\, cdt\, dr + \\ + \frac{2vr}{c}\,(\cos\varphi - \sin\varphi)\, cdt\, d\varphi + \frac{2v}{c}\, cdt\, dz - \\ - dr^2 - r^2 d\varphi^2 - dz^2. \quad (2)$$

Next we introduce the factor of the Earth's gravitational field in the same way as it is made in Schwarzschild metric (see §100 in *The Classical Theory of Fields* [29]) — the metric of a spherically symmetric gravitational field, produced by a sperical mass M in emptiness, which in the cylindrical coordinates is

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 d\varphi^2 - dz^2, \quad (3)$$

where we should take into account that fact that $\frac{2GM}{c^2 r}$ is small value, so we have

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - \\ - r^2 d\varphi^2 - dz^2. \quad (4)$$

Besides, we should take into account the factor of rotational motion of the observer, in common with the satellite, along its orbit around the Earth. We see how to do it in the example of a plane metric in the cylindrical coordinates

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\varphi^2 - dz^2, \quad (5)$$

where we change the reference frame to another one, which rotates relative to the initially reference frame with a constant angular velocity ω. By the applying the transformation of the coordinates $r' = r$, $\varphi' = \varphi + \omega t$, $x' = z$, we obtain ds^2 in the rotating reference frame*

$$ds^2 = \left(1 - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c}\, cdt\, d\varphi - dr^2 - \\ - r^2 d\varphi^2 - dz^2. \quad (6)$$

Following with the aforementioned steps†, we obtain the metric of the *local physical space* of a satellite-bound observer which takes all properties of such a space into account. This resulting metric is represented in formula (7).

This metric will be used by us in calculation for the density of the Earth microwave background, measured by an observer on board of a satellite of the Earth.

This metric is definitely curved due to two factors: non-zero gravitational potential $\mathrm{w} = c^2(1 - \sqrt{g_{00}}) \neq 0$ and the space non-holonomity $g_{0i} \neq 0$. Hence we are able to consider Einstein equations in such a space.

On the other hand this metric doesn't take into account that fact that the Earth microwave background, in common with the Earth, moves in a weak intergalactic field with a velocity of $\mathrm{v} = 365 \pm 18$ km/sec (as observational analysis indicates it). To calculate the accociated dipole anisotropy of the Earth microwave background, which is due to the motion, we should use such a space metric which takes this motion into account. To do it we take the metric (7) then apply Lorentz' transformations to the z-coordinate (we direct the z-axis with the motion of the Earth in the weak intergalactic field) and time with an obvious approximation of $\mathrm{v} \ll c$ and high order terms omitted: $z' = z + \mathrm{v}t$, $t' = t + \frac{\mathrm{v}z}{c^2}$. In other word, we "move" the whole local physical space of an earthy satellite-bound observer relative to the source of the weak intergalactic field. As a result the local physical space of such an observer and all physical fields connected to the Earth should experience a drift in the z-direction and a corresponding change the

*See §10.3 in [27], or §3.6 in [28] for detail.

†As known in Riemannian geometry, which is particular to metric geometries, a common metric can be deduced as a superposition of all the particular metrics each of whom takes a particular property of the common space into account.

local physically observed time that should has a sequel on the observed charachteristics of the Earth's microwave field.

The resulting metric we have obtained after the transformation is (8). We will use this metric in calculation for the anisotropy of the Earth microwave background measured by a satellite-bound observer.

3 The density of the Earth's microwave background at the 2nd Lagrange point

To calculate the density of a field (distributed matter) dependent from the properties of the space wherein this field is situated we should operate with Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta}\, R = -\,\kappa\, T_{\alpha\beta} + \lambda\, g_{\alpha\beta}\,, \tag{9}$$

the left side of which is for the space geometry, while the right side describes distributed matter (it is with the energy-momentum tensor of distributed matter and the λ-term which describes the distribution of physical vacuum).

Projection of the energy-momentum tensor $T_{\alpha\beta}$ onto the time line and spatial section of an observer's local physical space gives the properties of distributed matter observed by him [25–28]: the density of the energy of distributed matter $\rho = \frac{T_{00}}{g_{00}}$, the density of the momentum $J^i = \frac{c\,T_0^i}{\sqrt{g_{00}}}$, and the stress-tensor $U^{ik} = c^2 T^{ik}$. To express the first of these observable quantities through the observable properties of the local physical space is a task in our calculation.

To reach this task we should project the whole Einstein equations onto the ime line and spatial section of the metric space (7) with taking into account that fact that the energy-momentum tensor is of an electromagnetic field. The left side of the projected equations will be containing the observable properties of the local space of such an observer, while the right side will be containing the aforementioned observable properties of distributed matter (the Earth microwave background, in our case). Then we can express the density of the Earth microwave background ρ as a function of the observable properties of the local space.

Einstein's equations projected onto the time line and spatial section of a common case were obtained in the 1940's by Zelmanov [25–28], and are quite complicate in the left side (the observable properties of the local space). We therefore first should obtain the observable properties of the given space (7), then decide what propetries can be omitted from consideration in the framework of our problem.

According to the theory of physical observable quantities of the General Theory of Relativity [25–28], the observable properties of a space are the three-dimensional quantities which are invariant within the fixed spatial section of an observer (so-called *chronometrically invariant quantities*). Those are the three-dimensional metric tensor h_{ik}, the gravitational inertial force F_i, the angular velocity of the space rotation A_{ik} (known as the non-holonomity tensor), the space deformation tensor D_{ik}, the three-dimensional Christoffel symbols Δ^i_{kn}, and the three-dimensional curvature C_{iklj}, expressed through the gravitational potential $\mathrm{w} = c^2\,(1-\sqrt{g_{00}})$ and the linear velocity of the space rotation $v_i = -\,\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ (whose components are $v^i = -\,c\,g^{0i}\sqrt{g_{00}}$ and $v_i = h_{ik}\,v^k$):

$$h_{ik} = -\,g_{ik} + \frac{g_{0i}\,g_{0k}}{g_{00}} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \tag{10}$$

$$h^{ik} = -\,g^{ik}, \qquad h^i_k = -\,g^i_k = \delta^i_k\,, \tag{11}$$

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \tag{12}$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\,(F_i v_k - F_k v_i)\,, \tag{13}$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}, \qquad D^{ik} = -\,\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \tag{14}$$

$$D = h^{ik} D_{ik} = D^k_k = \frac{{}^*\partial \ln\sqrt{h}}{\partial t}, \quad h = \det\|h_{ik}\|\,, \tag{15}$$

$$\Delta^i_{jk} = \frac{1}{2}\,h^{im}\left(\frac{{}^*\partial h_{jm}}{\partial x^k} + \frac{{}^*\partial h_{km}}{\partial x^j} - \frac{{}^*\partial h_{jk}}{\partial x^m}\right), \tag{16}$$

$$\Delta^j_{jk} = \frac{{}^*\partial \ln\sqrt{h}}{\partial x^k}\,, \tag{17}$$

$$C_{lkij} = \frac{1}{4}\left(H_{lkij} - H_{jkil} + H_{klji} - H_{iljk}\right), \tag{18}$$

$$C_{kj} = C^{\dots i}_{kij\cdot} = h^{im} C_{kimj}\,, \quad C = C^j_j = h^{lj} C_{lj}\,. \tag{19}$$

