Charge of the Electron, and the Constants of Radiation According to J. A. Wheeler's Geometrodynamic Model

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This study suggests a mechanical interpretation of Wheller's model of the charge. According to the suggested interpretation, the oppositely charged particles are connected through the vortical lines of the current thus create a close contour "input-output" whose parameters determine the properties of the charge and spin. Depending on the energetic state of the system, the contour can be structurized into the units of the second and thirs order (photons). It is found that, in the framework of this interpretation, the charge is equivalent to the momentum. The numerical value of the unit charge has also been calculated proceeding from this basis. A system of the relations, connecting the charge to the constants of radiation (the Boltzmann, Wien, and Stefan-Boltzmann constants, and the fine structure constant) has been obtained: this give a possibility for calculating all these constants through the unit charge.

William Thomson (Baron Kelvin), the prominent physicist of the 19th century, said: "we can mean a phenomenon to be clearly understood only if a mechanical model of it has been constructed". It would be fine if the famous phrase would be actual in the nowadays as well. This however meets some difficulties, in particular — in the case of the electron, despite its spin has the dimension of mechanical angular momentum, and the charge is not (at least) a special "entity" or "electric substance".

In order to explain the properties of the electric charge, John A. Wheeler suggested his own concept of geometrodynamics. According to the concept, the charged microparticles are special points in the three-dimensional spatial surface of our world, connected to each other through "wormholes" — vortical tubes analogous to the lines of current working according to the "input-output" ("source-drain") principle, but in an additional dimension of space.

Is the fourth dimension still necessary in this case?

Suppose that the world, being an entity in the limits of the three-dimensional continuum, is a really *surface* which is *topologically non-unitary coherent* and *fractalized* upto the parameters of the micro-world bearing a fraction dimension of the numerical value upto three. In this case, it is easy to see that the Wheeler vortical tube is located "under the surface" of our world, thus is "invisible" to us, the fragments of the fractalized surface.

Meanwhile numerous specific properties of the microworld do not manifest themselves into it, or are manifested being *distorted*, as if they were projected into our world from an "additional" dimension. In particular, this should be true in the charge and spin of the electron, which can be considered according to the mechanistic scheme as the respective momentum of the vortical tube and the angular momentum with respect to its longitudinal axis. So forth we will consider, for brevity, the close contour crossing the surface X of the our world in the points, say, p^+ and e^-. In the framework of this scheme, a free charged particle is presented as a section of the open contour, or as a single-pole curl directed along the "additional" direction; the electron can be presented as an object activating the motion of the medium (electric current).

Let S be the sinus of an angle determining the projection of the momentum onto the surface X, and also the projection of the circulation velocity v (this is also, in the same time, the velocity of the rotation around the longitudinal axis of the contour) onto the chosen direction, say the axis $p^+ - e^-$. In this case, S^i characterizes the ratio of the projection of the velocity to the velocity itself ($i = 1, 2, 3$ depending on the orientation of the velocity vector).

Let, according to our initially suggestion, the charge be equivalent to the momentum, thus be Coulomb = kg×m/sec. Replace the elementary charge with the ultimate momentum of the electron, $m_e c$, in the formulae of Coulomb and Ampere. With taking this into account, in order to arrive at the numerical coincidence with the electric and magnetic forces (determined by the classical formulae), it is sufficient to introduce new formulae for the electric and magnetic contants, ε_0 and μ_0, as follows

$$\varepsilon_0 = \frac{m_e}{r_e} = 3.233\times10^{-16}\ [\text{kg/m}]\,, \tag{1}$$

$$\mu_0 = \frac{1}{c^2\,\varepsilon_0} = 0.0344\ [\text{N}^{-1}]\,, \tag{2}$$

where m_e is the mass of the electron, while c is the velocity of light. The quantity r_e means the classical radius of the electron, which is, in SI units,

$$r_e = \frac{10^{-7}\,e_0^2}{m_e}\,, \tag{3}$$

where e_0 is the charge of the electron.

