Thermoelastic Property of a Semi-Infinite Medium Induced by a Homogeneously Illuminating Laser Radiation

Ibrahim A. Abdallah*, Amin F. Hassan†, Ismail M. Tayel*

*Dept. of Mathematics, †Dept. of Physics, Helwan University, Helwan, 11795, Egypt
E-mail: iaawavelets@yahoo.com

The problem of thermoelasticity, based on the theory of Lord and Shulman with one relaxation time, is used to solve a boundary value problem of one dimensional semi-infinite medium heated by a laser beam having a temporal Dirac distribution. The surface of the medium is taken as traction free. The general solution is obtained using the Laplace transformation. Small time approximation analysis for the stresses, displacement and temperature are performed. The convolution theorem is applied to get the response of the system on temporally Gaussian distributed laser radiation. Results are presented graphically. Concluding that the small time approximation has not affected the finite velocity of the heat conductivity.

1 Introduction

The classical theory (uncoupled) of thermoelasticity based on the conventional heat conduction equation. The conventional heat conduction theory assumes that the thermal disturbances propagate at infinite speeds. This prediction may be suitable for most engineering applications but it is a physically unacceptable situation, especially at a very low temperature near absolute zero or for extremely short-time responses.

Biot [1] formulated the theory of coupled thermoelasticity to eliminate the shortcoming of the classical uncoupled theory. In this theory, the equation of motion is a hyperbolic partial differential equation while the equation of energy is parabolic. Thermal disturbances of a hyperbolic nature have been derived using various approaches. Most of these approaches are based on the general notion of relaxing the heat flux in the classical Fourier heat conduction equation, thereby, introducing a non Fourier effect.

The first theory, known as theory of generalized thermoelasticity with one relaxation time, was introduced by Lord and Shulman [2] for the special case of an isotropic body. The extension of this theory to include the case of anisotropic body was developed by Dhaliwal and Sherief [4].

In view of the experimental evidence available in favor of finiteness of heat propagation speed, generalized thermoelasticity theories are supposed to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes and/or short time intervals, like those occurring in laser units and energy channels.

The purpose of the present work is to study the thermoelastic interaction caused by heating a homogeneous and isotropic thermoelastic semi-infinite body induced by a Dirac pulse having a homogeneous infinite cross-section by employing the theory of thermo-elasticity with one relaxation time. The problem is solved by using the Laplace transform technique. Approximate small time analytical solutions to stress, displacement and temperature are obtained. The convolution theorem is applied to get the spatial and temporal temperature distribution induced by laser radiation having a temporal Gaussian distribution. At the end of this work we present the computed results obtained from the theoretical relations applied on a Cu target.

2 Formulation of the problem

We consider a thermoelastic, homogeneous, isotropic semi-infinite target occupying the region $z \geqslant 0$, and initially at uniform temperature T_0. The surface of the target $z = 0$ is heated homogeneously by a leaser beam and assumed to be traction free. The Cartesian coordinates (x, y, z) are considered in the solution and z-axis pointing vertically into the medium. The equation of motion in the absence of the body forces has the form

$$\sigma_{ji,j} = \rho\, \ddot{u}_i\,, \tag{1}$$

where σ_{ij} is the components of stress tensor, u_i is the components of displacement vector and ρ is the mass density. Due to the Lord and Shalman theory of coupled thermoelasticity [2] (L-S) who considered a wave-type heat equation by postulating a new law of heat conduction equation to replace the Fourier's law

$$\rho\, c_E \left(\frac{\partial T}{\partial t} + t_0 \frac{\partial^2 T}{\partial t^2}\right) + \\ + \gamma\, T_0 \operatorname{div}\left(\frac{\partial u}{\partial t} + t_0 \frac{\partial^2 u}{\partial t^2}\right) = k\, \nabla^2 T\,, \tag{2}$$

where T_0 is a uniform reference temperature, $\gamma{=}(3\lambda{+}2\mu)\alpha_t$, λ, and μ are Lame's constants. α_t is the linear thermal expansion coefficient, c_E is the specific heat at constant strain and k is the thermal conductivity. The boundary conditions:

$$\sigma_{zz} = 0, \quad z = 0\,, \tag{3}$$

$$-k\,\frac{dT}{dz} = A_0\,q_0\,\delta(t)\,, \quad z = 0\,, \tag{4}$$

where A_0 is an absorption coefficient of the material, q_0 is the intensity of the laser beam and $\delta(t)$ is the Dirac delta function [5]. The initial conditions:

$$\left.\begin{aligned} &T(z,0) = T_0 \\ &\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial t^2} = \frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial t^2} = 0\,, \text{ at } t = 0\,, \forall z \end{aligned}\right\}. \tag{5}$$

Due to the symmetry of the problem and the external applied thermal field, the displacement vector u has the components:

$$u_x = 0\,, \quad u_y = 0\,, \quad u_z = w(z,t)\,. \tag{6}$$

From equation (6) the strain components e_{ij}, and the relation of the strain components to the displacement read;

$$\left.\begin{aligned} &e_{xx} = e_{yy} = e_{xy} = e_{xz} = e_{yz} = 0 \\ &e_{zz} = \frac{\partial w}{\partial z} \\ &e_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i}) \end{aligned}\right\}. \tag{7}$$

