

The Length of the Day: A Cosmological Perspective

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We have found an empirical law for the variation of the length of the Earth's day with geologic time employing Wells's data. We attribute the lengthening of the Earth's day to the present cosmic expansion of the Universe. The prediction of law has been found to be in agreement with the astronomical and geological data. The day increases at a present rate of 0.002 sec/century. The length of the day is found to be 6 hours when the Earth formed. We have also found a new limit for the value of the Hubble constant and the age of the Universe.

1 Introduction

According to Mach's principle the inertia of an object is not a mere property of the object but depends on how much matter around the object. This means that the distant universe would affect this property. Owing to this, we would expect a slight change in the strength of gravity with time. This change should affect the Earth-Moon-Sun motion. It is found that the length of the day and the number of days in the year do not remain constant. From coral fossil data approximately 400 million years (m.y.) ago, it has been estimated that there were little over 400 days in a year at that time. It is also observed that the Moon shows an anomalous acceleration (Dickey, 1994 [1]). As the universe expands more and more matter appears in the horizon. The expansion of the universe may thus have an impact on the Earth-Moon-Sun motion. Very recently, the universe is found to be accelerating at the present time (Peebles, 1999 [2], Bahcall *et al.*, 1999 [3]). To account for this scientists suggested several models. One way to circumvent this is to allow the strength of gravity to vary slightly with time (Arbab, 2003 [4]). For a flat universe, where the expansion force is balanced by gravitational attraction force, this would require the universe to accelerate in order to avoid a future collapse. This can be realized if the strength of the gravitational attraction increases with time (Arbab, 1997 [5], 2003 [4]), at least during the present epoch (matter dominated). One appropriate secure way to do this is to define an effective Newton's constant, which embodies this variation while keeping the "bare" Newton's constant unchanged. The idea of having an effective constant, which shows up when a system is interacting with the outside world, is not new. For instance, an electron in a solid moves not with its "bare" mass but rather with an effective mass. This effective mass exhibits the nature of interaction in question. With the same token, one would expect a celestial object to interact (couple) with its effective constant rather than the normal Newton's constant, which describes the strength of gravity in a universe with constant mass. We, therefore, see that the ex-

pansion of the universe affects indirectly (through Newton's constant) the evolution of the Earth-Sun system. Writing an effective quantity is equivalent to having summed all perturbations (gravitational) affecting the system. With this minimal change of the ordinary Newton's constant to an effective one, one finds that Kepler's laws can be equally applicable to a perturbed or an unperturbed system provided the necessary changes are made. Thus one gets a rather modified Newton's law of gravitation and Kepler's laws defined with this effective constant while retaining their usual forms. In the present study, we have shown that the deceleration of the Earth rotation is, if not all, mainly a cosmological effect. The tidal effects of the Earth deceleration could, in principle, be a possible consequence, but the cosmological consequences should be taken seriously.

The entire history of the Earth has not been discovered so far. Very minute data are available owing to difficulties in deriving it. Geologists derived some information about the length of the day in the past from the biological growth rhythm preserved in the fossil records (e.g., bi-valves, corals, stromatolites, etc.). The first study of this type was made by the American scientist John Wells (1963 [6]), who investigated the variation of the number of days in the year from the study of fossil corals. He inferred, from the sedimentation layers of calcite made by the coral, the number of days in the year up to the Cambrian era. Due to the lack of a well-preserved records, the information about the entire past is severely hindered. The other way to discover the past rotation is to extrapolate the presently observed one. This method, however, could be very misleading.

