Derivation of Maxwell's Equations Based on a Continuum Mechanical Model of Vacuum and a Singularity Model of Electric Charges

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The main purpose of this paper is to seek a mechanical interpretation of electromagnetic phenomena. We suppose that vacuum is filled with a kind of continuously distributed material which may be called $\Omega(1)$ substratum. Further, we speculate that the $\Omega(1)$ substratum might behave like a fluid with respect to translational motion of large bodies through it, but would still posses elasticity to produce small transverse vibrations. Thus, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum. Furthermore, we speculate that electric charges are emitting or absorbing the $\Omega(1)$ substratum continuously and establish a fluidic source and sink model of electric charges. Thus, Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this mechanical model of vacuum and the singularity model of electric charges.

1 Introduction

Maxwell's equations in vacuum can be written as [1]

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{3}$$

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{4}$$

where $\mathbf{E}$ is the electric field vector, $\mathbf{B}$ is the magnetic induction vector, ρ_e is the density field of electric charges, $\mathbf{j}$ is the electric current density, ϵ_0 is the dielectric constant of vacuum, μ_0 is magnetic permeability of vacuum, t is time, $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$ is the Hamilton operator.

The main purpose of this paper is to derive the aforementioned Maxwell equations in vacuum based on a continuum mechanics model of vacuum and a singularity model of electric charges.

The motivation for this paper was looking for a mechanism of electromagnetic phenomena. The reasons why new mechanical models of electromagnetic fields are interesting may be summarized as follows.

First, there exists various electromagnetic phenomena which could not be interpreted by the present theories of electromagnetic fields, e.g., the spin of an electron [1, 2], the Aharonov-Bohm effect [3, 4], etc. New theories of of electromagnetic phenomena may consider these problems from new sides.

Second, there exists some inconsistencies and inner difficulties in Classical Electrodynamics, e.g., the inadequacy of the Liéenard-Wiechert potentials [5–7]. New theories of electromagnetic phenomena may overcome such difficulties.

Third, there exists some divergence problems in Quantum Electrodynamics [8]. By Dirac's words, "I must say that I am very dissatisfied with the situation, because this so-called good theory does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics". New theories of electromagnetic phenomena may open new ways to resolve such problems.

Fourth, since the quantum theory shows that vacuum is not empty and produces physical effects, e.g., the Casimir effect [9–12], it is valuable to reexamine the old concept of electromagnetic aether.

Fifth, from the viewpoint of reductionism, Maxwell's theory of electromagnetic fields can only be regarded as a phenomenological theory. Although Maxwell's theory is a field theory, the field concept is different from that of continuum mechenics [13–16] due to the absence of a medium. Thus, from the viewpoint of reductionism, the mechanism of electromagnetic phenomena is still remaining an unsolved problem of physics [17].

Sixth, one of the puzzles of physics is the problem of dark matter and dark energy (refer to, for instance, [18–26]). New theories of electromagnetic phenomena may provide new ideas to attack this problem.

Finally, one of the tasks of physics is the unification of the four fundamental interactions in the Universe. New theories of electromagnetic phenomena may shed some light on this puzzle.

To conclude, it seems that new considerations for electromagnetic phenomena is needed. It is worthy keeping an open mind with respect to all the theories of electromagnetic phenomena before the above problems been solved.

Now let us briefly review the long history of the mechanical interpretations of electromagnetic phenomena.

According to E. T. Whittaker [17], Descartes was the first person who brought the concept of aether into science by suggested mechanical properties to it. Descartes believed that every physical phenomenon could be interpreted in the framework of a mechanical model of the Universe. William Watson and Benjamin Franklin (independently) constructed the one-fluid theory of electricity in 1746 [17]. H. Cavendish attempted to explain some of the principal phenomena of electricity by means of an elastic fluid in 1771 [17]. Not contented with the above mentioned one-fluid theory of electricity, du Fay, Robert Symmer and C. A. Coulomb developed a two-fluid theory of electricity from 1733 to 1789 [17].

Before the unification of both electromagnetic and light phenomena by Maxwell in 1860's, light phenomena were independent studied on the basis of Descartes' views for the mechanical origin of Nature. John Bernoulli introduced a fluidic aether theory of light in 1752 [17]. Euler believed in an idea that all electrical phenomena are caused by the same aether that moves light. Furthermore, Euler attempted to explain gravity in terms of his single fluidic aether [17].

In 1821, in order to explain polarisation of light, A. J. Frensnel proposed an aether model which is able to transmit transverse waves. After the advent of Frensnel's successful transverse wave theory of light, the imponderable fluid theories were abandoned. In the 19th century, Frensnel's dynamical theory of a luminiferous aether had an important influence on the mechanical theories of Nature [17]. Inspired by Frensnel's luminiferous aether theory, numerous dynamical theories of elastic solid aether were established by Stokes, Cauchy, Green, MacCullagh, Boussinesq, Riemann and William Thomson. (See, for instance, [17]).

Thomson's analogies between electrical phenomena and elasticity helped to James Clark Maxwell to establish a mechanical model of electrical phenomena [17]. Strongly impressed by Faraday's theory of lines of forces, Maxwell compared the Faraday lines of forces with the lines of flow of a fluid. In 1861, in order to obtain a mechanical interpretation of electromagnetic phenomena, Maxwell established a mechanical model of a magneto-electric medium. The Maxwell magneto-electric medium is a cellular aether, looks like a honeycomb. Each cell of the aether consists of a molecular vortex surrounded by a layer of idle-wheel particles. In a remarkable paper published in 1864, Maxwell established a group of equations, which were named after his name later, to describe the electromagnetic phenomena.

In 1878, G. F. FitzGerald compared the magnetic force with the velocity in a quasi-elastic solid of the type first suggested by MacCullagh [17]. FitzGerald's mechanical model of such an electromagnetic aether was studied by A. Sommerfeld, by R. Reiff and by Sir J. Larmor later [17].