Thus, these constants get a clear physical meaning now. They characterize the vortical tube, because ε_0 has a dimension of its density per meter, while μ_0 is the quantity reciprocal to the centrifugal force which appears when the element of the vortical tube, whose mass is m_e, rotates with the radius r_e with the linear velocity c.

The contour's length can vary, depending on the energetic state of the system. Assume that its increase, according to the well-known analogy to hydrodynamics, results the decrease of the tube's radius upto an arbitrary numerical value r, and also the creation of the secondary and tertiary spiral structures, which fill the toroidal volume (the section of the torus is the same as the classical radius of the electron r_e).

Thus, the charge of a particle can be characterized by the projection of the longitudinal component of the momentum Mv onto the surface X, where the mass of the vortical tube (contour) is proportional to the tube's length, and is

$$M = \varepsilon_0 R = \varepsilon n^2 R_b\,, \tag{4}$$

where n is the leading quantum number, $R_b = \alpha^2 r_e$ is the Bohr 1st radius, while α is the reciprocal fine structure constant which is 137.036 (it will be shown below that α is also determined according to the suggested model).

Among the possible contours characterized by different masses and velocities, there is such a contour in which the energy of the unit charge (electron) reaches the maximal numerical value. We take into account that a potential, in the framework of the mechanistic "coulombless" system, corresponds to a velocity. Thus, in the case of this contour, we can write down

$$e\,v = m_e c^2 = E_{max}\,, \tag{5}$$

where e is the common charge, which is identical to the momentum (in contrast to its projection, the observed charge e_0). In this contour, we determine the standard unit of the potential (velocity) as follows

$$\mathrm{v} = \frac{m_e v^2}{e} = 1\ \text{[m/sec]}\,. \tag{6}$$

Thus we obtain, from (5) and (6),

$$v = c_p^{2/3}\,\mathrm{v}\,, \tag{7}$$

where the dimensionless velocity of light $c_p = \frac{c}{\mathrm{v}}$ has been introduced, and also

$$e = M v = m_e c_p^{2/3} c_p^{2/3}\,\mathrm{v}\,. \tag{8}$$

In other word, we see that the mass M of the contour is the same as $m_e c_p^{2/3} = 4.48{\times}10^5\, m_e$ that is close to the *summary mass of the bosons* W^+, W^-, Z^0.

We will refer to the contour as the *standard contour*. In it, the maximal energy of the "point-like" electron, $m_e c^2$, is the same as that of the current tube, $M v^2$. The numerical values of the charge and spin remain unchanged for any contour, and have a common component — the contour's momentum $M v$. It should be noted that, despite the dimension of electric charge corresponds to the dimension of momentum, it is not common to both entities, thus cannot be divided by the dimensions of mass and velocity.

The projection of the momentum, which is the *observed charge*, is

$$e_0 = m_e c_p^{4/3} S^i\,\mathrm{v}\,, \tag{9}$$

where, as is obvious, $i = 1$, while the complete momentum of the vortical tube (the Planck constant h) reduced to the radius of the electron can be determined as the vector recovered, on the basis of the projection, in the general way where $i = 3$. Thus

$$\frac{h}{r_e} = 2\pi\alpha m_e c = \frac{e_0}{S^3}\,. \tag{10}$$

Taking e_0 from (9), we obtain, through (10),

$$S = \frac{c_p^{1/6}}{\sqrt{2\pi\alpha}} = 0.881, \tag{11}$$

thus the *projective angle* is 61.82°, while the obtained numerical value of the observed charge $e_0 = 1.61{\times}10^{-19}$ kg/m×sec differs from the exact value (standard numerical value obtained in the experiments) for doles of the percent.