Here $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\,v_i\,\frac{{}^*\partial}{\partial t}$ are the chronometrically invariant differential operators, while

$$H^{\dots j}_{lki\cdot} = \frac{{}^*\partial \Delta^j_{il}}{\partial x^k} - \frac{{}^*\partial \Delta^j_{kl}}{\partial x^i} + \Delta^m_{il}\Delta^j_{km} - \Delta^m_{kl}\Delta^j_{im} \tag{20}$$

is Zelmanov's tensor constructed by him on the basis of the non-commutativity of the second chronometrically invariant derivatives of an arbitrary spatial vector taken in a given three-dimensional spatial section

$${}^*\nabla_i{}^*\nabla_k\, Q_l - {}^*\nabla_k{}^*\nabla_i\, Q_l = \frac{2A_{ik}}{c^2}\frac{{}^*\partial Q_l}{\partial t} + H^{\dots j}_{lki\cdot}\, Q_j\,, \tag{21}$$

where ${}^*\nabla_i\, Q_l = \frac{{}^*\partial Q_l}{\partial x^i} - \Delta^j_{ji}\, Q_l$ is the chronometrically invariant derivative of the vector (${}^*\nabla_i\, Q^l = \frac{{}^*\partial Q^l}{\partial x^i} + \Delta^j_{ji}\, Q^l$ respectively). The tensor $H^{\dots j}_{lki\cdot}$ was introduced by Zelmanov similarly to Schouten's tensor of the theory of non-holonomic manifolds [24] so that the three-dimensional curvature tensor C_{lkij} possesses all the algebraic properties of the Riemann-Christoffel curvature tensor in the spatial section.

We take the components of the fundamental metric tensor $g_{\alpha\beta}$ from the metric of the local physical space of a satellite-bound observer (7), then calculate the aforementioned observable quantities. In this calculation we take into account that fact that $\frac{2GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$ are in order of 10^{-9} near the surface of the Earth, and the values decrease with altitude. We therefore operate these terms according to the rules of small values. We also neglect all high order terms. We however cannot neglect $\frac{2GM}{c^2r}$ and $\frac{\omega^2r^2}{c^2}$ in $g_{00}=1-\frac{2GM}{c^2r}-\frac{\omega^2r^2}{c^2}$ when calculating the gravitational potential $\mathrm{w}=c^2(1-\sqrt{g_{00}})$ according to the rule of small values

$$\begin{aligned}\mathrm{w} = c^2\left(1-\sqrt{1-\frac{2GM}{c^2r}-\frac{\omega^2r^2}{c^2}}\right) = \\ = c^2\left\{1-\left(1+\frac{GM}{c^2r}+\frac{\omega^2r^2}{2c^2}\right)\right\}\end{aligned} \tag{22}$$

because these terms are multiplied by c^2. We also assume the linear velocity of the space rotation v to be small to the velocity of light c. We assume that v doesn't depend from the z-coordinate. This assumption is due to the fact that the Earth, in common with its space, moves relative to a weak intergalactic microwave background that causes the anisotropy of the Earth's microwave field.

As a result we obtain the substantially non-zero components of the characteristics of the space

$$\mathrm{w} = \frac{GM}{r}+\frac{\omega^2r^2}{2}\,, \tag{23}$$

$$\left.\begin{aligned} v_1 &= -v\left(\cos\varphi+\sin\varphi\right) \\ v_2 &= -r\left[v\left(\cos\varphi-\sin\varphi\right)-\omega r\right] \\ v_3 &= -v \end{aligned}\right\}, \tag{24}$$

$$\left.\begin{aligned} &F_1 = \left(\cos\varphi+\sin\varphi\right)v_t+\omega^2r-\frac{GM}{r^2} \\ &F_2 = r\left(\cos\varphi-\sin\varphi\right)v_t\,, \qquad F_3 = v_t \end{aligned}\right\}, \tag{25}$$

$$\left.\begin{aligned} &A_{12} = \omega r+\frac{1}{2}\big[\left(\cos\varphi+\sin\varphi\right)v_\varphi - \\ &\qquad\qquad - r\left(\cos\varphi-\sin\varphi\right)v_r\big] \\ &A_{23} = -\frac{v_\varphi}{2}\,, \qquad A_{13} = -\frac{v_r}{2} \end{aligned}\right\}, \tag{26}$$

$$\left.\begin{aligned} &h_{11}=h_{33}=1\,,\quad h_{22}=r^2,\quad h^{11}=h^{33}=1 \\ &h^{22}=\frac{1}{r^2}\,,\qquad h=r^2,\qquad \frac{\partial\ln\sqrt{h}}{\partial r}=\frac{1}{r} \end{aligned}\right\}, \tag{27}$$

$$\Delta^1_{22} = -r\,,\qquad \Delta^2_{12}=\frac{1}{r} \tag{28}$$

while all components of the tensor of the space deformation D_{ik} and the three-dimensional curvature C_{iklj} are negligible in the framework of the first order approximation (the four-dimensional Riemannian curvature isn't negligible).

The quantities v_r, v_φ, and v_t denote the partial derivatives of the linear velocity of the space rotation v by the respective coordinate and time. (Here $v_z=0$ according to the initially assumptions in the framework of our problem.)