Because of some dissatisfactions with the mechanical models of an electromagnetic aether and the success of the theory of electromagnetic fields, the mechanical world-view was removed with the electromagnetic world-view gradually. Therefore, the concepts of a luminiferous aether and an elastic solid aether were removed with the concepts of an electromagnetic aether or an electromagnetic field. This paradigm shift in scientific research was attributed to many scientists, including Faraday, Maxwell, Sir J. Larmor, H. A. Lorentz, J. J. Thomson, H. R. Hertz, Ludwig Lorenz, Emil Wiechert, Paul Drude, Wilhelm Wien, etc. (See, for instance, [17].)

In a remarkable paper published in 1905, Einstein abandoned the concept of aether [27]. However, Einstein's assertion did not cease the exploration of aether (refer to, for instance, [17,28–37,68,69]). Einstein changed his attitude later and introduced his new concept of aether [38, 39]. In 1979, A. A. Golebiewska-Lasta observed the similarity between the electromagnetic field and the linear dislocation field [28]. V. P. Dmitriyev have studied the similarity between the electromagnetism and linear elasticity since 1992 [32, 35, 37, 40]. In 1998, H. Marmanis established a new theory of turbulence based on the analogy between electromagnetism and turbulent hydrodynamics [34]. In 1998, D. J. Larson derived Maxwell's equations from a simple two-component solid-mechanical aether [33]. In 2001, D. Zareski gave an elastic interpretation of electrodynamics [36]. I regret to admit that it is impossible for me to mention all the works related to this field of history.

A. Martin and R. Keys [41–43] proposed a fluidic cosmonic gas model of vacuum in order to explain the physical phenomena such as electromagnetism, gravitation, Quantum Mechanics and the structure of elementary particles.

Inspired by the above mentioned works, we show that Maxwell's equations of electromagnetic field can be derived based on a continuum mechanics model of vacuum and a singularity model of electric charges.

2 Clues obtained from dimensional analysis

According to Descartes' scientific research program, which is based on his views about the mechanical origin of Nature, electromagnetic phenomena must be (and can be) interpreted on the basis of the mechanical motions of the particles of aether.

Therefore, all the physical quantities appearing in the theory of electromagnetic field should be mechanical quantities.

Thus, in order to construct a successful mechanical model of electromagnetic fields, we should undertake a careful dimensional analysis (refer to, for instance, [44]) for physical quantities in the theory of electromagnetism (for instance, electric field vector $\mathbf{E}$, magnetic induction vector $\mathbf{B}$, the density field of electric charges ρ_e, the dielectric constant of vacuum ϵ_0, the magnetic permeability of vacuum μ_0, etc.).

It is known that Maxwell's equations (1-4) in vacuum can also be expressed as [1]

$$\nabla^2\phi + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{A}) = -\frac{\rho_e}{\epsilon_0}\,, \qquad (5)$$

$$\nabla^2\mathbf{A} - \nabla(\nabla\cdot\mathbf{A}) - \mu_0\epsilon_0\frac{\partial}{\partial t}\left(\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}\right) = -\mathbf{j}\,, \qquad (6)$$

where ϕ is the scalar electromagnetic potential, **A** is the vector electromagnetic potential, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

In 1846, W. Thomson compared electric phenomena with elasticity. He pointed out that the elastic displacement **u** of an incompressible elastic solid is a possible analogy to the vector electromagnetic potential **A** [17].

Noticing the similarity between the Eq. (6) and the equation (39) of momentum conservation of elastic solids, it is natural to judge that vacuum is filled with a kind of elastic substratum. Further, we may say that the dimension of the electromagnetic vector potential **A** of such an elastic substratum is the same that of the displacement vector **u** of an elastic solid. Thus, the dimension of the vector electromagnetic potential **A** of the elastic substratum is $[L^0 M^0 T^0]$, where L, M and T stands for the dimensions of length, mass, and time, respectively. Therefore, we can determine the dimensions of the rest physical quantities of the theory of electromagnetism, for instance, the electric field vector **E**, the magnetic induction vector **B**, the electric charge q_e, the dielectric constant of vacuum ϵ_0, the magnetic permeability of vacuum μ_0, etc. For instance, the dimension of an electric charge q_e should be $[L^0 M^1 T^{-1}]$.

Inspired by this clue, we are going to produce, in the next Sections, an investigation in this direction.

3 A visco-elastic continuum model of vacuum

The purpose of this Section is to establish a visco-elastic continuum mechanical model of vacuum.

In 1845–1862, Stokes suggested that aether might behave like a glue-water jelly [45–47]. He believed that such an aether would act like a fluid on the transit motion of large bodies through it, but would still possessing elasticity to produce a small transverse vibration.

Following Stokes, we propose a visco-elastic continuum model of vacuum.

Assumption 1 *Suppose that vacuum is filled with a kind of continuously distributed material.*

In order to distinguish this material with other substratums, we may call this material as $\Omega(1)$ substratum, for convenience. Further, we may call the particles that constitute the $\Omega(1)$ substratum as $\Omega(1)$ particles (for convenience).

In order to construct a continuum mechanical theory of the $\Omega(1)$ substratum, we should take some assumptions based on the experimental data about the macroscopic behavior of vacuum.

Assumption 2 *We suppose that all the mechanical quantities of the $\Omega(1)$ substratum under consideration, such as the density, displacements, strains, stresses, etc., are piecewise continuous functions of space and time. Furthermore, we suppose that the material points of the $\Omega(1)$ substratum remain be in one-to-one correspondence with the material points before a deformation appears.*

Assumption 3 *We suppose that the material of the $\Omega(1)$ substratum under consideration is homogeneous, that is $\frac{\partial\rho}{\partial x} = \frac{\partial\rho}{\partial y} = \frac{\partial\rho}{\partial z} = \frac{\partial\rho}{\partial t} = 0$, where ρ is the density of the $\Omega(1)$ substratum.*

Assumption 4 *Suppose that the deformation processes of the $\Omega(1)$ substratum are isothermal. So we neglect the thermal effects.*

Assumption 5 *Suppose that the deformation processes are not influenced by the gradient of the stress tensor.*

Assumption 6 *We suppose that the material of the $\Omega(1)$ substratum under consideration is isotropic.*

Assumption 7 *We suppose that the deformaton of the $\Omega(1)$ substratum under consideration is small.*

Assumption 8 *We suppose that there are no initial stress and strain in the body under consideration.*

When the $\Omega(1)$ substratum is subjected to a set of external forces, the relative positions of the $\Omega(1)$ particles form the body displacement.

