Wave Particle Duality and the Afshar Experiment

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We analyze the experiment realized in 2003-2004 by S. Afshar et al. [1] in order to refute the principle of complementarity. We discuss the general meaning of this principle and show that contrarily to the claim of the authors Bohr's complementarity is not in danger in this experiment.

1 Introduction

In an interesting series of articles published few years ago Afshar and coworkers [1,2] reported an optical experiment in which they claimed to refute the well known N. Bohr principle of complementarity [3,4]. Obviously this result, if justified, would constitute a serious attack against the orthodox interpretation of quantum mechanics (known as the Copenhagen interpretation). This work stirred much debate in different journals (see for examples references [5–12]).

We think however that there are still some important misunderstandings concerning the interpretation of this experiment. In a preprint written originally in 2004 [5] (and following some early discussions with Afshar) we claimed already that the interpretation by Afshar *et al.* can be easily stated if we stay as close as possible from the texts written by Bohr. The aim of the present article (which was initially written in 2005 to precise a bit the thought developed in [5]) is to comment the interpretation discussed in [1]. We will in the following analyze the meaning of Bohr principle and show that far from disproving its content the experiment [1] is actually a complete confirmation of its general validity.

The difficulties associated with the understanding of this principle are not new and actually complementarity created troubles even in Einstein mind [3] so that we are here in good company. To summarize a bit emphatically Bohr's complementarity we here remind that this principle states that if one of a pair of non commuting observables of a quantum object is known for sure, then information about the second (complementary) is lost [3, 4, 15, 16]. This can be equivalently expressed as a kind of duality between different descriptions of the quantum system associated with different experimental arrangements which mutually exclude each other (read in particular [3,4]). Later in the discussion we will try to precise this definition but for the moment it is enough to illustrate the concepts by examples

Consider for instance the well known Young double-pinholes interference experiment made with photons. The discrete nature of light precludes the simultaneous observation of a same photon in the aperture plane and in the interference pattern: the photon cannot be absorbed twice. This is already a trivial manifestation of the principle of Bohr. Here it implies that the two statistical patterns associated with the wave in the aperture plane and its Fourier (i. e., momentum) transform require necessarily different photons for their recording. It is in that sense that each experiment excludes and completes reciprocally the other. In the case considered before the photon is absorbed during the first detection (this clearly precludes any other detection). However even a non-destructive solution for detection implying entanglement with other quantum systems has a radical effect of the same nature: the complementarity principle is still valid. For example, during their debate Bohr and Einstein [3] discussed an ideal *which-way* experiment in which the recoil of the slits is correlated to the motion of the photon. Momentum conservation added to arguments based on the uncertainty relations are sufficient to explain how such entanglement photon-slits can erase fringes [15–19]. It is also important for the present discussion to remind that the principle of complementarity has a perfidious consequence on the experimental meaning of trajectory and path followed by a particle. Indeed the unavoidable interactions existing between photons and detectors imply that a trajectory existing independently of any measurement process cannot be unambiguously defined. This sounds even like a tragedy when we consider once again the two-holes experiment. Indeed for Bohr this kind of experiments shows definitely the essential element of ambiguity which is involved in ascribing conventional physical attributes to quantum systems. Intuitively (i. e., from the point of view of classical particle dynamic) one would expect that a photon detected in the focal plane of the lens must have crossed only one of the hole 1 or 2 before to reach its final destination. However, if this is true, one can not intuitively understand how the presence of the second hole (through which the photon evidently did not go) forces the photon to participate to an interference pattern (which obviously needs an influence coming from both holes). Explanations to solve this paradox have been proposed by de Broglie, Bohm, and others using concepts such as empty waves or quantum potentials [20, 21]. However all these explanations are in agreement with Bohr principle (since they fully reproduce quantum predictions) and can not be experimentally distinguished. Bohr and Heisenberg proposed for all needed purposes a much more pragmatic and simpler answer: *don't bother*, the complementarity principle precludes the simultaneous observation of a photon trajectory and of an interference pattern. For Bohr [3]: *This point is of great logical consequence, since it is only the circumstance that we are presented with a choice of either tracing the path*

