

On the Neutrino Opera in the CNGS Beam

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In this brief paper, we solve the relativistic kinematics related to the intersection between a relativistic beam of particles (neutrinos, e.g.) and consecutive detectors. The gravitational effects are neglected, but the effect of the Earth rotation is taken into consideration under a simple approach in which we consider two instantaneous inertial reference frames in relation to the fixed stars: an instantaneous inertial frame of reference having got the instantaneous velocity of rotation (about the Earth axis of rotation) of the Cern at one side, the lab system of reference in which the beam propagates, and another instantaneous inertial system of reference having got the instantaneous velocity of rotation of the detectors at Gran Sasso at the other side, this latter being the system of reference of the detectors. Einstein's relativity theory provides a velocity of intersection between the beam and the detectors greater than the velocity of light in the empty space as derived in this paper, in virtue of the Earth rotation. We provide a simple calculation for the discrepancy between a correct measure for the experiment and a measure arising due to the effect derived in this paper.

1 Definitions and Assumptions

Consider a position vector for CERN in relation to the center of the Earth, vector \vec{C} , and a position vector for the Gran Sasso receptors in relation to the center of the earth, vector \vec{G} . Consider the angular velocity vector of the Earth along its axis of rotation, vector $\vec{\omega}$. The velocity of rotation of \vec{C} in relation to Earth's axis is given by $\vec{v}_C = \vec{\omega} \times \vec{C}$. Analogously, the velocity of rotation of \vec{G} in relation to Earth's axis is given by $\vec{v}_G = \vec{\omega} \times \vec{G}$. Consider a baseline \mathcal{L}_{CG} connecting \vec{C} and \vec{G} along the vector $\vec{G} - \vec{C}$; CERN's and Gran Sasso's latitudes (\downarrow_S^N), λ_C and λ_G , respectively, and CERN's and Gran Sasso's longitudes ($\leftarrow WE \rightarrow$), α_C and α_G , respectively.

Since the effect related to the velocity of the neutrinos depends on its own velocity at the completion of the calculation and on the rotation of the Earth, viz., such effect does not depend on the specific values of the lateral velocity (to be defined below) of the receptors, as we will see, we may consider some geometric assumptions to simplify the geometry related to the baseline path $\vec{G} - \vec{C}$ along \mathcal{L}_{CG} through the Earth.

Firstly, we will consider \vec{C} and \vec{G} having got the same latitude λ^* :

$$\lambda_C = \lambda_G = \lambda. \quad (1)$$

These latitudes would be important if the effect to be derived here was related to specific values of latitude, its fluctuations, systematic and/or statistical errors related to it etc., related to the six standard deviations that characterizes the claim related to the experiment. But that is not the case. Now, consider the plane Π , orthogonal to $\vec{\omega}$, that cross the Earth through the hypothetically common latitude containing the points \vec{C} and \vec{G} . Trace two lines pertaining to Π :

*The latitudes of CERN and Gran Sasso are, respectively: $46^{\text{deg}}14^{\text{min}}3^{\text{sec}}$ (N) and $42^{\text{deg}}28^{\text{min}}12^{\text{sec}}$ (N). The longitudes of CERN and Gran Sasso are, respectively: $6^{\text{deg}}3^{\text{min}}19^{\text{sec}}$ (E) and $13^{\text{deg}}33^{\text{min}}0^{\text{sec}}$ (E).

the line \mathcal{L}_{CA} , from the point \vec{C} to Earth's rotation axis, and the line \mathcal{L}_{GA} , from the point \vec{G} to Earth's rotation axis. \mathcal{L}_{CA} and \mathcal{L}_{GA} cross the rotation axis at the point \vec{A} . Also, trace the mediatriz line \mathcal{L}_{MA} , from the point \vec{A} to the point $\vec{M} = (1/2)(\vec{G} + \vec{C})$, equally dividing \mathcal{L}_{CG} . The angle between \mathcal{L}_{CA} and \mathcal{L}_{MA} equals the angle between \mathcal{L}_{MA} and \mathcal{L}_{GA} , being this angle given by:

$$\alpha = \frac{1}{2}(\alpha_G - \alpha_C). \quad (2)$$

Upon the previous remarks regarding the geometric simplifications, the same remarks hold for the radius of the Earth, i.e., we will consider the Earth as a sphere. Thus, the following relation holds:

$$|\vec{C}| = |\vec{G}| = R_E = \frac{R}{\cos \lambda}, \quad (3)$$

where R_E is the radius of the Earth, its averaged value $R_E = 6.37 \times 10^6 m$, and $R = |\vec{C} - \vec{A}| = |\vec{G} - \vec{A}|$.