The volume dilation e takes the form

$$e = e_{xx} + e_{yy} + e_{zz} = \frac{\partial w}{\partial z}\,. \tag{8}$$

The stress components are given by:

$$\left.\begin{aligned} &\sigma_{xx} = \lambda e - \gamma(T - T_0) \\ &\sigma_{yy} = \lambda e - \gamma(T - T_0) \\ &\sigma_{zz} = 2\mu\,\frac{\partial w}{\partial z} + \lambda e - \gamma(T - T_0) \end{aligned}\right\}, \tag{9}$$

where

$$\left.\begin{aligned} &\sigma_{xy} = 0 \\ &\sigma_{xz} = 0 \\ &\sigma_{yz} = 0 \end{aligned}\right\}. \tag{10}$$

The equation of motion (1) will be reduces to

$$\sigma_{zz,z} + \sigma_{xz,x} + \sigma_{yz,y} = \rho\,\ddot{u}_z\,. \tag{11}$$

Substituting from (9) and (10) into the last equation and using $\theta = T - T_0$ we get,

$$(2\mu + \lambda)\,\frac{\partial^2 w}{\partial z^2} - \gamma\,\frac{\partial \theta}{\partial z} = \rho\,\frac{\partial^2 w}{\partial t^2}\,, \tag{12}$$

where θ is the temperature change above a reference temperature T_0. Differentiating (12) with respect to z and using (8), we obtain

$$(2\mu + \lambda)\,\frac{\partial^2 e}{\partial z^2} - \gamma\,\frac{\partial^2 \theta}{\partial z^2} = \rho\,\frac{\partial^2 e}{\partial t^2}\,. \tag{13}$$

The energy equation can be written in the form:

$$\left.\begin{aligned} &\left(\frac{\partial}{\partial t} + t_0\,\frac{\partial^2}{\partial t^2}\right)(\rho\,c_E\,\theta + \gamma T_0\,e) = k\nabla^2\,T \\ &\nabla^2 \equiv \frac{\partial^2}{\partial z^2} \end{aligned}\right\}. \tag{14}$$

For convenience, the following non-dimensional quantities are introduced

$$\left.\begin{aligned} &z^* = c_1\eta z\,, \quad w^* = c_1\eta w\,, \quad t^* = c_1^2\eta t \\ &t_0^* = c_1^2\eta t_0\,, \quad \sigma_{ij}^* = \frac{\sigma_{ij}}{\mu}\,, \quad \theta^* = \frac{T - T_0}{T_0} \\ &\eta = \frac{\rho\,c_E}{k}\,, \quad c_1^2 = \sigma_{ij}^* = \frac{\lambda + 2\mu}{\rho} \end{aligned}\right\}. \tag{15}$$

Substituting from (15) into (12) we get after dropping the asterisks and adopting straight forward manipulation

$$\left.\begin{aligned} &\nabla^2 e - g_1\nabla^2\theta = \frac{\partial^2 e}{\partial t^2} \\ &\nabla^2\theta = \left(\frac{\partial}{\partial t} + t_0\,\frac{\partial^2 e}{\partial t^2}\right)(\theta + g_2 e) \end{aligned}\right\}, \tag{16}$$

where $g_1 = \frac{\gamma T_0}{(2\mu+\lambda)}$ and $g_2 = \frac{\gamma}{\rho\,c_E}$.

Substituting from (15) into (9) we get,

$$\left.\begin{aligned} &\sigma_{xx} = \sigma_{yy} = \beta e - \lambda_1\theta \\ &\sigma_{zz} = \alpha e - \lambda_1\theta \end{aligned}\right\}, \tag{17}$$

where $\alpha = \frac{\gamma(2\mu+\lambda)}{\mu}$, $\beta = \frac{\lambda}{\mu}$ and $\lambda_1 = \frac{\gamma T_0}{\mu}$. We now introduce the Laplace transform defined by the formula:

$$\bar{f}(z,s) = \int_0^\infty e^{-st} f(z,t)\,dt\,. \tag{18}$$

Applying (18) to both sides of equation (16) we get,

$$(\nabla^2 - s^2)\,\bar{e} - g_1\nabla^2\,\bar{\theta} = 0\,, \tag{19}$$

$$(\nabla^2 - s(1 + t_0 s))\,\bar{\theta} - s\,(1 + t_0 s)\,g_2\,\bar{e} = 0\,. \tag{20}$$

Eliminating $\bar{\theta}$ and $\bar{e}$ between equation (19) and (20) we get the following fourth-order differential equations satisfied by $\bar{e}$ and $\bar{\theta}$; respectively

$$(\nabla^4 - A\nabla^2 + C)\,\bar{e} = 0\,, \tag{21}$$

$$(\nabla^4 - A\nabla^2 + C)\,\bar{\theta} = 0\,, \tag{22}$$

with $A = s^2 + s(1+t_0 s)(1+g_1 g_2)$ and $C = s^3(1+t_0 s)$. One can solve these fourth order ordinary differential equations by using e^{-kz} and finding the roots of the inditial equation

$$k^4 - Ak^2 + C = 0\,, \tag{23}$$

suppose that k_i $(i = 1, 2)$ are the positive roots, then the solution of (23) for $z \geqslant 0$ and $k_i > 0$ are; respectively

$$\bar{e}(z, s) = \sum_{i=1}^{2} A_i \, e^{-k_i z} \tag{24}$$

and

$$\bar{\theta}(z, s) = \sum_{i=1}^{2} A_i' \, e^{-k_i z}, \tag{25}$$

where $A_i = A_i(s)$ and $A_i' = A_i'(s)$ are some parameters depending only on s and k_i are functions of s. Substituting by (24) and (25) into (20) we get the relation,