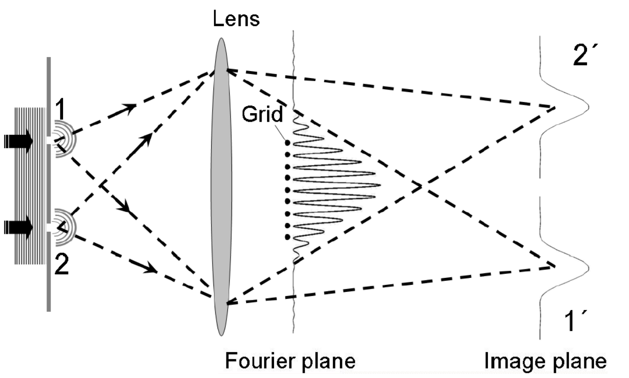

Fig. 1: The experiment described in [1]. Photons coming from pinholes 1 and 2 interfere in the back-focal plane of a lens (Fourier plane) whereas they lead to two isolated narrow spots in the image plane (the image plane is such that its distance p' to the lens is related to the distance p between the lens and the apertures screen by $1/p + 1/p' = 1/f$, where f is the focal length). The wire grid in the back focal plane, distant of f from the lens, is passing through the minima of the interference pattern. The subsequent propagation of the wave is consequently not disturbed by the grid.

of a particle or observing interference effects, which allows us to escape from the paradoxical necessity of concluding that the behaviour of an electron or a photon should depend on the presence of a slit in the diaphragm through which it could be proved not to pass. From such an analysis it seems definitively that Nature resists to deeper experimental investigation of its ontological level. As summarized elegantly by Brian Greene [22]: *Like a Spalding Gray soliloquy, an experimenter's bare-bones measurement are the whole show. There isn't anything else. According to Bohr, there is no backstage.* In spite of its interest it is however not the aim of the present article to debate on the full implications of such strong philosophical position.

2 Complementarity versus the experiments

2.1 A short description of the Afshar *et al.* experiment

The experiment reported in [1] (see Fig. 1) is actually based on a modification of a *gedanken* experiment proposed originally by Wheeler [23]. In the first part of their work, Afshar *et al.* used an optical lens to image the two pinholes considered in the Young interference experiment above mentioned. Depending of the observation plane in this microscope we can then obtain different complementary information.

If we detect the photons in the focal plane of the lens (or equivalently just in front of the lens [24]) we will observe, i.e, after a statistical accumulation of photon detection events, the interference fringes. However, if we record the particles in the image plane of the lens we will observe (with a sufficiently high numerical aperture) two sharp spots 1' and 2' images of the pinholes 1 and 2. Like the initial Young two-holes experiment this example illustrates again very well the principle of Bohr. One has indeed complete freedom for measuring the photon distribution in the image plane instead of detecting the fringes in the back focal plane. However, the two kinds of measurements are mutually exclusive: a single photon can participate only to one of these statistical patterns.

In the second and final part of the experiment, Afshar *et al.* included a grid of thin absorbing wires located in the interference fringes plane. Importantly, in the experiment the wires must be located at the minimum of the interference pattern in order to reduce the interaction with light. In the following we will consider a perfect interference profile (with ideal unit visibility $V = (I_{max} - I_{min})/(I_{max} + I_{min}) = 1$) to simplify the discussion. If additionally the geometrical cross section of each wire tends ideally to zero then the interference behavior will, at the limit, not be disturbed and the subsequent wave propagation will be kept unchanged. This implies that the photon distributions 1' and 2', located in the image plane optically conjugated with the aperture plane, are not modified by the presence, or the absence, of the infinitely thin wire grid. Naturally, from practical considerations an infinitely thin dielectric wire is not interacting with light and consequently produces the same (null) effect whatever its location in the light path (minimum or maximum of the interference for example). In order to provide a sensible probe for the interference pattern, necessary for the aim of the experiment considered, we will suppose in the following idealized wires which conserve a finite absorbtion efficiency and this despite the absence of any geometrical transversal extension. We will briefly discuss later what happens with spatially extended scattering wires with finite cross section, but this point is not essential to understand the essential of the argumentation. With such wires, and if we close one aperture (which implies that there is no interference fringes and thus that a finite field impinges on the wires) the scattering and absorbtion strongly affect the detection behavior in the image plane. As it is seen experimentally [1, 2] the scattering by the wire grid in general produces a complicated diffraction pattern and not only an isolated narrow peak in 1' or 2' as it would be without the grid.

