IN MEMORIAM OF NIKIAS STAVROULAKIS

On the Field of a Spherical Charged Pulsating Distribution of Matter

Nikias Stavroulakis*

In the theory of the gravitational field generated by an isotropic spherical mass, the spheres centered at the origin of $\mathbb{R}^3$ are non-Euclidean objects, so that each of them possesses a curvature radius distinct from its Euclidean radius. The classical theory suppresses this distinction and consequently leads to inadmissible errors. Specifically, it leads to the false idea that the field of a pulsating source is static. In a number of our previous publications (see references), we have exposed the inevitable role that the curvature radius plays and demonstrated that the field generated by a pulsating not charged spherical course is dynamical. In the present paper we prove that the curvature radius plays also the main role in the description of the gravitational field generated by a charged pulsating source.

1 Introduction

The manifold underlying the field generated by an isolated spherical distribution of matter is the space $\mathbb{R} \times \mathbb{R}^3$, considered with the product topology of four real lines. In fact, the distribution of matter is assumed to be located in a system represented topologically by the space $\mathbb{R}^3$ and moreover to every point of $\mathbb{R}^3$ there corresponds the real line described by the time coordinate t (or rather ct). In the general case, the investigation of the gravitational field by means of the Einstein equations is tied up with great mathematical difficulty. In order to simplify the problem, we confine ourselves to the case when the spherical distribution of matter is isotropic. The term "*isotropic*" refers classically to the action of the rotation group SO(3) on $\mathbb{R}^3$ and the corresponding invariance of a class of metrics on $\mathbb{R}^3$. But in our case we have to deal with a *space-time* metric on $\mathbb{R} \times \mathbb{R}^3$, so that its invariance must be conceived with respect to another group defined by means of SO(3) and acting on $\mathbb{R} \times \mathbb{R}^3$. This necessity leads to the introduction of the group SΘ(4), which consist of the matrices

$$\begin{pmatrix} 1 & 0_H \\ 0_V & A \end{pmatrix}$$

with $0_H = (0, 0, 0)$, $0_V = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $A \in SO(3)$. It is also convenient to introduce the larger group $\Theta(4)$ consisting of the matrices of the same form for which $A \in O(3)$.

From the general theory [10] of the SΘ(4)-invariant and Θ(4)-invariant tensor fields on $\mathbb{R} \times \mathbb{R}^3$, we deduce the explicit form of an SΘ(4)-invariant space-time metric to be

$$ds^2 = \left[f\left(t, \|x\|\right) dt + f_1\left(t, \|x\|\right)(xdx)\right]^2 - l_1^2(t, \|x\|)dx^2 - $$

$$- \frac{l^2\left(t, \|x\|\right) - l_1^2\left(t, \|x\|\right)}{\|x\|^2}(xdx)^2$$

and the condition $l(t, 0) = l_1(t, 0)$ is satisfied, which is also Θ(4)-invariant. The functions that appear in it, result from the functions of two variables

$$f(t, u)\,, \quad f_1(t, u)\,, \quad l(t, u)\,, \quad l_1(t, u)\,,$$

assumed to be C^∞ on $\mathbb{R} \times [0, +\infty[$, if we replace u by the norm

$$\|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

However, since the norm $\|x\|$ is not differentiable at the origin of $\mathbb{R}^3$, the functions

$$f(t, \|x\|)\,, \quad f_1(t, \|x\|)\,, \quad l(t, \|x\|)\,, \quad l_1(t, \|x\|)$$

are not either. So, without appropriate conditions on these functions in a neighborhood of the origin, the curvature tensor and hence the gravitational field, will present a singularity at the origin of $\mathbb{R}^3$, which would not have any physical meaning. In order to avoid the singularity, our functions must be **smooth functions of the norm** in the sense of the following definition:

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*Professor Dr. Nikias Stavroulakis, born on the island of Crete on October 6, 1921, passed away in Athens, Greece, on December 20, 2009. A handwritten manuscript of this paper was found on his desk by his daughter Eleni, who gave it to Dr. Ioannis M. Roussos, Professor of Mathematics at Hamline University, Saint Paul, Minnesota, compatriot scientific collaborator and closed friend of her father, to fill in some gaps, rectify some imperfections existing in the manuscript and submit it for publication to *Progress in Physics*. At this point Dr. I. M. Roussos wishes to express that he considers it a great honor to himself the fact that his name will remain connected with this great and original scientist. This is a continuation of the 5 most recent research papers that have appeared in this journal since 2006, but as we shall see at the end of this paper, very unfortunately Professor Stavroulakis has left it unfinished. Some of the claimed final conclusions are still pending. We believe that an expert on this subject matter and familiar with the extensive work of Professor Stavroulakis, on the basis of the material provided here and in some of his previous papers, will be able to establish these claims easily. No matter what, these 6 papers make up his swan-song on his pioneering research on gravitation and relativity.

Definition 1. Let $\phi(t, u)$ be a function C^∞ on $\mathbb{R} \times [0, \infty[$. (This implies that the function $\phi\,(t, \|x\|)$ is C^∞ with respect to the coordinates $t,\ x_1,\ x_2,\ x_3$ on $\mathbb{R}\times\left[\mathbb{R}^3 \times \left(\mathbb{R}^3 - \{(0,\,0,\,0)\}\right)\right]$.) Then the function $\phi(t, u)$ will be called **smooth function of the norm**, if every derivative

$$\frac{\partial^{p_0+p_1+p_2+p_3}\,\phi\,(t, \|x\|)}{\partial t^{p_0}\partial x_1^{p_1}\partial x_2^{p_2}\partial x_3^{p_3}}$$

at the point $(t,\ x) \in \mathbb{R}^3 \times \left(\mathbb{R}^3 - \{(0,\,0,\,0)\}\right)$ tends to a definite value, as $(x_1,\ x_2,\ x_3) \to (0,\ 0,\ 0)$.

The following Theorem characterizes the smooth functions of the norm:

Theorem 1. Let $\phi(t, u)$ be a C^∞ function on $\mathbb{R} \times [0, \infty[$. Then $\phi\,(t, \|x\|)$ is a smooth function of the norm if and only if the right derivatives of odd order

$$\left.\frac{\partial^{2s+1}\phi\,(t, u)}{\partial u^{2s+1}}\right|_{u=0}$$

vanish for every value of t.

We will not need this theorem in the sequel, because we confine ourselves to the gravitational field outside the spherical source, so that we have to do exclusively with functions whose restrictions to a compact neighborhood of the origin of $\mathbb{R}^3$ are not taken into account.

This is why we also introduce two important functions on account of their geometrical and physical significance. Namely:

$$h\,(t, \|x\|) = \|x\|\ f_1\,(t, \|x\|)$$

and

$$g\,(t, \|x\|) = \|x\|\ l_1\,(t, \|x\|)\ ,$$

although, considered globally on $\mathbb{R}\times\mathbb{R}^3$, they are not smooth functions of the norm. Then if we set $\|x\| = \rho$, we can conveniently rewrite the space-time metric in the form

$$ds^2 = \left[f dt + \frac{h}{\rho}(x dx)\right]^2 - \left(\frac{g}{\rho}\right)^2 dx^2 - \frac{1}{\rho^2}\left[l^2 - \left(\frac{g}{\rho}\right)^2\right](x dx)^2 \quad (1)$$

under the condition $|h| \leqslant l$, as explained in [4]. We recall, [2], that with this metric, the field generated by a spherical charged, pulsating in general, distribution of matter, is determined by the system of equations

$$Q_{00} + \frac{\upsilon^2}{g^4} f^2 = 0\,, \quad (2)$$

$$Q_{01} + \frac{\upsilon^2}{g^4}\frac{fh}{\rho} = 0\,, \quad (3)$$

$$Q_{11} + \frac{\upsilon^2}{g^2\rho^2} = 0\,, \quad (4)$$

$$Q_{11} + \rho^2 Q_{22} + \frac{\upsilon^2}{g^4}(-l^2 + h^2) = 0\,, \quad (5)$$

where $\upsilon^2 = \frac{k}{c^4}\,\varepsilon^2$, ε being the charge of the source.