$$A_i' = \frac{s(1+t_0 s) g_2}{k_i^2 - s(1+t_0 s)} A_i, \tag{26}$$

while Laplace transform of Equation (8) and integration w.r.t. z we obtain

$$\bar{w}(z, s) = -\sum_{i=1}^{2} \frac{A_i}{k_i} e^{-k_i z}. \tag{27}$$

Substituting from Equation (24) and Equation (26) into (17) we get the stresses,

$$\begin{aligned}\sigma_{xx} = \sigma_{yy} = \sum_{i=1}^{2} A_i e^{-k_i z} \times \\ \times \frac{\beta(k_i^2 - s(1+t_0 s)) - s(1+t_0 s)\lambda_1 g_2}{k_i^2 - s(1+t_0 s)}.\end{aligned} \tag{28}$$

$$\begin{aligned}\sigma_{zz} = \sum_{i=1}^{2} A_i e^{-k_i z} \times \\ \times \frac{\alpha(k_i^2 - s(1+t_0 s)) - s(1+t_0 s)\lambda_1 g_2}{k_i^2 - s(1+t_0 s)}.\end{aligned} \tag{29}$$

Therefore it is easy to determine A_i and A_i' for $i = 1, 2$

$$A_1 = \frac{-A_0 q_0 (k_1^2 - s(1+t_0 s)) B_1(s)}{g_2 s(1+t_0 s)[-k_1 B_2(s) + k_2 B_3(s)]}, \tag{30}$$

$$A_2 = \frac{A_0 q_0 (k_2^2 - s(1+t_0 s)) B_1(s)}{g_2 s(1+t_0 s)[-k_1 B_2(s) + k_2 B_3(s)]}, \tag{31}$$

$$A_1' = \frac{-A_0 q_0 B_1(s)}{[-k_1 B_2(s) + k_2 B_3(s)]}, \tag{32}$$

$$A_2' = \frac{A_0 q_0 B_1(s)}{[-k_1 B_2(s) + k_2 B_3(s)]}, \tag{33}$$

where $B_1(s) = \alpha(k_2^2 - s(1+t_0 s))(\alpha + \lambda_1 g_2)$, $B_2(s) = \alpha k_2^2 - s(1+t_0 s)(\alpha + \lambda_1 g_2)$, and also $B_3(s) = \alpha k_1^2 - s(1+t_0 s)(\alpha + \lambda_1 g_2)$.

3 Small time approximation

We now determine inverse transforms for the case of small values of time (large values of s). This method was used by Hetnarski [6] to obtain the fundamental solution for the coupled thermelasticity problem and by Sherief [7] to obtain the fundamental solution for generalized thermoelasticity with two relaxation times for point source of heat. k_1 and k_2 are the positive roots of the characteristic equation (23), given by

$$\begin{aligned}k_1 = \Bigg(\frac{s}{2}\Big[s + (1+t_0 s)(1+\epsilon) + \\ + \sqrt{s^2 + 2s(\epsilon-1)(1+t_0 s) + (1+t_0 s)^2(1+\epsilon)^2}\Big]\Bigg)^{\frac{1}{2}},\end{aligned} \tag{34}$$

$$\begin{aligned}k_2 = \Bigg(\frac{s}{2}\Big[s + (1+t_0 s)(1+\epsilon) + \\ - \sqrt{s^2 + 2s(\epsilon-1)(1+t_0 s) + (1+t_0 s)^2(1+\epsilon)^2}\Big]\Bigg)^{\frac{1}{2}},\end{aligned} \tag{35}$$

where $\epsilon = g_1 g_2 = \frac{\alpha_t^2 (3\lambda + 2\mu)^2 T_0}{\rho c_E (2\mu + \lambda)}$. Setting $v = \frac{1}{s}$, equations (34) and (35) can be expressed in the following from

$$k_i = v^{-1} [f_i(v)]^{\frac{1}{2}}, \qquad i = 1, 2, \tag{36}$$

where

$$\begin{aligned}f_1(v) = \frac{1}{2}\Big[1 + (v+t_0)(1+\epsilon) + \\ + \sqrt{1 + 2(\epsilon-1)(v+t_0) + (v+t_0)^2(1+\epsilon)^2}\Big],\end{aligned} \tag{37}$$

$$\begin{aligned}f_2(v) = \frac{1}{2}\Big[1 + (v+t_0)(1+\epsilon) - \\ - \sqrt{1 + 2(\epsilon-1)(v+t_0) + (v+t_0)^2(1+\epsilon)^2}\Big].\end{aligned} \tag{38}$$

Expanding $f_1(v)$ and $f_2(v)$ in the Maclaurin series around $v = 0$ and consider only the first four terms, can be written $f_i(v)$ $(i = 1, 2)$ as

$$f_i(v) = a_{i0} + a_{i1} v + a_{i2} v^2 + a_{i3} v^3, \qquad i = 1, 2, \tag{39}$$