In such conditions, the absence of absorbtion by the wires when the two apertures are open is a clear indication of the existence of the interference fringes zeros, i.e., of a wave-like character, and this even if the photon is absorbed in the image plane in 1' or 2'. Following Afshar *et al.*, this should be considered as a violation of complementarity since the same photons have been used for recording *both* the 'path' and the wave-like information. The essential questions are however what we mean precisely here by path and wave-like information and what are the connections of this with the definition of complementarity. As we will see hereafter it is by finding a clear answer to these questions that the paradox and the contradictions with Bohr's complementarity are going to vanish.

2.2 The wave-particle duality mathematical relation

At that stage, it is important to point out that the principle of complementarity is actually a direct consequence of the mathematical formalism of quantum mechanics and of its statistical interpretation [4]. It is in particular the reason why the different attempts done by Einstein to refute complementarity and the Heisenberg uncertainty relations always failed: the misinterpretations resulted indeed from a non-cautious introduction of classical physics in the fully consistent quantum mechanic formalism [3]. For similar reasons here we show that a problem since Afshar *et al.* actually mixed together, i.e imprudently, argumentations coming from classical and quantum physics. We will show that this mixing results into an apparent refutation of the complementarity principle.

After this remark we now remind that a simple mathematical formulation of complementarity exists in the context of two path interferometry [25–28]. For example in the Young double-apertures experiment considered previously the field amplitudes C_1 and C_2 associated with the two narrow apertures, separated by the distance d, allow us to define the wave function in the two-apertures plane by:

$$\psi(x) \sim C_1\delta(x - d/2) + C_2\delta(x + d/2). \quad (1)$$

From this formula one can easily introduce the "distinguishability"

$$K = \frac{||C_1|^2 - |C_2|^2|}{|C_1|^2 + |C_2|^2}. \quad (2)$$

This quantity can be physically defined by recording the photons distribution in the aperture plane and constitutes an observable measure of the "path" distinguishability (see however section 3.3). The interpretation of K is actually clear, and in particular if $K = 0$ each apertures play a symmetrical role, whereas if $K = 1$ one of the two apertures is necessarily closed. Naturally, like in the Afshar experiment, K can also be measured by recording photons in the image plane of the lens in 1' and 2'. Equations (1) and (2) are still valid, with the only differences that: i) we have now a diffraction spot (like an Airy disk) instead of a Dirac distribution in equation (1), and ii) that the spatial variables are now magnified by the lens.

Instead of the spatial representation one can also consider the Fourier transform corresponding to the far field interference pattern recorded at large distance of the two-slits screen:

$$\psi(k) \sim C_1 \cdot e^{ikd/2} + C_2 \cdot e^{-ikd/2}. \quad (3)$$

Such a wave is associated with an oscillating intensity in the k-space given by

$$I(k) \sim 1 + V\cos(kd + \chi) \quad (4)$$

where $\chi = \arg(C_1) - \arg(C_2)$ and V is the fringe visibility

$$V = \frac{2|C_1| \cdot |C_2|}{|C_1|^2 + |C_2|^2}. \quad (5)$$

This quantity is also a physical observable which can defined by recording the photons in the far-field, or, like in the Afshar *et al.* first experiment, by recording the photons fringes in the back focal plane of the lens (the back focal plane is the plane where the momentum distribution $\hbar k$ is experimentally and rigorously defined [16]). Like it is for K, the meaning of V is also very clear: if $V = 1$ both apertures must play a symmetrical role, whereas if $V = 0$ only one aperture is open.

A direct mathematical consequence of equations (2) and (5) is the relation

$$V^2 + K^2 = 1, \quad (6)$$

which expresses the duality [25, 26] between the two mathematical measures K and V associated with the two mutually exclusive (i.e., complementary) experiments in the direct and Fourier space respectively. A particularly important application of equation (6) concerns which-path experiments. In such experiments, we wish to observe the interference pattern, and to find through each hole each photon is going through. As we explained before, a photon can not be observed twice, and this represents in general a fatal end for such expectations. There is however an important exception in the particular case with only one aperture open (i.e., $K = 1$). Indeed, in such case it is not necessary to record the photon in the aperture plane to know its path since if it is detected (in the back focal plane) it necessarily means that it went through the opened aperture. Of course, from equation (6) we have in counterpart $V = 0$, which means that fringes are not possible.