We consider the projected Einstein equations in complete form, published in [25–28]

$$\left.\begin{aligned} &\frac{{}^*\partial D}{\partial t}+D_{jl}D^{jl}+A_{jl}A^{lj}+\left({}^*\nabla_j-\frac{1}{c^2}F_j\right)F^j = \\ &= -\frac{\kappa}{2}\left(\rho c^2+U\right)+\lambda c^2 \\ &{}^*\nabla_j\left(h^{ij}D-D^{ij}-A^{ij}\right)+\frac{2}{c^2}F_jA^{ij}=\kappa J^i \\ &\frac{{}^*\partial D_{ik}}{\partial t}-\left(D_{ij}+A_{ij}\right)\left(D^j_k+A^{\cdot j}_{k\cdot}\right)+DD_{ik}+ \\ &+3A_{ij}A^{\cdot j}_{k\cdot}+\frac{1}{2}\left({}^*\nabla_iF_k+{}^*\nabla_kF_i\right)-\frac{1}{c^2}F_iF_k- \\ &-c^2C_{ik}=\frac{\kappa}{2}\left(\rho c^2h_{ik}+2U_{ik}-Uh_{ik}\right)+\lambda c^2h_{ik} \end{aligned}\right\}. \tag{29}$$

We withdraw the λ-term, the space deformation D_{ik}, and the three-dmensional curvature C_{iklj} from consideration. We also use the aforemonetioned assumptions on small values and high order terms that reduce the chronometrically invariant differential operators to the regular differential operators: $\frac{{}^*\partial}{\partial t}=\frac{\partial}{\partial t}$, $\frac{{}^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}$. As a result of all these, the projected Einstein equations take the simplified form

$$\left.\begin{aligned} &\frac{\partial F^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}F^i-A_{ik}A^{ik}=-\frac{\kappa}{2}\left(\rho c^2+U\right) \\ &\frac{\partial A^{ik}}{\partial x^k}+\frac{\partial\ln\sqrt{h}}{\partial x^k}A^{ik}=-\kappa J^i \\ &2A_{ij}A^{\cdot j}_{k\cdot}+\frac{1}{2}\left(\frac{\partial F_i}{\partial x^k}+\frac{\partial F_k}{\partial x^i}-2\Delta^m_{ik}F_m\right)= \\ &\qquad =\frac{\kappa}{2}\left(\rho c^2h_{ik}+2U_{ik}-Uh_{ik}\right) \end{aligned}\right\}. \tag{30}$$

We substitute hereto the obtained observable characheristics of the local physical space of a satellite-bound observer. Because the value v is assumed to be small, we neglect not only the square of it, but also the square of its derivative and the products of the derivatives.

The Einstein equations (30) have been written for a space filled with an arbitrary matter, which is described by the energy-momentum tensor written in the common form $T_{\alpha\beta}$. In other word, the distributed matter can be the superposition of an electromagnetic field, dust, liquid or other matter. Concerning our problem, we consider only an electromagnetic field. As known [29], the energy-momentum tensor $T_{\alpha\beta}$ of any electromagnetic field should satisfy the condition $T=\rho c^2-U$. We therefore assume that the right side of the Einstein equations contains the energy-momentum tensor of only an electromagnetic field (no dust, liquid, or other matter distributed near the Earth). In other word we should mean, in the right side,

$$\rho c^2 = U\,. \tag{32}$$

Besides, because all measurement in the framework of our problem are processed by an observer on board of a satel-

$$\left.\begin{aligned}
&-2\omega^2-2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_{\varphi}}{r}+2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=-\kappa\rho c^2\\
&\frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right)+\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_{\varphi}}{r^2}-\frac{v_{r\varphi}}{r}\right)\right]=-\kappa J^1\\
&\frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_{\varphi}}{r^3}-\frac{v_{r\varphi}}{r^2}\right)+\left(\cos\varphi-\sin\varphi\right)\frac{v_{rr}}{r}\right]=-\kappa J^2\\
&\frac{1}{2}\left(v_{rr}+\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right)=-\kappa J^3\\
&2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_{\varphi}}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}=\kappa U_{11}\\
&\frac{r^2}{2}\left[\left(\cos\varphi+\sin\varphi\right)\frac{v_{t\varphi}}{r^2}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{tr}}{r}\right]=\kappa U_{12}\\
&\omega\frac{v_{\varphi}}{r}+\frac{1}{2}v_{tr}=\kappa U_{13}\\
&2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_{\varphi}}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=\kappa\frac{U_{22}}{r^2}\\
&\frac{r^2}{2}\left(\frac{v_{t\varphi}}{r^2}-2\omega\frac{v_r}{r}\right)=\kappa U_{23}\\
&\kappa U_{33}=0
\end{aligned}\right\} \qquad (31)$$

lite, we should also take into account the weightlessness condition

$$\frac{GM}{r^2}=\omega^2 r\,. \qquad (33)$$

As a result, we obtain the system of the projected Einstein equations (30) in the form (31) which is specific to the real physical space of a satellite-bound observer.

In other word, that fact that we used the conditions (32) and (33) means that our theoretical calculation targets measurement of an electromagnetic field in the weightlessness state in an orbit of the Earth.

We are looking for the quantity ρ as a function of the properties of the space from the first (scalar) equation of the Einstein equantions (31). This isn't a trivial task, because the aforementioned scalar Einstein equation

$$\begin{aligned}
\kappa\rho c^2 &= 2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_{\varphi}}{r}-\\
&-2\omega\left(\cos\varphi-\sin\varphi\right)v_r-\left(\cos\varphi+\sin\varphi\right)v_{tr}-\\
&-\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}
\end{aligned} \qquad (34)$$

contains the distribution functions of the linear velocity of the space rotation (the functions v_r, v_{φ}, and v_t), which are unknown. We therefore should first find the functions.

According to our asumption, $\rho c^2=U$. Therefore $\kappa\rho c^2$ and κU are the same in the framework of our problem. We calculate the quantity

$$\kappa U=\kappa h^{ik}U_{ik}=\kappa\left(U_{11}+\frac{U_{22}}{r^2}+U_{33}\right) \qquad (35)$$

as the sum of the 5th and the 8th equations of the system of the Einstein equations (31) with taking into account that fact that, in our case, $U_{33}=0$ (as seen from the 10th equation, with $\rho c^2=U$). We obtain

$$\begin{aligned}
\kappa U &= 4\omega^2+4\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_{\varphi}}{r}-\\
&-4\omega\left(\cos\varphi-\sin\varphi\right)v_r+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\\
&+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}\,.
\end{aligned} \qquad (36)$$

Subtracting $\kappa\rho c^2$ (34) from κU (36) then equalizing the result to zero, according to the electromagnetic field condition $\rho c^2=U$, we obtain the geometrization condition for the electromagnetic field

$$\begin{aligned}
&\omega^2+\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_{\varphi}}{r}-\omega\left(\cos\varphi-\sin\varphi\right)v_r+\\
&+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=0\,.
\end{aligned} \qquad (37)$$

With this condition, all the components of the energy-momentum tensor of the field $T_{\alpha\beta}$ (the right side of the Einstein equations) are expressed in only the properties of the space (the left side of the Einstein equations). Hence we have geometrized the electromagnetic field. This is an important result: earlier only isotropic electromagnetic fields (they are satisfying Rainich's condition and Nordtvedt-Pagels condition) were geometrized.

