

Derivation of the Newton's Law of Gravitation Based on a Fluid Mechanical Singularity Model of Particles

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The main purpose of this paper is to seek a mechanical interpretation of gravitational phenomena. We suppose that the universe may be filled with a kind of fluid which may be called the $\Omega(0)$ substratum. Thus, the inverse-square law of gravitation is derived by methods of hydrodynamics based on a sink flow model of particles. The first feature of this theory of gravitation is that the gravitational interactions are transmitted by a kind of fluidic medium. The second feature is the time dependence of gravitational constant G and gravitational mass. The Newton's law of gravitation is arrived if we introduce an assumption that G and the masses of particles are changing so slowly that they can be treated as constants.

1 Introduction

The Newton's law of gravitation can be written as

$$\mathbf{F}_{21} = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{21}, \quad (1)$$

where m_1 and m_2 are the masses of two particles, r is the distance between the two particles, G is the gravitational constant, \mathbf{F}_{12} is the force exerted on the particle with mass m_2 by the particle with mass m_1 , $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the particle with mass m_1 to the particle with mass m_2 .

The main purpose of this paper is to derive the Newton's law of gravitation by means of fluid mechanics based on sink flow model of particles.

The motive of this paper is to seek a mechanism of gravitational phenomena. The reasons why new models of gravity are interesting may be summarized as follows.

Firstly, there exists some astronomical phenomena that could not be interpreted by the present theories of gravitation, for instance, the Titius-Bode law [1]. New theories of gravity may view these problems from new angles.

Secondly, whether the gravitational constant G depends on time and space is still unknown [2–8]. It is known that the gravitational constant G is a constant in the Newton's theory of gravitation and in theory of general relativity.

Thirdly, the mechanism of the action-at-a-distance gravitation remains an unsolved problem in physics for more than 300 years [9–11]. Although theory of general relativity is a field theory of gravity [12], the concept of field is different from that of continuum mechanics [13–16] because of the absence of a continuum in theory of general relativity. Thus, theory of general relativity can only be regarded as a phenomenological theory of gravity.

Fourthly, we do not have a satisfactory quantum theory of gravity presently [17–21]. One of the challenges in theoretic-

all physics is to reconcile quantum theory and theory of general relativity [17, 22]. New theories of gravity may open new ways to solve this problem.

Fifthly, one of the puzzles in physics is the problem of dark matter and dark energy [23–31]. New theories of gravity may provide new methods to attack this problem [24, 25].

Finally, we do not have a successful unified field theory presently. Great progress has been made towards an unification of the four fundamental interactions in the universe in the 20th century. However, gravitation is still not unified successfully. New theories of gravity may shed some light on this puzzle.

To conclude, it seems that new considerations on gravitation is needed. It is worthy keeping an open mind with respect to all the theories of gravity before the above problems been solved.

Now let us briefly review the long history of mechanical interpretations of gravitational phenomena. Many philosophers and scientists, such as Laozi [32], Thales, Anaximenes, believed that everything in the universe is made of a kind of fundamental substance [9]. Descartes was the first to bring the concept of aether into science by suggesting that it has mechanical properties [9]. Since the Newton's law of gravitation was published in 1687 [33], this action-at-a-distance theory was criticized by the French Cartesian [9]. Newton admitted that his law did not touch on the mechanism of gravitation [34]. He tried to obtain a derivation of his law based on Descartes' scientific research program [33]. Newton himself even suggested an explanation of gravity based on the action of an aetherial medium pervading the space [34, 35]. Euler attempted to explain gravity based on some hypotheses of a fluidic aether [9].

In a remarkable paper published in 1905, Einstein abandoned the concept of aether [36]. However, Einstein's assertion did not cease the explorations of aether [9, 37–46]. Einstein changed his view later and introduced his new con-

cept of ether [47, 48]. I regret to admit that it is impossible for me to mention all the works related to this field in history. Adolphe Martin and Roy Keys [49–51] proposed a fluidic cosmic gas model of vacuum to explain the physical phenomena such as electromagnetism, gravitation, quantum mechanics and the structure of elementary particles.