This dilemma, can not be solved by considering less invasive methods, like those using entanglement between the photon and an other quantum system or an internal degree of freedom (such as polarization or spins). To see that we consider a wave function $|\Psi\rangle$ describing the entanglement between the photon and these others quantum variables defining a which-path detector. We write

$$|\Psi\rangle = \int [C_1\delta(x - d/2)|x\rangle|\gamma_1\rangle + C_2\delta(x + d/2)|x\rangle|\gamma_2\rangle]dx$$
$$= \int [C_1 \cdot e^{ikd/2}|k\rangle|\gamma_1\rangle + C_2 \cdot e^{-ikd/2}|k\rangle|\gamma_2\rangle]dp \quad (7)$$

where $|\gamma_1\rangle$ and $|\gamma_2\rangle$ are the quantum state of the which-path detector if the photon is going through the aperture 1 or 2. Consider now the kind of information one can extract from $|\Psi\rangle$. First, by averaging (tracing) over the detector degrees of freedom we can define the total probability $P(x) = \mathrm{Tr}[\hat{\rho}|x\rangle\langle x|]$ of detecting a photon in the aperture plane in x by

$$P(x) \propto |C_1|^2\langle\gamma_1|\gamma_1\rangle(\delta(x - d/2))^2 + |C_2|^2\langle\gamma_2|\gamma_2\rangle(\delta(x + d/2))^2. \quad (8)$$

with $\hat{\rho} = |\Psi\rangle\langle\Psi|$ is the total density matrix. By analogy with equation (2) the total distinguishability is then defined by

$$K = \frac{||C_1|^2\langle\gamma_1|\gamma_1\rangle - |C_2|^2\langle\gamma_2|\gamma_2\rangle|}{|C_1|^2\langle\gamma_1|\gamma_1\rangle + |C_2|^2\langle\gamma_2|\gamma_2\rangle}. \quad (9)$$

Same as for equations (3-5) we can define the total probability to detect a photon of (transverse) wave vector k by

$$P(k) = \mathrm{Tr}[\hat{\rho}|k\rangle\langle k|] \propto 1 + V \cos(kx + \phi), \quad (10)$$

where the visibility V is written

$$V = \frac{2|C_1| \cdot |C_2| \cdot |\langle\gamma_1|\gamma_2\rangle|}{|C_1|^2\langle\gamma_1|\gamma_1\rangle + |C_2|^2\langle\gamma_2|\gamma_2\rangle}. \quad (11)$$

By combining V and K we deduce immediately $K^2 + V^2 = \eta^2 \leq 1$ with

$$\eta^2 = 1 - \frac{4|C_1|^2 \cdot |C_2|^2 \cdot (\langle\gamma_1|\gamma_1\rangle\langle\gamma_2|\gamma_2\rangle - |\langle\gamma_1|\gamma_2\rangle|^2)}{(|C_1|^2\langle\gamma_1|\gamma_1\rangle + |C_2|^2\langle\gamma_2|\gamma_2\rangle)^2} \quad (12)$$

and where the inequality results from the Cauchy-Schwartz relation $\langle\gamma_1|\gamma_1\rangle\langle\gamma_2|\gamma_2\rangle - |\langle\gamma_1|\gamma_2\rangle|^2 \geq 0$.
However, we can remark that by tracing over the degrees of freedom associated with the detector we did not consider a which-path experiment but simply decoherence due to entanglement. In order to actually realize such a which-path experiment we need to calculate the joint probability associated with a recording of the photon in the state $|x\rangle$ (or $|k\rangle$) in coincidence with a measurement of the detector in the eigenstate $|\lambda\rangle$ corresponding to one of its observable. These joint probabilities read $P(x, \lambda) = \mathrm{Tr}[\hat{\rho}|x\rangle\langle x||\lambda\rangle\langle\lambda|]$ and $P(k, \lambda) = \mathrm{Tr}[\hat{\rho}|k\rangle\langle k||\lambda\rangle\langle\lambda|$ with

$$\begin{aligned} P(x, \lambda) &\propto |C_1|^2|\langle\lambda|\gamma_1\rangle|^2(\delta(x - d/2))^2 \\ &+|C_2|^2|\langle\lambda|\gamma_2\rangle|^2(\delta(x + d/2))^2 \\ P(k, \lambda) &=\propto 1 + V_\lambda \cos(kx + \phi_\lambda). \end{aligned} \quad (13)$$