To find the distribution functions of v, we consider the conservation law $\nabla_{\sigma}T^{\alpha\sigma}=0$, expressed in terms of the phys-

$$\left.\begin{aligned}&(\cos\varphi-\sin\varphi)\left(\frac{v_{tr\varphi}}{r}-\frac{v_{t\varphi}}{r^2}\right)+\omega\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_{r\varphi}}{r}-\frac{v_{\varphi}}{r^2}\right)-\\&-\omega\left(\cos\varphi-\sin\varphi\right)v_{rr}+\left(\cos\varphi+\sin\varphi\right)v_{trr}=0\\&(\cos\varphi+\sin\varphi)\left(\frac{v_{tr\varphi}}{r^2}-\frac{v_{t\varphi}}{r^3}\right)+\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_{t\varphi\varphi}}{r^3}+\frac{v_{tr}}{r^2}\right)+\\&+\omega\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_{\varphi\varphi}}{r^3}+\frac{v_r}{r^2}\right)+\omega\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_{\varphi}}{r^3}-\frac{v_{r\varphi}}{r^2}\right)=0\end{aligned}\right\}\qquad(41)$$

ical observed quantities [25–28]

$$\left.\begin{aligned}&\frac{{}^*\partial\rho}{\partial t}+D\rho+\frac{1}{c^2}D_{ij}U^{ij}+\\&\qquad+\left({}^*\nabla_i-\frac{1}{c^2}F_i\right)J^i-\frac{1}{c^2}F_iJ^i=0\\&\frac{{}^*\partial J^k}{\partial t}+2\left(D^k_i+A^{\cdot k}_{i\cdot}\right)J^i+\\&\qquad+\left({}^*\nabla_i-\frac{1}{c^2}F_i\right)U^{ik}-\rho F^k=0\end{aligned}\right\}\qquad(38)$$

which, under the assumptions specific in our problem, is

$$\left.\begin{aligned}&\frac{\partial J^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}J^i=0\\&\frac{\partial J^k}{\partial t}+2A^{\cdot k}_{i\cdot}J^i+\frac{\partial U^{ik}}{\partial x^i}+\Delta^k_{im}U^{im}+\\&\qquad+\frac{\partial\ln\sqrt{h}}{\partial x^i}U^{ik}-\rho F^k=0\end{aligned}\right\}.\qquad(39)$$

The first (scalar) equation of the system of the conservation equations (39) means actually that the chronometrically invariant derivative of the vector J^i is zero

$${}^*\nabla_iJ^i=\frac{\partial J^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}J^i=0\,,\qquad(40)$$

i.e. the flow of the vector J^i (the flow of the density of the field momentum) is constant. So, the first equation of (39) satisfies identically as ${}^*\nabla_iJ^i=0$.

The rest three (vectorial) equations of the system (39), with the properties of the local space of a satellite-bound observer and the components of the energy-momentum tensor substituted (the latest should be taken from the Einstein equations), take the form (41). As seen, only first two equations still remaining meaningful, while the third of the vectorial equations of conservation vanishes becoming the identity zero equals zero.

In other word, we have obtained the equations of the conservation law specific to the real physical space of a satellite-bound observer.

Let's suppose that the function v has the form

$$v=T(t)\,r\,e^{i\varphi},\qquad(42)$$

hence the partial derivatives of this function are

$$\left.\begin{aligned}&v_r=T\,e^{i\varphi} && v_{\varphi}=i\,T\,r\,e^{i\varphi}\\&v_{tr}=\dot{T}\,e^{i\varphi} && v_{t\varphi}=i\,\dot{T}\,r\,e^{i\varphi}\\&v_{rr}=0 && v_{trr}=0\\&v_{tr\varphi}=i\,\dot{T}\,e^{i\varphi} && v_{t\varphi\varphi}=-\dot{T}\,r\,e^{i\varphi}\\&v_{\varphi\varphi}=-T\,r\,e^{i\varphi} && v_{r\varphi}=i\,T\,e^{i\varphi}\end{aligned}\right\}.\qquad(43)$$

After the functions substituted into the equations of the conservation law (41), we see that the equations satisfy identically. Hence $v=T(t)\,r\,e^{i\varphi}$ is exact solution of the conservation equations with respect to v.

Now we need to find only the unknown function $T(t)$. This function can be found from the electromagnetic field condition $\rho c^2=U$ expressed by us through the properties of the space itself as the formula (37).

We assume that the satellite, on board of which the observer is located, displaces at small angle along its orbit during the process of his observation. This is obvious assumption, because the very fast registration process for a single photon. Therefore φ is small value in the framework of our problem. Hence in concern of the formula (37), we should mean $\cos\varphi\simeq 1+\varphi$ and $\sin\varphi\simeq\varphi$. We also take into account only real parts of the function v and its derivatives. (This is due to that fact that a real instrument processes measurement with only real quantities.) Concerning those functions which are contained in the formula (37), all these means that

$$\left.\begin{aligned}&v=T\,r\,(1+\varphi)\\&v_r=T\,(1+\varphi) && \frac{v_{\varphi}}{r}=-T\,\varphi\\&v_{tr}=\dot{T}\,(1+\varphi) && \frac{v_{t\varphi}}{r}=-\dot{T}\,\varphi\end{aligned}\right\}.\qquad(44)$$

Substituting these into (37), we obtain

$$(1+2\varphi)\,\dot{T}-(1+2\varphi)\,\omega T+\omega^2=0\,,\qquad(45)$$

or, because $\varphi=\omega t$ and ω is small value (we also neglect the terms which order is higher than ω^2),

$$\dot{T}-\omega T=-\frac{\omega^2}{1+2\omega t}=-\omega^2\left(1-2\omega t\right)=-\omega^2.\qquad(46)$$

This is a linear differential equation of the first order

$$\dot{y}+f(t)\,y=g(t)\qquad(47)$$

whose exact solution is (see Part I, Chapter I, §4.3 in Erich Kamke's reference book [30])

$$y = e^{-F}\left(y_0 + \int_{t_0=0}^{t} g(t)\, e^{F} dt\right), \tag{48}$$

where

$$F(t) = \int f(t)\, dt\,. \tag{49}$$

We substitute $f = -\omega$ and $g = -\omega^2$. So we obtain, for small values of ω,

$$e^{F} = e^{\int -\omega dt} = e^{-\omega t}, \qquad e^{-F} = e^{\omega t}, \tag{50}$$

hence the function y is

$$y = e^{\omega t}\left(y_0 - \omega^2 \int_{t_0=0}^{t} e^{-\omega t} dt\right) = $$

$$= e^{\omega t}\left[y_0 + \omega\left(e^{-\omega t} - 1\right)\right]. \tag{51}$$

We assume the numerical value of the function $y = T(t)$ to be zero at the initial moment of observation: $y_0 = T_0 = 0$. As a result we obtain the solution for the function $T(t)$:

$$T = \omega\left(1 - e^{\omega t}\right). \tag{52}$$

Applying this solution, we can find a final formula for the density of the energy of the Earth's microwave background $W = \rho c^2$ observed by a satellite-bound observer.