2 The model

Recently, we proposed a cosmological model for an effective Newton's constant (Arbab, 1997 [5]) of the form

$$G_{\text{eff}} = G_0 \left(\frac{t}{t_0} \right)^\beta, \quad (1)$$

where the subscript “0” denotes the present value of the quantity: G_0 is the normal (bare) Newton’s constant and t_0 is the present age of the Universe. Here G_{eff} includes all perturbative effects arising from all gravitational sources. We remark here that G_0 does not vary with time, but other perturbations induce an effect that is parameterized in G_{eff} in the equation of motion. Thus, we don’t challenge here any variation in the normal Newton’s constant G_0 . We claim that such a variation can not be directly measured as recently emphasized by Robin Booth (2002 [7]). It can only be inferred from such analysis. We remark here that β is not well determined ($\beta > 0$) by the cosmological model. And since the dynamics of the Earth is determined by Newton’s law of gravitation any change in G would affect it. This change may manifest itself in various ways. The length of day may attributed to geological effects which are in essence gravitational. The gravitational interaction should be described by Einstein’s equations. We thus provide here the dynamical reasons for these geological changes. We calculate the total effect of expansion of the universe on the Earth dynamics.

The Kepler’s 2nd law of motion for the Earth-Sun system, neglecting the orbit eccentricity, can be written as

$$G_{\text{eff}}^2 [(M + m)^2 m^3] T_{\text{eff}} = 2\pi L_{\text{eff}}^3, \quad (2)$$

where m , M are the mass of the Earth and the Sun respectively; L_{eff} is the orbital angular momentum of the Earth and T_{eff} is the period (year) of the Earth around the Sun at any time in the past measured by the days in that time. T_{eff} defines the number of days (measured at a given time) in a year at the epoch in which it is measured. This is because the length of day is not constant but depends on the epoch in which it is measured. Since the angular momentum of the Earth about the Sun hasn’t changed, the length of the year does not change. We however measure the length of the year by the number of days which are not fixed. The length of the year in seconds (atomic time) is fixed. Thus one can still use Kepler’s law as in (2) (which generalizes Kepler’s laws) instead of adding other perturbations from the nearby bodies to the equation of motion of the Earth. We, however, incorporate all these perturbations in a single term, viz. G_{eff} . Part of the total effect of the increase of length of day could show up in geological terms. We calculate here the total values affecting the Earth dynamics without knowing exactly how much the contribution of each individual components.

The orbital angular momentum of the Earth (around the Sun) is nearly constant. From equation (2), one can write

$$T_{\text{eff}} = T_0 \left(\frac{G_0}{G_{\text{eff}}} \right)^2, \quad (3)$$

where $T_0 = 365$ days and $G_0 = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

Equations (1) and (3) can be written as

$$T_{\text{eff}} = T_0 \left(\frac{t_0}{t_0 - t_p} \right)^{2\beta}, \quad (4)$$

where t_0 is the age of the universe and t_p is the time measured from present time backward. This equation can be casted in the form

$$x = \ln \left(\frac{T_{\text{eff}}}{T_0} \right) = 2\beta \ln \left(\frac{t_0 - t_p}{t_0} \right), \quad (5)$$

or equivalently,

$$t_0 = \frac{t_p}{(1 - \exp(-x/2\beta))}. \quad (6)$$

To reproduce the data obtained by Wells for the number of days in a year (see Table. 1), one would require $\beta = 1.3$ and $t_0 \approx 11 \times 10^9$ years. This is evident since, from (Arbab, 2003 [4]) one finds the Hubble constant is related to the age of the Universe by the relation,

$$t_0 = \left(\frac{2 + \beta}{3} \right) H_0^{-1} = 1.1 H_0^{-1}, \quad (7)$$

and the effective Newton’s constant would vary as

$$G_{\text{eff}} = G_0 \left(\frac{t_0 - t_p}{t_0} \right)^{1.3}. \quad (8)$$

This is an interesting relation, and it is the first time relation that constrained the age of the Universe (or Hubble constant) from the Earth rotation. However, the recent Hipparcos satellite results (Chaboyer *et al.*, 1998 [8]) indicate that the age of the universe is very close to 11 billion years. Hence, this work represent an unprecedented confirmation for the age of the universe. One may attribute that the Earth decelerated rotation is mainly (if not only) due to cosmic expansion that shows up in tidal deceleration. Thus, this law could open a new channel for providing valuable information about the expansion of the Universe. The Hubble constant in this study amounts to $H_0 = 97.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$. However, the Hubble constant is considered to lie in the limit, $50 \text{ km s}^{-1} \text{ Mpc}^{-1} < H_0 < 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Higher values of H_0 imply a fewer normal matter, and hence a lesser dark matter. This study, therefore, provides an unprecedented way of determining the Hubble constant. Astronomers usually search into the space to collect their data about the Universe. This well determined value of β is crucial to the predictions of our cosmological model in Arbab, 2003 [4]. We notice that the gravitational constant is doubled since the Earth was formed (4.5 billion years ago).