Indeed, the aim of such entanglement with a degree of freedom $|\lambda\rangle$ (produced for example by inserting polarization converters like quarter or half wave-plates just after the apertures [32]) is to generate a wave function

$$\psi_\lambda(x) \sim C_{1,\lambda}\delta(x - d/2) + C_{2,\lambda}\delta(x + d/2) \quad (14)$$

with either $C_{1,\lambda}$ or $C_{2,\lambda}$ (but not both) equal to zero. A subsequent projection on $|\lambda\rangle$ will reveal the path information. However, from the duality relation given by equation (5) applied to $\psi_\lambda(x)$ it is now obvious that we did not escape from the previous conclusion. Indeed, while the photon was not destroyed by the entanglement with the which-path detector, we unfortunately only obtained path distinguishability ($K_\lambda = 1$) at the expense of losing the interference behavior ($V_\lambda = 0$).
From all these experiments, it is clear that the discreteness of photon, and more generally of every quantum object, is the key element to understand complementarity. This was evident without entanglement, since the only way to observe a particle is to destroy it. However, even the introduction of a 'which-path' quantum state $|\lambda\rangle$ does not change the rule of the game, since at the end of journey we necessarily need to project, that is to kill macroscopically, the quantum system. This fundamental fact, was already pointed out many times by Bohr in his writings when he considered the importance of separating the macroscopic world of the observer from the microscopic quantum system observed, and also when he insisted on the irreversible act induced by the observer on the quantum system during any measurement process [4].

Let now return to the interpretation of Afshar *et al.* experiments. In the configuration with the lens and without the grid, we have apparently a new aspect of the problem since the fringes occur in a plane located before the imaging plane. Contrarily to the which-path experiments above mentioned, where the destructive measurements occurred in the interference plane, we have a priori here the freedom to realize a 'fringes-interaction free-experiment' which aim is to observe the fringes without detecting the particle in the back focal plane whereas the destructive measurement will occur in the image plane (i.e., in 1' or 2'). The role of the grid is expected to provide such information necessary for the interference reconstruction. Due to the absence of disturbance by the grid, Afshar *et al.* logically deduce that the field equals zero at the wires locations. If we *infer* the existence of an interference pattern with visibility V we must have

$$V = \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} = \frac{(I_{max} - 0)}{(I_{max} + 0)} = 1, \quad (15)$$

since $I_{min} = 0$. This means that we can obtain the value of the visibility only from the two assumptions that (i) the form of the profile should be a 'cos' function given by equation (4), and that (ii) no photon have been absorbed by the wires. Finally in this experiment, we record the photons in the area 1' (or 2') and consequently we have at the same time the path information. Importantly, following Afshar *et al.* we here only consider one image spot 1' or 2' (since each photon impinges one only one of these two regions) and we deduce therefore $K = 1$. Together with the interference visibility $V = 1$ this implies

$$K^2 + V^2 = 2, \quad (16)$$

in complete contradiction with the bound given by equation (6).

In the previous analysis we only considered the infinitely thin wires to simplify the discussion. Actually, this is however the only experimental configuration in which the Afshar experiment is easily analyzable since it is only in such case that the duality relation can be defined. Indeed, scattering by the wire always results into complicated diffraction pattern in the image plane and the simple mathematical derivation [25–28] leading to equations 2, 5, and 6 is not possible. We will then continue to consider the idealized case of the infinitely thin wires in the rest of the paper since it is this ideal limit that the authors of [1]wanted obviously to reach.

3 The rebuttal: Inference and Complementarity

3.1 Duality again

There are several reasons why the analysis by Afshar *et al.* actually fails. First, from a mathematical point of view it is not consistent to write $K^2 + V^2 = 2$. Indeed, in all the experiments previously discussed (excluding the Afshar experiments) it was necessary to consider statistics on all the recorded photons in order to observe either the interference or the path information (in the case were entanglement was involved only the photons tagged by $|\lambda\rangle$ have to be considered). Same here, if one consider all the detected photons one will deduce $K = 0$ and equation (6) will be respected. Actually, this results directly from the experimental method considered by the authors of [1]. Indeed, if somebody is accepting the existence of an interference pattern he or she needs to know the complete distribution 1' *and* 2' recorded in the image plane. This is necessary in order to deduce that the wire grid didn't caused any disturbances on the propagation. Indeed, the disturbance could have no consequence in 1' but yet have some effects in 2'. Consequently, ignoring 2' does not allow us to deduce that the experiment with the grid is interaction-free. For this reason, it is unjustified to write $K = 1$, that is to consider only one half of the detected photon population, while we actually need both pinhole images to deduce the value of V (this is also in agreement with the obvious fact that an interference pattern requires the two apertures 1 and 2 opened for its existence).