First, we substitute the distribution functions of v (44) into the initially formula for ρc^2 (34) which is originated from the scalar Einstein equation. Assuming $\cos\varphi \simeq 1 + \varphi$ and $\sin\varphi \simeq \varphi$, we obtain

$$\kappa\rho c^2 = -2\omega^2 - 2\omega T(1 + 2\varphi) - (1 + 2\varphi)\,\dot{T}\,. \tag{53}$$

Then we do the same substitution into the geometrization condition (37) which is originated from the Einstein equations, and is necessary to be applied to our case due to that fact that we have only an electromagnetic field distributed in the space ($\rho c^2 = U$ in the right side of the Einstein equations, as for any electromagnetic field). After algebra the geometrization condition (37) takes the form

$$\omega^2 - \omega T\,(1 + 2\varphi) + (1 + 2\varphi)\,\dot{T} = 0\,. \tag{54}$$

We express $(1 + 2\varphi)\,\dot{T} = \omega T\,(1 + 2\varphi) - 4\omega^2$ from this formula, then substitute it into the previous expression (53) with taking into accout that fact that the angle φ is a small value. As a result, we obtain

$$\rho c^2 = \frac{3\omega}{\kappa}\,(\omega - T) = \frac{3\omega^2}{\kappa}\left[1 - \left(1 - e^{\omega t}\right)\right]. \tag{55}$$

Expanding the exponent into the series $e^{\omega t} = 1 + \omega t + \frac{1}{2}\,\omega^2 t^2 + \ldots \simeq 1 + \omega t$ and taking into account that fact that ω is small value*, we arrive to the final formula for calculation the density of the energy of the Earth's microwave background observed on board of a satellite

$$\rho c^2 = \frac{3\omega^2}{\kappa}\,, \tag{56}$$

which is obviously dependent on altitude from the surface of the Earth due to that fact that $\omega = \sqrt{GM_{\oplus}/R^3}$.

With this final formula (55), we calculate the ratio between the density of the Earth's microwave background expected to be measured at different altitudes from the surface of the Earth. According to this formula, the ratio between the density at the altitude of the COBE orbit ($R_{\rm COBE} = 6{,}370 + 900 = 7{,}270$ km) and that at the altitude of a U2 aeroplane ($R_{\rm U2} = 6{,}370 + 25 = 6{,}395$ km) should be

$$\frac{\rho_{\rm COBE}}{\rho_{\rm U2}} = \frac{R^3_{\rm U2}}{R^3_{\rm COBE}} \simeq 0.68\,, \tag{57}$$

the ratio between the density at the 2nd Lagrange point ($R_{\rm L2} = 1.5$ million km) and that at the COBE orbit should be

$$\frac{\rho_{\rm L2}}{\rho_{\rm COBE}} = \frac{R^3_{\rm COBE}}{R^3_{\rm L2}} \simeq 1.1\times10^{-7}, \tag{58}$$

and the ratio between the density at the 2nd Lagrange point and that at the altitude of a U2 aeroplane should be

$$\frac{\rho_{\rm L2}}{\rho_{\rm U2}} = \frac{R^3_{\rm U2}}{R^3_{\rm L2}} \simeq 7.8\times10^{-8}. \tag{59}$$

As a result of our calculation, processed on the basis of the General Theory of Relativity, we see that a microwave background field which originates in the Earth (the Earth microwave background) should have almost the same density at the position of a U2 aeroplane and the COBE satellite. However, at the 2nd Lagrange point (1.5 million km from the Earth, the point of location of the WMAP satellite and the planned PLANCK satellite), the density of the background should be only $\sim 10^{-7}$ of that registered either by the U2 aeroplane or by the COBE satellite.

4 The anisotropy of the Earth's microwave background at the 2nd Lagrange point

We consider the anisotropy of the Earth's microwave background which is due to the rapid motion of the source of this field (the Earth) in a weak intergalactic microwave field†. From views of physics this means that photons, the mediators for electromagnetic radiation, being radiated by the source of the field (the Earth) should experience a carrying in the direc-

*The quantity $\omega = \sqrt{GM_{\oplus}/R^3}$, the frequency of the rotation of the Earth space for an observer existing in the weightless state, takes its maximum numerical value at the equator of the Earth's surface, where $\omega = 1.24\times10^{-3}$ sec^{-1}, and decreases with altitude above the surface.

†As observatonal analysis indicates it, the Earth moves in the weak intergalactic field with a velocity of $\mathrm{v} = 365\pm18$ km/sec in the direction of the anisotropy.

tion whereto the Earth flies in the weak intergalactic field. From mathematical viewpoint this problem can be formulated as a shift of the trajectories experienced by photons of the Earth's microwave field in the direction of this motion.

A light-like free particle, e.g. a free photon, moves along isotropic geodesic trajectories whose four-dimensional (general covariant) equations are [25–28]

$$\frac{dK^\alpha}{d\sigma} + \Gamma^\alpha_{\mu\nu} K^\mu \frac{dx^\nu}{d\sigma} = 0\,, \tag{60}$$

where $K^\alpha = \frac{\Omega}{c}\frac{dx^\alpha}{d\sigma}$ is the four-dimensional wave vector of the photon (the vector satisfies the condition $K_\alpha K^\alpha = 0$ which is specific to any isotropic vector), Ω is the proper cyclic frequency of the photon, while $d\sigma$ is the three-dimensional chronometrically invariant (observable) spatial interval determined as $d\sigma^2 = \left(-g_{ik} + \frac{g_{0i}\,g_{0k}}{g_{00}}\right) dx^i dx^k = h_{ik}\, dx^i dx^k$. The quantity $d\sigma$ is chosen as a parameter of differentiation along isotropic geodesics, because along them the four-dimensional interval is zero $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$ while $d\sigma = c d\tau \neq 0$ (where $d\tau$ is the interval of the physical observable time determined as $d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\, dx^i$).

In terms of the physical observables, the isotropic geodesic equations are represented by their projections on the time line and spatial section of an observer [25–28]

$$\left.\begin{aligned} &\frac{d\Omega}{d\tau} - \frac{\Omega}{c^2} F_i c^i + \frac{\Omega}{c^2} D_{ik} c^i c^k = 0 \\ &\frac{d}{d\tau}\left(\Omega c^i\right) + 2\Omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right) c^k - \\ &\qquad - \Omega F^i + \Omega \Delta^i_{kn} c^k c^n = 0 \end{aligned}\right\} \tag{61}$$

where $c^i = \frac{dx^i}{d\tau}$ is the three-dimensional vector of the observable velocity of light (the square of the vector satisfies $c_k c^k = h_{ik}\, c^i c^k = c^2$ in the spatial section of the observer). The first of the equations (the scalar equation) represents the law of energy for the particle, while the vectorial equation is the three-dimensional equation of its motion.