Regarding the function Q_{00}, Q_{01}, Q_{11}, Q_{22}, they occur in the definition of the Ricci tensor $R_{\alpha\beta}$ related to (1) and are given by:

$$R_{00} = Q_{00}\,, \quad R_{0i} = R_{i0} = Q_{01} x_i\,,$$
$$R_{ii} = Q_{11} + Q_{22} x_i^2\,, \quad R_{ij} = Q_{22} x_i x_j\,,$$

where $i,\ j = 1, 2, 3$ and $i \neq j$.

This been said, before dealing with the solutions of the equations of gravitation, we have to clarify the questions related to the boundary conditions at finite distance.

Let S_m be the sphere be the sphere bounding the matter. S_m is an isotropic non-Euclidean sphere, characterized therefore by its radius and its curvature-radius, which, in the present situation, are both time dependent. Let us denote them by $\sigma(t)$ and $\zeta(t)$ respectively. Since the internal field extends to the external one through the sphere S_m, the non-stationary (dynamical) states outside the pulsating source are brought about by the radial deformations of S_m, which are defined by the motions induced by the functions $\sigma(t)$ and $\zeta(t)$. Consequently these functions are to be identified with the boundary conditions at finite distance.

How is the time occurring in the functions $\sigma(t)$ and $\zeta(t)$ defined? Since the sphere S_m is observed in a system of reference defined topologically by the space $\mathbb{R}^3$, the time t must be conceived in the same system. But the latter is not known *metrically* in advance (i.e., before solving the equations of gravitation) and moreover it is time dependent. Consequently the classical method of special relativity is not applicable to the present situation. It follows that the first principles related to the notion of time must be introduced axiomatically in accordance to the very definition of SΘ(4)-invariant metric. Their physical justification will be sought a-posteriori on the basis of results provided by the theory itself.

This been said, the introduction of the functions $\sigma(t)$ and $\zeta(t)$ is implicitly related to another significant notion, namely the notion of synchronization in S_m. If S_1 denotes the unit sphere

$$S_1 = \{\ \alpha \in \mathbb{R}^3 \quad | \quad \|\alpha\| = 1\}\,,$$

the equation of S_m *at each distance* t is written as

$$x = \alpha\sigma(t)\,.$$

So the assignment of the value t at every point of S_m defines both the radius $\sigma(t)$ and the "*simultaneous events*"

$$\{\,[t,\ \alpha\sigma(t)] \quad | \quad \alpha \in S_1\,\}\,.$$

What do we mean exactly by saying that two events A and B in S_m are simultaneous? The identity of values of time at A and B does not imply by itself that we have to do with simultaneous events. The simultaneity is ascertained by the fact that the value of time in question corresponds to a definite position of the advancing spherical gravitational disturbance which is propagated radially and isotropically according to the very definition of the SΘ(4)-invariant metric.

If $\sigma'(t) = \zeta'(t) = 0$ on a compact interval of time $[t_1, t_2]$, no propagation of gravitational disturbances takes place in the external space during $[t_1, t_2]$ (at least there is no diffusion of disturbances), so that the gravitational radiation outside the matter depends on the derivatives $\sigma'(t)$ and $\zeta'(t)$. It follows that we may identify the pair $[\sigma'(t), \zeta'(t)]$ with the gravitational disturbance emitted radially from the totality of the points of S_m at the instant t. We assume that this gravitational disturbance is propagated as a spherical wave and reaches the totality of any of the spheres

$$S_\rho = \{\, x \in \mathbb{R}^3 \quad | \quad \|x\| = \rho > \sigma(t) \,\}$$

outside the matter, in consideration, at another instant.