The charge of the "point-like" electron in the region X, we will denote as e_x, is substituted into the formulae of Coulomb and Ampere: under ε_0 and μ_0 assumed in the model, it consist a very small part of e_0, which is

$$e_x = m_e c = \frac{e_0}{c_p^{1/3} S} = \frac{e_0}{590}\,. \tag{12}$$

The *main standard quantum number* can be expressed through the mass M of the contour and its density per one meter (the electric constant ε_0)

$$n_s = \sqrt{\frac{m_e c_p^{2/3}}{\varepsilon_0 R_b}} = \frac{c_p^{1/3}}{\alpha} = 4.884; \tag{13}$$

the contour's size is $R_s = n_s^2 R_b = 1.26{\times}10^{-9}$ m.

The *number of the ordered structural units* z of the contour (we will refer to them as *photons*, for brevity) is determined, for an arbitrary quantum number, by the ratio between the full length of the contour and the length of the wave λ

$$z = \frac{n^2 R_b \left(\frac{r_e}{r}\right)}{\lambda}\,, \tag{14}$$

where

$$\lambda = \frac{W}{R_\infty}\,, \tag{15}$$

Rydberg's constant is expressed as

$$R_\infty = \frac{1}{4\pi\alpha^3 r_e}\,, \tag{16}$$

while Balmer's formula is

$$W = \frac{m^2 n^2}{m^2 - n^2}\,, \tag{17}$$

where $n, m = 1, 2, \dots$ Here the ratio of the radii $\frac{r_e}{r}$ takes into account the increase of the length of the "stretched" contour in the case where the spiral structures of the second and third orders are created. Because $\varepsilon_0 = const$ and $\mu_0 = const$, in the case of arbitrary r and v the formulae (1) and (2) lead to

$$\frac{r_e}{r} = \left(\frac{c}{v}\right)^2. \tag{18}$$

We obtain the *velocity* v and *radius* r *of the vortical tube of the contour*, in the general case, from the condition of constancy of the momentum which is true for any contour having an arbitrary quantum number n. We obtain

$$M v = m_e c_p^{4/3} \mathrm{v} = n^2 R_b \varepsilon_0 v\,, \tag{19}$$

wherefrom, substituting the extended formulae of R_b and ε_0, and taking (18) into account, we obtain

$$v = \frac{c_p^{1/3} c}{(\alpha n)^2}\,, \tag{20}$$

$$r = \frac{c_p^{2/3} r_e}{(\alpha n)^4}\,. \tag{21}$$

As a result, with (15) and (16) taken into account, and having the velocity v replaced with its projection vS^i, we obtain the number of the photons in the arbitrary contour

$$z = \frac{n^6 \alpha^3}{4\pi W c_p^{2/3} S^{2i}}\,. \tag{22}$$

In particular, consider the standard contour (denote it by the index s). In the unitary transfer in it from n_s to $n_s + 1$, we obtain: $W_s = 76.7$, $\lambda_s = 7.0{\times}10^{-6}$ m, $v_s = 4.48{\times}10^5$ m/sec, $r_s = 6.3{\times}10^{-21}$ m, while the number of the photos z_s being calculated under $i = 2$ is close to $\alpha = 137$.

Thus, given a "standard" photon, the following relation

$$\frac{R_s}{r_e} = \frac{r_e}{r_s} = c_p^{2/3} = 448000 \tag{23}$$

is reproduced (that is specific to an atom).

The *Boltzmann, Wien, and Stefan-Boltzmann constants*, k, b, and σ, can be determined connecting the energy of the section of the contour in the region X taken per one photon, E_z, i.e. the energy of the structural unit, with the energy of the heat motion E_t (the average energy of the radiating oscillator) in the case of a specific particular conditions.

We express E_z and E_t as follows

$$E_z = \frac{e_x v S}{z}\,, \tag{24}$$

$$E_t = k\,T\,. \tag{25}$$

The numerical value of E_z decreases with the increase of the quantum number so that, with a numerical value of n, it becomes equal E_t taken with the wavelength λ of the photon emitted by a black body whose temperature is that of the scale unit

$$E_z = E_t \quad \text{under} \quad T = 1^\circ\ [\mathrm{K}]\,. \tag{26}$$

With decreasing n, the numerical value of E_z increases faster than E_t. Assume that, with taking (23) into account, the following ratio

$$(E_z)_s = z E_t \quad \text{under} \quad T = T_s \tag{27}$$

is true for the standard contour.