The terms $\frac{F_i}{c^2}$ and $\frac{D_{ik}}{c^2}$ are negligible in the framework of our assumption. We obtain, from the scalar equation of (61), that the proper frequency of the photons, registered by the observer, is constant. In such a case the vectorial equations of isotropic geodesics (61), written in component notation, are

$$\left.\begin{aligned} &\frac{dc^1}{d\tau} + 2\left(D^1_k + A^{\cdot 1}_{k\cdot}\right) c^k - F^1 + \Delta^1_{22}\, c^2 c^2 + \\ &\qquad + 2\Delta^1_{23}\, c^2 c^3 + \Delta^1_{33}\, c^3 c^3 = 0 \\ &\frac{dc^2}{d\tau} + 2\left(D^2_k + A^{\cdot 2}_{k\cdot}\right) c^k - F^2 + 2\Delta^2_{12}\, c^1 c^2 + \\ &\qquad + 2\Delta^2_{13}\, c^1 c^3 + \Delta^2_{33}\, c^3 c^3 = 0 \\ &\frac{dc^3}{d\tau} + 2\left(D^3_k + A^{\cdot 3}_{k\cdot}\right) c^k - F^3 + \Delta^3_{11}\, c^1 c^1 + \\ &\qquad + 2\Delta^3_{12}\, c^1 c^2 + 2\Delta^3_{13}\, c^1 c^3 + \\ &\qquad + \Delta^3_{22}\, c^2 c^2 + 2\Delta^3_{23}\, c^2 c^3 = 0 \end{aligned}\right\}, \tag{62}$$

where $c^1 = \frac{dr}{d\tau}$, $c^2 = \frac{d\varphi}{d\tau}$, and $c^3 = \frac{dz}{d\tau}$, while $\frac{d}{d\tau} = \frac{^*\partial}{\partial t} + \mathrm{v}^i \frac{^*\partial}{\partial x^i}$.

We direct the z-axis of our cylindrical coordinates along the motion of the Earth in the weak intergalactic field. In such a case the local physical space of a satellite-bound observer is described by the metric (8). We therefore will solve the isotropic geodesic equations in the metric (8).

The metric (7) we used in the first part of the problem is a particular to the metric (8) in a case, where $\mathrm{v} = 0$. Therefore the solution $v = T(t)\, r\, e^{i\varphi}$ (42) we have obtained for the metric (7) is also lawful for the generatized metric (8). We therefore calculate the observable characteristics of the space with taking this function into account. As earlier, we take into account only real part of the function $e^{i\varphi} = \cos\varphi + i\sin\varphi \simeq \simeq (1+\varphi) + i\,\varphi$. We also take into account the derivatives of this function (43) and the function $T = \omega\,(1 - e^{\omega t})$ we have found earlier (52).

As well as in the first part of the problem, we assume φ to be small value so that $\cos\varphi \simeq 1 + \varphi$ and $\sin\varphi \simeq \varphi$. Because ω is small value too, we neglect $\omega^2\varphi$ terms. Aside for these, we take the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ into account in calculation for the gravitational inertial force. It should be noted that the weightlessness condition is derived from the derivative of the gravitational potential $\mathrm{w} = c^2\,(1 - \sqrt{g_{00}})$. We therefore cannot mere substitute the weightlessness condition into $g_{00} = 1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}$ taken from the metric (8). We should first calculate $\mathrm{w} = c^2\,(1 - \sqrt{g_{00}})$, then take derivative of it by the respective coordinate that is required in the formula for the gravitational inertial force F_i (12). Only then the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ is lawful to be substituted.

Besides these, we should take into account that fact that the anisotropy of a field is a second order effect. We therefore cannot neglect the terms divided by c^2. This is in contrast to the first part of the problem, where we concerned only a first order effect. As a result the space deformation and the three-dimensional curvature, neglected in the first part, now cannot be neglected. We however take into account only the space deformation D_{ik}. The three-dimensional curvature C_{iklj} isn't considered here due to the fact that this quantity isn't contained in the equations of motion.

In the same time, in the framework of our assumption for a weak gravitational field and a low speed of the space rotation, $\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t} \simeq \frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{^*\partial}{\partial t} \simeq \frac{\partial}{\partial x^i}$.

Applying all these conditions to the definitions of v_i, h_{ik}, F_i, A_{ik}, D_{ik}, and Δ^i_{km}, given in Page 7, we obtain substantially non-zero components of the characteristics of the space whose metric is (8):

$$\mathrm{w} = \frac{GM}{r} + \frac{\omega^2 r^2}{2} - v\mathrm{v}\,, \tag{63}$$

$$\left.\begin{aligned} &v_1 = \omega^2 t r \\ &v_2 = \omega r^2 (\omega t + 1) \\ &v_3 = \omega^2 t r \end{aligned}\right\}, \tag{64}$$

$$\left.\begin{aligned} &\ddot{r} - \omega^2\left(t - \frac{r\,\mathrm{v}}{c^2}\right)\dot{z} + \omega^2\left(r - \mathrm{v}t\right) + \frac{\omega^2\mathrm{v}\,t}{c^2}\,\dot{z}^2 = 0 \\ &\ddot{\varphi} + 2\omega\left(1 + \frac{\omega t}{2}\right)\frac{\dot{r}}{r} + \frac{\omega^2\mathrm{v}}{c^2}\,\dot{z} + \omega^2 + \frac{2\omega\mathrm{v}\left(1 + \frac{\omega t}{2}\right)}{c^2 r}\,\dot{r}\dot{z} = 0 \\ &\ddot{z} + \omega^2\left(t + \frac{r\,\mathrm{v}}{c^2}\right)\dot{r} + \frac{2\omega^2\mathrm{v}\,r}{c^2}\,\dot{z} + \omega^2 r + \frac{\omega^2\mathrm{v}\,t}{c^2}\,\dot{r}^2 + \frac{2\omega^2\mathrm{v}\,t}{c^2}\,\dot{r}\dot{z} = 0 \end{aligned}\right\} \tag{70}$$

$$\dot{r}^2 + \frac{2\omega^2 r\,\mathrm{v}\,t}{c^2}\,\dot{r}\dot{z} + \left(1 - \frac{2\omega^2 r\,\mathrm{v}\,t}{c^2}\right)\dot{z}^2 = c^2 \tag{71}$$

$$\left.\begin{aligned} F_1 &= -\omega^2\left(r - \mathrm{v}t\right) \\ F_2 &= -\omega^2 r^2 \\ F_3 &= -\omega^2 r \end{aligned}\right\}, \tag{65}$$

