A Bipolar Model of Oscillations in a Chain System for Elementary Particle Masses

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The philosophical idea of a bipolar nature (the Chinese "Yin and Yang") is combined with the mathematical formalism of a fractal scaling model originally published by Müller in this journal. From this extension new rules for the calculation of proton and electron resonances via continued fractions are derived. The set of the 117 most accurately determined elementary particle masses (all with error < 0.13%) was expressed through this type of continued fractions. Only one outlier was found, in all other cases the numerical errors were smaller than the standard deviation. Speaking in terms of oscillation properties, the results suggest that the electron is an inverted or mirrored oscillation state of the proton and vice versa. A complete description of elementary particle masses by the model of oscillations in a chain system is only possible when considering both, proton and electron resonances.

1 Introduction

The mass distribution of elementary particles is still an unsolved mystery of physics. According tho the Standard Model, mass is given by arbitrary variable couplings to the Higgs boson, and the coupling is then adequately adjusted to reproduce the experimentally observed mass.

However, the particle mass spectrum is not completely chaotic, and some groupings are clearly visible. Several attempts have already been made to obtain equations to describe regularities in the set of elementary particle masses.

For instance Greulich [1] calculated the masses of all fundamental elementary particles (those with a lifetime $> 10^{-24}$ seconds) with an inaccuracy of approximately 1% using the equation

$$\frac{m_{particle}}{m_{electron}} = \frac{N}{2\alpha},$$

where α is the fine structure constant (= 1/137.036), and N is an integer variable.

Paasch [2] assigned each elementary particle mass a position on a logarithmic spiral. As a result, particles then accumulate on straight lines.

A study from India [3] revealed a tendency for successive mass differences between particles to be close to an integer multiple or integer fraction of 29.315 MeV. The value 29.315 MeV is the mass difference between a muon and a neutral pion.

Even more recently Boris Tatischeff published a series of articles [4–8] dealing with fractal properties of elementary particle masses. He even predicted tentatively the masses of some still unobserved particles [5].

An other fractal scaling model was used in a previous article of the present author [9], and a set of 78 accurately measured elementary particle masses was expressed in the form of continued fractions. This underlying model was originally published by Müller [10–12], and its very basic idea is to treat all protons as fundamental oscillators connected through the physical vacuum. This leads to the idea of a chain of equal harmonic proton oscillators with an associated logarithmic spectrum of eigenfrequencies which can be expressed through continued fractions. Particle masses are interpreted as proton resonance states and expressed in continued fraction form. However, the results obtained in reference [9] were not completely satisfying since around 14% of the masses were outliers, i.e. could not be reproduced by this model.

A more recent article [13] revealed that electron resonance states exist analogously which serves now as the basis for further extensions of Müller's model. From this starting point, the present article proposes a new version of the model developed with the objective to reproduce all elementary particle masses.

2 Data sources and computational details

Masses of elementary particles (including the proton and electron reference masses) were taken from the Particle Data Group website [14] and were expressed in GeV throughout the whole article. An electronic version of these data is available for downloading. Quark masses were eliminated from the list because it has not been possible to isolate quarks.

Some of the listed particle masses are extremely accurate and others have a quite high measurement error. Figure 1 shows an overview of the particle masses and their standard deviations (expressed in % of the particle mass). It can be roughly estimated that more or less 60% of the particles have a standard deviation (SD) below 0.13%; this set of excellent measurements consists of 117 particles and only this selection of very high quality data was used for the numerical analysis and extension of Müller's model.

Fig. 1: Overview of particle masses on the logarithmic number line together with their standard deviations expressed in % of the mass. Note that a few particles with very low or high mass or percentage error were omitted for clarity (e.g. electron, muon, proton, gauge bosons).

For consistency with previous articles on this topic, the following abbreviations and conventions for the numerical analysis hold:

Calculation method:

The considered particle mass is transformed into a continued fraction according to the equations

$$\ln \frac{m_{particle}}{m_{electron}} = p + S, \qquad \ln \frac{m_{particle}}{m_{proton}} = p + S,$$

where p is the phase shift and S is the continued fraction (e is Euler's number)

$$S = n_0 + \cfrac{e}{n_1 + \cfrac{e}{n_2 + \cfrac{e}{n_3 + ...}}}. \tag{1}$$

The continued fraction representation $p+S$ is abbreviated as $[p; n_0 \mid n_1, n_2, n_3, \dots]$, where the free link n_0 is allowed to be $0, \pm 3, \pm 6, \pm 9 \dots$ and all partial denominators n_i can take the values $e{+}1, -e{-}1, \pm 6, \pm 9, \pm 12 \dots$. In the tables these abbreviations were marked with P or E, in order to indicate proton or electron resonance states.

For practical reasons only 18 partial denominators were determined. Next, the particle mass was repeatedly calculated from the continued fraction, every time considering one more partial denominator. As soon as the calculated mass value (on the linear scale) was in the interval "mass ± standard deviation", no further denominators were considered and the resulting fractions are displayed in the tables. In some rare cases, this procedure provides a mass value just a little inside the interval and considering the next denominator would match the measured value almost exactly. In such cases this denominator is then additionally given in brackets.

Table 1: Continued fraction representations of the lepton masses (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
electron	$5.10998910 \times 10^{-4} \pm 1.3 \times 10^{-11}$ P [x; -6 | 12, -6]	 1.21×10^{-15}
μ^-	$1.05658367 \times 10^{-1} \pm 4.0 \times 10^{-9}$ P [x; 0 | -6, -9, -e-1, 12, -6, -15] E [-x; 3 | e+1, e+1, -e-1, e+1, 9, -48, e+1, -e-1]	 2.45×10^{-10} 3.06×10^{-9}
τ^-	$1.77682 \pm 1.6 \times 10^{-4}$ P [0; 0 | e+1, 6, -e-1, e+1, -e-1, -e-1] P [x; 3 | -e-1, -e-1, 231] E [0; 9 | -e-1, 6, -e-1, -e-1, -6] E [-x; 6 | 6, e+1, -45]	 4.52×10^{-5} 2.50×10^{-6} 6.81×10^{-5} 1.92×10^{-5}

Table 2: Continued fraction representations of the boson masses (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
W^+	$8.0399 \times 10^{1} \pm 2.3 \times 10^{-2}$ E [0; 12 | -81, e+1, (24)]	 3.23×10^{-5}
Z^0	$9.11876 \times 10^{1} \pm 2.1 \times 10^{-3}$ P [x; 6 | 9, -e-1, -15, -e-1, e+1] E [0; 12 | 30, -6, (12)]	 1.01×10^{-3} 7.23×10^{-4}

The numerical error is always understood as the absolute value of the difference between the measured particle mass and the mass calculated from the corresponding continued fraction representation.