From (3) and (8) one finds the effective number of days in the year (T_{eff}) to be

$$T_{\text{eff}} = T_0 \left(\frac{t_0}{t_0 - t_p} \right)^{2.6}, \quad (9)$$

and since the length of the year is constant, the effective length of the day (D_{eff}) is given by

$$D_{\text{eff}} = D_0 \left(\frac{t_0 - t_p}{t_0} \right)^{2.6}, \quad (10)$$

so that

$$T_0 D_0 = T_{\text{eff}} D_{\text{eff}}. \quad (11)$$

Time*	65	136	180	230	280	345	405	500	600
solar days/year	371.0	377.0	381.0	385.0	390.0	396.0	402.0	412.0	424.0

*Time is measured in million years (m.y.) before present.

Table 1: Data obtained from fossil corals and radiometric time (Wells, 1963 [6]).

Time*	65	136	180	230	280	345	405	500	600
solar days/year	370.9	377.2	381.2	385.9	390.6	396.8	402.6	412.2	422.6
length of solar day (hr)	23.6	23.2	23.0	22.7	22.4	22.1	21.7	21.3	20.7

Time*	715	850	900	1200	2000	2500	3000	3560	4500
solar days/year	435.0	450.2	456.0	493.2	615.4	714.0	835.9	1009.5	1434.0
length of solar day (hr)	20.1	19.5	19.2	17.7	14.2	12.3	10.5	8.7	6.1

*Time is measured in million years (m.y.) before present.

Table 2: Data obtained from our empirical law: equations (9) and (10).

We see that the variation of the length of day and month is a manifestation of the changing conditions (perturbation) of the Earth which are parameterized as a function of time (t) only. Thus, equation (7) guarantees that the length of the year remains invariant.

3 Discussion

The Wells's fossil data is shown in Table 1 and our corresponding values are shown in Table 2. In fact, the length of the year does not change, but the length of the day was shorter than now in the past. So, when the year is measured in terms of days it seems as if the length of the year varies. Sonett *et al.* (1996 [9]) have shown that the length of the day 900 m.y. ago was 19.2 hours, and the year contained 456 days. Our law gives the same result (see Table 2). Relying on the *law of spin isochronism* Alfvén and Arrhenius (1976 [10]) infer for the primitive Earth a length of day of 6 hours (p.226). Using coral as a clock, Poropudas (1991 [11], 1996 [12]) obtained an approximate ancient time formula based on fossil data. His formula shows that the number of days in the year is 1009.77 some 3.556 b.y. ago. Our law shows that this value corresponds rather to a time 3.56 b.y. ago, and that the day was 8.7 hours. He suggested that the day to be 5–7 hours at approximately 4.5 b.y. ago. Ksanfomality (1997 [13]) has shown that according to the principle of isochronism all planets had an initial period of rotation between 6–8 hours. However, our model gives a value of 6 hours (see Table 2). Berry and Baker (1968 [14]) have suggested that laminae, ridges and troughs, and bands on present day and Cretaceous bivalve shells are growth increments of the day and month, respectively. By counting the number of ridges and troughs they therefore find that the year contains 370.3 days in the late Cretaceous. According to the latest research by a group of Chinese scientists (Zhu *et al.* [15]), there were 15 hours in one day, more than 540 days, in a year from a study of *stromatolite* samples. We however remark that according to our law that when the day was 15 hours there were 583 days in a year 1.819 billion years

ago. The difference in time could be due to dating of their rock.