2 Defining two Instantaneous Inertial Reference Frames

The relativistic kinematics will run in the plane Π previously defined. The line \mathcal{L}_{CG} will define an axis: Ox , with the origin O at the point \vec{C} , being the unitary vector of the axis Ox , \hat{e}_x , given by:

$$\hat{e}_x = \frac{\vec{G} - \vec{C}}{|\vec{G} - \vec{C}|}. \quad (4)$$

Now, define the Oz axis contained in the Π plane such that its unitary vector, \hat{e}_z , is given by:

$$\hat{e}_z = -\hat{e}_x \times \frac{\vec{\omega}}{|\vec{\omega}|}. \quad (5)$$

The axis Oy is trivially obtained with its unitary vector being given by:

$$\hat{e}_y = \hat{e}_z \times \hat{e}_x. \quad (6)$$

Now, we define the system at the Gran Sasso's detectors, $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$, such that its origin \tilde{O} is at the point \tilde{C} , being the unitary vector of the axis $\tilde{O}\tilde{x}$, the same $\hat{e}_x \equiv \hat{e}_{\tilde{x}}$. The axis $\tilde{O}\tilde{z}$ is parallel to Oz , with the same unitary vector $\hat{e}_z \equiv \hat{e}_{\tilde{z}}$, with analogous reasoning to obtain the axis $\tilde{O}\tilde{y}$ and its unitary vector $\hat{e}_{\tilde{y}} \equiv \hat{e}_y$. In other words, $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ is the parallel pure translation of $Oxyz$ from \tilde{C} (CERN) to \tilde{C} (Gran Sasso).

To define the two instantaneous inertial reference frames to accomplish, simply, the effect of the Earth rotation, we, firstly, write down the rotation velocities of the points \tilde{C} and \tilde{C} about Earth's rotation axis, i.e., we write down the rotation velocities of (CERN) and (Gran Sasso) about Earth's axis. For CERN, the rotation velocity \vec{v}_C reads:

$$\vec{v}_C = \vec{\omega} \times \vec{C} = \omega R \hat{e}_\phi = \omega R (\cos \alpha \hat{e}_x - \sin \alpha \hat{e}_z), \quad (7)$$

where the auxiliar unitary vector has been the azimuthal ϕ -versor of the spherical coordinates, the spherical coordinates with origin at the center of the Earth with its equatorial dextrogyre plane Ξ such that $\vec{\omega} \cdot \vec{\xi} = \vec{0} \forall \vec{\xi} \in \Xi$. For Gran Sasso, the rotation velocity \vec{v}_G reads:

$$\vec{v}_G = \vec{\omega} \times \vec{G} = \omega R \hat{e}_{\tilde{\phi}} = \omega R (\cos \alpha \hat{e}_{\tilde{x}} + \sin \alpha \hat{e}_{\tilde{z}}), \quad (8)$$

where $\hat{e}_{\tilde{\phi}}$ is the azimuthal ϕ -versor previously defined, but now at Gran Sasso.

We see via the eqs. (7) and (8) that both the frames of reference, $Oxyz$ and $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$, are instantaneously under a null relative translation through the common axis $Ox \equiv \tilde{O}\tilde{x}$, and under a reverse translation through their respective parallel axes $Oz \parallel \tilde{O}\tilde{z}$. We will *inertially* consider this quite instantaneous* effect of the reverse translation (Newton's first law will hold, we will instantaneously neglect the gravitational field through the neutrino travel to Gran Sasso, as well as the weak characteristic for neutrino interactions with matter) via the following approach:

- We will consider a system of reference $O_C x_C y_C z_C$ that exactly coincides with $Oxyz$ at the instant $t_C = 0$, but with the following constant velocity of translation in relation to the fixed stars: $\vec{v}_\epsilon = \omega R \cos \alpha \hat{e}_{x_C} - \omega R \sin \alpha \hat{e}_{z_C} = \omega R \cos \alpha \hat{e}_x - \omega R \sin \alpha \hat{e}_z = \vec{v}_C$, such that the neutrino travel will be in this inertial referential. The subscript ϵ is to asseverate this referential is being considered for the neutrino travel during the entire process (emission→detection), but with $\epsilon \approx 0$ in the sense given in the previous footnote [$\epsilon \equiv \delta t_\nu / T \ll 1$]. Considered this, we will drop the subscript C in $O_C x_C y_C z_C$, for the sake of economy of notation, and rename it simply as $Oxyz$, although this latter is not the original one;

*The time spent by a neutrino beam to accomplish the race from \tilde{C} to \tilde{C} , δt_ν , obey $\delta t_\nu / T \ll 1$, where T is the period of Earth's rotation about its axis, thus quite instantaneous in relation to the Earth daily kinematics.

- We will consider a system of reference $O_G x_G y_G z_G$ that exactly coincides with $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ at the instant $t_G = t_C = 0^\dagger$, but with the following velocity of translation in relation to the fixed stars: $\vec{u}_\epsilon = \omega R \cos \alpha \hat{e}_{x_G} + \omega R \sin \alpha \hat{e}_{z_G} = \omega R \cos \alpha \hat{e}_{\tilde{x}} + \omega R \sin \alpha \hat{e}_{\tilde{z}} = \vec{v}_G$. Considered this, we will drop the subscript G in $O_G x_G y_G z_G$, for the sake of economy of notation, and rename it simply as $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$, although this latter is not the original one;
- We will consider a system of reference travelling with the beam of neutrinos, but this will be explained in the next section.

3 From CERN to the Flux through the Gran Sasso Detectors

From now on, we model the lattice (strips, emulsion, cintillators etc) distribution through the Grand Sasso's detectors from the perspective of an $Oxyz^\ddagger$ observer with the following characteristics:

- The average proper (no Lorentz contraction in $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$) displacement of detectors along $\tilde{O}\tilde{x}$ is $d_{\tilde{O}\tilde{x}}$;
- The average proper (no Lorentz contraction in $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$) displacement of detectors along $-\tilde{O}\tilde{y}$ is $d_{\tilde{O}\tilde{y}}$;
- The average proper (no Lorentz contraction in $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$) displacement of detectors along $-\tilde{O}\tilde{z}$ is $d_{\tilde{O}\tilde{z}}$;
- The detectors in $Oxyz$ will be abstracted to a tridimensional $d_{0x} \times d_{0y} \times d_{0z}$ orthogonally spaced lattice falling upward [see the eqs. (7) and (8)] at the velocity $\vec{v}_G - \vec{v}_C = 2\omega R \sin \alpha \hat{e}_z$, being the basis vectors of these sites given by $\{\vec{d}_{0x} = d_{0x} \hat{e}_x, \vec{d}_{0y} = -d_{0y} \hat{e}_y, \vec{d}_{0z} = -d_{0z} \hat{e}_z\}$, where $\{\hat{e}_x, \hat{e}_y, \hat{e}_z\}$ is the canonical spacelike 3D euclidian orthonormal basis of $Oxyz$.
- We will neglect relativistic (Einstein's) effects related to the movement of the lattice of detectors, the movement of $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ in $Oxyz$, as previously stated, but such effects will become important in the referential of the neutrino beam (to be defined below).

Now, we define the neutrino frame of reference $O'x'y'z'$ in the canonical configuration with the frame of reference $Oxyz$, i.e., coincident origins at $t = t' = 0$ keeping the space-like parallelism of the axes $x \equiv x', y \equiv y'$ and $z \equiv z'$ and

[†]The relativistic effects between the systems of reference at CERN and at Gran Sasso related to time synchronization is being neglected due to the order of magnitude related to the velocities due to the Earth rotation and due to the magnitude of the gravitational field as previously stated. Furthermore, we are undressing these effects between these systems at \tilde{C} and \tilde{C} to asseverate the relevant relativistic effects that will lead to the neutrino velocity will raise in virtue of relativistic motion in relation to the detectors in Gran Sasso, as we will see.