In order to avoid machine based rounding errors, numerical values of continued fractions were always calculated using the the Lenz algorithm as indicated in reference [15].

Outliers:

A particle mass is considered as an outlier (i.e. does not fit into the here extended Müller model) when its mass, as calculated from the corresponding continued fraction representation provides a value outside the interval "particle mass ± standard deviation".

3 Results and discussion

3.1 Fundamental philosophical idea

Chinese philosophy is dominated by the concept of "Yin and Yang" describing an indivisible whole of two complementary effects (male–female, day–night, good–bad, etc.). This means that everything has two opposite poles, and both poles are necessary to understand the whole thing (e.g. male can only be understood completely because female also exists as the opposite).

Table 3: Continued fraction representations of the light unflavored mesons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
π^+	$1.3957018 \times 10^{-1} \pm 3.5 \times 10^{-7}$ P [x; 0 | -18, 6, 6, (-117)] E [0; 6 | -6, -e-1, e+1, -e-1, 48]	 7.67×10^{-10} 1.68×10^{-7}
π^0	$1.349766 \times 10^{-1} \pm 6.0 \times 10^{-7}$ E [0; 6 | -6, -6, -6, 6, -e-1]	 2.49×10^{-7}
η^0	$5.47853 \times 10^{-1} \pm 2.4 \times 10^{-5}$ P [0; 0 | -6, e+1, -e-1, 6, -e-1, 12] E [0; 6 | e+1, -e-1, e+1, -6, -e-1, e+1, (24)]	 6.52×10^{-7} 2.51×10^{-7}
$\rho(770)^{0,+}$	$7.7549 \times 10^{-1} \pm 3.4 \times 10^{-4}$ P [0; 0 | -15, e+1, (-174)]	 1.73×10^{-7}
$\omega(782)^0$	$7.8265 \times 10^{-1} \pm 1.2 \times 10^{-4}$ P [0; 0 | -15, (243)] E [-x; 6 | -6, -6, e+1, -9, (135)]	 2.10×10^{-7} 4.51×10^{-11}
$\eta'(958)^0$	$9.5778 \times 10^{-1} \pm 6.0 \times 10^{-5}$ P [0; 0 | 132, (30)] E [-x; 6 | -12, -e-1, -6, (-24)]	 6.81×10^{-7} 4.66×10^{-7}
$\phi(1020)^0$	$1.019455 \pm 2.0 \times 10^{-5}$ P [0; 0 | 33, -12, e+1]	 4.92×10^{-6}
$f_2(1270)^0$	$1.2751 \pm 1.2 \times 10^{-3}$ P [0; 0 | 9, -21] P [x; 3 | -e-1, e+1, -6, (36)] E [-x; 6 | 39, -e-1]	 3.84×10^{-4} 1.87×10^{-5} 3.78×10^{-4}
$f_1(1285)^0$	$1.2818 \pm 6.0 \times 10^{-4}$ P [0; 0 | 9, -9, -6] P [x; 3 | -e-1, e+1, -6, -e-1, e+1] E [-x; 6 | 36, -6]	 2.46×10^{-5} 9.88×10^{-5} 1.20×10^{-4}
$a_2(1320)^{0,+}$	$1.3183 \pm 5.0 \times 10^{-4}$ P [0; 0 | 9, -e-1, e+1, -e-1, e+1, -e-1, e+1] P [x; 3 | -e-1, e+1, 186] E [-x; 6 | 27, -e-1, e+1, -e-1]	 4.50×10^{-4} 5.66×10^{-6} 2.98×10^{-4}
$f_1(1420)^0$	$1.4264 \pm 9.0 \times 10^{-4}$ P [0; 0 | 6, 6, -6, (-39)] P [x; 3 | -e-1, 6, 24] E [-x; 6 | 15, -15]	 1.64×10^{-6} 9.34×10^{-5} 3.29×10^{-5}
$\rho_3(1690)^{0,+}$	$1.6888 \pm 2.1 \times 10^{-3}$ P [0; 0 | e+1, e+1, -e-1, (-51)] P [x; 3 | -e-1, -6, -e-1, e+1] E [0; 9 | -e-1, e+1, 12]	 1.95×10^{-5} 5.29×10^{-4} 8.78×10^{-4}

Table 4: Continued fraction representations of masses of the strange mesons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
K^+	$4.93677 \times 10^{-1} \pm 1.6 \times 10^{-5}$ P [0; 0 | -e-1, -6, e+1, 45] E [0; 6 | e+1, -e-1, -e-1, 15, -e-1] E [-x; 6 | -e-1, e+1, e+1, 6, e+1, -6]	 5.65×10^{-7} 6.96×10^{-6} 4.04×10^{-6}
K^0, K^0_S, K^0_L	$4.97614 \times 10^{-1} \pm 2.4 \times 10^{-5}$ E [-x; 6 | -e-1, e+1, e+1, -e-1, -e-1, e+1, e+1]	 4.73×10^{-6}
$K^*(892)^+$	$8.9166 \times 10^{-1} \pm 2.6 \times 10^{-4}$ P [0; 0 | -54, e+1] E [-x; 6 | -9, -6, 6]	 6.63×10^{-5} 6.13×10^{-5}
$K^*(892)^0$	$8.9594 \times 10^{-1} \pm 2.2 \times 10^{-4}$ P [0; 0 | -60, e+1, -e-1] E [-x; 6 | -9, -e-1, -6]	 1.47×10^{-4} 5.48×10^{-5}
$K_2^*(1430)^+$	$1.4256 \pm 1.5 \times 10^{-3}$ P [0; 0 | 6, 6, -6] P [x; 3 | -e-1, 6, 30] E [-x; 6 | 15, -21]	 7.56×10^{-4} 1.08×10^{-4} 1.40×10^{-4}
$K_2^*(1430)^0$	$1.4324 \pm 1.3 \times 10^{-3}$ P [0; 0 | 6, 6, 6] P [x; 3 | -e-1, 6, 9, (-e-1)] E [-x; 6 | 15, -6, e+1, (36)]	 3.72×10^{-4} 6.31×10^{-4} 5.37×10^{-7}