2 Propagation function and canonical metric

A detailed study of the propagation process appears in the paper [2]. It is shown that the propagation of gravitation from a spherical pulsating source is governed by a function $\pi(t,\rho)$, termed **propagation function**, such that

$$\frac{\partial \pi(t,\rho)}{\partial t} > 0\,, \quad \frac{\partial \pi(t,\rho)}{\partial \rho} \leqslant 0\,, \quad \rho \geqslant \sigma(t)\,, \quad \pi[t, \sigma(t)] = t\,.$$

If the gravitational disturbance reaches the sphere

$$S_\rho = \{\, x \in \mathbb{R}^3 \quad | \quad \|x\| = \rho > \sigma(t) \,\}$$

at the instant t, then

$$\tau = \pi(t,\rho)$$

is the instant of its radial emission from the totality of the sphere S_m.

Among the infinity of possible choices for $\pi(t,\rho)$, we distinguish principally the one obtained in the limit case where $h = l$. Then $\pi(t,\rho)$ reduces to the time coordinate, denoted by τ, in the sphere that bounds the matter and the space-time metric takes the so-called **canonical form**

$$ds^2 = \left[f(t,\rho)d\tau + l(\tau,\rho)\frac{(xdx)}{\rho} \right]^2 - \left[\left[\frac{g(\tau,\rho)}{\rho} \right]^2 dx^2 + \left[l^2(\tau,\rho) - \left[\frac{g(\tau,\rho)}{\rho} \right]^2 \right] \frac{(xdx)^2}{\rho^2} \right]. \tag{6}$$

Any other Θ(4)-invariant metric is derived from (6) if we replace τ by a conveniently chosen propagation function $\pi(t,\rho)$. It follows that the general form of a Θ(4)-invariant metric outside the matter can be written as follows:

$$ds^2 = \left[f\,[\pi(t,\rho),\rho]\, \frac{\partial \pi(t,\rho)}{\partial t}\, dt + \left(f\,[\pi(t,\rho),\rho]\, \frac{\partial \pi(t,\rho)}{\partial t} + l\,[\pi(t,\rho),\rho] \right) \frac{(xdx)}{\rho} \right]^2 - \tag{7}$$
$$- \left[\left(\frac{g[\pi(t,\rho),\rho]}{\rho} \right)^2 dx^2 + \left(l^2[\pi(t,\rho),\rho] - \left(\frac{g[\pi(t,\rho),\rho]}{\rho} \right)^2 \right) \frac{(xdx)^2}{\rho^2} \right].$$

We do not need to deal with the equations of gravitation related to (7). Their solution follows from that of the equations of gravitation related to (6), if we replace in it τ by the general propagation function $\pi(t,\rho)$. Each permissible propagation function is connected with a certain conception of time, so that, the infinity of possible propagation functions introduces an infinity of definitions of time with respect to (7). So, the notion of time involved in (7) is not quite clear.

Our study of the gravitational field must begin necessarily with the canonical form (6). Although the conception of time related to (6) is unusual, it is easily definable and understandable. The time in the bounding the matter sphere S_m as well as in any other sphere S_ρ outside the matter is considered as a time synchronization according to what has been said previously. But of course this synchronization cannot be extended radially. Regarding the time along the rays, it is defined by the radial motion of photons. The motion of a photon emitted radially at the instant τ_0 from the sphere S_m will be defined by the equation $\tau = \tau_0$. If we label this photon with indication τ_0, then as it travels to infinity, it assigns the value of time τ_0 to every point of the corresponding ray. **The identity values of τ along this ray does not mean "synchronous events"**. This conception of time differs radically from the one encountered in special relativity.

3 The equations related to (2.1)

Since $h = l$ the equations (2), (3), (4) and (5) are greatly simplified to:

$$Q_{00} + \frac{v^2}{g^4} f^2 = 0\,, \tag{8}$$

$$\rho\, Q_{01} + \frac{v^2}{g^4} fl = 0\,, \tag{9}$$

$$\rho^2 Q_{11} + \frac{v^2}{g^2} = 0\,, \tag{10}$$

$$Q_{11} + \rho^2 Q_{22} = 0\,. \tag{11}$$

Regarding the functions Q_{00}, Q_{01}, Q_{11} and Q_{22}, they are already known, [3], to be:

$$Q_{00} = \frac{1}{l}\frac{\partial^2 f}{\partial\tau\partial\rho} - \frac{f}{l^2}\frac{\partial^2 f}{\partial\rho^2} + \frac{f}{l^2}\frac{\partial^2 l}{\partial\tau\partial\rho} + \frac{2}{g}\frac{\partial^2 g}{\partial\tau^2} - \frac{f}{l^3}\frac{\partial l}{\partial\tau}\frac{\partial l}{\partial\rho} + \frac{f}{l^3}\frac{\partial f}{\partial\rho}\frac{\partial l}{\partial\rho} + \frac{2f}{l^2 g}\frac{\partial l}{\partial\tau}\frac{\partial g}{\partial\rho} - \frac{2f}{l^2 g}\frac{\partial f}{\partial\rho}\frac{\partial g}{\partial\rho} - \tag{12}$$
$$- \frac{2}{fg}\frac{\partial f}{\partial\tau}\frac{\partial g}{\partial\tau} - \frac{2}{lg}\frac{\partial l}{\partial\tau}\frac{\partial g}{\partial\tau} + \frac{2}{lg}\frac{\partial f}{\partial\rho}\frac{\partial g}{\partial\tau} - \frac{1}{fl}\frac{\partial f}{\partial\tau}\frac{\partial f}{\partial\rho}\,,$$

$$\rho Q_{01}=\frac{\partial}{\partial\tau}\left[\frac{1}{fl}\frac{\partial(fl)}{\partial\rho}\right]-\frac{\partial}{\partial\rho}\left(\frac{1}{l}\frac{\partial f}{\partial\rho}\right)+\frac{2}{g}\frac{\partial^2 g}{\partial\tau\partial\rho}-\frac{2}{lg}\frac{\partial f}{\partial\rho}\frac{\partial g}{\partial\rho}\,, \quad (13)$$

$$\rho^2 Q_{11} = -1-\frac{2g}{fl}\frac{\partial^2 g}{\partial\tau\partial\rho}+\frac{g}{l^2}\frac{\partial^2 g}{\partial\rho^2}-\frac{2}{fl}\frac{\partial g}{\partial\tau}\frac{\partial g}{\partial\rho}- -\frac{g}{l^3}\frac{\partial l}{\partial\rho}\frac{\partial g}{\partial\rho}+\frac{1}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{g}{fl^2}\frac{\partial f}{\partial\rho}\frac{\partial g}{\partial\rho}\,, \quad (14)$$

$$Q_{11}+\rho^2 Q_{22}=\frac{2}{g}\left[\frac{\partial^2 g}{\partial\rho^2}-\frac{\partial g}{\partial\rho}\frac{1}{fl}\frac{\partial(fl)}{\partial\rho}\right]. \quad (15)$$

From (8) and (9) we deduce the equation

$$lQ_{00}-f\rho\,Q_{01}=0\,, \quad (16)$$

which is easier to deal with than (8) on account of the identity

$$lQ_{00}-f\rho\,Q_{01}=\frac{2l}{g}\frac{\partial^2 g}{\partial\tau^2}+\frac{2f}{lg}\frac{\partial l}{\partial\tau}\frac{\partial g}{\partial\rho}-\frac{2l}{fg}\frac{\partial f}{\partial\tau}\frac{\partial g}{\partial\tau}- \quad (17)$$

$$-\frac{2}{g}\frac{\partial l}{\partial\tau}\frac{\partial g}{\partial\tau}+\frac{2}{g}\frac{\partial f}{\partial\rho}\frac{\partial g}{\partial\tau}-\frac{2f}{g}\frac{\partial^2 g}{\partial\tau\partial\rho}$$

which follows from (12) and (13).