Using (12), (20), and (22), we modify (24) then re-write (26) and (27) for n and n_s assuming that the most large contour has been contracted into a tertiary structure

$$\frac{AW}{n^8} = k\,T\,, \qquad i = 3, \quad T = 1^\circ\ [\mathrm{K}]\,, \tag{28}$$

$$\frac{A_s W_s}{n_s^8} = k\,T_s\,z\,, \qquad i = 2. \tag{29}$$

where $A = 4\pi S^{2i} n_s^5 e_0 \mathrm{v}$. Taking into account that $\frac{A}{A_s} = S^2$ and also

$$1^\circ\ [\mathrm{K}] = \frac{bR_\infty}{W}\,, \tag{30}$$

$$T_s = \frac{bR_\infty}{W_s} \tag{31}$$

where Wien's constant is

$$b = T\lambda\,, \tag{32}$$

we obtain, from the common solution of (28) and (29),

$$\frac{n^4}{W} = \frac{S\,z^{1/2} n_s^4}{W_s}\,. \tag{33}$$

Assume $z = z_s = 137$. Taking (17) into account, we calculate, for the transfer from n to $n+1$: $n = 39.7$, $W = 32470$, $\lambda = 0.0030$ m, *Wien's constant* $b = 0.0030$ m×K. From (28), we obtain *Boltzmann's constant* $k = 1.38{\times}10^{-23}$ J/K. According (22), we obtain the *number of the photons* of the contour: $z = 117840$ under $i = 3$.

The number of the photons z of the given contour is very close to the numerical value of $2\pi\alpha^2$. This result does not follow from the initially assumptions, thus is absolutely independent. So, the presence of the secondary and tertiary structures has been confirmed. That is, there are three specific contours: the contour of the 1st order (the Bohr 1st radius, $n = 1$; the contour of the 2nd order (the standard contour, $n = 4.884$, containing α structural units, the photons); and the contour of the 3rd order ($n \approx 40$, containing $2\pi\alpha^2$ photons).

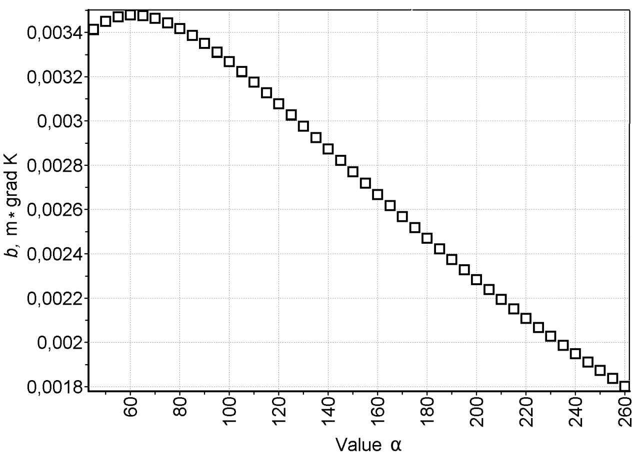

Fig. 1: Dependency of Wien's constant b on the reciprocal value of the fine structure constant α.

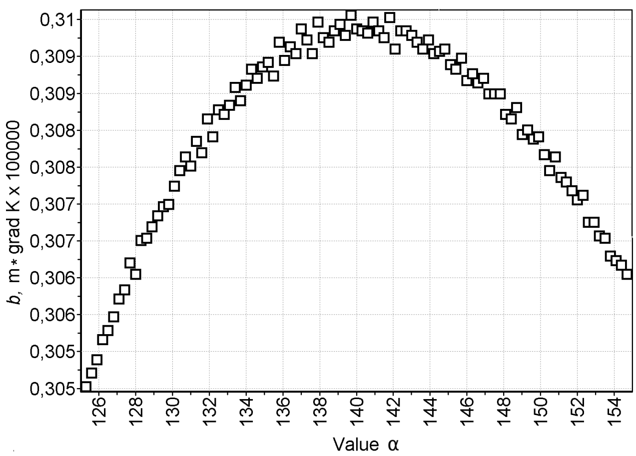

Fig. 2: Results of the numerical differentiation of the function $b(\alpha)$ in the region of the second singular point (inflection of the $b(\alpha)$ arc). The ordinate axis means the speed of the change of the parameter b.