$$\left.\begin{aligned} A_{12} &= \omega\, r\left(1 + \frac{\omega t}{2}\right) \\ A_{23} &= 0 \\ A_{13} &= \frac{\omega^2 t}{2} \end{aligned}\right\}, \tag{66}$$

$$\left.\begin{aligned} &h_{11} = 1\,, \qquad h_{13} = \frac{\omega^2\mathrm{v}\,t\,r}{c^2} \\ &h_{22} = r^2, \qquad h_{23} = \frac{\omega\, r^2\mathrm{v}\left(1 + \omega t\right)}{c^2} \\ &h_{33} = 1 - \frac{2\omega^2\mathrm{v}\,t\,r}{c^2} \\ &h^{11} = 1\,, \qquad h^{13} = -\frac{\omega^2\mathrm{v}\,t\,r}{c^2} \\ &h^{22} = \frac{1}{r^2}\,, \qquad h^{23} = -\frac{\omega\mathrm{v}\left(1 + \omega t\right)}{c^2} \\ &h^{33} = 1 + \frac{2\omega^2\mathrm{v}\,t\,r}{c^2} \\ &h = r^2\left(1 + \frac{2\omega^2\mathrm{v}\,t\,r}{c^2}\right) \end{aligned}\right\}, \tag{67}$$

$$\left.\begin{aligned} &D_{13} = \frac{\omega^2 r\,\mathrm{v}}{2\,c^2}\,, \qquad D_{23} = \frac{\omega^2 r^2\mathrm{v}}{2\,c^2} \\ &D_{33} = \frac{\omega^2 r\,\mathrm{v}}{c^2}\,, \qquad D = \frac{\omega^2 r\,\mathrm{v}}{c^2} \end{aligned}\right\}, \tag{68}$$

$$\left.\begin{aligned} &\Delta^1_{22} = -r\,, \qquad \Delta^1_{23} = -\frac{\omega\, r\,\mathrm{v}}{c^2}\left(1 + \frac{\omega t}{2}\right) \\ &\Delta^1_{33} = \frac{\omega^2\mathrm{v}\,t}{c^2}\,, \qquad \Delta^2_{12} = \frac{1}{r} \\ &\Delta^2_{13} = \frac{\omega\mathrm{v}}{c^2 r}\left(1 + \frac{\omega t}{2}\right), \quad \Delta^3_{11} = \frac{\omega^2\mathrm{v}\,t}{c^2} \\ &\Delta^3_{12} = \frac{\omega^2 r\,\mathrm{v}\,t}{2c^2}\,, \qquad \Delta^3_{13} = -\frac{\omega^2\mathrm{v}\,t}{c^2} \end{aligned}\right\}, \tag{69}$$

where we present only those components of Christoffel's symbols which will be used in the geodesic equations (equations of of motion).

After substitution of the components, the vectorial equations of isotropic geodesic (62) take the form (70). The condition $h_{ik}\,c^i c^k = c^2$ — a chronometrically invariant expression of the condition $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$, which is specific to isotropic trajectories — takes the form (71).

We consider a light beam (a couple of photons) travelling from the Earth along the radial direction r. Therefore, looking for anisotropy in the distribution of the photons' trajectories in the field, we are interested to solve only the third isotropic geodesic equation of (70), which is the equation of motion of a photon along the z-axis orthogonal to the light beam's direction r.

Before to solve the equation, a few notes on our assumptions should be made.

First, because the Earth moves relative to the weak microwave background with a velocity v^{i} along the z-direction, only $\mathrm{v}^3 = \dot{z}$ of the components v^{i} is non-zero. Besides that, as easy to see from our previous considerations, we should mean $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t} \simeq \frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial z} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}v_3\frac{{}^*\partial}{\partial t} \simeq \frac{\partial}{\partial z} = 0$. Hence, we apply $\frac{d}{d\tau} = \frac{{}^*\partial}{\partial t} + \mathrm{v}^3\frac{{}^*\partial}{\partial z} = \frac{d}{\partial t}$ to our calculation.

Second, the orbital velocity of a satellite of the Earth, ∼8 km/sec, is much lesser than the velocity of light. We therefore assume that a light beam doesn't sense the orbital motion of such a satellite. The coordinate φ in the equations of isotropic geodesics is related to the light beam (a couple of single photons), not the rotation of the reference space of a satellite bound observer. Hence, we assume $\varphi = \mathrm{const}$ in our calculation, i.e. $c^2 = \frac{d\varphi}{dt} = \dot{\varphi} = 0$.

Third, we are talking about the counting for signle photons in a detector which is located on board of a satellite. The process of the measurement is actually instant. In other word, the measurement is processed very close the moment $t_0 = 0$. Hence we assume $\dot{z} = 0$ in our calculation, while the acceleration $\ddot{z}$ can be non-zero in the z-direction orthogonally to the initially r-direction of such a photon.

Fourth, we apply the relations $\dot{r} = c$ and $r = ct$ which are obvious for such a photon. If such a photon, travelling initially in the r-direction, experiences a shift to the z-direction (the direction of the motion of the Earth relative to the weak intergalactic field), the distribution of photons of the Earth's microwave field has an anisotropy to the z-direction.

After taking all the factors into account, the third equation of the system (70), which is the equation of motion of a single photon in the z-direction, takes the simple form

$$\ddot{z} + \omega^2 \left(ct + \frac{r\,\mathrm{v}}{c} \right) + \omega^2 \left(r + \mathrm{v}t \right) = 0 \tag{72}$$

which, due to the weightlessness condition $\frac{GM}{r^2} = \omega^2 r$ and the condition $r = ct$, is

$$\ddot{z} + \frac{2GM_{\oplus}}{c^2 t^2} \left(1 + \frac{\mathrm{v}}{c} \right) = 0\,. \tag{73}$$

Integration of this equation gives

$$\dot{z} = \frac{2GM_{\oplus}}{cr} \left(1 + \frac{\mathrm{v}}{c} \right) = \dot{z}' + \Delta z'. \tag{74}$$

The first term of the solution (74) manifests that fact that such a photon, initially launched in the r-direction (radial direction) in the gravitational field of the Earth, is carried into the z-direction by the rotation of the space of the Earth. The second term, $\Delta z'$, manifests the carriage of the photon into the z-direction due to the motion of the Earth in this direction through the weak intergalactic field.

As a result we obtain the carriage of the three-dimensional vector of the observable velocity of light from the initially r-direction to the z-direction, due to the common motion of the space of the Earth in the point of observation:

$$\frac{\Delta \dot{z}'}{\dot{z}'} = \frac{\mathrm{v}}{c}\,. \tag{75}$$

Such a carriage of a photon radiated from the Earth's surface, being applied to a microwave background generated by oceanic water, reveals the anisotropy associated with the dipole component of the microwave background.