Inspired by the aforementioned thoughts and others [52–56], we show that the Newton's law of gravitation is derived based on the assumption that all the particles are made of singularities of a kind of ideal fluid.

During the preparation of the manuscript, I noticed that John C. Taylor had proposed an idea that the inverse-square law of gravitation may be explained based on the concept of source or sink [65].

2 Forces acting on sources and sinks in ideal fluids

The purpose of this section is to calculate the forces between sources and sinks in inviscid incompressible fluids which is called ideal fluids usually.

Suppose the velocity field \mathbf{u} of an ideal fluid is irrotational, then we have [16, 54–59],

$$\mathbf{u} = \nabla \phi, \quad (2)$$

where ϕ is the velocity potential, $\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$ is the Hamilton operator.

It is known that the equation of mass conservation of an ideal fluid becomes Laplace's equation [54–59],

$$\nabla^2 \phi = 0, \quad (3)$$

where ϕ is velocity potential, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator.

Using spherical coordinates (r, θ, φ) , a general form of solution of Laplace's equation (3) can be obtained by separation of variables as [56]

$$\phi(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta), \quad (4)$$

where A_l and B_l are arbitrary constants, $P_l(x)$ are Legendre's function of the first kind which is defined as

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l. \quad (5)$$

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then these isolated points are called singularities usually.

Definition 1 Suppose there exists a singularity at point $P_0 = (x_0, y_0, z_0)$. If the velocity field of the singularity at point $P = (x, y, z)$ is

$$\mathbf{u}(x, y, z, t) = \frac{Q}{4\pi r^2} \hat{\mathbf{r}}, \quad (6)$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$, $\hat{\mathbf{r}}$ denotes the unit vector directed outward along the line from the singularity to the point $P = (x, y, z)$, then we call this singularity a source if $Q > 0$ or a sink if $Q < 0$. Q is called the strength of the source or sink.

Suppose a static point source with strength Q locates at the origin $(0, 0, 0)$. In order to calculate the volume leaving the source per unit time, we may enclose the source with an arbitrary spherical surface S with radius a . A calculation shows that

$$\oint_S \mathbf{u} \cdot \mathbf{n} dS = \oint_S \frac{Q}{4\pi a^2} \hat{\mathbf{r}} \cdot \mathbf{n} dS = Q, \quad (7)$$

where \mathbf{n} denotes the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) . Equation (7) shows that the strength Q of a source or sink evaluates the volume of the fluid leaving or entering a control surface per unit time.

From (4), we see that the velocity potential $\phi(r, \theta)$ of a source or sink is a solution of Laplace's equation $\nabla^2 \phi = 0$.

Theorem 2 Suppose (1) there exists an ideal fluid (2) the ideal fluid is irrotational and barotropic, (3) the density ρ is homogeneous, that is $\partial \rho / \partial x = \partial \rho / \partial y = \partial \rho / \partial z = \partial \rho / \partial t = 0$, (4) there are no external body forces exerted on the fluid, (5) the fluid is unbounded and the velocity of the fluid at the infinity is approaching to zero. Suppose a source or sink is stationary and is immersed in the ideal fluid. Then, there is a force

$$\mathbf{F}_Q = -\rho Q \mathbf{u}_0 \quad (8)$$

exerted on the source by the fluid, where ρ is the density of the fluid, Q is the strength of the source or the sink, \mathbf{u}_0 is the velocity of the fluid at the location of the source induced by all means other than the source itself.

Proof Only the proof of the case of a source is needed. Let us select the coordinates that is attached to the static fluid at the infinity.