In order to describe the deformation of the $\Omega(1)$ substratum, let us introduce a Cartesian coordinate system $\{o, x, y, z\}$ or $\{o, x_1, x_2, x_3\}$ which is static relative to the $\Omega(1)$ substratum. Now we may introduce a definition to the displacement vector **u** of every point in the $\Omega(1)$ substratum:

$$\mathbf{u} = \mathbf{r} - \mathbf{r}_0, \tag{7}$$

where $\mathbf{r}_0$ is the position of the point before the deformation, while $\mathbf{r}$ is the position after the deformation.

The displacement vector may be written as $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ or $\mathbf{u} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$, where **i**, **j**, **k** are three unit vectors directed along the coordinate axes.

The gradient of the displacement vector **u** is the relative displacement tensor $u_{i,j} = \frac{\partial u_i}{\partial x_j}$.

We decompose the tensor $u_{i,j}$ into two parts, the symmetric ε_{ij} and the skew-symmetric Ω_{ij} (refer to, for instance, [14, 48, 49])

$$u_{i,j} = \frac{1}{2}(u_{i,j} + u_{j,i}) + \frac{1}{2}(u_{i,j} - u_{j,i}) = \varepsilon_{ij} + \Omega_{ij}\,, \tag{8}$$

$$2\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})\,, \qquad \Omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i})\,. \tag{9}$$

The symmetric tensor ε_{ij} manifests a pure deformation of the body at a point, and is known the strain tensor (refer to, for instance, [14,48,49]). The matrix form and the component notation of the strain tensor ε_{ij} are

$$\varepsilon_{ij} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{pmatrix}, \tag{10}$$

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}. \tag{11}$$

The strain-displacements equations come from Eq. (10)

$$\left.\begin{aligned} \varepsilon_{11} &= \frac{\partial u}{\partial x}, & \varepsilon_{12} = \varepsilon_{21} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \varepsilon_{22} &= \frac{\partial v}{\partial y}, & \varepsilon_{23} = \varepsilon_{32} &= \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \varepsilon_{33} &= \frac{\partial w}{\partial z}, & \varepsilon_{31} = \varepsilon_{13} &= \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \end{aligned}\right\}. \tag{12}$$

For convenience, we introduce the definitions of the mean strain deviator ε_m and the strain deviator e_{ij} as

$$\varepsilon_m = \frac{1}{3}\left(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}\right), \tag{13}$$

$$e_{ij} = \varepsilon_{ij} - \varepsilon_m = \begin{pmatrix} \varepsilon_{11}-\varepsilon_m & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22}-\varepsilon_m & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}-\varepsilon_m \end{pmatrix}. \tag{14}$$

When the $\Omega(1)$ substratum deforms, the internal forces arise due to the deformation. The component notation of the stress tensor σ_{ij} is

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}. \tag{15}$$

For convenience, we introduce the definitions of mean stress σ_m and stress deviator s_{ij} as

$$\sigma_m = \frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right), \tag{16}$$

$$s_{ij} = \sigma_{ij} - \sigma_m = \begin{pmatrix} \sigma_{11}-\sigma_m & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\sigma_m & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\sigma_m \end{pmatrix}. \tag{17}$$

Now let us turn to study the constitutive relation.

An elastic Hooke solid responds instantaneously with respect to an external stress. A Newtonian viscous fluid responds to a shear stress by a steady flow process.

In 19th century, people began to point out that fact that some materials showed a time dependence in their elastic response with respect to external stresses. When a material like pitch, gum rubber, polymeric materials, hardened cement and even glass, is loaded, an instantaneous elastic deformation follows with a slow continuous flow or creep.

Now this time-dependent response is known as viscoelasticity (refer to, for instance, [50–52]). Materials bearing both instantaneous elastic elasticity and creep characteristics are known as viscoelastic materials [51, 52]. Viscoelastic materials were studied long time ago by Maxwell [51–53], Kelvin, Voigt, Boltzamann [51, 52, 54], etc.

Inspired by these contributors, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum.

It is natural to say that the constitutive relation of the $\Omega(1)$ substratum may be a combination of the constitutive relations of the Hooke-solid and the Newtonian-fluid.

For the Hooke-solid, we have the generalized Hooke law as follows (refer to, for instance, [14, 48, 49, 55]),

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\theta\delta_{ij}, \quad \varepsilon_{ij} = \frac{\sigma_{ij}}{2G} - \frac{3\nu}{Y}\sigma_m\delta_{ij}, \tag{18}$$

where δ_{ij} is the Kronecker symbol, σ_m is the mean stress, where Y is the Yang modulus, ν is the Poisson ratio, G is the shear modulus, λ is Lamé constant, θ is the volume change coefficient. The definition of θ is $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The generalized Hooke law Eq. (18) can also be written as [55]

$$s_{ij} = 2G\,e_{ij}, \tag{19}$$

where s_{ij} is the stress deviator, e_{ij} is the strain deviator.

For the Newtonian-fluid, we have the following constitutive relation

$$\frac{de_{ij}}{dt} = \frac{1}{2\eta}s_{ij}, \tag{20}$$

where s_{ij} is the stress deviator, $\frac{de_{ij}}{dt}$ is the strain rate deviator, η is the dynamic viscocity.

The $\Omega(1)$ substratum behaves like the Hooke-solid during very short duration. We therefore differentiate both sides of Eq. (19), then obtain

$$\frac{de_{ij}}{dt} = \frac{1}{2G}\frac{ds_{ij}}{dt}. \tag{21}$$

A combination of Eq. (21) and Eq. (20) gives

$$\frac{de_{ij}}{dt} = \frac{1}{2\eta}s_{ij} + \frac{1}{2G}\frac{ds_{ij}}{dt}. \tag{22}$$

We call the materials behaving like Eq. (22) "Maxwell-liquid" since Maxwell established such a constitutive relation in 1868 (refer to, for instance, [50–53]).