[‡]From now on, we are working with the inertial frames (in relation to the fixed stars) defined above, viz., from now on: $Oxyz$ means $O_C x_C y_C z_C$ (see the two final paragraphs of the previous section); $\tilde{O}\tilde{x}\tilde{y}\tilde{z}$ means $O_G x_G y_G z_G$ (see the two final paragraphs of the previous section).

boosted with velocity $v_\nu \hat{e}_x$ in relation to the $Oxyz$ frame. Consider a neutrino beam entering the block of detectors in Gran Sasso in the $Oxyz$ frame of reference. The beam passes a lattice of detectors stated above, being these detectors raining upward with velocity $\vec{v}_G - \vec{v}_C = 2\omega R \sin \alpha \hat{e}_z$ through the beam in the $Oxyz$ frame of reference. A horizontal neutrino beam, along Ox , may contact a horizontal [vertical means along Oz , upward means in the \hat{e}_z direction, and horizontal means parallel to the xy plane] lattice of detectors parallelly raining upward in virtue of the Earth rotation as discussed before (raining upward with velocity $\vec{v}_G - \vec{v}_C = 2\omega R \sin \alpha \hat{e}_z$). Once an interaction occurs between the horizontal beam and consecutively located detectors in this horizontal lattice, this interaction is simultaneous in the $Oxyz$ (rigorously $Oxyzt$, but the context is clear here) world, implying non-simultaneity for these raindrops of detectors in the $O'x'y'z'$ world. The distribution of these raindrops of detectors must have, *instantaneously* at t' in $O'x'y'z'$ world, the following characteristics:

- The displacement between two consecutive raindrops of detectors correlated to the respective simultaneous ones in $Oxyz$, these latter displaced by the proper distance $x_{i+1} - x_i = d_{0x}$ along Ox and belonging to the falling upward xy plane of detectors in $Oxyz$, is given by:

$$\begin{aligned} x'_{i+1}(t') - x'_i(t') &= \gamma^{-1} (x_{i+1} - x_i) \\ &= \gamma^{-1} d_{0x}, \end{aligned} \quad (9)$$

being $\gamma = 1/\sqrt{1 - v_\nu^2/c^2}$, c the speed of light in the empty space, v_ν the speed of the neutrino, whose velocity is along the \hat{e}_x direction in the $Oxyz$ world (the beam of neutrinos is at rest in its referential $O'x'y'z'$, as previously seen).

- The displacement between two consecutive raindrops of detectors correlated to the respective simultaneous ones in $Oxyz$, these latter displaced by the proper distance $z_i - z_{i+1} = 0$ along Oz and belonging to the falling upward xy plane of raining detectors in $Oxyz$, is given by:

$$z'_{i+1}(t') - z'_i(t') = 2 \frac{v_\nu d_{0x}}{c^2} \omega R \sin \alpha. \quad (10)$$

- The vertical distance between consecutive (consecutive but inclined in the $O'z'y'z'$ world; the parallel to xy planes of detectors parallelly raining upward in $Oxyz$ become inclined in $O'x'y'z'$) raining planes of detectors Π'_i and Π'_{i+1} , $\forall i$, remains the same d_{0z} distance, the distance between consecutive parallelly raining planes of detectors. The raining upward planes turn out to be inclined in relation to the $x'y'$ plane of the neutrino world $O'x'y'z'$ by the angle:

$$\theta = \pi - \arctan \left(2 \frac{\gamma v_\nu}{c^2} \omega R \sin \alpha \right). \quad (11)$$