We set the origin of the coordinates at the location of the source. We surround the source by an arbitrary small spherical surface S . The surface S is centered at the origin of the coordinates with radius r . The outward unit normal to the surface S is denoted by \mathbf{n} . Let $\tau(t)$ denotes the mass system of fluid enclosed in the volume between the surface S and the source at time t . Let \mathbf{F}_Q denotes the hydrodynamic force exerted on the source by the mass system τ , then a reaction of this force must act on the the fluid enclosed in the mass system τ . Let \mathbf{F}_S denotes the hydrodynamic force exerted on the mass system τ due to the pressure distribution on the surface S , \mathbf{K} denotes momentum of the mass system τ .

As an application of the Newton's second law of motion to the mass system τ , we have

$$\frac{D\mathbf{K}}{Dt} = -\mathbf{F}_Q + \mathbf{F}_S, \quad (9)$$

where D/Dt represents the material derivative in the lagrangian system [16, 54–59]. The expressions of the momentum \mathbf{K} and the force \mathbf{F}_S are

$$\mathbf{K} = \iiint_{\tau} \rho \mathbf{u} dV, \quad \mathbf{F}_S = \oint_S (-p) \mathbf{n} dS, \quad (10)$$

where the first integral is volume integral, the second integral is surface integral, \mathbf{n} denotes the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) .

Since the velocity field is irrotational, we have the following relation

$$\mathbf{u} = \nabla \phi, \quad (11)$$

where ϕ is the velocity potential.

According to Ostrogradsky–Gauss theorem (see, for instance, [54–56, 58, 59]), we have

$$\iiint_{\tau} \rho \mathbf{u} dV = \iiint_{\tau} \rho \nabla \phi dV = \oint_S \rho \phi \mathbf{n} dS. \quad (12)$$

Note that the mass system τ does not include the singularity at the origin. According to Reynolds’ transport theorem [54–56, 58, 59], we have

$$\frac{D}{Dt} \iiint_{\tau} \rho \mathbf{u} dV = \frac{\partial}{\partial t} \iiint_V \rho \mathbf{u} dV + \oint_S \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dS, \quad (13)$$

where V is the volume fixed in space which coincide with the mass system $\tau(t)$ at time t , that is $V = \tau(t)$.

Then, using (13), (10) and (12), we have

$$\frac{D\mathbf{K}}{Dt} = \oint_S \rho \frac{\partial \phi}{\partial t} \mathbf{n} dS + \oint_S \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dS. \quad (14)$$

According to Lagrange–Cauchy integral [54–56, 58, 59], we have

$$\frac{\partial \phi}{\partial t} + \frac{(\nabla \phi)^2}{2} + \frac{p}{\rho} = f(t), \quad (15)$$

where $f(t)$ is an arbitrary function of time t . Since the velocity \mathbf{u} of the fluid at the infinity is approaching to zero, and noticing (4), $\phi(t)$ must be of the following form

$$\phi(r, \theta, t) = \sum_{l=0}^{\infty} \frac{B_l(t)}{r^{l+1}} P_l(\cos \theta), \quad (16)$$

where $B_l(t), l \geq 0$ are functions of time t . Thus, we have the following estimations at the infinity of the velocity field

$$\phi = O\left(\frac{1}{r}\right), \quad \frac{\partial \phi}{\partial t} = O\left(\frac{1}{r}\right), \quad r \rightarrow \infty, \quad (17)$$

where $\varphi(x) = O(\psi(x)), x \rightarrow a$ stands for $\overline{\lim}_{x \rightarrow a} |\varphi(x)| / \psi(x) = k, (0 \leq k < +\infty)$.