Eq. (22) is valid only in the case of infinitesimal deformation because the presence of the derivative with respect to time. Oldroyd recognized that we need a special definition for the operation of derivation, in order to satisfy the principle of material frame indifference or objectivity [51, 56]. Unfortunately, there is no unique definition of such a differential operation fulfil the principle of objectivity presently [51].

As an enlightening example, let us recall the description [50] for a simple shear experiment. We suppose

$$\frac{d\sigma_t}{dt} = \frac{\partial\sigma_t}{\partial t}, \qquad \frac{de_t}{dt} = \frac{\partial e_t}{\partial t}, \tag{23}$$

where σ_t is the shear stress, e_t is the shear strain.

Therefore, Eq. (22) becomes

$$\frac{\partial e_t}{\partial t} = \frac{1}{2\eta}\sigma_t + \frac{1}{2G}\frac{\partial\sigma_t}{\partial t}. \tag{24}$$

Integration of Eq. (24) gives

$$\sigma_t = e^{-\frac{G}{\eta}t}\left(\sigma_0 + 2G\int_0^t \frac{de_t}{dt}\, e^{\frac{G}{\eta}}\, dt\right). \tag{25}$$

If the shear deformation is kept constant, i.e. $\frac{\partial e_t}{\partial t} = 0$, we have

$$\sigma_t = \sigma_0\, e^{-\frac{G}{\eta}t}. \tag{26}$$

Eq. (26) shows that the shear stresses remain in the Maxwell-liquid and are damped in the course of time.

We see that $\frac{\eta}{G}$ must have the dimension of time. Now let us introduce the following definition of Maxwellian relaxation time τ

$$\tau = \frac{\eta}{G}. \tag{27}$$

Therefore, using Eq. (27), Eq. (22) becomes

$$\frac{s_{ij}}{\tau} + \frac{ds_{ij}}{dt} = 2G\,\frac{de_{ij}}{dt}. \tag{28}$$

Now let us introduce the following hypothesis

Assumption 9 *Suppose the constitutive relation of the $\Omega(1)$ substratum satisfies Eq. (22).*

Now we can derive the the equation of momentum conservation based on the above hypotheses 9.

Let T be a characteristic time scale of an observer of the $\Omega(1)$ substratum. When the observer's time scale T is of the same order that the period of the wave motion of light, the Maxwellian relaxation time τ is a comparigly large number. Thus, the first term of Eq. (28) may be neglected. Therefore, the observer concludes that the strain and the stress of the $\Omega(1)$ substratum satisfy the generalized Hooke law.

The generalized Hooke law (18) can also be written as [14, 55]

$$\left.\begin{aligned}
\sigma_{11} &= \lambda\theta + 2G\varepsilon_{11}\\
\sigma_{22} &= \lambda\theta + 2G\varepsilon_{22}\\
\sigma_{33} &= \lambda\theta + 2G\varepsilon_{33}\\
\sigma_{12} &= \sigma_{21} = 2G\varepsilon_{12} = 2G\varepsilon_{21}\\
\sigma_{23} &= \sigma_{32} = 2G\varepsilon_{23} = 2G\varepsilon_{32}\\
\sigma_{31} &= \sigma_{13} = 2G\varepsilon_{31} = 2G\varepsilon_{13}
\end{aligned}\right\}, \tag{29}$$

where $\lambda = \frac{Y\nu}{(1+\nu)(1-2\nu)}$ is Lamé constant, θ is the volume change coefficient. By its definition, $\theta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$.

The following relationship are useful

$$G = \frac{Y}{2(1+\nu)}, \qquad K = \frac{Y}{3(1-2\nu)}, \tag{30}$$

where K is the volume modulus.

It is known that the equations of the momentum conservation are (refer to, for instance, [14, 48, 49, 55, 57, 58]),

$$\frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} + \frac{\partial\sigma_{13}}{\partial z} + f_x = \rho\,\frac{\partial^2 u}{\partial t^2}, \tag{31}$$

$$\frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{22}}{\partial y} + \frac{\partial\sigma_{23}}{\partial z} + f_y = \rho\,\frac{\partial^2 v}{\partial t^2}, \tag{32}$$

$$\frac{\partial\sigma_{31}}{\partial x} + \frac{\partial\sigma_{32}}{\partial y} + \frac{\partial\sigma_{33}}{\partial z} + f_z = \rho\,\frac{\partial^2 w}{\partial t^2}, \tag{33}$$

where f_x, f_y and f_z are three components of the volume force density **f** exerted on the $\Omega(1)$ substratum.

The tensor form of the equations (31-33) of the momentum conservation can be written as

$$\sigma_{ij,j} + f_i = \rho\,\frac{\partial^2 u_i}{\partial t^2}. \tag{34}$$

Noticing Eq. (29), we write Eqs. (31-33) as

$$2G\left(\frac{\partial\varepsilon_{11}}{\partial x} + \frac{\partial\varepsilon_{12}}{\partial y} + \frac{\partial\varepsilon_{13}}{\partial z}\right) + \lambda\frac{\partial\theta}{\partial x} + f_x = \rho\,\frac{\partial^2 u}{\partial t^2}, \tag{35}$$

$$2G\left(\frac{\partial\varepsilon_{21}}{\partial x} + \frac{\partial\varepsilon_{22}}{\partial y} + \frac{\partial\varepsilon_{23}}{\partial z}\right) + \lambda\frac{\partial\theta}{\partial y} + f_y = \rho\,\frac{\partial^2 v}{\partial t^2}, \tag{36}$$

$$2G\left(\frac{\partial\varepsilon_{31}}{\partial x} + \frac{\partial\varepsilon_{32}}{\partial y} + \frac{\partial\varepsilon_{33}}{\partial z}\right) + \lambda\frac{\partial\theta}{\partial z} + f_z = \rho\,\frac{\partial^2 w}{\partial t^2}. \tag{37}$$