Indeed, let's derive these facts. Firstly, instantaneously at t in $Oxyz$, two consecutive raindrops* Ox along, are time delayed in $O'x'y'z' \times \{t'\}$ world by the amount:

$$\begin{aligned} t'_{i+1} - t'_i &= \gamma \left(t - \frac{v_\nu}{c^2} x_{i+1} \right) - \gamma \left(t - \frac{v_\nu}{c^2} x_i \right) \\ &= -\gamma \frac{v_\nu}{c^2} (x_{i+1} - x_i) = -\gamma \frac{v_\nu}{c^2} d_{0x}, \end{aligned} \quad (12)$$

in virtue of the Lorentz transformations $(x, t) \rightarrow (x', t')$. Here, we see a detection that occurs at the position x_{i+1} pertaining to the horizontal lattice of detectors in Gran Sasso, at the plane $\tilde{x}\tilde{y}$ within the block of detectors in Gran Sasso, hence more internal, (remember $\tilde{x}\tilde{y} \parallel xy$), *must occur earlier* than the detection at the position x_i in the frame of reference of the beam of neutrinos, and the i -raindrop is late in relation to the $(i+1)$ -raindrop. Hence, backwarding the t'_i clocks down to the t'_{i+1} instant (backwarding the movie, maybe better: backwarding the neutrino's opera), i.e., comparing the non-simultaneous events in the beam of neutrinos frame, the event $i+1$ occurring when the $i+1$ -raindrop crosses the beam of neutrinos and the event i when the i -raindrop crosses the beam of neutrinos (remember these events are simultaneous in $Oxyz$) previously to infer the instantaneous (at t'_{i+1}) position of the i -raindrop when the $i+1$ raindrop crosses the beam of neutrinos at the instant $t'_{i+1} < t'_i$ in the $O'x'y'z't'$ frame, the i -raindrop must move the amounts (backwarding the movie from the instant t'_i at which the i -raindrop crosses the beam of neutrinos in the $O'x'y'z't'$ world to the non-simultaneous instant $t'_{i+1} < t'_i$ at which the $i+1$ -raindrop crosses the beam of neutrinos in the $O'x'y'z't'$ world): $\delta z'$ downward and $\delta x'$ to the right, being these amounts given by:

$$\begin{aligned} \delta z' &= \left(\frac{2\omega R \sin \alpha}{\gamma} \right) \times \left(-\gamma \frac{v_\nu}{c^2} d_{0x} \right) = -\frac{2\omega d_{0x} v_\nu R \sin \alpha}{c^2}; \\ \delta x' &= (-v_\nu) \times \left(-\gamma \frac{v_\nu}{c^2} d_{0x} \right) = \frac{v_\nu^2 \gamma d_{0x}}{c^2}, \end{aligned} \quad (13)$$

since $(-v_\nu \hat{e}_x + (2\omega R \sin \alpha / \gamma) \hat{e}_z)$ is the velocity of raindrops in $O'x'y'z'$, obtained from the Lorentz transformations $L(\vec{u})$ for the 3-velocities of the Gran Sasso lattice block of sensors, the raining raindrops lattice of sensors, from the $Oxyz$ to the beam of neutrinos frame $O'x'y'z'$:

$$(0, 0, 2\omega R \sin \alpha) |_{Oxyz} \xrightarrow{L(\vec{u})} (-v_\nu, 0, 2\omega R \sin \alpha / \gamma) |_{O'x'y'z'}. \quad (14)$$

But, at t , the i -raindrop and the $(i+1)$ -raindrop have got the same z coordinate, since they are in a xy plane, and, since the $z \rightarrow z'$ Lorentz map is identity, these raindrops must have the same z' coordinate at their respective transformed instants

*From now on, we will call raindrops the detectors in the lattice of detectors within the block of detectors at Gran Sasso. Thus, raindrops \equiv detectors within the lattice of detectors defined at the beginning of this section; 1 raindrop \equiv 1 detector within the lattice of detectors within the block of detectors at Gran Sasso.

(of course, since at *each* of these transformed instants, different instants in $O'x'y'z'$ in virtue of the non-simultaneity in this frame, the z' coordinate will read the same, since these raindrops will cross the beam and the beam has the same coordinate z' in its own frame of reference, viz., the beam is parallel to $O'x'$). Hence, backwarding t'_i clocks down to the the t'_{i+1} instant, one concludes that the $\delta z'$ in the eq. (13) is the instantaneous, at same t' , height shift between consecutive raindrops that simultaneously cross the beam of neutrinos in $Oxyz$. The $x \rightarrow x'$ Lorentz map is not identity, implying one must calculate the $x'_{i+1} - x'_i$ shift at the $Oxyz$ instantaneous t :

$$\begin{aligned} x'_{i+1}(t) - x'_i(t) &= \gamma(x_{i+1} - v_v t) - \gamma(x_i - v_v t) = \gamma(x_{i+1} - x_i) \\ &= \gamma d_{0x}. \end{aligned} \quad (15)$$