On account of (15), the equation (11) gives

$$\frac{\partial}{\partial\rho}\left(\frac{1}{fl}\frac{\partial g}{\partial\rho}\right)=0$$

whence

$$\frac{1}{fl}\frac{\partial g}{\partial\rho}=\beta=\text{function of }\ \tau$$

and, more explicitly,

$$\frac{\partial g(\tau,\rho)}{\partial\rho}=\beta(\tau)\,f(\tau,\rho)\,l(\tau,\rho)\,.$$

We contend that the function $\beta(\tau)$ cannot vanish. In fact, if $\beta(\tau_0)=0$ for some value τ_0 of τ, then

$$\frac{\partial g(\tau_0,\rho)}{\partial\rho}=0$$

from which it follows that

$$g(\tau_0,\rho)=\text{constant}\,.$$

This condition is un-physical: Since a photon traveling radially to infinity, assigns the values of time τ_0 to every point of a ray, this condition implies that the curvature radius $g(\tau_0,\rho)$ is constant outside the matter at the instant τ_0. Consequently $\beta(\tau)\neq 0$, so that

either $\beta(\tau)>0$ or $\beta(\tau)<0$ for every value of τ.

But, since $f(\tau,\rho)>0$ and $l(\tau,\rho)>0$, the condition $\beta(\tau)<0$ implies

$$\frac{\partial g(\tau,\rho)}{\partial\rho}<0$$

and so the curvature radius $g(\tau,\rho)$ is a strictly decreasing function of ρ. **This last conclusion is also un-physical**.

Consequently $\beta(\tau)>0$ for every τ, so that we can define the positive function

$$\alpha=\alpha(\tau)=\frac{1}{\beta(\tau)}$$

and write

$$fl=\alpha\frac{\partial g}{\partial\rho}$$

and so

$$f=\frac{\alpha}{l}\frac{\partial g}{\partial\rho}\,. \quad (18)$$

Consequently

$$\frac{\partial f}{\partial\rho}=-\frac{\alpha}{l^2}\frac{\partial l}{\partial\rho}\frac{\partial g}{\partial\rho}+\frac{\alpha}{l}\frac{\partial^2 g}{\partial\rho^2} \quad (19)$$

and inserting this expression into (14) we obtain

$$\rho^2 Q_{11} = -1-\frac{2g}{\alpha\frac{\partial g}{\partial\rho}}\frac{\partial^2 g}{\partial\tau\partial\rho}+\frac{2g}{l^2}\frac{\partial^2 g}{\partial\rho^2}- -\frac{2}{\alpha}\frac{\partial g}{\partial\tau}-\frac{2g}{l^3}\frac{\partial l}{\partial\rho}\frac{\partial g}{\partial\rho}+\frac{1}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^2.$$

On account of (10), we can deduce that

$$0=\left(\rho^2 Q_{11}+\frac{v^2}{g^2}\right)\frac{\partial g}{\partial\rho}=$$

$$=-\frac{\partial g}{\partial\rho}-\frac{2g}{\alpha}\frac{\partial g^2}{\partial\tau\partial\rho}+\frac{2g}{l^2}\frac{\partial^2 g}{\partial\rho^2}\frac{\partial g}{\partial\rho}-\frac{2}{\alpha}\frac{\partial g}{\partial\tau}\frac{\partial g}{\partial\rho}-$$

$$-\frac{2g}{l^3}\frac{\partial l}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{1}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^3+\frac{v^2}{g^2}\frac{\partial g}{\partial\rho}=$$

$$=\frac{\partial}{\partial\rho}\left[-g-\frac{2g}{\alpha}\frac{\partial g}{\partial\tau}+\frac{g}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{v^2}{g}\right],$$

whence

$$-g-\frac{2g}{\alpha}\frac{\partial g}{\partial\tau}+\frac{g}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{v^2}{g}=-2\mu=\text{function of }\ \tau$$

and so

$$\frac{\partial g}{\partial\tau}=\frac{\alpha}{2}\left[-1+\frac{2\mu}{g}-\frac{v^2}{g^2}+\frac{1}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^2\right]. \quad (20)$$