There is an other equivalent way to see why the choice $K = 0$ is the only one possible. Indeed, having measured in the image plane the two distributions 1' and 2' with intensity $|C_1|^2$ and $|C_2|^2$ we can, by applying the laws of optics, propagate backward in time the two converging beams until the interference plane (this was done by Afshar *et al.*). In this plane equation (4) and (5), which are a direct consequence of these above mentioned optical laws, are of course valid. Since we have $|C_1|^2 = |C_2|^2$, we deduce (from equations (2) and (5)) that $K = 0$ and $V = 1$ in full agreement with the duality relation (6). It is important to remark that since the phase of C_1 and C_2 are not know from the destructive measurements in the image plane, we cannot extrapolate the value of $\chi = \arg(C_1) - \arg(C_2)$. However, the presence of the grid give us access to this missing information since it provides the points where $I(k) = 0$ (for example if $I(\pi/d) = 0$ then $\chi = 2\pi \cdot N$ with N =0, 1, ...). We can thus define completely the variable V and χ without recording any photon in the Fourier plane. It is clear, that this would be impossible if the duality condition $K^2 + V^2 = 1$ was not true since this relation is actually a direct consequence of the law of optics used in our derivations as well as in the one by Afshar *et al.*.

To summarize the present discussion, we showed that Afshar *et al.* reasoning is obscured by a misleading interpretation of the duality relation given by equation (6). We however think that this problem is not so fundamental for the discussion of the experiment. Actually, we can restate the complete reasoning without making any reference to this illusory violation of equation (6). After doing this we think that the error in the deductions by Afshar *et al.* should become very clear. Let then restate the story:

A) First, we record individuals photons in the regions 1' and 2'. We can then keep a track or a list of each detection event, so that, for each photon, we can define its 'path' information. However, this individual property of each photon is not entering in conflict with the statistical behavior, which in the limit of large number, give us the two narrow distribution in 1' and 2'. That is, the value $K = 0$ is not in conflict with the existence of a which-path information associated with each photon. This situation differs strongly from usual which-path experiments in which the path detection, or tagging, is done *before* the interference plane. As we explained before in these experiments the value $K = 1$ was a necessary consequence of the preselection procedure done on the photon population. This point also means that we have to be very prudent when we use the duality relation in experimental situations different from the ones for which a consensus has already been obtained.

B) Second, we apply the laws of optics backward in time to deduce the value of the visibility V. Inferring the validity of such optical laws we can even reconstruct completely the interference profile thanks to the presence of wire grid.

C) Finally, we can check that indeed $K^2+V^2 = 1$ in agreement with the duality relation.

Having elucidated the role of the duality relation, the question that we have still to answer is what are the implications of this experiment for complementarity. What has indeed been shown by Afshar *et al.* is that each photon detected in the image plane is associated with a wave behavior since none of them crossed the wires. Using the laws of optics backward in time allow us to deduce the precise shape of intensity profile in the back focal plane but this is a theoretical inference and actually not a measurement. We will now show that this is the weak point.

3.2 Classical versus quantum inferences

In classical physics, such an inference (i.e., concerning interference) is of no consequence since we can always, at least in principle, imagine a test particle or detector to check the validity of our assumptions concerning the system. However, in quantum mechanics we are dealing with highly sensitive systems and this modify the rules of the game.