Using Eq. (12), Eqs. (35-37) can also be expressed by means of the displacement **u**

$$\left.\begin{aligned}
G\nabla^2 u + (G+\lambda)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + f_x = \rho\,\frac{\partial^2 u}{\partial t^2}\\
G\nabla^2 v + (G+\lambda)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + f_y = \rho\,\frac{\partial^2 v}{\partial t^2}\\
G\nabla^2 w + (G+\lambda)\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + f_z = \rho\,\frac{\partial^2 w}{\partial t^2}
\end{aligned}\right\}. \tag{38}$$

The vectorial form of the aforementioned equations (38) can be written as (refer to, for instance, [14, 48, 49, 55, 57, 58]),

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla(\nabla\cdot\mathbf{u}) + \mathbf{f} = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2}. \tag{39}$$

When no body force in the $\Omega(1)$ substratum, Eqs. (39) reduce to

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla(\nabla\cdot\mathbf{u}) = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2}. \tag{40}$$

From Long's theorem [48, 59], there exist a scalar function ψ and a vector function **R** such that **u** is represented by

$$\mathbf{u} = \nabla\psi + \nabla\times\mathbf{R} \tag{41}$$

and ψ and **R** satisfy the following wave equations

$$\nabla^2\psi - \frac{1}{c_l}\frac{\partial^2\psi}{\partial t^2} = 0, \tag{42}$$

$$\nabla^2\mathbf{R} - \frac{1}{c_t}\frac{\partial^2\mathbf{R}}{\partial t^2} = 0, \tag{43}$$

where c_l is the velocity of longitudinal waves, c_t is the velocity of transverse waves. The definitions of these two elastic wave velocities are (refer to, for instance, [48, 49, 57, 58]),

$$c_l = \sqrt{\frac{\lambda + 2G}{\rho}}\,, \quad c_t = \sqrt{\frac{G}{\rho}}\,. \tag{44}$$

ψ and $\mathbf{R}$ is usually known as the scalar displacement potential and the vector displacement potential, respectively.

4 Definition of point source and sink

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then, usually, these isolated points are called velocity singularities. Point sources and sinks are examples of such velocity singularities.

Assumption 10 *Suppose there exists a singularity at a point $P_0 = (x_0, y_0, z_0)$ in a continuum. If the velocity field of the singularity at a point $P = (x, y, z)$ is*

$$\mathbf{v}(x, y, z, t) = \frac{Q}{4\pi r^2}\,\hat{\mathbf{r}}, \tag{45}$$

where $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$, $\hat{\mathbf{r}}$ is the unit vector directed outward along the line from the singularity to this point $P = (x, y, z)$, we call such a singularity a point source in the case of $Q > 0$ or a point sink in the case of $Q < 0$. Here Q is called the strength of the source or sink.

Suppose that a static point source with the strength Q locates at the origin $(0, 0, 0)$. In order to calculate the volume leaving the source per unit of time, we may enclose the source with an arbitrary spherical surface S of the radius a. Calculation shows that

$$\oiint_S \mathbf{u}\cdot\mathbf{n}\,dS = \oiint_S \frac{Q}{4\pi a^2}\,\hat{\mathbf{r}}\cdot\mathbf{n}\,dS = Q\,, \tag{46}$$

where $\mathbf{n}$ is the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z). Equation (46) shows that the strength Q of a source or sink evaluates the volume of the fluid leaving or entering a control surface per unit of time.

For the case of continuously distributed point sources or sinks, it is useful to introduce a definition for the volume density ρ_s of point sources or sinks. The definition is

$$\rho_s = \lim_{\triangle V\to 0} \frac{\triangle Q}{\triangle V}\,, \tag{47}$$

where $\triangle V$ is a small volume, $\triangle Q$ is the sum of the strengthes of all the point sources or sinks in the volume $\triangle V$.

5 A point source and sink model of electric charges

The purpose of this Section is to propose a point source and sink model of electric charges.

Let T be the characteristic time of a observer of an electric charge in the $\Omega(1)$ substratum. We may suppose that the observer's time scale T is very large to the Maxwellian relaxation time τ. So the Maxwellian relaxation time τ is a relatively small, and the stress deviator s_{ij} changes very slow. Thus, the second term in the left side of Eq. (28) may be neglected. For such an observer, the constitutive relation of the $\Omega(1)$ substratum may be written as

$$s_{ij} = 2\eta\,\frac{de_{ij}}{dt}\,. \tag{48}$$

The observer therefore concludes that the $\Omega(1)$ substratum behaves like a Newtonian-fluid on his time scale.

In order to compare fluid motions with electric fields, Maxwell introduced an analogy between sources or sinks and electric charges [17].

Einstein, Infeld and Hoffmann introduced an idea by which all particles may be looked as singularities in fields [60, 61].

Recently [62], we talked that the universe may be filled with a kind fluid which may be called "tao". Thus, Newton's law of gravitation is derived by methods of hydrodynamics based on a point sink flow model of particles.

R. L. Oldershaw talked that hadrons may be considered as Kerr-Newman black holes if one uses appropriate scaling of units and a revised gravitational coupling factor [63].

Inspired by the aforementioned works, we introduce the following

Assumption 11 *Suppose that all the electric charges in the Universe are the sources or sinks in the $\Omega(1)$ substratum. We define such a source as a negative electric charge. We define such a sink as a positive electric charge. The electric charge quantity q_e of an electric charge is defined as*

$$q_e = -\,k_Q \rho\, Q\,, \tag{49}$$

where ρ is the density of the $\Omega(1)$ substratum, Q is called the strength of the source or sink, k_Q is a positive dimensionless constant.

A calculation shows that the mass m of an electric charge is changing with time as

$$\frac{dm}{dt} = -\,\rho\, Q = \frac{q_e}{k_Q}\,, \tag{50}$$

where q_e is the electric charge quantity of the electric charge.

We may introduce a hypothesis that the masses of electric charges are changing so slowly relative to the time scaler of human beings that they can be treaten as constants approximately.