This shift is related to different instants, t'_i, t'_{i+1} , in the beam of neutrinos frame. Thus, backwarding t'_i clocks down to the the t'_{i+1} instant (backwarding the movie to observe the earlier t'_{i+1} instantaneous), this amount given by the eq. (15) is reduced by the amount $\delta x'$ given by eq. (13):

$$\begin{aligned} x'_{i+1}(t') - x'_i(t') &= \gamma d_{0x} - \gamma d_{0x} \frac{v_v^2}{c^2} = \gamma d_{0x} \left(1 - \frac{v_v^2}{c^2}\right) \\ &= \gamma^{-1} d_{0x}. \end{aligned} \quad (16)$$

The first eq. (13) gives the eq. (10), since eq. (13) gives the z' position of the i -raindrop at the previous instant t'_{i+1} before the i -raindrop crosses the beam of neutrinos in the $O'x'y'z'$, therefore:

$$z'_i(t'_{i+1}) = z'_v - \frac{2\omega d_{0x} v_v R \sin \alpha}{c^2}, \quad (17)$$

where z'_v is a constant z' coordinate of the beam of neutrinos in its own frame; and, since the z' position of the $(i+1)$ -raindrop at the t'_{i+1} instant is z'_v (due to the very fact the $(i+1)$ -raindrop crosses the beam at the instant t'_{i+1} in the $O'x'y'z'$ world), one has $z'_{i+1}(t'_{i+1}) = z'_v$, from which, with the eq. (17), one has got:

$$z'_{i+1}(t'_{i+1}) - z'_i(t'_{i+1}) = 2 \frac{v_v d_{0x}}{c^2} \omega R \sin \alpha, \quad (18)$$

reaching the eq. (10). The non-instantaneous displacement (non-instantaneous in $O'x'y'z'$) given by eq. (15) is the distance between two successive non-instantaneous interactions with the beam, raindrops marks assigned upon the beam in $O'x'y'z'$. This fact is easy to understand, as these instantaneously assigned marks (instantaneous in $Oxyz$) would become splayed in $O'x'y'z'$, since the beam turns out to be contracted in $Oxyz$ due to Lorentz contraction. Also, one shall infer that eq. (16) gives the t' instantaneous displacement of falling upward raindrops along $O'x'$. The reason why the distance between consecutive raindrops marks γd_{0x} are bigger than the contracted distance $\gamma^{-1} d_{0x}$ of the two consecutive falling raindrops is explained by the non-simultaneity between these raindrops when touching the proper beam in the

$O'x'y'z'$ world, straightforwardly seen by the inclination (the horizontal planes of raindrops in $Oxyz$ inclines in $O'x'y'z'$) between the raindrop plane containing these two consecutive raindrops in $O'x'y'z'$ and the proper plane $\Pi'_v \parallel x'y'$ containing the neutrinos beam in $O'x'y'z'$; i.e., when the first sensor raindrop crosses the beam, assigning the first interaction, the second travels an amount $\delta x'$ to the left given by the second eq. (13) before crossing the beam, assigning the second interaction. A xy instantaneous falling upward plane of sensors within the block of sensors at Gran Sasso containing raindrops in $Oxyz$ world becomes an inclined instantaneous falling upward plane in $O'x'y'z'$ world, being the inclination, eq. (11), easily derived from eqs. (16) and (18):

$$\tan(\pi - \theta) = \frac{\delta z'(t')}{x'_{i+1}(t') - x'_i(t')} = 2 \frac{\gamma v_v}{c^2} \omega R \sin \alpha, \quad (19)$$

giving the eq. (11).