Applying (15) at the infinity and using (17), we have $|\mathbf{u}| \rightarrow 0, \partial \phi / \partial t \rightarrow 0$ and $p = p_{\infty}$, where p_{∞} is a constant. Thus, $f(t) = p_{\infty} / \rho$. Therefore, according to (15), we have

$$p = p_{\infty} - \rho \frac{\partial \phi}{\partial t} - \frac{\rho (\mathbf{u} \cdot \mathbf{u})}{2}. \quad (18)$$

Using (10) and (18), we have

$$\mathbf{F}_S = \oint_S \rho \frac{\partial \phi}{\partial t} \mathbf{n} dS + \oint_S \frac{\rho (\mathbf{u} \cdot \mathbf{u}) \mathbf{n}}{2} dS. \quad (19)$$

Using (9), (14), (19), we have

$$\mathbf{F}_Q = \oint_S \left[\frac{1}{2} \rho (\mathbf{u} \cdot \mathbf{u}) \mathbf{n} - \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) \right] dS. \quad (20)$$

Now let us calculate this velocity \mathbf{u} in order to obtain \mathbf{F}_Q . Since the velocity field induced by the source Q is (6), then according to the superposition principle of velocity field of ideal fluids, the velocity on the surface S is

$$\mathbf{u} = \frac{Q}{4\pi r^2} \mathbf{n} + \mathbf{u}_0, \quad (21)$$

where \mathbf{n} denotes the unit vector directed outward along the line from the origin of the coordinates to the field point (x, y, z) . Using (20) and (21), we have

$$\mathbf{F}_Q = \rho \oint_S \left[-\frac{Q^2}{32\pi^2 r^4} \mathbf{n} + \frac{1}{2} (\mathbf{u}_0 \cdot \mathbf{u}_0) \mathbf{n} - \frac{Q}{4\pi r^2} \mathbf{u}_0 - (\mathbf{u}_0 \cdot \mathbf{n}) \mathbf{u}_0 \right] dS. \quad (22)$$

Since the radius r can be arbitrarily small, the velocity \mathbf{u}_0 can be treated as a constant in the integral of (22). Thus, (22) turns out to be

$$\mathbf{F}_Q = -\rho \oint_S \frac{Q}{4\pi r^2} \mathbf{u}_0 dS. \quad (23)$$

Since again \mathbf{u}_0 can be treated as a constant, (23) turns out to be (8). This completes the proof. \square

Remark Lagally [52], Landweber and Yih [53, 54], Faber [55] and Currie [56] obtained the same result of Theorem 2 for the special case where the velocity field is steady.

Theorem 2 only considers the situation that the sources or sinks are at rest. Now let us consider the case that the sources or sinks are moving in the fluid.

Theorem 3 Suppose the presuppositions (1), (2), (3), (4) and (5) in Theorem 2 are valid and a source or a sink is moving in the fluid with a velocity \mathbf{v}_s , then there is a force

$$\mathbf{F}_Q = -\rho Q (\mathbf{u}_f - \mathbf{v}_s) \quad (24)$$

is exerted on the source by the fluid, where ρ is the density of the fluid, Q is the strength of the source or the sink, \mathbf{u}_f is the

velocity of the fluid at the location of the source induced by all means other than the source itself.

Proof The velocity of the fluid relative to the source at the location of the source is $\mathbf{u}_f - \mathbf{v}_s$. Let us select the coordinates that is attached to the source and set the origin of the coordinates at the location of the source. Then (24) can be arrived following the same procedures in the proof of Theorem 2. \square

Applying Theorem 3 to the situation that a source or sink is exposed to the velocity field of another source or sink, we have:

Corollary 4 Suppose the presuppositions (1), (2), (3), (4) and (5) in Theorem 2 are valid and a source or a sink with strength Q_2 is exposed to the velocity field of another source or sink with strength Q_1 , then the force \mathbf{F}_{21} exerted on the singularity with strength Q_2 by the velocity field of the singularity with strength Q_1 is

$$\mathbf{F}_{21} = -\rho Q_2 \frac{Q_1}{4\pi r^2} \hat{\mathbf{r}}_{21} + \rho Q_2 \mathbf{v}_2, \quad (25)$$

where $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the singularity with strength Q_1 to the singularity with strength Q_2 , r is the distance between the two singularities, \mathbf{v}_2 is the velocity of the source with strength Q_2 .