Recently, McNamara and Awramik (1992 [16]) have concluded, from the study of *stromatolite*, that at about 700 m.y. ago the number of days in a year was 435 days and the length of the day was 20.1 hours. In fact, our model shows that this value corresponds more accurately to 715 m.y. ago. Vanyo and Awramik (1985 [17]) has investigated *stromatolite*, that is 850 m.y. old, obtained a value between 409 and 485 days in that year. Our law gives 450 days in that year and 19.5 hours in that day. This is a big success for our law. Here we have gone over all data up to the time when the Earth formed. We should remark that this is the first model that gives the value of the length of the day for the entire geologic past time.

The present rate of increase in the length of the day is 0.002 m/s/century. Extrapolating this astronomically determined lengthening of the day since the seventeenth century leads to 371 days in the late Cretaceous (65 m.y. ago) Pannella (1972 [18]). The slowing down in the rotation is not uniform; a number of irregularities have been found. This conversion of Earth's rotational energy into heat by tidal friction will continue indefinitely making the length of the day longer. In the remote past the Earth must have been rotating very fast. As the Earth rotational velocity changes, the Earth will adjust its self to maintain an equilibrium (shape) compatible with the new situation. In doing so, the Earth should have experienced several geologic activities. Accordingly, one would expect that the tectonic movements (plate's motion) to be attributed to this continued adjustment.

We plot the length of day (in hours) against time (million years back) in Fig. (1). We notice here that a direct extrapolation of the present deceleration would bring the age of the Earth-Moon system t a value of 3.3 billion years. We observe that the plot deviates very much from straight line. The plot curves at two points which I attribute the first one to emergence of water in huge volume resulting in slowing down the rotation of the Earth's spin. The second point is when water becomes abundant and its rate of increase becomes steady.

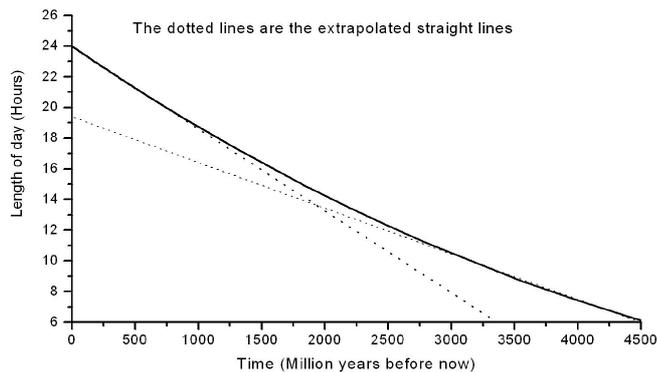


Fig. 1: The variation of length of day versus geological time.

These two points correspond to 1100 m.a. and 3460 m.a., and their corresponding lengths of day are 18.3 and 8.9 hours, respectively. As the origin of life is intimately related to existence of water, we may conclude that life has started since 3.4 billion years ago, as previously anticipated by scientists.

4 Conclusion

We have constructed a model for the variation of length of the day with time. It is based on the idea of an effective Newton's constant as an effective coupling representing all gravitational effects on a body. This variation can be traced back over the whole history of the Earth. We obtained an empirical law for the variation of the length of the day and the number of days in a year valid for the entire past Earth's rotation. We have found that the day was 6 hours when the Earth formed. These data pertaining to the early rotation of the Earth can help paleontologists to check their data with this findings. The change in the strength of gravity is manifested in the way it influences the growth of biological systems. Some biological systems (rythmites, tidalites, etc.) adjust their rhythms with the lunar motion (or the tide). Thus any change in the latter system will show up in the former. These data can be inverted and used as a geological calendar. The data we have obtained for the length of the day and the number of days in the year should be tested against any possible data pertaining to the past's Earth rotation. Our empirical law has been tested over an interval as far back as 4500 m.y. and is found to be in consistency with the experimental data so far known. In this work we have arrived at a generalized Kepler's laws that can be applicable to our ever changing Earth-Moon-Sun system.

Acknowledgements

I wish to thank the University of Khartoum for providing research support for this work, and the Abdus salam International Center for Theoretical Physics (ICTP) for hospitality where this work is carried out.

Submitted on September 21, 2008 / Accepted on September 26, 2008

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