4 Faster than Light Effects in Gran Sasso

To understand the effect, first, consider two sensors, say i -raindrop and $(i+1)$ -raindrop. If these sensors are constructed to tag the instants, t_{i+1} and t_i , at which two events are registered at their exact locations and a team of physicists obtains the time variation interval by $t_{i+1} - t_i$, being $x_{i+1} - x_i$ the distance between these sensors, one would have:

$$\frac{\delta x}{\delta t} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i} = \infty, \quad (20)$$

for simultaneous events ($t_i = t_{i+1}$), if one expects a signal is travelling between the sensors. Furthermore, if one expects a privileged direction along which the signal should travel from the i -raindrop (first) to the $(i+1)$ raindrop (later), if the $(i+1)$ -raindrop registered a signal before the i -raindrop, violating the expected sequential direction of detections, one would say the signal would have been registered from the future to the past direction. In the previous section the instantaneous events in the $Oxyz$ became non-instantaneous in the beam frame of reference, and the internal register within the Gran Sasso block along the direction $Ox \equiv \vec{O}\hat{x}$, at the position x_{i+1} registered the interaction with the beam at the same instant the internal register at the position x_i registered, since these events were hypothetically simultaneous in $Oxyz$, in virtue of the Earth rotation. From the point of view of the neutrino beam, these registers occurred in the order: x'_{i+1} before, x_i later, due to the inclination of the raindrops planes in virtue of the Earth rotation. We are forced to conclude the rotation of the Earth may provide a kinematics of intersection between beams and sequential sensors that may led to the conclusion the sensors are registering time intervals related to quasi-simultaneous events that are cintilated by different particles at different positions almost at the same time, leading to an erroneous conclusion that the signal would have travelled between the sensors generating the time tag data. E.g.,

suppose two ideal clocks, perfect ones, gedanken ones, that register the instants: t_i at which a beam of neutrinos enters the block of raindrop sensors in Gran Sasso and t_o at which this beam of neutrinos emerges from the block. Let d_B be the length travelled through the block. One team of physicists will measure the velocity of the beam by $d_B/(t_o - t_i)$ with no use of data from the sensors within the block. Another team will perform the calculation from the data obtained from a sequence of sensors (raindrops) located Ox along. This second team may obtain registers at different positions x_{i+1} and x_i related to the lateral intersection between these sensors and the beam entirely into the block of sensors but with the beam travel not entirely accomplished through the block. The data of this second team would be mistaken, since the registers at the different locations x_{i+1} and x_i would not have been made by the same neutrino, implying the clocks at x_{i+1} and x_i would be registering two quasi-simultaneous events *not related to a same neutrino*, concluding erroneously that the time variation between these events was so small that the particle that generated these events would be travelling with a velocity greater than c .

Einstein's theory of relativity does not avoid velocities greater than the light in the empty space, but avoids *an unique* particle propagating with velocity greater than the velocity of light in the empty space. To infer that a velocity greater than c may arise from the discussion through this brief article, consider the velocity two *different* raindrops interact with the beam of neutrinos in the beam $O'x'y'z'$ frame of reference. These events are non-simultaneous in the beam frame as previously discussed, but the beam crosses two successive interactions with a propagation that is faster than c , since the distance between two successive interactions along the beam in the beam frame of reference is given by the eq. (15), γd_{0x} , being the time spent given by the eq. (12), $(\gamma v_y d_{0x})/c^2$. Thus, the 2-propagation $\mathcal{V}'_{(i+1) \rightarrow (i)}$ (the number 2 to denote two bodies are related to a single propagation velocity):

$$\mathcal{V}'_{(i+1) \rightarrow (i)} = \frac{c}{v_y} c \Rightarrow \mathcal{V}'_{(i+1) \rightarrow (i)} > c. \quad (21)$$

As asseverated this is not a propagation of a single particle, but a ratio between the covered distance along the beam in the beam frame and the time interval spent to interact, non-simultaneously, with two sequential but distinct sensors (raindrops x_{i+1} and x_i). Of course, if $v_y \rightarrow 0$, these distinct interactions will tend to become simultaneous, leading to the result discussed at the beginning of this section (eq. 20). It follows that is not difficult to conclude that the time elapsed between two distinct sensors must be related to just an unique particle if one is intended to use their time tags for velocity computations.