In physics we can find a number of analogous dualities, for instance: positive and negative charges, north and south magnetic poles, particles and antiparticles, emission and absorption of quanta, destructive and constructive interference of waves, nuclear fusion and fission, and in the widest sense also Newton's principle "action = reaction".

From these observations an interesting question arises: does such a duality also exist in the model of oscillations in a chain system, and how must this model be extended to make the "Yin-Yang" obvious and visible?

Applying this idea to Müller's model, it must be claimed that the fundamental spectrum of proton resonances must have an opposite, an anti-oscillation or inverted oscillation spectrum. What could it be?

We know that these proton master-oscillations are stable, so the theorized counter-oscillations must belong to a particle with similar lifetime than the proton. Consequently the electron is the only particle that could be a manifestation of such an inverted oscillation.

Now the concept of an inverted oscillation must be translated into a mathematical equation. According to Müller's standard model, we can express the electron mass as a proton resonance and the proton mass as an electron resonance:

$$\ln \frac{m_{electron}}{m_{proton}} = p + \mathrm{S_p},$$

$$\ln \frac{m_{proton}}{m_{electron}} = p + \mathrm{S_e},$$

where p is the phase shift (with value 0 or 1.5) and S the continued fraction as discussed in previous papers (given in equation (1)). Obviously for $p \neq 0$, $\mathrm{S_p} \neq \mathrm{S_e}$, and this is the starting point for the further modification of the model. We have to adjust the phase shift (when different from zero) in such a way that both continued fractions become opposite in the sense of oscillation information. This means that the denominators of $\mathrm{S_p}$ and $\mathrm{S_e}$ must be the same, but with opposite sign. If

$$\mathrm{S_p} = \mathrm{n_0} + \cfrac{\mathrm{e}}{\mathrm{n_1} + \cfrac{\mathrm{e}}{\mathrm{n_2} + \cfrac{\mathrm{e}}{\mathrm{n_3} + \ldots}}},$$

then must hold for $\mathrm{S_e}$:

$$\mathrm{S_e} = -\mathrm{n_0} + \cfrac{\mathrm{e}}{-\mathrm{n_1} + \cfrac{\mathrm{e}}{-\mathrm{n_2} + \cfrac{\mathrm{e}}{-\mathrm{n_3} + \ldots}}}.$$

Mathematically it is now obvious that one equation must be modified by a minus sign and we have to write:

$$\ln \frac{m_{electron}}{m_{proton}} = p + \mathrm{S_p}, \qquad (2)$$

$$\ln \frac{m_{proton}}{m_{electron}} = -p + \mathrm{S_e}, \qquad (3)$$

However, this is not yet a complete set of rules to find new continued fraction representations of the proton and electron; in order to arrive at a conclusion, it is absolutely necessary to develop further physical ideas.

Idea 1 – Length of continued fractions

The resulting continued fractions $\mathrm{S_p}$ and $\mathrm{S_e}$ should be short. A previous article already suggested that short fractions are associated with stability [9]. However, the fractions must not be too short. The fundamental oscillators must be represented by the simplest variant of a chain of oscillators. This is a single mass hold via two massless flexible strings between two motionless, fixed walls. This setup leads to 3 parameters determining the eigenfrequency of the chain, the mass value and the two different lengths of the strings. Consequently the continued fraction also should have 3 free parameters (the free link and two denominators). This idea solves the conceptual problem of a "no information oscillation". When expressing the electron mass as a proton resonance, then $\ln \frac{m_{electron}}{m_{proton}} = p + S$, and p must not have values determining S as zero or any other integer number ($\pm 3, \pm 6, \pm 9$...). In such a case no continued fraction can be written down, and the oscillation would not have any property.

Idea 2 – Small denominators

According to Müller's theory, a high positive or negative denominator locates the data point in a fluctuating zone. Consequently the considered property should be difficult to be kept constant. From all our observations, it is highly reasonable to believe that proton and electron masses are constant even over very long time scales. Therefore their masses cannot be located too deep inside a fluctuation zone. In this study, the maximum value of the denominators was tentatively limited to ± 18.

Idea 3 – The free link

The calculation

$$\ln \frac{m_{electron}}{m_{proton}} \approx -7.51$$

Table 5: Continued fraction representations of masses of the charmed, and charmed strange mesons (x = -1.75083890054)