where the coefficients of the first four terms are given by

$$\left.\begin{aligned}
a_{10} &= \frac{1 + (1+\epsilon)\, t_0 + \sqrt{1 + 2(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0^2}}{2} \\
a_{20} &= \frac{1 + (1+\epsilon)\, t_0 - \sqrt{1 + 2(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0^2}}{2} \\
a_{11} &= \frac{1}{2}\left[(1+\epsilon) - \frac{(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0}{\sqrt{1 + 2(\epsilon-1) t_0 + (1+\epsilon)^2\, t_0^2}}\right] \\
a_{21} &= \frac{1}{2}\left[(1+\epsilon) + \frac{(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0}{\sqrt{1 + 2(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0^2}}\right] \\
a_{12} &= \frac{\epsilon}{[1 + 2(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0^2]^{\frac{3}{2}}} \\
a_{22} &= -\frac{\epsilon}{[1 + 2(\epsilon-1)\, t_0 + (1+\epsilon)^2\, t_0^2]^{\frac{3}{2}}} \\
a_{13} &= \frac{-\epsilon(-1 + \epsilon + (1+\epsilon)^2\, t_0)}{[1 + 2(\epsilon-1) t_0 + (1+\epsilon)^2\, t_0^2]^{\frac{3}{2}}} \\
a_{23} &= \frac{\epsilon(-1 + \epsilon + (1+\epsilon)^2\, t_0)}{[1 + 2(\epsilon-1) t_0 + (1+\epsilon)^2\, t_0^2]^{\frac{3}{2}}}
\end{aligned}\right\}. \tag{40}$$

Next, we expand $[f_i(v)]^{\frac{1}{2}}$ in the Maclaurin series around $v = 0$ and retaining the first three terms, we obtain finally the expressions for k_1 and k_2 which can be written in the form

$$k_i = v^{-1}\left(b_{i0} + b_{i1}v + b_{i2}v^2\right), \qquad i = 1, 2\,, \tag{41}$$

where

$$b_{i0} = \sqrt{a_{i0}}\,,$$

$$b_{i1} = \frac{a_{i1}}{2\sqrt{a_{i0}}}\,,$$

and

$$b_{i2} = \frac{1}{8a_{i0}^{\frac{3}{2}}(9a_{i2}a_{i0} - a_{i0}^2)}\,.$$

Consider k_i to be written as

$$k_i = b_{i0}s + b_{i1}, \quad i = 1, 2\,. \tag{42}$$

Applying Maclaurin series expansion around $v = 0$ of the following expressions;

$$\frac{1}{k_i}A_i\,, \qquad \frac{s(1+t_0 s)g_2}{k_i^2 - s(1+t_0 s)}A_i\,,$$

$$\left[\frac{\beta(k_i^2 - s(1+t_0 s)) - s(1+t_0 s)\lambda_1 g_2}{k_i^2 - s(1+t_0 s)}\right]A_i\,,$$

$$\left[\frac{\alpha(k_i^2 - s(1+t_0 s)) - s(1+t_0 s)\lambda_1 g_2}{k_i^2 - s(1+t_0 s)}\right]A_i\,,$$

$$i = 1, 2\,.$$

We find that $\bar\theta$, $\bar w$, $\bar\sigma_{xx}$, $\bar\sigma_{yy}$, and $\bar\sigma_{zz}$ can be written in the following form

$$\begin{aligned}\bar\theta = &\left(\frac{c_{\theta 0}}{s} + \frac{c_{\theta 1}}{s^2} + \frac{c_{\theta 2}}{s^3}\right)e^{-k_1 z} + \\ &+ \left(\frac{c_{\theta 3}}{s} + \frac{c_{\theta 4}}{s^2} + \frac{c_{\theta 5}}{s^3}\right)e^{-k_2 z}\,,\end{aligned} \tag{43}$$

$$\begin{aligned}\bar w = &\left(\frac{c_{w0}}{s^2} + \frac{c_{w1}}{s^3} + \frac{c_{w2}}{s^4}\right)e^{-k_1 z} + \\ &+ \left(\frac{c_{w3}}{s^2} + \frac{c_{w4}}{s^3} + \frac{c_{w5}}{s^4}\right)e^{-k_2 z},\end{aligned} \tag{44}$$

$$\begin{aligned}\bar\sigma_{xx} = \bar\sigma_{yy} = &\left(\frac{c_{\sigma 0}}{s} + \frac{c_{\sigma 1}}{s^2} + \frac{c_{\sigma 2}}{s^3}\right)e^{-k_1 z} + \\ &+ \left(\frac{c_{\sigma 3}}{s} + \frac{c_{\sigma 4}}{s^2} + \frac{c_{\sigma 5}}{s^3}\right)e^{-k_2 z},\end{aligned} \tag{45}$$

$$\begin{aligned}\bar\sigma_{zz} = &\left(\frac{c_{z0}}{s} + \frac{c_{z1}}{s^2} + \frac{c_{z2}}{s^3}\right)e^{-k_1 z} + \\ &+ \left(\frac{c_{z3}}{s} + \frac{c_{z4}}{s^2} + \frac{c_{z5}}{s^3}\right)e^{-k_2 z},\end{aligned} \tag{46}$$