Boltzmann's constant can be expressed also through the parameters of the standard contour

$$k = \frac{n_s e_0 \mathrm{v}}{\alpha T_s} = 1.38{\times}10^{-23} \ [\mathrm{J/K}] \,, \qquad (34)$$

where $T_s = \frac{b}{\lambda_s} = 414.7°$ K.

Formula (34) can be transformed so that

$$\frac{k T}{n_s} = m_e \left(c_p^{2/3} \mathrm{v}\right)^2 \frac{S}{\alpha} \,, \qquad (35)$$

i.e. *given the standard contour, the energy of the radiating oscillator per the contour's quantum number is equal to the energy of the internal rotation of the "point-like" electron taken per the number of the structural units of the contour*.

It is interesting to compare the Planck entropy of the photon, S_h, to the entropy of the part of the contour related to the single photon, S_z, within the region X. The Planck entropy remains constant

$$S_h = \frac{E_h}{T} = \frac{hc}{\lambda T} = \frac{hc}{b} = 6.855{\times}10^{-23} \ [\mathrm{J/K}] \,, \qquad (36)$$

while S_z decreases rapidly with the increase of the leading quantum number

$$S_z = \frac{E_z}{T} = \frac{AW}{n^8 T} = \frac{AW^2}{n^8 b R_\infty} \,. \qquad (37)$$

Equalizing S_h to S_z, and expanding the formulae for h, R_∞, and A for the case of the ionization of the atom (that means the transfer from n to $m \to \infty$ under $W \to \infty$), we obtain, under $i = 1 \ldots 3$,

$$n_\infty = \sqrt[4]{8\pi n_s^3 S^{2i+1}} = 6.7 \ \ldots \ 5.9 \,. \qquad (38)$$

Because the common direction of the physical processes to the increase of entropy, thermodynamics prefers, with $n > 6.7$, that the structural units of the contour exist separately from each other, i.e. are the photons. It is probably, this result verifies the identity of the contour's structural units to the photons, and also manifests one of the causes of that the stable atoms have no more electronic shells than 6 or 7.

The *Stefan-Boltzmann constant* can be expressed as the projection of the unit energy of the heat motion per one photon and the unit square of the standard contour, i.e. as $k\Delta T S/(\alpha n_s^4 R_b^2)$, where $\Delta T = 1°$K, and reduced to the unit of time and the unit of temperature (in the respective exponent). As a result, we obtain $\sigma = 5.56{\times}10^{-8}$ [W/m^{2}K^4].

The obtained formulae (34), (35), and (39) are actually definitions. They completely confirm the existence of the special standard contour.

Despite the fine structure constant was used in the calculation (the constant itself is meant to be derived from e_0 and h), the calculation was processed in independent way. Besides, assuming that α and all other quantities dependent on it (r_e, S, e_0, n_s, z, k, b) are variables, we can determine the numerical value of α according to the location of the second singular point (inflection of the $b(\alpha)$ arc in Fig. 1), where the change of b is proportional to the quantum number. Numerical differentiation, Fig. 2, manifest the numerical value of α within the boundaries 137–140 and, hence, it manifests the numerical values of all other parameters (for instance, k and z in Fig. 3 and Fig. 4 respectively). That is, finally, in order to calculate all the parameters we only need: the *mass of the electron*, the *velocity of light*, the *units of velocity and temperature*, and the *assumption that E_z is proportional to E_t in the standard contour*.