<table>
<tr><th>Particle</th><th>Mass ± SD [GeV]
Continued fraction representation(s)</th><th>Numerical error [GeV]</th></tr>
<tr><td>D^+</td><td>$1.86957 \pm 1.6 \times 10^{-4}$
P [0; 0 | e+1, 12, 27]
E [0; 9 | -e-1, 9, 39]
E [-x; 6 | 6, -213]</td><td>
2.92×10^{-5}
5.45×10^{-6}
1.95×10^{-8}</td></tr>
<tr><td>D^0</td><td>$1.86480 \pm 1.4 \times 10^{-4}$
P [0; 0 | e+1, 12, -e-1, -6]
E [0; 9 | -e-1, 9, -12, e+1]
E [-x; 6 | 6, 129]</td><td>
1.03×10^{-4}
1.29×10^{-4}
6.40×10^{-6}</td></tr>
<tr><td>$D^*(2007)^0$</td><td>$2.00693 \pm 1.6 \times 10^{-4}$
P [0; 0 | e+1, -18, -e-1, e+1, -e-1]
P [x; 3 | -6, 6, 15]
E [0; 9 | -e-1, -78]
E [-x; 6 | 6, -e-1, 6, e+1, 6]</td><td>
8.59×10^{-5}
7.91×10^{-5}
3.94×10^{-5}
2.21×10^{-5}</td></tr>
<tr><td>$D^*(2010)^+$</td><td>$2.01022 \pm 1.4 \times 10^{-4}$
P [0; 0 | e+1, -18, (-102)]
P [x; 3 | -6, 6, 6, (-21)]
E [0; 9 | -e-1, -63, (6)]
E [-x; 6 | 6, -e-1, 6, -12]</td><td>
4.53×10^{-7}
5.72×10^{-6}
3.23×10^{-7}
1.62×10^{-4}</td></tr>
<tr><td>$D_1(2420)^0$</td><td>$2.4213 \pm 6.0 \times 10^{-4}$
P [0; 0 | e+1, -e-1, 6, -e-1, 6]
P [x; 3 | -9, -102]
E [0; 9 | -6, e+1, -e-1, 9, -e-1]
E [-x; 6 | e+1, 27, e+1, -e-1, e+1]</td><td>
4.56×10^{-4}
3.68×10^{-6}
4.37×10^{-4}
4.10×10^{-4}</td></tr>
<tr><td>$D_2^*(2460)^0$</td><td>$2.4626 \pm 7.0 \times 10^{-4}$
P [0; 0 | e+1, -e-1, e+1, 18, -9]
E [0; 9 | -6, e+1, -15]
E [-x; 6 | e+1, 348]</td><td>
5.21×10^{-6}
9.58×10^{-5}
1.02×10^{-5}</td></tr>
<tr><td>$D_2^*(2460)^+$</td><td>$2.4644 \pm 1.9 \times 10^{-3}$
P [0; 0 | e+1, -e-1, e+1, 24]
P [x; 3 | -9, -e-1, -e-1, (e+1, 18)]
E [0; 9 | -6, e+1, -18]
E [-x; 6 | e+1, 663]</td><td>
7.45×10^{-5}
2.40×10^{-6}
1.14×10^{-4}
1.95×10^{-6}</td></tr>
<tr><td>D_s^+</td><td>$1.96845 \pm 3.3 \times 10^{-4}$
P [0; 0 | e+1, -54, (-e-1, -15)]
P [x; 3 | -6, e+1, 6, (-63)]
E [0; 9 | -e-1, 42, e+1, -e-1]
E [-x; 6 | 6, -e-1, -e-1, -6]</td><td>
3.13×10^{-7}
6.81×10^{-7}
2.34×10^{-4}
2.00×10^{-4}</td></tr>
<tr><td>D_s^{*+}</td><td>$2.1123 \pm 5.0 \times 10^{-4}$
P [x; 3 | -6, -12, -e-1]
E [0; 9 | -e-1, -9, 6, -e-1, (-18, -45)]
E [-x; 6 | e+1, e+1, -e-1, e+1, -e-1,
e+1, -e-1]</td><td>
4.00×10^{-4}
3.42×10^{-9}

4.70×10^{-4}</td></tr>
<tr><td>$D_{s0}^*(2317)^+$</td><td>$2.3178 \pm 6.0 \times 10^{-4}$
P [0; 0 | e+1, -e-1, -27]
E [0; 9 | -e-1, -e-1, e+1, -e-1, -39]</td><td>
4.57×10^{-4}
1.50×10^{-5}</td></tr>
<tr><td>$D_{s1}(2460)^+$</td><td>$2.4595 \pm 6.0 \times 10^{-4}$
P [0; 0 | e+1, -e-1, e+1, 12, (15)]
P [x; 3 | -9, -6, e+1, (-9)]
E [0; 9 | -6, e+1, -12, e+1, (12)]
E [-x; 6 | e+1, 189]</td><td>
1.19×10^{-6}
5.71×10^{-5}
4.66×10^{-6}
5.06×10^{-5}</td></tr>
<tr><td>$D_{s1}(2536)^+$</td><td>$2.53528 \pm 2.0 \times 10^{-4}$
P [0; 0 | e+1, -e-1, e+1, -e-1, e+1,
e+1, -6]
E [0; 9 | -6, 6, -36]
E [-x; 6 | e+1, -21, e+1, -e-1, (-e-1)]</td><td>

3.89×10^{-5}
1.87×10^{-5}
1.88×10^{-5}</td></tr>
<tr><td>$D_{s2}^*(2573)^+$</td><td>$2.5726 \pm 9.0 \times 10^{-4}$
P [x; 3 | -12, e+1, 15]
E [0; 9 | -6, 9, 6]</td><td>
8.95×10^{-5}
2.24×10^{-4}</td></tr>
</table>

Table 6: Continued fraction representations of masses of the bottom mesons (including strange and charmed mesons) (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
B^+	$5.27917 \pm 2.9 \times 10^{-4}$ P [x; 3 | 6, -9, 6, 6]	8.81×10^{-5}
B^0	$5.27950 \pm 3.0 \times 10^{-4}$ P [x; 3 | 6, -9, 6, (33)]	4.56×10^{-6}
$B^{*0,+}$	$5.3251 \pm 5.0 \times 10^{-4}$ P [x; 3 | 6, -6, -6, e+1, e+1]	1.09×10^{-4}
$B_2^*(5747)^{0,+}$	$5.743 \pm 5.0 \times 10^{-3}$ E [0; 9 | 9, -e-1, -12]	2.95×10^{-4}
B_s^0	$5.3663 \pm 6.0 \times 10^{-4}$ P [x; 3 | 6, -6, e+1, e+1, (9)]	4.93×10^{-6}
B_s^{*0}	$5.4154 \pm 1.4 \times 10^{-3}$ P [x; 3 | 6, -e-1, -e-1, 12]	2.19×10^{-5}
$B_{s2}^*(5840)^0$	$5.8397 \pm 6.0 \times 10^{-4}$ P [x; 3 | e+1, e+1, -e-1, e+1, -9, (-6)]	4.08×10^{-5}
B_c^+	$6.277 \pm 6.0 \times 10^{-3}$ P [x; 3 | e+1, 6, -153] E [0; 9 | 6, 6, -e-1, e+1, (63)]	 1.21×10^{-5} 1.71×10^{-6}

leads to a value between the principal nodes -6 and -9. From this is follows that in the continued fractions, the free link n_0 can only take the values ±6 and ±9.