where

$$\left.\begin{aligned}
c_{\theta 0} &= \frac{y_1}{f_0} = 0.00002466\\
c_{\theta 1} &= \frac{y_2}{f_0} - \frac{f_1 y_1}{f_0^2} = 0.000666\\
c_{\theta 2} &= \frac{y_3}{f_0} + \frac{f_1^2 y_1}{f_0^3} - \frac{f_2 y_1 + f_1 y_2}{f_0^2} = -0.911471\\
c_{\theta 3} &= \frac{y_1^*}{f_0} = 0.705\\
c_{\theta 4} &= \frac{y_2^*}{f_0} - \frac{f_1 y_1^*}{f_0^2} = -1.7696\\
c_{\theta 5} &= \frac{y_3^*}{f_0} + \frac{f_1^2 y_1^*}{f_0^3} - \frac{f_2 y_1^* + f_1 y_2^*}{f_0^2} = 50.6493\\
c_{w0} &= \frac{A_1}{R_0} = -0.0007519\\
c_{w1} &= -\frac{R_1 A_1}{R_0^2} + \frac{A_2}{R_0} = 0.18\\
c_{w2} &= \frac{R_1^2 A_1}{R_0^3} - \frac{R_2 A_1 + R_1 A_1}{R_0^2} + \frac{A_3}{R_0} = 26.90\\
c_{w3} &= \frac{A_1^*}{R_0} = 0.000106\\
c_{w4} &= \frac{-R_1^* A_1^*}{R_0^{*2}} + \frac{A_2^*}{R_0^*} = -0.000493\\
c_{w5} &= \frac{R_1^{*2} A_1^*}{R_0^{*3}} - \frac{R_2^* A_1^* + R_1^* A_1^*}{R_0^{*2}} + \frac{A_3^*}{R_0^*} = 194.0138\\
c_{\sigma 0} &= \frac{x_1}{\eta_1} = 0.001511\\
c_{\sigma 1} &= \frac{x_2}{\eta_1} - \frac{\eta_2 x_1}{\eta_1^2} = -0.03623\\
c_{\sigma 2} &= \frac{x_1}{\eta_1} - \frac{\eta_2 x_2}{\eta_1^2} - \frac{x_1 \eta_3}{\eta_1^2} = -54.064\\
c_{\sigma 3} &= \frac{x_1^*}{\eta_1} = -0.002985\\
c_{\sigma 4} &= \frac{x_2^*}{\eta_1} - \frac{\eta_2 x_1^*}{\eta_1^2} = 0.07314\\
c_{\sigma 5} &= \frac{x_1^*}{\eta_1} - \frac{\eta_2 x_2^*}{\eta_1^2} - \frac{x_1^* \eta_3}{\eta_1^2} = 53.02\\
c_{z0} &= \frac{L_1}{\eta_1} = 0.003015\\
c_{z1} &= \frac{L_2}{\eta_1} - \frac{\eta_2 L_1}{\eta_1^2} = -0.0722\\
c_{z2} &= \frac{L_1}{\eta_1} - \frac{\eta_2 L_2}{\eta_1^2} - \frac{L_1 \eta_3}{\eta_1^2} = -107.88\\
c_{z3} &= \frac{L_1^*}{\eta_1} = -0.003\\
c_{z4} &= \frac{L_2^*}{\eta_1} - \frac{\eta_2 L_1^*}{\eta_1^2} = 0.0722\\
c_{z5} &= \frac{L_1^*}{\eta_1} - \frac{\eta_2 L_2^*}{\eta_1^2} - \frac{L_1^* \eta_3}{\eta_1^2} = 107.88
\end{aligned}\right\}\,. \tag{47}$$

From equation (39), we obtain

$$e^{-k_1 z} = e^{-(b_{10}s+b_{11})z} = e^{-b_{11}z}e^{-b_{10}sz}\,,$$

and

$$e^{-k_2 z} = e^{-(b_{20}s+b_{21})z} = e^{-b_{21}z}e^{-b_{20}sz}.$$

Applying the inverse Laplace transform for equations (43, 44, 45, 46) we get θ, w, σ_{xx}, σ_{yy} and σ_{zz} in the following form

$$\theta = e^{-b_{11}z}\,\Theta_1 H(t-b_{10}z) + e^{-b_{21}z}\,\Theta_2 H(t-b_{20}z)\,, \tag{48}$$

where

$$\Theta_1 = \left[c_{\theta 0} + c_{\theta 1}(t-b_{10}z) + \frac{c_{\theta 2}}{2}(t-b_{10}z)^2\right],$$

$$\Theta_2 = \left[c_{\theta 3} + c_{\theta 4}(t-b_{20}z) + \frac{c_{\theta 5}}{2}(t-b_{20}z)^2\right],$$

and also

$$w = e^{-b_{11}z}\,W_1 H(t-b_{10}z) + e^{-b_{21}z}\,W_2 H(t-b_{20}z)\,, \tag{49}$$

where

$$W_1 = \left[c_{w0}(t-b_{10}z) + \frac{c_{w1}(t-b_{10}z)^2}{2} + \frac{c_{w2}(t-b_{10}z)^3}{6}\right],$$

$$W_2 = \left[c_{w3}(t-b_{20}z) + \frac{c_{w4}(t-b_{20}z)^2}{2} + \frac{c_{w5}(t-b_{20}z)^3}{6}\right],$$

and also

$$\sigma_{xx} = \sigma_{yy} = \\ = e^{-b_{11}z}\,\Sigma_1 H(t-b_{10}z) + e^{-b_{21}z}\,\Sigma_2 H(t-b_{20}z)\,, \tag{50}$$

where

$$\Sigma_1 = \left[c_{\sigma 0} + c_{\sigma 1}(t-b_{10}z) + c_{\sigma 2}\frac{(t-b_{10}z)^2}{2}\right],$$

$$\Sigma_2 = \left[c_{\sigma 3} + c_{\sigma 4}(t-b_{20}z) + c_{\sigma 5}\frac{(t-b_{20}z)^2}{2}\right],$$

and also

$$\sigma_{zz} = e^{-b_{11}z} Z_1 H(t-b_{10}z) + e^{-b_{21}z} Z_2 H(t-b_{20}z)\,, \tag{51}$$

where

$$Z_1 = \left[c_{z0} + c_{z1}(t-b_{10}z) + c_{z2}\frac{(t-b_{10}z)^2}{2}\right],$$

$$Z_2 = \left[c_{z3} + c_{z4}(t-b_{20}z) + c_{z5}\frac{(t-b_{20}z)^2}{2}\right],$$