For the case of continuously distributed electric charges, it is useful to introduce the following definition of the volume density ρ_e of electric charges

$$\rho_e = \lim_{\triangle V\to 0} \frac{\triangle q_e}{\triangle V}\,, \tag{51}$$

where $\triangle V$ is a small volume, $\triangle q_e$ is the sum of the strengthes of all the electric charges in the volume $\triangle V$.

From Eq. (47), Eq. (49) and Eq. (51), we have

$$\rho_e = -\,k_Q \rho \rho_s\,. \tag{52}$$

6 Derivation of Maxwell's equations in vacuum

The purpose of this Section is to deduce Maxwell's equations based on the aforementioned visco-elastic continuum model of vacuum and the singularity model of electric charges.

Now, let us deduce the continuity equation of the $\Omega(1)$ substratum from the mass conservation. Consider an arbitrary volume V bounded by a closed surface S fixed in space. Suppose that there are electric charges continuously distributed in the volume V. The total mass in the volume V is

$$M = \iiint_V \rho\, dV\,, \tag{53}$$

where ρ is the density of the $\Omega(1)$ substratum.

The rate of the increase of the total mass in the volume V is

$$\frac{\partial M}{\partial t} = \frac{\partial}{\partial t}\iiint_V \rho\, dV\,. \tag{54}$$

Using the Ostrogradsky–Gauss theorem (refer to, for instance, [16, 64–67]), the rate of the mass outflow through the surface S is

$$\oiint_S \rho\,(\mathbf{u}\cdot\mathbf{n})\, dS = \iiint_V \nabla\cdot(\rho\,\mathbf{u})\, dV\,, \tag{55}$$

where $\mathbf{v}$ is the velocity field of the $\Omega(1)$ substratum.

The definition of the velocity field $\mathbf{v}$ is

$$v_i = \frac{\partial u_i}{\partial t}\,, \quad \text{or} \quad \mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}\,. \tag{56}$$

Using Eq. (52), the rate of the mass created inside the volume V is

$$\iiint_V \rho\rho_s\, dV = \iiint_V -\frac{\rho_e}{k_Q}\, dV. \tag{57}$$

Now according to the principle of mass conservation, and making use of Eq. (54), Eq. (55) and Eq. (57), we have

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \iiint_V -\frac{\rho_e}{k_Q}\, dV - \iiint_V \nabla\cdot(\rho\,\mathbf{v})\, dV. \tag{58}$$

Since the volume V is arbitrary, from Eq. (58) we have

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\,\mathbf{v}) = -\frac{\rho_e}{k_Q}\,. \tag{59}$$

According to Assumption 3, the $\Omega(1)$ substratum is homogeneous, that is $\frac{\partial\rho}{\partial x} = \frac{\partial\rho}{\partial y} = \frac{\partial\rho}{\partial z} = \frac{\partial\rho}{\partial t} = 0$. Thus, Eq. (59) becomes

$$\nabla\cdot\mathbf{v} = -\frac{\rho_e}{k_Q\,\rho}\,. \tag{60}$$

According to Assumption 11 and Eq. (50), the masses bearing positive electric charges are changing since the strength of a sink evaluates the volume of the $\Omega(1)$ substratum entering the sink per unit of time. Thus, the momentum of a volume element $\triangle V$ of the $\Omega(1)$ substratum containing continuously distributed electric charges, and moving with an average speed $\mathbf{v}_e$, changes. The increased momentum $\triangle\mathbf{P}$ of the volume element $\triangle V$ during a time interval $\triangle t$ is the decreased momentum of the continuously distributed electric charges contained in the volume element $\triangle V$ during a time interval $\triangle t$, that is,

$$\triangle\mathbf{P} = \rho\,(\rho_s \triangle V \triangle t)\,\mathbf{v}_e = -\frac{\rho_e}{k_Q}\triangle V \triangle t\,\mathbf{v}_e\,. \tag{61}$$

Therefore, the equation of momentum conservation Eq. (39) of the $\Omega(1)$ substratum should be changed as

$$G\nabla^2\mathbf{u} + (G+\lambda)\nabla(\nabla\cdot\mathbf{u}) + \mathbf{f} = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}\,. \tag{62}$$

In order to simplify the Eq. (62), we may introduce an additional assumption as

Assumption 12 *We suppose that the $\Omega(1)$ substratum is almost incompressible, or we suppose that θ is a sufficient small quantity and varies very slow in the space so that it can be treaten as $\theta = 0$.*

From Assumption 12, we have

$$\nabla\cdot\mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \theta = 0\,. \tag{63}$$

Therefore, the vectorial form of the equation of momentum conservation Eq. (62) reduces to the following form

$$G\nabla^2\mathbf{u} + \mathbf{f} = \rho\,\frac{\partial^2\mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}\,. \tag{64}$$

According to the Stokes-Helmholtz resolution theorem (refer to, for instance, [48, 57]), which states that every sufficiently smooth vector field may be decomposed into irrotational and solenoidal parts, there exist a scalar function ψ and a vector function $\mathbf{R}$ such that $\mathbf{u}$ is represented by

$$\mathbf{u} = \nabla\psi + \nabla\times\mathbf{R}. \tag{65}$$

Now let us introduce the definitions

$$\nabla\phi = k_E\,\frac{\partial}{\partial t}(\nabla\psi)\,, \quad \mathbf{A} = k_E\,\nabla\times\mathbf{R}\,, \tag{66}$$

$$\mathbf{E} = -\,k_E\,\frac{\partial\mathbf{u}}{\partial t}\,, \quad \mathbf{B} = k_E\,\nabla\times\mathbf{u}\,, \tag{67}$$

where ϕ is the scalar electromagnetic potential, $\mathbf{A}$ is the vector electromagnetic potential, $\mathbf{E}$ is the electric field intensity, $\mathbf{B}$ is the magnetic induction, k_E is a positive dimensionless constant.