Idea 4 – Effect of canceling denominators

Elementary particles can be divided in two groups: the vast majority with an extremely short half-life, and a small set with comparable longer lifetime. When analyzing the more stable particles with Müller's standard model, already a striking tendency can be discovered that especially the sum of the free link and the first denominators tends to be zero.

Examples:

The τ can be interpreted as proton resonance and the full continued fraction representation, as calculated by the computer is: P [0; 0 | e+1, 6, -e-1, e+1, -e-1, -e-1, (6)]. Note that in the end, every determination of a continued fraction results in an infinite periodical alternating sequence of the denominators e+1 and -e-1, which is always omitted here. Without significantly changing the mass value, the fraction can be rewritten: P [0; 0 | e+1, 6, -e-1, e+1, -e-1, -e-1, (e+1, -6)], and then the sum of all denominators equals zero.

The full continued fraction for the charged pion is:
E [0; 6 | -6, -e-1, e+1, -e-1, 48, (-e-1, 6, -24, e+1, -e-1, 12)]. It can be seen that the free link and the first 3 denominators cancel successively. Then this changes. A minimal manipulation leads to:
E [0; 6 | -6, -e-1, e+1, -e-1, 48, (-e-1, 6, -48, e+1, -6, e+1)].

The full continued fraction for the neutral pion is:
E [0; 6 | -6, -6, -6, 6, -e-1, (12, -12, e+1, -e-1, e+1, 45, 6)]. Here we have only to eliminate the 11^{th} denominator (45) and the sum equals zero.

The full continued fraction for the η^0 is:
P [0; 0 | -6, e+1, -e-1, 6, -e-1, 12, (-9, -12, -e-1, e+1, -e-1, -e-1, e+1, -e-1, e+1, e+1)].
Again the first 4 denominators form a zero sum, then the 7^{th} denominator (-9) interrupts this canceling. Without significant change of the numerical value, this fraction could be shortened and rewritten: P [0; 0 | -6, e+1, -e-1, 6, -e-1, 12, (-12, e+1)].

When interpreting η^0 as electron resonance, again adding the free link to the first 5 denominators gives zero:
E [0; 6 | e+1, -e-1, e+1, -6, -e-1, e+1, (24)]. We can add and rewrite: E [0; 6 | e+1, -e-1, e+1, -6, -e-1, e+1, (24, -e-1, -24)].

A completely different case is the neutron; here the con-

Table 7: Continued fraction representations of masses of the $c\bar{c}$ mesons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
$\eta_c(1S)^0$	$2.9803 \pm 1.2 \times 10^{-3}$ P [x; 3 | -30, e+1, -e-1] E [0; 9 | -9, e+1, (-216)] E [-x; 6 | e+1, -e-1, 18, -e-1, e+1]	 6.56×10^{-5} 7.34×10^{-7} 8.84×10^{-4}
$J/\psi(1S)^0$	$3.096916 \pm 1.1 \times 10^{-5}$ E [-x; 6 | e+1, -e-1, e+1, 6, -e-1, e+1, 6, e+1, (-18)]	1.19×10^{-8}
$\chi_{c0}(1P)^0$	$3.41475 \pm 3.1 \times 10^{-4}$ P [x; 3 | 63, e+1, (57)] E [0; 9 | -15, e+1, -e-1, (-12)]	 6.99×10^{-8} 9.48×10^{-6}
$\chi_{c1}(1P)^0$	$3.51066 \pm 7.0 \times 10^{-5}$ no continued fraction found	outlier
$h_c(1P)^0$	$3.52541 \pm 1.6 \times 10^{-4}$ P [x; 3 | 36, 6, (-24)]	1.94×10^{-6}
$\chi_{c2}(1P)^0$	$3.55620 \pm 9.0 \times 10^{-5}$ P [x; 3 | 33, -9, e+1, -e-1, e+1] E [0; 9 | -18, 21, -e-1]	 7.52×10^{-5} 5.36×10^{-5}
$\eta_c(2S)^0$	$3.637 \pm 4.0 \times 10^{-3}$ E [0; 9 | -21, (66)]	5.00×10^{-6}
$\psi(2S)^0$	$3.68609 \pm 4.0 \times 10^{-5}$ E [0; 9 | -24, e+1, e+1, e+1, e+1]	6.30×10^{-6}
$\psi(3770)^0$	$3.77292 \pm 3.5 \times 10^{-4}$ E [0; 9 | -30, e+1, (-12)]	5.90×10^{-5}
$\chi_{c2}(2P)^0$	$3.9272 \pm 2.6 \times 10^{-3}$ P [x; 3 | 15, -27] E [0; 9 | -51, -9, e+1]	 1.47×10^{-4} 1.10×10^{-4}
$\psi(4040)^0$	$4.0390 \pm 1.0 \times 10^{-3}$ P [x; 3 | 12, e+1, -e-1, (495)] E [0; 9 | -108, -e-1]	 3.14×10^{-8} 5.66×10^{-4}
$\psi(4160)^0$	$4.1530 \pm 3.0 \times 10^{-3}$ P [x; 3 | 12, -e-1, -e-1, (6)] E [0; 9 | 915]	 1.88×10^{-5} 1.36×10^{-5}
$\psi(4415)^0$	$4.421 \pm 4.0 \times 10^{-3}$ P [x; 3 | 9, 81] E [0; 9 | 42, -6]	 4.82×10^{-5} 3.64×10^{-4}