In quantum mechanics it is common to say that the wave function represents the catalog of all the potentiality accessible to the system. Due to the very nature of this theory there are however some (complementary) pages which can not be read at the same time without contradictions. In the Afshar experiment, we do not have indeed the slightest experimental proof that the observed photons did participate to the "cos"

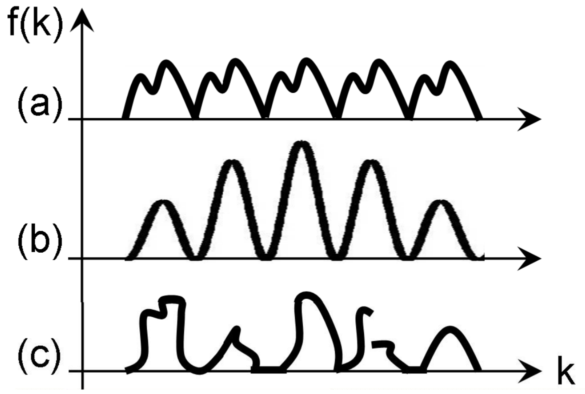

Fig. 2: Different possible intensity profiles in the Fourier plane. Each profile $f(k)$ obeys to the condition $f(k) = 0$ on the wires. (a) A continuous periodic function. (b) The diffractive interference profile predicted by quantum mechanics. (c) A discontinuous profile intensity. Each profile is 'apriori' equiprobable for an observer which has no knowledge in optics and quantum mechanics.

interference pattern given by equations (3) and (4). Furthermore, by detecting the photons in the image plane, we only know from the experiment that the photons never crossed the wires but this is not sufficient to rebuild objectively the complete interference pattern.

We can go further in this direction by using information theory. Indeed, from the point of view of the information theory of Gibbs [33], Shannon [34], and Jaynes [35], every interference patterns, such that $I(k) = 0$ on the wires, are equiprobable (see Fig. 2). However, there are an infinity of such profiles, so that our information is rather poor. More precisely, let write $\rho[f(k)]$ the functional giving the density of probability associated with the apriori likelihood of having the interference profile $f(k)$ located in an infinitely small (functional) volume $\mathcal{D}[f(k)]$. We write $\Sigma[f(x)]$ the space of all this interference profiles obeying to the condition $f(k) = 0$ on the wires. We have thus $\rho[f(k)] = 1/\Sigma$ (equiprobability) for the function f contained in Σ, and $\rho[f(k)] = 0$ for the function outside Σ (that are functions which do not satisfy the requirements $f(k) = 0$ on the wires). The Shannon entropy [33–35] $S[f(x)]$ associated with this distribution is given by

$$S[f(x)] = -\int_{(\Sigma)} \mathcal{D}[f(k)]\rho[f(k)] \ln(\rho[f(k)]) \\ = \ln(\Sigma[f(k)]) \to +\infty, \quad (17)$$

which expresses our absence of objective knowledge concerning $f(k)$. In this reasoning, we used the concept of probability taken in the Bayesian sense, that is in the sense of decision-maker theory used for example by poker players. For an observer which do not have any idea concerning quantum mechanics and the laws of optics, this equiprobability is the most reasonable guess if he wants only to consider the photons he actually detected. Of course, by considering a different experiment, in which the photons are recorded in the Fourier plane, the observer might realize what is actually the interference pattern. However (and this is essential for understanding the apparent paradox discussed in reference 1) it will be only possible by considering different recorded photons in full agreement with the principle of complementarity.

Let now summarize a bit our analysis. We deduced that in the experiment discussed in [1] the photons used to *measure* objectively the interference pattern i.e. to calculate the visibility $V = 1$ are not the same than those used to *measure* the distribution in the image plane and calculate the distinguishability $K = 0$. This is strictly the same situation than in the original two-holes experiment already mentioned. It is in that sense that the relationship (6) represents indeed a particular formulation of complementarity [25–28]. Actually (as we already commented before) the value $V = 1$ obtained in [1] does not result from a measurement but from an extrapolation. Indeed, from their negative measurement Afshar et al. recorded objectively $I_{min} = 0$. If we suppose that there is a hidden sinusoidal interference pattern in the plane of the wires we can indeed write

$$V = (I_{max} - I_{min}) / (I_{max} + I_{min}) = I_{max}/I_{max} = 1. \quad (18)$$

However to prove experimentally that such sinusoidal interference pattern actually exists we must definitively record photons in the rest of the wires plane. This is why the experiment described in [1] does not constitutes a violation of complementarity.

It is finally interesting to remark that similar analysis could be easily done already in the Young two-holes experiment. Indeed, suppose that we record the photon interference fringes after the holes. We can thus measure $V = 1$. However, if we suppose that the sinusoidal oscillation of the intensity results from the linear superposition of waves coming from holes 1 and 2 then from equation 5 we deduce $|C_1|^2 + |C_2|^2 - 2|C_1||C_2| = 0$ i. e., $|C_1| = |C_2|$. From equation 2 this implies $K = 0$. Reasoning like Afshar *et al.* we could be tempted to see once again a violation of complementarity since we deduced the distinguishability without disturbing the fringes! However, we think that our previous analysis sufficiently clarified the problem so that paradoxes of that kind are now naturally solved without supplementary comments.