As seen from the obtained formula (75), such a carriage of a photon into the z-direction, doesn't depend on the path travelled by such a photon in the radial direction r from the Earth. In other word, the anisotropy associated with the dipole component of the Earth microwave background shouldn't be dependent on altitude from the surface of the Earth: the anisotropy of the Earth microwave background should be the same if measured on board a U2 aeroplane (25 km), at the orbit of the COBE satellite (900 km), and at the 2nd Langrange point (the WMAP satellite and PLANCK satellite, 1.5 million km from the Earth).

Acknowledgement

We are very grateful to Pierre-Marie Robitaille for consultations and valuable comments.

Submitted on January 04, 2008 / Accepted on January 07, 2008
First published online on January 07, 2008 / Corrected on February 11, 2008

References

1. Borissova L. and Rabounski D. On the nature of the microwave background at the Lagrange 2 Point. Part II. *Progress in Physics*, 2007, v. 4, 84–95.
2. Penzias A. A. and Wilson R. W. A measurement of excess antenna temperature at 4080 Mc/s. *Astrophys. J.*, 1965, v. 1, 419–421.
3. Robitaille P-M.L. Nuclear magnetic resonance and the age of the Universe. *Bulletin of the American Physical Society*, APS Centennial Meeting, Atlanta, Georgia, March 20–26, 1999, Section BC19.14, http://flux.aps.org/meetings/YR99/CENT99/abs/S560014.html
4. Robitaille P-M.L. The MAP satellite: a powerful lesson in thermal physics. *Bulletin of the American Physical Society*, APS Northwest Section Spring Meeting 2001, Seattle, Washington, May 25–26, 2001, Section F4.004, http://flux.aps.org/meetings/YR01/NWS01/abs/S120004.html
5. Robitaille P.-M. WMAP: a radiological analysis. *Progress in Physics*, 2007, v. 1, 3–18.
6. Robitaille P.-M. On the origins of the CMB: insight from the COBE, WMAP and Relikt-1 satellites. *Progress in Physics*, 2007, v. 1, 19–23.
7. Robitaille P.-M. On the Earth Microwave Background: absorption and scattering by the atmosphere. *Progress in Physics*, 2007, v. 3, 3–4.
8. Robitaille P.-M. On the nature of the microwave background at the Lagrange 2 Point. Part I. *Progress in Physics*, 2007, v. 4, 74–83.
9. Robitaille P.-M. and Rabounski D. COBE and the absolute assignment of the CMB to the Earth. *Bulletin of the American Physical Society*, 2007, v. 52, no. 1, 2007 APS March Meeting, Denver, Colorado, March 5–9, 2007, http://meetings.aps.org/link/BAPS.2007.MAR.L20.7
10. Robitaille P.-M., Borissova L., and Rabounski D. On the microwave signal at the second Lagrange point. *Bulletin of the American Physical Society*, 2007, v. 52, no. 13, 74th APS Annual Meeting of the Southeastern Section, Nashville, Tennessee, November 8–10, 2007, http://meetings.aps.org/Meeting/SES07/Event/73969
11. Conklin E. K. Velocity of the Earth with respect to the Cosmic Background Radiation. *Nature*, 1969, v. 222, 971.
12. Henry P. S. Isotropy of the 3 K Background. *Nature*, 1971, v. 231, 516–518.
13. Corey B. E. and Wilkinson D. T. A measurement of the Cosmic Microwave Background Anisotropy at 19 GHz. *Bulletin of the American Astronomical Society*, 1976, v. 8, 351.
14. Smoot G. F., Gorenstein M. V. and Muller R. A. Detection of anisotropy in the Cosmic Blackbody Radiation. *Phys. Rev. Lett.*, 1977, v. 39, 898–901.
15. Lineweaver C.H. The CMB Dipole: The most recent measurement and some history. In: *Microwave Background Anistropies. Proceedings of the XVIth Moriond Astrophysics Meeting*, Les Arcs, Savoie, France, March 16–23, 1996, Gif-sur-Yvette: Editions Frontieres, 1997; (see also arXiv: astro-ph/9609034).

16. Smoot G.F., et al. Preliminary results from the COBE differential microwave interferometers: large angular scale isotropy of the Cosmic Microwave Background. *Astrophys. J.*, 1991, v. 371, L1–L5.
17. Boggess N.W., et al. The COBE mission: its design and perfomance two years after launch. *Astrophys. J.*, 1992, v. 397, 420–429.
18. Page L., et al. The optical design and characterization of the Microwave Anisotropy Probe. *Astrophys. J.*, 2003, v. 585, 566–586.
19. Bennett C.L., et al. The Microwave Anisotropy Probe mission. *Astrophys. J.*, 2003, v. 583(1), 1–23.
20. Bennett C.L., et al. First-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: preliminary maps and basic results. *Astrophys. J. Suppl. Ser.*, 2003, v. 148(1), 1–27.
21. Rabounski D. The relativistic effect of the deviation between the CMB temperatures obtained by the COBE satellite. *Progress in Physics*, 2007, v. 1, 19–21.
22. Borissova L. Preferred spatial directions in the Universe: a General Relativity approach. *Progress in Physics*, 2006, v. 4, 51–58.
23. Borissova L. Preferred spatial directions in the Universe. Part II. Matter distributed along orbital trajectories, and energy produced from it. *Progress in Physics*, 2006, v. 4, 59–64.
24. Schouten J. A. und Struik D. J. Einführung in die neuren Methoden der Differentialgeometrie. Noordhoff, Groningen, 1938.
25. Zelmanov A. L. Chronometric invariants: on deformations and the curvature of the accompanying space. Dissertation thesis, 1944. American Research Press, Rehoboth (NM), 2006, 236 pages.
26. Zelmanov A. L. Chronometric invariants and accompanying frames of reference in the General Theory of Relativity. *Soviet Physics Doklady*, MAIK Nauka/Interperiodica (distributed by Springer), 1956, v. 1, 227–230 (translated from *Doklady Akademii Nauk URSS*, 1956, v. 107, no. 6, 815–818).
27. Zelmanov A. L. and Agakov V. G. Elements of the General Theory of Relativity. Nauka, Moscow, 1988, 236 pages.
28. Zelmanov A. Zelmanov's lectures on General Relativity. American Research Press, Rehoboth (NM), 2008 (*in print*).
29. Landau L. D. and Lifshitz E. M. The classical theory of fields. 4th edition, Butterworth–Heinemann, 1980, 428 pages.
30. Kamke E. Differentialgleichungen: Lösungsmethoden und Lösungen. Chelsea Publishing Co., New York, 1959.