and $H(t-b_{i0}z)$ is Heaviside's unit step functions. By using the convolution theorem $h(t) = \int_0^t f(\tau)g(t-\tau)d\tau$ for (48), (49), (50) and (51) we obtain under the assumption that $f(\tau) = e^{-\frac{(t_b-\tau)^2}{\varphi^2}}$; which represents the time behavior of the intensity of the laser radiation, where t_b is the time at which $f(\tau)$ has maximum. Here φ is the time at which the intensity of the laser radiation reduces to $\frac{1}{e}$

$$\begin{aligned}\theta = e^{-b_{11}z}&\left[\left(c_{\theta 0}+c_{\theta 1}(t-b_{10}z)+c_{\theta 2}\frac{(t-b_{10}z)^2}{2}+\right.\right.\\ &+c_{\theta 2}\frac{(t-b_{10}z)^2}{2}+\left.\frac{\varphi^2 c_{\theta 2}}{4}\right)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-c_{\theta 2}\frac{t\varphi}{4}e^{-\frac{t^2}{\varphi^2}}+\\ &+\left.(c_{\theta 1}+c_{\theta 2}(t-b_{10}z))\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)\right]+\\ &+e^{-b_{21}z}\left[\left(c_{\theta 3}+c_{\theta 4}(t-b_{10}z)+c_{\theta 5}\frac{(t-b_{10}z)^2}{2}+\right.\right.\\ &+\left.\frac{\varphi^2 c_{\theta 5}}{4}\right)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-c_{\theta 2}\frac{t\varphi}{4}e^{-\frac{t^2}{\varphi^2}}+\\ &+\left.(c_{\theta 4}+c_{\theta 5}(t-b_{10}z))\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)\right],\end{aligned} \tag{52}$$

$$\begin{aligned}w = e^{-b_{11}z}&\left[\left(c_{w0}(t-b_{10}z)+\frac{c_{w1}}{2}\left((t-b_{10}z)^2+\frac{\varphi^2}{2}\right)+\right.\right.\\ &+\left.c_{w2}\left(\frac{(t-b_{10}z)^3}{6}+\frac{\varphi^2}{4}\right)\right)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-\\ &-\left(c_{w0}+c_{w1}(t-b_{10}z)-\frac{\varphi^2 c_{w2}}{12}\left(\varphi^2-(t^2-\varphi^2)e^{-\frac{t^2}{\varphi^2}}\right)+\right.\\ &+\left.\frac{c_{w2}}{2}(t-b_{10}z)^2\right)\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)-\\ &-\left.\frac{1}{4}(c_{w1}+c_{w2}(t-b_{10}z))t\varphi^2 e^{-\frac{t^2}{\varphi^2}}\right]+\\ &+e^{-b_{21}z}\left[\left(c_{w3}(t-b_{20}z)+\frac{c_{w4}}{2}\left((t-b_{10}z)^2+\frac{\varphi^2}{2}\right)+\right.\right.\\ &+\left.c_{w5}\left(\frac{(t-b_{10}z)^3}{6}+\frac{\varphi^2}{4}\right)\right)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-\\ &-\left(c_{w3}+c_{w4}(t-b_{10}z)-\frac{\varphi^2 c_{w5}}{12}\left(\varphi^2-(t^2-\varphi^2)e^{-\frac{t^2}{\varphi^2}}\right)+\right.\\ &+\left.\frac{c_{w5}}{2}(t-b_{10}z)^2\right)\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)-\\ &-\left.\frac{1}{4}(c_{w4}+c_{w5}(t-b_{10}z))t\varphi^2 e^{-\frac{t^2}{\varphi^2}}\right],\end{aligned} \tag{53}$$

$$\begin{aligned}\sigma_{zz} = e^{-b_{11}z}&\left[\left(c_{z0}+c_{z1}(t-b_{10}z)+\frac{c_{z2}}{2}(t-b_{10}z)^2+\right.\right.\\ &+\left.\frac{\varphi^2 c_{z2}}{4}\right)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-\frac{c_{z2}}{4}t\varphi e^{-\frac{t^2}{\varphi^2}}+\\ &+\left.(c_{z1}+c_{z2}(t-b_{10}z)\right)\frac{\varphi^2}{2}\left.\left(1-e^{-\frac{t^2}{\varphi^2}}\right)\right]+\\ &+e^{-b_{11}z}\left[\left(c_{z3}+c_{z4}(t-b_{10}z)+\frac{c_{z5}}{2}(t-b_{10}z)^2+\right.\right.\\ &+\left.\frac{\varphi^2 c_{z5}}{4}\right)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-\frac{c_{z5}}{4}t\varphi e^{-\frac{t^2}{\varphi^2}}+\\ &+\left.(c_{z4}+c_{z5}(t-b_{10}z))\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)\right],\end{aligned} \tag{54}$$

$$\sigma_{xx} = \sigma_{yy} = e^{-b_{11}z}\Bigg[\Bigg(c_{\sigma 0}+c_{\sigma 1}(t-b_{10}z)+ \\ + \frac{c_{\sigma 2}}{2}(t-b_{10}z)^2+\frac{\varphi^2 c_{\sigma 2}}{4}\Bigg)\frac{\sqrt{\pi}}{2}\varphi erf\left(\frac{t}{\varphi}\right)-\frac{c_{\sigma 2}\varphi}{4}te^{-\frac{t^2}{\varphi^2}}+ \\ +(c_{\sigma 1}+c_{\sigma 2}(t-b_{10}z))\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)\Bigg]+ \\ +e^{-b_{21}z}\Bigg[\Bigg(c_{\sigma 3}+c_{\sigma 4}(t-b_{20}z)+\frac{c_{\sigma 5}}{2}((t-b_{10}z)^2+ \\ +\frac{c_{\sigma 5}\varphi^2}{4}\Bigg)\varphi\frac{\sqrt{\pi}}{2}erf\left(\frac{t}{\varphi}\right)-\frac{\varphi c_{\sigma 5}}{4}te^{-\frac{t^2}{\varphi^2}}+ \\ +(c_{\sigma 4}+c_{\sigma 5}(t-b_{10}z))\frac{\varphi^2}{2}\left(1-e^{-\frac{t^2}{\varphi^2}}\right)\Bigg]. \quad (55)$$