3 Derivation of inverse-square-law of gravitation

Since quantum theory shows that vacuum is not empty and has physical effects, e.g., the Casimir effect [45, 60–62], it is valuable to probe vacuum by introducing the following hypotheses:

Assumption 5 Suppose the universe is filled by an ideal fluid named $\Omega(0)$ substratum; the ideal fluid fulfil the conditions (2), (3), (4), (5) in Theorem 2.

This fluid may be named $\Omega(0)$ substratum in order to distinguish with Cartesian aether. Following Einstein, Infeld and Hoffmann, who introduced the idea that particles may be looked as singularities in fields [63, 64], and noticing (25), it is nature to introduce the following:

Assumption 6 All the microscopic particles were made up of a kind of elementary sinks of $\Omega(0)$ substratum. These elementary sinks were created simultaneously. The initial masses and the strengths of the elementary sinks are the same.

We may call these elementary sinks as monads.

Suppose a particle with mass m is composed of N monads. Then, according to Assumption 6, we have:

$$m_0(t) = m_0(0) + \rho q_0 t, \quad (26)$$

$$Q = -Nq_0, \quad m(t) = Nm_0(t) = -\frac{Q}{q_0} m_0(t), \quad (27)$$

$$\frac{dm_0}{dt} = \rho q_0, \quad \frac{dm}{dt} = -\rho Q, \quad (28)$$

where $m_0(t)$ is the mass of monad at time t , $-q_0(q_0 > 0)$ is the strength of a monad, $m(t)$ is the mass of a particle at time

t , Q is the strength of the particle, N is the number of monads that make up the particle, ρ is the density of the $\Omega(0)$ substratum, $t \geq 0$.

From (28), we see that the mass m_0 of a monad is increasing since q_0 evaluates the volume of the $\Omega(0)$ substratum fluid entering the monad per unit time. From (28), we also see that the mass of a monad or a particle is increasing linearly.

Based on Assumption 5 and Assumption 6, the motion of a particle is determined by:

Theorem 7 The equation of motion of a particle is

$$m(t) \frac{d\mathbf{v}}{dt} = \frac{\rho q_0}{m_0(t)} m(t) \mathbf{u} - \frac{\rho q_0}{m_0(t)} m(t) \mathbf{v} + \mathbf{F}, \quad (29)$$

where $m_0(t)$ is the mass of monad at time t , $-q_0$ is the strength of a monad, $m(t)$ is the mass of a particle at time t , \mathbf{v} is the velocity of the particle, \mathbf{u} is the velocity of the $\Omega(0)$ substratum at the location of the particle induced by all means other than the particle itself, \mathbf{F} denotes other forces.

Proof Applying the Newton's second law and Theorem 3 to this particle, we have $m d\mathbf{v}/dt = -\rho Q (\mathbf{u} - \mathbf{v}) + \mathbf{F}$. Noticing (27), we get (29). \square

Formula (29) shows that there exists a universal damping force

$$\mathbf{F}_d = -\frac{\rho q_0}{m_0} m \mathbf{v} \quad (30)$$

exerted on each particle.

Now let us consider a system consists of two particles. Based on Assumption 6, applying Theorem 7 to this system, we have:

Corollary 8 Suppose there is a system consists of two particles and there are no other forces exerted on the particles, then the equations of motion of this system are

$$m_1 \frac{d\mathbf{v}_1}{dt} = -\frac{\rho q_0}{m_0} m_1 \mathbf{v}_1 - \frac{\rho q_0^2}{4\pi m_0^2} \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{12} \quad (31)$$

$$m_2 \frac{d\mathbf{v}_2}{dt} = -\frac{\rho q_0}{m_0} m_2 \mathbf{v}_2 - \frac{\rho q_0^2}{4\pi m_0^2} \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{21}, \quad (32)$$

where $m_{i=1,2}$ is the mass of the particles, $\mathbf{v}_{i=1,2}$ is the velocity of the particles, m_0 is the mass of a monad, $-q_0$ is the strength of a monad, ρ is the density of the $\Omega(0)$ substratum, $\hat{\mathbf{r}}_{12}$ denotes the unit vector directed outward along the line from the particle with mass $m_2(t)$ to the particle with mass $m_1(t)$, $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the particle with mass $m_1(t)$ to the particle with mass $m_2(t)$.