It is interesting that more precise numerical value of α arrives under the condition that m and n approach to infinity in the function $b(\alpha)$ and Balmer's formula (17). Thus Balmer's formula becomes $W = \frac{n^3}{2}$ under infinite large distance between the charges, that meets the determination of the textbook nu-

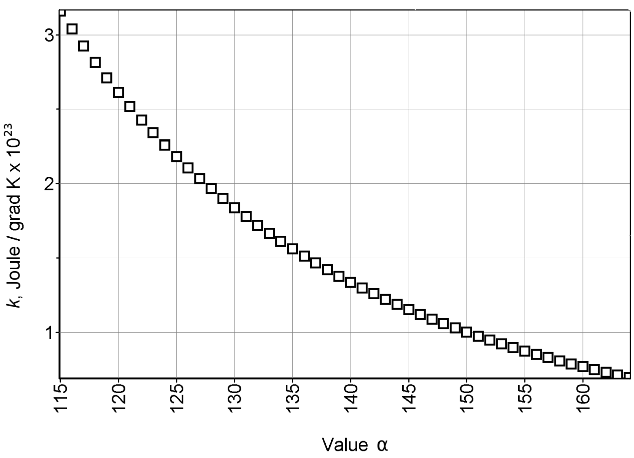

Fig. 3: Dependency of Boltzmann's constant k on the reciprocal value of the fine structure constant α.

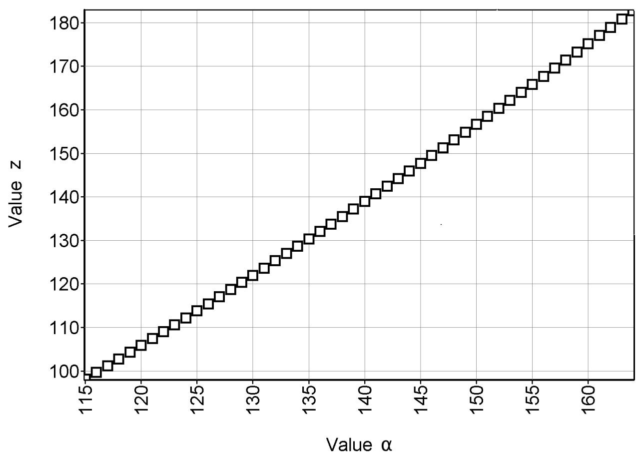

Fig. 4: Dependency of the number of the photons z of the standard contour on the reciprocal value of the fine structure constant α.

merical value of α. In the same time, we can obtain the exact numerical value of the charge from formula (1), by substituting α determined from the function $b(\alpha)$, Fig. 2.

Note that the validity of the suggested model is confirmed by that significant fact that the quantity kT, which is the unit of the work done by the structural unit of an ideal gas (this quantity is also interpreted, in the theory of heat radiation, as the energy of an elementary oscillator), is connected here with the charge of the electron. A connexion between Planck's constant and the quantity kT was found, as is known, in already a century ago by Max Planck, through the formula of the blackbody radiation. This formula is proportional to

$$\frac{1}{\lambda}\,\frac{1}{\lambda}\,\frac{1}{\lambda\,(e^{C/\lambda}-1)}\,,$$

where C is a constant. Taking all that has been obtained in our study, we understand follows. The first term here manifests the decrease of the intensity of the radiation with the increase of the wavelength of the photon. The second term manifests the decrease of the number of the photons per the unit of the full length of the contour. The third term manifests the change of the length of the contour itself, which reaches a contant with the increase of λ thus the Planck formula transforms into the Rayleigh-Jeans formula. With small numerical values of λ, the contour compresses upto the size of the photon. This gives an explanation to the decrease of the radiation power on high frequencies.

In the end, it should be noted that the properties of the charge are, of course, not limited by Wheeler's model in its mechanistic interpretation suggested here. Meanwhile, the unexpected relation between the charge of the electron and the molecular kinetic properties of the atoms and molecules manifests additional connexions between the elementary particles and macro-particles, thus this fact needs to be more studied in the future.

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