4 Computation and discussions

We have calculated the spatial temperature, displacement and stress θ, w, σ_{xx}, σ_{yy} and σ_{zz} with the time as a parameter for a heated target with a spatial homogeneous laser radiation having a temporally Gaussian distributed intensity with a width of (10E-3 s). We have performed the computation for the physical parameters $T_0 = 293$ K, $\rho = 8954$ Kg/m^3,

$$A = 0.01, \quad c_E = 383.1 \text{ J/kgK},$$

$$\varphi = 10^{-3} \text{ s}, \quad \epsilon = g_1 g_2 = 0.01680089,$$

$$\alpha_t = 1.78(10^{-5}) \text{ K}^{-1}, \quad k = 386 \text{ W/mK},$$

$$\lambda = 7.76(10^{10}) \text{ kg/m sec}^2, \quad \mu = 3.86(10)^{10} \text{ kg/m sec}^2$$

and

$$t_0 = 0.02 \text{ sec}$$

for Cu as a target. We obtain the results displayed in the following figures.

Considering surface absorption the obtained results in Figure 1 show the temperature θ, Figure 2 display the temporal temperature distribution and the temporal behavior of the laser radiation, Figure 3 for the displacement w, Figure 4 for the stress σ_{zz} and Figure 5 for the stresses σ_{xx} and σ_{yy}.

The coupled system of differential equations describing the thermoelasticity treated through the Laplace transform of a temporally Dirac distributed laser radiation illuminating homogeneous a semi-infinite target and absorbed at its irradiated surface. Since the system is linear the response of the system on the Dirac function was convoluted with a temporally Gaussian distributed laser radiation. The theoretical obtained results were applied on the Cu target. Figure 1 illustrates the calculated spatial distribution of the temperature per unit intensity at different values of the time parameter ($t = 0.005, 0.007, 0.01, 0.015$, and 0.02). From the curves it is evident that the temperature has a finite velocity expressed through the strong gradient of the temperature which moves deeper in the target as the time increases.

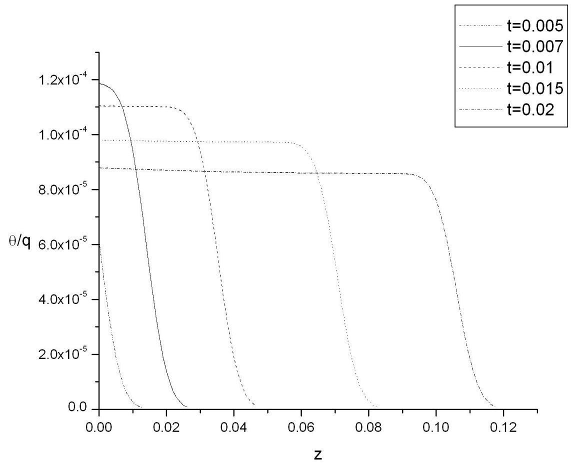

Fig. 1: The temperature distribution θ per unit intensity versus z with the time as a parameter.

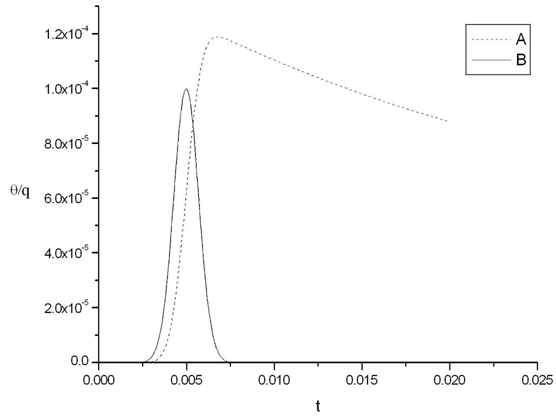

Fig. 2: (A) The temporal temperature distribution θ per unit intensity form the. (B) The temporal behavior of the laser radiation which is assumed to have a Gaussian distribution with width $\varphi = 10^{-3} s$.

Figure 2 represent the calculated front temporal temperature distribution per unit intensity (curve A); as a result of the temporal behavior of the laser radiation which is assumed to have a Gaussian distribution with a width equals to (10E-3 s) (curve B). From the figure it is evident that the temperature firstly increases with increasing the time this can be attributed to the increased absorbed energy which over compensates the heat losses given by the heat conductivity inside the material. As the absorbed power equals the conducted one inside the material the temperature attains its maximum value. the maximum of the temperature occurs at later time than the maximum of the radiation this is the result of the heat conductivity of Cu and the relatively small gradient of the temperature in the vicinity of $z = 0$ as seen from Figure 1. After the radiation becomes week enough such that it can not compensate the diffused power inside the material the temperature decreases monotonically with increasing time.