To continue our discussion we need the following derivatives obtained by direct computation:

$$\frac{\partial^2 g}{\partial\tau\partial\rho}=\alpha\left[-\frac{\mu}{g^2}\frac{\partial g}{\partial\rho}+\frac{\upsilon^2}{g^3}\frac{\partial g}{\partial\rho}-\frac{1}{l^3}\frac{\partial l}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{1}{l^2}\frac{\partial g}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}\right], \quad (21)$$

$$\frac{\partial^3 g}{\partial\tau\partial\rho^2}=\alpha\left[\frac{2\mu}{g^3}\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{\mu}{g^2}\frac{\partial^2 g}{\partial\rho^2}-\frac{3\upsilon^2}{g^4}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{\upsilon^2}{g^3}\frac{\partial^2 g}{\partial\rho^2}+\right.$$
$$+\frac{3}{l^4}\left(\frac{\partial l}{\partial\rho}\right)^2\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{1}{l^3}\frac{\partial^2 l}{\partial\rho^2}\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{4}{l^3}\frac{\partial l}{\partial\rho}\frac{\partial l}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}+$$
$$\left.+\frac{1}{l^2}\left(\frac{\partial^2 g}{\partial\rho^2}\right)^2+\frac{1}{l^2}\frac{\partial g}{\partial\rho}\frac{\partial^3 g}{\partial\rho^3}\right]. \quad (22)$$

Consider now the equation (9). Since

$$\frac{\partial}{\partial\tau}\left[\frac{1}{fl}\frac{\partial(fl)}{\partial\rho}\right]=\frac{\partial}{\partial\tau}\left(\frac{\frac{\partial^2 g}{\partial\rho^2}}{\frac{\partial g}{\partial\rho}}\right)=\frac{\frac{\partial g}{\partial\rho}\frac{\partial^3 g}{\partial\tau\partial\rho^2}-\frac{\partial^2 g}{\partial\rho^2}\frac{\partial^2 g}{\partial\tau\partial\rho}}{\left(\frac{\partial g}{\partial\rho}\right)^2}$$

by taking into account (21) and (22), we find after some computations

$$\frac{\partial}{\partial\tau}\left[\frac{1}{fl}\frac{\partial(fl)}{\partial\rho}\right]=\alpha\left[\frac{2\mu}{g^3}\frac{\partial g}{\partial\rho}-\frac{3\upsilon^2}{g^4}\frac{\partial g}{\partial\rho}+\frac{3}{l^4}\left(\frac{\partial l}{\partial\rho}\right)^2\frac{\partial g}{\partial\rho}-\right.$$
$$\left.-\frac{3}{l^3}\frac{\partial l}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}-\frac{1}{l^3}\frac{\partial^2 l}{\partial\rho^2}\frac{\partial g}{\partial\rho}+\frac{1}{l^2}\frac{\partial g^3}{\partial\rho^3}\right].$$

On the other hand (19) leads to the relation

$$\frac{\partial}{\partial\rho}\left[\frac{1}{l}\frac{\partial f}{\partial\rho}\right]=\alpha\left[\frac{3}{l^4}\left(\frac{\partial l}{\partial\rho}\right)^2\frac{\partial g}{\partial\rho}-\frac{1}{l^3}\frac{\partial^2 l}{\partial\rho^2}\frac{\partial g}{\partial\rho}-\frac{3}{l^3}\frac{\partial l}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}+\frac{1}{l^2}\frac{\partial^3 g}{\partial\rho^3}\right].$$

Moreover by (21)

$$\frac{2}{g}\frac{\partial^2 g}{\partial\tau\partial\rho}=\alpha\left[-\frac{2\mu}{g^3}\frac{\partial g}{\partial\rho}+\frac{2\upsilon^2}{g^4}\frac{\partial g}{\partial\rho}-\frac{2}{l^3 g}\frac{\partial l}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{2}{l^2 g}\frac{\partial g}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}\right]$$

and

$$\frac{2}{lg}\frac{\partial f}{\partial\rho}\frac{\partial g}{\partial\rho}=\alpha\left[-\frac{2}{l^3 g}\frac{\partial l}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{2}{l^2 g}\frac{\partial g}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}\right].$$

Inserting these expressions into (13) we find, after cancelations,

$$\rho Q_{01}=-\alpha\frac{\upsilon^2}{g^4}\frac{\partial g}{\partial\rho}$$

so that

$$\rho\, Q_{01}+\frac{\upsilon^2}{g^4}fl=-\alpha\frac{\upsilon^2}{g^4}\frac{\partial g}{\partial\rho}+\frac{\upsilon^2}{g^4}\alpha\frac{\partial g}{\partial\rho}=0\,.$$

Consequently the equation (9) is verified.