Ignoring the damping forces in (32), we have:

Corollary 9 Suppose (1) $\mathbf{v}_{i=1,2} \ll \mathbf{u}_{i=1,2}$, where \mathbf{v}_i is the velocity of the particle with mass m_i , \mathbf{u}_i is the velocity of the $\Omega(0)$ substratum at the location of the particle with mass m_i induced by the other particle, (2) there are no other forces exerted on the particles, then the force $\mathbf{F}_{21}(t)$ exerted on the

particle with mass $m_2(t)$ by the velocity field of $\Omega(0)$ substratum induced by the particle with mass $m_1(t)$ is

$$\mathbf{F}_{21}(t) = -G(t) \frac{m_1(t)m_2(t)}{r^2} \hat{\mathbf{r}}_{21}, \quad (33)$$

where $G = \rho q_0^2 / (4\pi m_0^2(t))$, $\hat{\mathbf{r}}_{21}$ denotes the unit vector directed outward along the line from the particle with mass $m_1(t)$ to the particle with mass $m_2(t)$, r is the distance between the two particles.

Corollary 9 is coincide with the Newton's inverse-square-law of gravitation (1) except for two differences. The first difference is that $m_{i=1,2}$ are constants in the Newton's law (1) while in (1) while in Corollary are functions of time t . The second difference is that G is a t . The second difference is that G is a constant in the Newton's

Let us now introduce an assumption that G and the masses of particles are changing so slowly relative to the time scale of human beings that they can be treated as constants approximately. Thus, the Newton's law (1) of gravitation may be considered as a result of Corollary 9 based on this assumption.

4 Superposition principle of gravitational field

The definition of gravitational field \mathbf{g} of a particle with mass m is $\mathbf{g} = \mathbf{F}/m_{test}$, where m_{test} is the mass of a test point mass, \mathbf{F} is the gravitational force exerted on the test point mass by the gravitational field of the particle with mass m . Based on Theorem 7 and Corollary 9, we have

$$\mathbf{g} = \frac{\rho q_0}{m_0} \mathbf{u}, \quad (34)$$

where ρ is the density of the $\Omega(0)$ substratum, m_0 is the mass of a monad, q_0 is the strength of a monad, \mathbf{u} is the velocity of the $\Omega(0)$ substratum at the location of the test point mass induced by the particle mass m . From (34), we see that the superposition principle of gravitational field is deduced from the superposition theorem of the velocity field of ideal fluids.

5 Time dependence of gravitational constant G and mass

According to Assumption 6 and Corollary 9, we have we have

$$G = \frac{\rho q_0^2}{4\pi m_0^2(t)}, \quad (35)$$

where $m_0(t)$ is the mass of monad at time t , $-q_0$ is the strength of a monad, ρ is the density of the $\Omega(0)$ substratum. The time dependence of gravitational mass can be seen from (35) and (28).

6 Conclusion

We suppose that the universe may be filled with a kind of fluid which may be called the $\Omega(0)$ substratum. Thus, the inverse-

square law of gravitation is derived by methods of hydrodynamics based on a sink flow model of particles. There are two features of this theory of gravitation. The first feature is that the gravitational interactions are transmitted by a kind of fluidic medium. The second feature is the time dependence of gravitational constant and gravitational mass. The Newton's law of gravitation is arrived if we introduce an assumption that G and the masses of particles are changing so slowly that they can be treated as constants. As a byproduct, it is shown that there exists a universal damping force exerted on each particle.

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