From Eq. (65), Eq. (66) and Eq. (67), we have

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}\,, \quad \mathbf{B} = \nabla\times\mathbf{A} \tag{68}$$

and

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{69}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{70}$$

Based on Eq. (66) and noticing that

$$\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}), \tag{71}$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A}), \tag{72}$$

and $\nabla \cdot \mathbf{u} = 0$, $\nabla \cdot \mathbf{A} = 0$, we have

$$k_E \nabla^2 \mathbf{u} = \nabla^2 \mathbf{A}. \tag{73}$$

Therefore, using Eq. (73), Eq. (64) becomes

$$\frac{G}{k_E} \nabla^2 \mathbf{A} + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \tag{74}$$

Using Eq. (72), Eq. (74) becomes

$$-\frac{G}{k_E} \nabla \times (\nabla \times \mathbf{A}) + \mathbf{f} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \tag{75}$$

Now using Eq. (68), Eq. (75) becomes

$$-\frac{G}{k_E} \nabla \times \mathbf{B} + \mathbf{f} = -\frac{\rho}{k_E} \frac{\partial \mathbf{E}}{\partial t} - \frac{\rho_e \mathbf{v}_e}{k_Q}. \tag{76}$$

It is natural to say that there are no other body forces or surface forces exerted on the $\Omega(1)$ substratum. Thus, we have $\mathbf{f} = 0$. Therefore, Eq. (76) becomes

$$\frac{k_Q G}{k_E} \nabla \times \mathbf{B} = \frac{k_Q \rho}{k_E} \frac{\partial \mathbf{E}}{\partial t} + \rho_e \mathbf{v}_e. \tag{77}$$

Now let us introduce the following definitions

$$\mathbf{j} = \rho_e \mathbf{v}_e, \qquad \epsilon_0 = \frac{k_Q \rho}{k_E}, \qquad \frac{1}{\mu_0} = \frac{k_Q G}{k_E}. \tag{78}$$

Therefore, Eq. (77) becomes

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}. \tag{79}$$

Noticing Eq. (67) and Eq. (78), Eq. (60) becomes

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\epsilon_0}. \tag{80}$$

Now we see that Eq. (69), Eq. (70), Eq. (79) and Eq. (80) coincide with Maxwell's equations (1–4).

7 Mechanical interpretation of electromagnetic waves

It is known that, from Maxwell's equations (1-4), we can obtain the following equations (refer to, for instance, [1])

$$\nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{\epsilon_0} \nabla \rho_e + \mu_0 \frac{\partial \mathbf{j}}{\partial t}, \tag{81}$$

$$\nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{H}}{\partial t^2} = -\frac{1}{\mu_0} \nabla \times \mathbf{j}. \tag{82}$$

Eq. (81) and Eq. (82) are the electromagnetic wave equations with sources in the $\Omega(1)$ substratum. In the source free region where $\rho_e = 0$ and $\mathbf{j} = 0$, the equations reduce to the following equations

$$\nabla^2 \mathbf{E} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \tag{83}$$

$$\nabla^2 \mathbf{H} - \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0. \tag{84}$$

Eq. (83) and Eq. (84) are the electromagnetic wave equations without the sources in the $\Omega(1)$ substratum.

From Eq. (83), Eq. (84) and Eq. (78), we see that the velocity c_0 of electromagnetic waves in vacuum is

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{G}{\rho}}. \tag{85}$$

From Eq. (44) and Eq. (85), we see that the velocity c_0 of electromagnetic waves in the vacuum is the same as the velocity c_t of the transverse elastic waves in the $\Omega(1)$ substratum.

Now we may regard electromagnetic waves in the vacuum as the transverse waves in the $\Omega(1)$ substratum. This idea was first introduced by Frensnel in 1821 [17].

8 Conclusion

We suppose that vacuum is not empty and may be filled with a kind continuously distributed material called $\Omega(1)$ substratum. Following Stokes, we propose a visco-elastic constitutive relation of the $\Omega(1)$ substratum. Following Maxwell, we propose a fluidic source and sink model of electric charges. Thus, Maxwell's equations in vacuum are derived by methods of continuum mechanics based on this continuum mechanical model of vacuum and the singularity model of electric charges.

9 Discussion

Many interesting theoretical, experimental and applied problems can be met in continuum mechanics, Classical Electrodynamics, Quantum Electrodynamics and also other related fields of science involving this theory of electromagnetic phenomena. It is an interesting task to generalize this theory of electromagnetic phenomena in the static $\Omega(1)$ substratum to the case of electromagnetic phenomena of moving bodies.