It remains to examine the equation (16), which amounts to transform the expression (17). In principle, we need the derivatives

$$\frac{\partial^2 g}{\partial\tau^2} \quad \text{and} \quad \frac{\partial f}{\partial\tau}$$

expressed by means of l and g.

First we consider the expression of $\frac{\partial^2 g}{\partial\tau^2}$ resulting from the derivative of (20) with respect to τ and then replace in it the $\frac{\partial g}{\partial\tau}$ and $\frac{\partial^2 g}{\partial\tau\partial\rho}$, given by their expressions (20) and (21). We get:

$$2\frac{\partial^2 g}{\partial\tau^2}=\frac{d\alpha}{d\tau}\left[-1+\frac{2\mu}{g}-\frac{\upsilon^2}{g^2}+\frac{1}{l^2}\left(\frac{\partial g}{\partial\rho}\right)^2\right]+$$
$$+\alpha\left[-\frac{2}{l^3}\frac{\partial l}{\partial\tau}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{2}{g}\frac{d\mu}{d\tau}\right]+$$
$$+\alpha^2\left[\frac{\mu}{g^2}-\frac{2\mu^2}{g^3}+\frac{3\mu\upsilon^2}{g^4}-\frac{3\mu}{l^2 g^2}\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{\upsilon^2}{g^3}-\frac{\upsilon^4}{g^5}+\right.$$
$$\left.+\frac{3\upsilon^2}{l^2 g^3}\left(\frac{\partial g}{\partial\rho}\right)^2-\frac{2}{l^5}\left(\frac{\partial g}{\partial\rho}\right)^3\frac{\partial l}{\partial\rho}+\frac{2}{l^4}\left(\frac{\partial g}{\partial\rho}\right)^2\frac{\partial^2 g}{\partial\rho^2}\right].$$

On the other hand taking the derivative of

$$f=\frac{\alpha}{l}\frac{\partial g}{\partial\rho}$$

with respect to τ and then, in the resulting expression, replace the expression of $\frac{\partial^2 g}{\partial\tau\partial\rho}$ given by equation (21) , we obtain

$$\frac{\partial f}{\partial\tau}=\frac{d\alpha}{d\tau}\frac{1}{l}\frac{\partial g}{\partial\rho}-\frac{\alpha}{l^2}\frac{\partial l}{\partial\tau}\frac{\partial g}{\partial\rho}+$$
$$+\frac{\alpha^2}{l}\left[-\frac{\mu}{g^2}\frac{\partial g}{\partial\rho}+\frac{\upsilon^2}{g^3}\frac{\partial g}{\partial\rho}-\frac{1}{l^3}\frac{\partial l}{\partial\rho}\left(\frac{\partial g}{\partial\rho}\right)^2+\frac{1}{l^2}\frac{\partial g}{\partial\rho}\frac{\partial^2 g}{\partial\rho^2}\right].$$

So, we have already obtained f, $\frac{\partial f}{\partial\tau}$, $\frac{\partial f}{\partial\rho}$, $\frac{\partial^2 g}{\partial\tau\partial\rho}$, $\frac{\partial^2 g}{\partial\tau^2}$, by means of l and g. Inserting them into (21), we get after some computations and several cancelations the relation

$$lQ_{00}-f\rho Q_{01}=\frac{2\alpha l}{g^2}\frac{d\mu}{d\tau}\,,$$

so that the equation (16) is written as

$$\frac{2\alpha l}{g^2}\frac{d\mu}{d\tau}=0$$

which implies

$$\frac{d\mu}{d\tau}=0$$

and so μ is a constant.

Later on we will prove that this constant is identified with the mass that produces the gravitational field...*

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*At this point, Professor Dr Nikias Stavroulakis laid the pen down for good, before writing the proofs of this latter important assertion and some of the claims found in the abstract and thus completing this very interesting work. — I. M. Roussos.

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