5 The Consequence of the Effect

A simple calculation provides the discrepancy obtained by the set of CNGS detections intended to obtain the correct velocity

of the neutrino particle announced few days ago. Let v_c be the correct value for the neutrino's velocity, δS_1 , the distance between the CERN and the point at which the neutrino enters the block of detectors at Gran Sasso, δS_2 the length of the block of detectors to be internally covered by the neutrino at Gran Sasso, δt_1 the elapsed time spent by the neutrino to cover δS_1 , δt_2 the elapsed time spent by the neutrino to cover δS_2 , δt_f a fake elapsed time due to the effect previously discussed, and v_f a fake velocity that would arise from an erroneous measure for the elapsed time through δS_2 . Hence:

$$v_c = \frac{\delta S_1 + \delta S_2}{\delta t_1 + \delta t_2}, \quad (22)$$

$$v_f = \frac{\delta S_1 + \delta S_2}{\delta t_1 + \delta t_f}. \quad (23)$$

A simple calculation gives:

$$\frac{v_f}{v_c} = 1 + \frac{\delta t_2 - \delta t_f}{\delta t_1 + \delta t_f} \Rightarrow \frac{v_f - v_c}{v_c} = \frac{\delta t_2 - \delta t_f}{\delta t_1 + \delta t_f}. \quad (24)$$

One should not write $v_f = \delta S_2/\delta t_f$, since, as previously discussed, δt_f is not related to a propagation of a particle, but to the time elapsed between two detectors in the same xy plane in the $Oxyz$ frame of reference. If a sequence of cintillations within the block of detectors are generated by different neutrinos due to the effect previously discussed, and this sequence is interpreted as a path traced by a single neutrino, the measure of the distance covered within the block of detectors at Gran Sasso would encapsulate an error for each estimated path having got the effect encrusted within it. This distance, an erroneous one, is δS_2 , although this distance may be defined as a correct one for purposes of comparison with a case in which (hypothetically) a neutrino travelled this distance with the correct velocity v_c . Obviously, the δt_f would not be related to the time spent to cover this distance, since, in a case in which the effect, as previously explained, was generated due to simultaneous time tagging at two different sensors (raindrops) due to two different neutrinos in the $Oxyz$ reference frame, one would have $\delta t_f = 0$ for a continuous laterally traced path in virtue of the Earth rotation, from which $\delta S_2 \neq 0$. δS_2 would arise, under the effect discussed in this paper, from a path misinterpretation. But, once one defines δS_2 as the distance to be covered in a comparison case with a neutrino used to cover it with the correct velocity v_c :

$$\delta t_2 = \frac{\delta S_2}{v_c}, \quad (25)$$

turns out to be the correct elapsed time. From the eq. (24), one reaches:

$$v_f (\delta t_1 + \delta t_f) - v_c \delta t_1 = v_c \delta t_2. \quad (26)$$

If the computation is done taken into consideration the erroneous *elapsed* time δt_f , being this $\delta t_f = 0$ in a case of

simultaneity due to the effect previously discussed, the eq. (26) reads:

$$v_f \delta t_1 - v_c \delta t_1 = v_c \delta t_2, \quad (27)$$

and the effect turns out to increase the fake velocity in relation to the correct velocity in a manner in which the neutrino with the fake velocity would be winning the race by an amount of distance given by $\delta S_2 = v_c \delta t_2$, accomplished the path δS_1 , from the eq. (27). Hence, eq. (24) turns out to read:

$$\frac{v_f - v_c}{v_c} = \frac{\delta t_2}{\delta t_1} = \frac{\delta S_2}{v_c \delta S_1} = \frac{\delta S_2}{\delta S_1}. \quad (28)$$

With the values [1]:

$$\frac{v_f - v_c}{v_c} = [2.48 \pm 0.28 \text{ (stat.)} \pm 0.30 \text{ (sys.)}] \times 10^{-5}, \quad (29)$$

and:

$$\delta S_1 = 733 \times 10^3 m, \quad (30)$$

we reach for the discrepancy between the covered distances:

$$\delta S_2 = [18.2 \pm 2.05 \text{ (stat.)} \pm 2.20 \text{ (sys.)}] m. \quad (31)$$

6 Conclusion

We conclude the relativistic effect discussed here in virtue of the Earth rotation may lead to a misinterpretation of the elapsed time within the block of detectors at Gran Sasso.

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