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References

1. Jackson J. D. Classical Electrodynamics. Wiley, New York, 1963.
2. Landau L. D. and Lifshitz E. M. Non-relativistic theory of Quantum Mechanics. Translated from the Russian by J.B. Sykes and J.S. Bell., Pergamon, London, 1958.
3. Aharonov Y. A. and Bohm D. *Phys. Rev.*, 1959, v. 115, 485.
4. Chambers R. G. *Phys. Rev. Lett.*, 1960, v. 5, 3.
5. Landau L. D. and Lifshitz E. M. The classical theory of fields. Translated from the Russian by Morton Hamermesh, Pergamon Press, New York, 1975.
6. Whitney C. K. *Hadronic J.*, 1988, v. 11, 257.
7. Chubykalo A. E. and Smirnov-Rueda R. *Phys. Rev. E*, 1996, v. 53, 5373.
8. Dirac P. A. M. Direction in Physics, Wiley, New York, 1978.
9. Lamoreaux S. K. *Rep. Prog. Phys.*, 2005, v. 68, 201.
10. Intravaia F. and Lambrecht A. *Phys. Rev. Lett.*, 2005, v. 94, 110404.
11. Guo J. G. and Zhao Y. P. *Journal of Microelectromechanical Systems*, 2004, v. 13, 1027.
12. Davies P. C. W. *J. Opt. B: Quantum Semiclass. Opt.*, 2005, v. 7, S40.
13. Truesdell C. The elements of continuum mechanics. Springer-Verlag, New York, 1966.
14. Fung Y. C. A first course in continuum mechanics. Prentice-Hall, London, 1977.
15. Eringen A. C. The elements of continuum mechanics. Robert E. Krieger Pub. Co., Huntington, 1980.
16. Landau L. D. and Lifshitz E. M. Fluid mechanics. Translated from the Russian by J.B. Sykes and W.H. Reid., Pergamon, New York, 1987.
17. Whittaker E. A history of the theories of aether and electricity. Thomas Nelson and Sons Ltd., London, 1953.
18. Ellis J. *Philos. Trans. R. Soc. London, A*, 2003, v. 361, 2607.
19. Linder E. V. *Phys. Rev. D*, 2004, v. 70, 023511.
20. Bernardeau F., *Rep. Prog. Phys.*, 2003, v. 66, 691.
21. Bergstrom L., Bringmann T., Eriksson M., and Gustafsson M. Phys. Rev. Lett., 2005, v. 94, 131301.
22. Beacom J. F., Bell N. F., and Bertone G. *Phys. Rev. Lett.*, 2005, v. 94, 171301.
23. Akerib D. S., et al. *Phys. Rev. Lett.*, 2006, v. 96, 011302.
24. Feng J. L., Su S. F., and Takayama F. *Phys. Rev. Lett.*, 2006, v. 96, 151802.
25. Fayet P. and Hooper D. *Phys. Rev. Lett.*, 2006, v. 96, 211302.
26. Carena M., Hooper D., and Skands P. *Phys. Rev. Lett.*, 2006, v. 97, 051801.
27. Einstein A. *Ann. Phys.*, 1905, v. 17, 891.
28. Golebiewska-Lasta A. A. *Int. J. Engng. Sic.*, 1979, v. 17, 441.
29. Barut A. *Found. Phys.*, 1988, v. 18, 95.
30. Oldershaw R. L. *International Journal of Theoretical Physics*, 19891, v. 28, 669.
31. Oldershaw R. L. *International Journal of Theoretical Physics*, 1989, v. 28, 1503. .
32. Dmitriyev V. P. *Mechanics of Solids*, 1992, v. 26, 60.
33. Larson D. J. *Physics Essays*, 1998, v. 11, 524.
34. Marmanis H. *Phys. Fluids*, 1998, v. 10, 1428.
35. Dmitriyev V. P. *Nuovo Cimento*, 1998, v. 111A, 501.
36. Zareski D. *Found. Phys. Lett.*, 2001, v. 14, 447.
37. Dmitriyev V. P. *Am. J. Phys.*, 2003, v. 71, 952.
38. Einstein A. Aether and the theory of relativity. 1920. Translated in: *Sidelights on Relativity*, Dover, 1983.
39. Kostro L. Einstein and the ether. Apeiron Publishers, Montreal, 2000.
40. Dmitriyev V. P. *Meccanica*, 2004, v. 39, 511.
41. Martin A. Gravitation in a gaseous ether. In: *Einstein and Poincaré: the physical vacuum*, Valeri V. Dvoeglazov ed., Apeiron Publ., Montreal, 2005.
42. Martin A. The electron as an extended structure in cosmonic gas. In: *What is the Electron?*, Volodimir Simulik ed., Apeiron Publ., Montreal, 2005.
43. Martin A. and Keys C. The ether revisited. In: *Frontiers of Fundamental Physics*, Volodimir Simulik ed., Plenum, New York, 1994.
44. Tan Q.-M. Dimensional analysis. (*In Chinese*), University of Science and Technology of China Press, Beijing, 2005.
45. Stokes G. G. *Phil. Mag.*, 1845, v. 27, 9.
46. Stokes G. G. *Trans. Camb. Phil. Soc.*, 1849, v. 9, 1.
47. Stokes G. G. *Brit. Assoc. Reports*, 1862, v. 32, 253.
48. Eringen A. C. Elastodynamics. Academic Press, New York, 1975.
49. Landau L. D. and Lifshitz E. M. Theory of elasticity. Translated from the Russian by J.B. Sykes and W.H. Reid., Pergamon, New York, 1986.
50. Reiner M. Lectures on theoretical rheology. North-Holland, 1960.
51. Christensen R. M. Theory of viscoelasticity: an introduction. Academic Press, New York, 1982.
52. Joseph D. D. Fluid dynamics of viscoelastic liquids. Springer-Verlag, New York, 1990.
53. Maxwell J. C. *Phil. Mag.*, 1868, v. 35, 129, 185.

54. Boltzamann I. *Sitzungsber. Math. Naturwiss. Kl. Kaiserl. Akad. Wiss.*, 1874, v. 70, 275.
55. Wang R., Xiong Z. H., and Huang W. B. Foundations of theory of plasticity. (*In Chinese*), Science Press, Beijing, 1982.
56. Oldroyd J. G. *Proc. R. Soc., London, Ser. A*, 1950, v. 200, 523.
57. Pao Y.-H. and Mow C.-C. Diffraction of elastic waves and dynamic stress concentrations. Crane, Russak & Company Inc. (US), 1973.
58. Pao Y.-H. *J. Appl. Mech*, 1983, v. 12.
59. Long C. F. *Acta Mech.*, 1967, v. 3, 371.
60. Einstein A. Infeld L., and Hoffman B. *Ann. Math*, 1938, v. 39, 65.
61. Einstein A. and Infeld L. *Ann. Math*, 1940, v. 41, 455.
62. Wang X.-S. arXiv: physics/0506062.
63. Oldershaw R. L. Hadrons as kerr-newman black holes. Preprints, http://www.amherst.edu/∼rloldershaw, 2007.
64. Kochin N. E., Kibel I. A., and Roze. N. V. Theoretical hydrodynamics. Translated from the fifth Russian ed., Interscience Publishers, New York, 1964.
65. Yih C.-S. Fliud mechanics. McGraw-Hill Book Company, New York, 1969.
66. Faber T. E. Fluid dynamics for physicists. Cambridge University Press, Cambridge, 1995.
67. Currie I. G. Fundamental mechanics of fluids. Cambridge University Press, 3rd ed., 2003.
68. Zelmanov A. Chronometric invariants. American Research Press, 2006.
69. Rabounski D. *Progress in Physics*, 2005, v. 2, 15–30.