Table 8: Continued fraction representations of masses of the $b\bar{b}$ mesons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
$\Upsilon(1S)^0$	$9.46030 \pm 2.6 \times 10^{-4}$ P [0; 3 | -e-1, -12, -87] E [-x; 9 | -e-1, e+1, -12, -63]	 1.02×10^{-5} 9.33×10^{-6}
$\chi_{b0}(1P)^0$	$9.8594 \pm 5.0 \times 10^{-4}$ E [0; 9 | e+1, -e-1, -e-1, e+1, -9, -e-1] E [-x; 9 | -e-1, e+1, 6, -e-1, 6, e+1]	 1.96×10^{-4} 3.40×10^{-4}
$\chi_{b1}(1P)^0$	$9.8928 \pm 4.0 \times 10^{-4}$ E [0; 9 | e+1, -e-1, -e-1, 6, (-75)] E [-x; 9 | -e-1, e+1, e+1, e+1, -12]	 4.52×10^{-6} 3.00×10^{-4}
$\chi_{b2}(1P)^0$	$9.9122 \pm 4.0 \times 10^{-4}$ P [0; 3 | -e-1, -6, e+1, 15, -e-1] E [0; 9 | e+1, -e-1, -e-1, 12, -6] E [-x; 9 | -e-1, e+1, e+1, 6]	 8.26×10^{-5} 1.07×10^{-5} 2.21×10^{-6}
$\Upsilon(2S)^0$	$1.002326 \times 10^1 \pm 3.1 \times 10^{-4}$ P [0; 3 | -e-1, -e-1, -e-1, e+1, -75] E [0; 9 | e+1, -e-1, -6, e+1, e+1, e+1, (-18)] E [-x; 9 | -e-1, e+1, e+1, -e-1, 6, -e-1, e+1, -e-1, e+1]	 1.86×10^{-6} 1.28×10^{-6} 2.49×10^{-4}
$\chi_{b0}(2P)^0$	$1.02325 \times 10^1 \pm 6.0 \times 10^{-4}$ P [0; 3 | -e-1, -e-1, 327] E [0; 9 | e+1, -e-1, -30] E [-x; 9 | -e-1, 6, -e-1, -e-1, -e-1, -e-1]	 1.29×10^{-6} 9.85×10^{-5} 2.80×10^{-4}
$\chi_{b1}(2P)^0$	$1.02555 \times 10^1 \pm 5.0 \times 10^{-4}$ P [0; 3 | -e-1, -e-1, 30] E [0; 9 | e+1, -e-1, -54] E [-x; 9 | -e-1, 6, -6, e+1, -e-1, -6]	 2.78×10^{-4} 4.85×10^{-4} 8.02×10^{-5}
$\chi_{b2}(2P)^0$	$1.02686 \times 10^1 \pm 5.0 \times 10^{-4}$ P [0; 3 | -e-1, -e-1, 21, -e-1, 9] E [0; 9 | e+1, -e-1, -93] E [-x; 9 | -e-1, 6, -6, 9, (-12)]	 1.11×10^{-5} 2.07×10^{-5} 4.33×10^{-6}
$\Upsilon(3S)^0$	$1.03552 \times 10^1 \pm 5.0 \times 10^{-4}$ P [0; 3 | -e-1, -e-1, 6, e+1, 6] E [-x; 9 | -e-1, 6, -30, -e-1]	 3.94×10^{-5} 1.75×10^{-4}
$\Upsilon(4S)^0$	$1.05794 \times 10^1 \pm 1.2 \times 10^{-3}$ P [0; 3 | -e-1, -e-1, e+1, -e-1, e+1, -15] E [0; 9 | e+1, -e-1, 6, e+1, 21]	 9.28×10^{-5} 4.37×10^{-5}
$\Upsilon(10860)^0$	$1.0876 \times 10^1 \pm 1.1 \times 10^{-2}$ E [0; 9 | e+1, -e-1, e+1, 24]	 8.32×10^{-5}
$\Upsilon(11020)^0$	$1.1019 \times 10^1 \pm 8.0 \times 10^{-3}$ P [0; 3 | -6, e+1, -e-1, 6, e+1] E [0; 9 | e+1, -e-1, e+1, -6, (-18)]	 3.60×10^{-3} 3.89×10^{-5}

tinued fraction is: P [0; 0 | 1974, -e-1, -e-1, (-24)]. As the first denominator is very high, the following denominators can make only minor changes of the numerical value of the fraction. So here it would be easily possible adding denominators to force the sum to be zero. Actually many particle representations fall in that category, so from looking only at these examples, the fundamental idea of a vanishing sum of denominators does not come out at all.

Hypothesis:
From all these examples we can theorize that for a permanently stable particle such as the proton and electron, the sum of the free link and all partial denominators must be zero.

3.2 Rules for constructing continued fractions

With these physical ideas, we can express the proton and electron through a very limited set of 10 pairs of continued fractions (Table 12), which can all be written down. For every continued fraction, the phase shift p can be calculated, so that equations (2) and (3) hold. Then, new rules for the interpretation of elementary particle masses can be derived. First, a mass can be either a proton or an electron resonance, and second, this newly found phase shift must now be considered.

When interpreting particle masses as proton resonance states we write (x is the new phase shift):

$$\ln \frac{m_{particle}}{m_{proton}} = (0 \text{ or } x) + S \tag{4}$$

and for electron resonances holds:

$$\ln \frac{m_{particle}}{m_{electron}} = (0 \text{ or } -x) + S. \tag{5}$$

The basic rule that the phase shift can be zero, is fundamental and will not be changed.

Now for every of these 10 different phase shifts, the new model must be checked. We have to find out to what extent other elementary particles are compatible to one of these 10 new versions of the model and still accumulate in spectral nodes. There is a set of 18 particle masses, which cannot be expressed as proton or electron resonances with phase shift zero; these are: μ^-, K^0, B^+, B^0, $B^{*0,+}$, B_s^0, $B_s{*}^0$, $B_{s2}{*}(5840)^0$, $J/\psi(1S)^0$, $\chi_{c1}(1P)^0$, $h_c(1P)^0$, $\Lambda(1520)^0$, Σ^0, $\Sigma(1385)^+$, Ξ^-, Λ_c^+, $\Sigma_b{*}^{0,+}$ and $\Sigma_b{*}^-$. The question is now: which of the 10 possible phase shifts can reproduce these 18 masses best, with the lowest number of outliers?