Figure 3 shows the calculated spatial displacement per unit intensity at different times($0.01, 0.015$ and 0.02). The

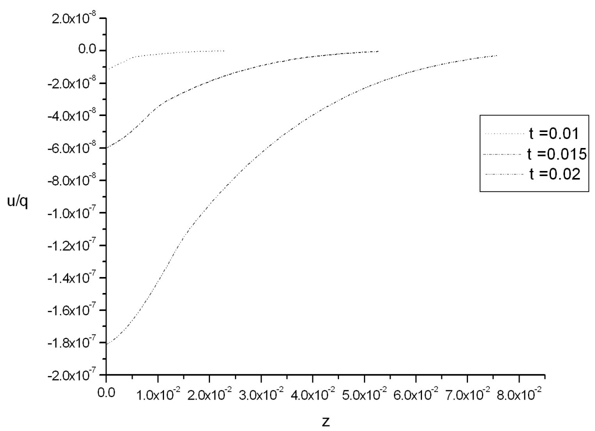

Fig. 3: The displacement distribution u per unit intensity versus z with the time as a parameter.

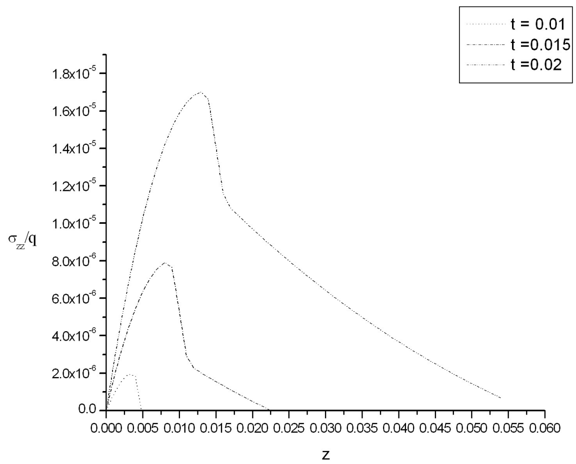

Fig. 4: The stress σ_{zz} distribution per unit intensity versus z with the time as a parameter.

displacement increases monotonically with time. It attains smaller gradient with increasing z. Both effects can be attributed to the temperature behavior. The negative displacement results from the co-ordinate system which is located at the front surface with positive direction of the z-axis pointing down words.

Figure 4 illustrates the spatial distribution of stress σ_{zz} per unit intensity at the times (0.01, 0.015 and 0.02). Since, $\sigma_{zz} = \alpha e - \lambda_1 \theta$, thus from Figure 3 σ_{zz} attains maxima at the locations for which the gradient of the displacement exhibits maxima and this is practically at the same points for which σ_{zz} is maximum. The calculations showed that σ_{xx} and σ_{yy} have the same behavior as σ_{zz}.

5 Results and conclusions

The thermoelasticity problem formulated by a coupled linear system of partial differential equations was discussed. The system was decoupled to provide a fourth order linear differential equations which were solved analytically using Laplace

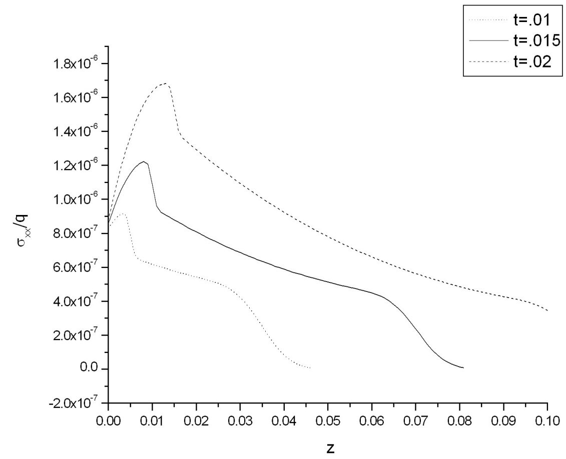

Fig. 5: The stress distribution σ_{xx} and σ_{yy} per unit intensity versus z with the time as a parameter.

transform. The small time approximation analysis was performed for the solution of temperature, displacement and for the stresses; showing that the finite velocity of the temperature described by the D.Es system was not affected by the small time approximation.

Acknowledgements

The authors want to thanks Dr. Ezzat F. Honin for his valuable discussions and comments.

Submitted on June 25, 2008 / Accepted on August 15, 2008

References

1. Biot M. Thermoelasticity and irreversible thermo-dynamics. *J. Appl. Phys.*, 1956, v. 27, 240–253.
2. Lord H. and Shulman Y. A generalized dynamical theory of thermoelasticity. *J. Mech. Phys. Solid.*, 1967, v. 15, 299–309.
3. Bondi H. Negative mass in General Relativity. *Review of Modern Physics*, 1957, v. 29 (3), 423–428.
4. Dhaliwal R. and Sherief H. Generalized thermoelasticity for anisotropic media. *Quatr. Appl. Math.*, 1980, v. 33, 1–8.
5. Hassan A.F., El-Nicklawy M.M., El-Adawi M.K., Nasr E.M., Hemida A.A. and Abd El-Ghaffar O.A. Heating effects induced by a pulsed laser in a semi-infinite target in view of the theory of linear systems. *Optics & Laser Technology*, 1996, v. 28 (5), 337–343.
6. Hetnarski R. Coupled one-dimensional thermal shock problem for small times. *Arch. Mech. Stosow.*, 1961, v. 13, 295–306.
7. Sherief H. and Dhaliwal R. Generalized one-dimensional thermal shock problem for small times. *J. Thermal Stresses.*, 1981, v. 4, 407–420.