3.3 The objectivity of trajectory in quantum mechanics

At the end of section 2.1 we shortly pointed that the concept of trajectory is a key issue in the analysis of the experiment reported in reference 1. This was also at the core of most commentaries (e.g. references [6–14]) concerning the work by Afshar *et al.*. As a corollary to the previous analysis we will now make a brief comment concerning the concept of path and trajectory in quantum mechanics since we think that a lot of confusion surrounds this problem. This is also important because Afshar *et al.* claimed not only that they can

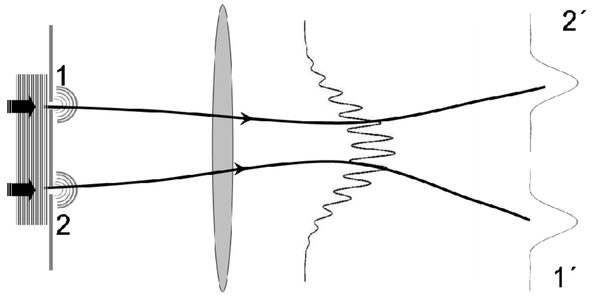

Fig. 3: Illustration of the counterintuitive paths followed by photons if we accept the ontological interpretation given by de Broglie and Bohm. The photons coming from aperture 1 or 2 reach the 'wrong' detector 2' or 1'.

circumvent complementarity but that additionally they determine the *path* chosen by the particle. Following here an intuitive assumption they accepted that with the two pinholes open a photon trajectory (if trajectory there is) connects necessarily a pinhole to its optical image like it is in geometrical optics. They called that intuition (probably in analogy with what occurs in classical physics) a 'consequence of momentum conservation'. However, the meaning of momentum and trajectory is not the same in quantum and classical mechanics. Actually, as it was realized by several physicists the connection 1 to 1' and 2 to 2' is a strong hypothesis which depends of our model of (hidden) reality and which can not in general be experimentally tested (read for example [29, 36]).

Actually nothing in this experiment with two holes forbids a photon coming from one pinhole to go in the *wrong* detector associated with the second pinhole. This is the case for example in the *hidden variable* theory of de Broglie-Bohm in which every photons coming from the aperture 1 (respectively 2) is reaching the wrong image spot 2'(respectively 1') [29, 36] as shown in figure 3. This is counter intuitive but not in contradiction with experiments since we can not objectively test such hidden variable model [36]. In particular closing one pinhole will define unambiguously the path followed by the particle. However this is a different experiment and the model shows that the trajectories are modified (in general non locally) by the experimental context. The very existence of a model like the one of de Broglie and Bohm demonstrates clearly that in the (hidden) quantum reality a trajectory could depend of the complete context of the experiment. For this reason we must be very prudent and conservative when we interpret an experiment: Looking the image of a pinhole recorded in a statistical way by a cascade of photon will not tell us from which pinhole an individual photon come from but only how many photons crossed this pinhole. In counterpart of course we can not see the fringes and the complementarity principle of Bohr will be, as in every quantum experiment, naturally respected. It is thus in general dangerous to speak unambiguously of a which-path experiment and this should preferably be avoided from every discussions limited to empirical facts. As claimed by Bohr the best empirical choice is in such conditions to accept that *it is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature* [4].

4 Conclusion

To conclude, in spite of some claims we still need at least two complementary experiments in order to exploit the totality of the phenomenon in Young-like interferometers. Actually, as pointed out originally by Bohr, we can not use information associated with a same photon event to reconstruct in a statistical way (i.e. by a accumulation of such events) the two complementary distributions of photons in the image plane of the lens and in the interference plane. The presence of the wires inserted in reference 1 does not change anything to this fact since the information obtained by adding the wires is too weak and not sufficient to rebuild objectively (i. e. , unambiguously from experimental data) the whole interference pattern. The reasoning of Afshar *et al.* is therefore circular and the experiment is finally in complete agreement with the principle of complementarity.

Submitted on September 12, 2010 / Accepted on September 15, 2010

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