By trial and error it was found that there is indeed such a "best possibility", providing only one outlier:

$$\ln \frac{m_{electron}}{m_{proton}} = x + (-6) + \cfrac{e}{12 + \cfrac{e}{-6}} \tag{6}$$

$$\ln \frac{m_{proton}}{m_{electron}} = -x + 6 + \cfrac{e}{-12 + \cfrac{e}{6}}. \tag{7}$$

The phase shift x equals -1.75083890054 and the numerical errors are very small (see Tables 1 and 9).

Tables 1 to 11 show the continued fraction representations for the considered data set (117 particles, 107 different masses) All possible fractions are given for both, proton and electron resonances with the phase shifts 0 and ±x. For completeness, Table 12 displays the 10 alternative continued fraction representations together with the calculated phase shifts and the number of outliers when trying to reproduce the aforementioned set of 18 masses.

A single outlier is a very satisfying result when comparing to 14% outliers, which have been found with the standard version of Müller's model [9]. Since the spectra of electron and proton resonances overlap, most particles can even expressed as both, proton and electron resonances. This demonstrates that it makes only sense to analyze high accuracy data, otherwise easily a continued fraction representation can be found.

As expected, the principle of "Yin and Yang" has not been found anymore in this set of particles. There are no other pairs of particles with opposite oscillation information. It seems to be that this fundamental concept is only applicable to longterm stable systems or processes. Further research on other data sets should confirm this.

3.3 Model discussion

Is the principle of "Yin and Yang" really necessary to obtain continued fraction representations for most elementary particle masses? The critical reader could argue that alone the additional consideration of electron resonances greatly enhances the chances to express particle masses via standard continued fractions (with phase shift 0 and 3/2). This is true, however, the author has found that the 14% outliers were very little reduced when considering such additional electron resonances. So another phase shift is definitively required.

But, are the electron resonances really necessary? Would it not be possible to write only

$$\ln \frac{m_{particle}}{m_{proton}} = (0 \text{ or } p) + S \qquad (8)$$

where p is just any other phase shift different from the standard value 3/2 (between 0 and ±3)? This was exactly the author's first attempt to modify Müller's model. It was found that such phase shift does not exist.

For that reason the problem can only be solved through a new physical or philosophical idea. Every good physical theory consists of two parts, equivalent to a soul and a body. The soul represents a fundamental physical law or a philosophical principle, while always mathematics is the body.

From this viewpoint the author is particularly satisfied having found the "Yin-Yang" principle as an adequate extension of the proton resonance concept. It clearly justifies the importance of electron resonances and distinguishes the model from numerology.

Regarding the selection of the appropriate phase shift, a very critical reader could note that there is only one outlier difference between

$$\ln \frac{m_{electron}}{m_{proton}} = [x_1; -9 \mid -9, 18] \qquad (2 \text{ outliers})$$

and the best variant

$$\ln \frac{m_{electron}}{m_{proton}} = [x_2; -6 \mid 12, -6], \qquad (1 \text{ outlier})$$

Table 9: Continued fraction representations of masses of the N, Δ, Λ, Σ, Ξ and Ω baryons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
p^+	$9.38272013 \times 10^{-1} \pm 2.3 \times 10^{-8}$ E [-x; 6 | -12, 6]	 2.22×10^{-12}
n^0	$9.39565346 \times 10^{-1} \pm 2.3 \times 10^{-8}$ P [0; 0 | 1974, -e-1, -e-1, (-24)]	 7.85×10^{-11}
$\Delta(1232)^{-,0,+,++}$	$1.2320 \pm 1.0 \times 10^{-3}$ P [0; 0 | 9, e+1, -e-1, e+1] P [x; 3 | -e-1, e+1, -e-1, 6, e+1, -e-1] E [-x; 6 | 75]	 4.29×10^{-4} 7.12×10^{-4} 8.61×10^{-4}
Λ^0	$1.115683 \pm 6.0 \times 10^{-6}$ P [0; 0 | 15, e+1, 15, -6]	 9.92×10^{-8}
$\Lambda(1405)^0$	$1.4051 \pm 1.3 \times 10^{-3}$ P [0; 0 | 6, e+1] P [x; 3 | -e-1, 6, -e-1, -e-1]	 2.50×10^{-5} 6.44×10^{-4}
$\Lambda(1520)^0$	$1.5195 \pm 1.0 \times 10^{-3}$ P [x; 3 | -e-1, 15, e+1] E [-x; 6 | 12, -e-1, e+1, -e-1]	 5.71×10^{-4} 4.36×10^{-4}
Σ^+	$1.18937 \pm 7.0 \times 10^{-5}$ P [0; 0 | 12, -6, e+1, -e-1, 6]	 5.70×10^{-6}
Σ^0	$1.192642 \pm 2.4 \times 10^{-5}$ E [-x; 6 | 606]	 1.24×10^{-5}
Σ^-	$1.197449 \pm 3.0 \times 10^{-5}$ P [0; 0 | 12, -e-1, 6, -e-1, e+1, -e-1, (93)] E [-x; 6 | 321, -e-1]	 5.89×10^{-9} 1.22×10^{-5}
$\Sigma(1385)^+$	$1.3828 \pm 4.0 \times 10^{-4}$ E [-x; 6 | 18, -15 (-e-1)]	 8.96×10^{-5}
$\Sigma(1385)^0$	$1.3837 \pm 1.0 \times 10^{-3}$ P [0; 0 | 6, e+1, -e-1, e+1, -e-1] E [-x; 6 | 18, -12, (e+1, 60)]	 6.88×10^{-4} 2.95×10^{-8}
$\Sigma(1385)^-$	$1.3872 \pm 5.0 \times 10^{-4}$ P [0; 0 | 6, e+1, -e-1, e+1, e+1] E [-x; 6 | 18, -6, e+1]	 3.03×10^{-4} 1.66×10^{-4}
Ξ^0	$1.31486 \pm 2.0 \times 10^{-4}$ P [0; 0 | 9, -e-1, e+1, -6] P [x; 3 | -e-1, e+1, -93] E [-x; 6 | 27, -9, e+1]	 1.42×10^{-4} 2.86×10^{-5} 1.53×10^{-4}
Ξ^-	$1.32171 \pm 7.0 \times 10^{-5}$ P [x; 3 | -e-1, e+1, 45, e+1]	 5.35×10^{-5}
$\Xi(1530)^0$	$1.53180 \pm 3.2 \times 10^{-4}$ P [0; 0 | 6, -6, (165)] E [0; 9 | -e-1, e+1, -e-1, e+1, -e-1, -e-1, -e-1]	 1.35×10^{-6} 5.19×10^{-5}
$\Xi(1530)^-$	$1.5350 \pm 6.0 \times 10^{-4}$ P [0; 0 | 6, -6, 9, (-12)] P [x; 3 | -e-1, 21, 6] E [0; 9 | -e-1, e+1, -e-1, e+1, -6, (-54)]	 1.09×10^{-5} 1.01×10^{-4} 1.18×10^{-6}
Ω^-	$1.67245 \pm 2.9 \times 10^{-4}$ P [0; 0 | e+1, e+1, -e-1, e+1, -e-1, -e-1] P [x; 3 | -e-1, 9, e+1, -9] E [0; 9 | -e-1, e+1, 48]	 1.09×10^{-4} 1.50×10^{-4} 1.23×10^{-4}

Table 10: Continued fraction representations of masses of the charmed baryons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
Λ_c^+	$2.28646 \pm 1.4 \times 10^{-4}$	
	E [-x; 6 | e+1, 6, 9, -e-1, (-e-1)]	8.64×10^{-6}
$\Lambda_c(2595)^+$	$2.5954 \pm 6.0 \times 10^{-4}$	
	P [x; 3 | -12, 9, e+1]	5.64×10^{-4}
	E [0; 9 | -6, 15, (66)]	1.23×10^{-6}
	E [-x; 6 | e+1, -12, e+1, -9]	1.13×10^{-4}
$\Sigma_c(2455)^{++}$	$2.45403 \pm 1.8 \times 10^{-4}$	
	P [x; 3 | -9, -6, -39]	8.51×10^{-7}
	E [0; 9 | -6, e+1, -9, e+1, -e-1]	2.02×10^{-5}
	E [-x; 6 | e+1, 105]	7.84×10^{-5}
$\Sigma_c(2455)^+$	$2.4529 \pm 4.0 \times 10^{-4}$	
	P [0; 0 | e+1, -e-1, e+1, 6, e+1, -e-1, e+1]	2.84×10^{-4}
	P [x; 3 | -9, -6, -9]	1.07×10^{-4}
	E [-x; 6 | e+1, 96]	1.02×10^{-4}
$\Sigma_c(2455)^0$	$2.45376 \pm 1.8 \times 10^{-4}$	
	P [x; 3 | -9, -6, -24]	3.06×10^{-5}
	E [0; 9 | -6, e+1, -9, e+1, -e-1, e+1]	1.48×10^{-4}
	E [-x; 6 | e+1, 102, (e+1)]	8.05×10^{-5}
$\Sigma_c(2520)^{++}$	$2.5184 \pm 6.0 \times 10^{-4}$	
	P [0; 0 | e+1, -e-1, e+1, -e-1, -18]	1.44×10^{-4}
	E [0; 9 | -6, 6, -e-1, e+1, (18)]	1.05×10^{-5}
	E [-x; 6 | e+1, -27, e+1, (6)]	1.68×10^{-5}
$\Sigma_c(2520)^+$	$2.5175 \pm 2.3 \times 10^{-3}$	
	P [0; 0 | e+1, -e-1, e+1, -e-1, -15, e+1]	1.01×10^{-4}
	E [0; 9 | -6, 6, -e-1, e+1, (-6)]	7.02×10^{-5}
	E [-x; 6 | e+1, -27]	4.20×10^{-4}
$\Sigma_c(2520)^0$	$2.5180 \pm 5.0 \times 10^{-4}$	
	P [0; 0 | e+1, -e-1, e+1, -e-1, -15]	2.46×10^{-4}
	E [0; 9 | -6, 6, -e-1, e+1, (-21)]	8.75×10^{-6}
	E [-x; 6 | e+1, -27, 6]	2.10×10^{-5}
Ξ_c^+	$2.4678 \pm 4.0 \times 10^{-4}$	
	P [0; 0 | e+1, -e-1, e+1, 60]	5.29×10^{-5}
	P [x; 3 | -9, -e-1, -e-1, e+1, e+1]	2.82×10^{-4}
	E [0; 9 | -6, e+1, -33]	8.89×10^{-6}
	E [-x; 6 | e+1, -933]	1.45×10^{-6}
Ξ_c^0	$2.47088 \pm 3.4 \times 10^{-4}$	
	P [0; 0 | e+1, -e-1, e+1, -162]	2.87×10^{-7}
	P [x; 3 | -9, -e-1, -9, (-9)]	1.73×10^{-5}
	E [0; 9 | -6, e+1, -141]	6.33×10^{-6}
	E [-x; 6 | e+1, -294]	5.91×10^{-6}
$\Xi_c'^+$	$2.5756 \pm 3.1 \times 10^{-3}$	
	P [x; 3 | -12, e+1, 6]	4.33×10^{-4}
	E [0; 9 | -6, 9, e+1]	1.02×10^{-3}
	E [-x; 6 | e+1, -12, -e-1, e+1]	1.40×10^{-3}
$\Xi_c'^0$	$2.5779 \pm 2.9 \times 10^{-3}$	
	P [x; 3 | -12, e+1, e+1]	5.26×10^{-4}
	E [-x; 6 | e+1, -12, -e-1]	8.16×10^{-4}
$\Xi_c(2645)^{0,+}$	$2.6459 \pm 5.0 \times 10^{-4}$	
	P [x; 3 | -12, -e-1, 9]	7.47×10^{-6}
	E [0; 9 | -6, -39, (-330)]	1.13×10^{-8}
	E [-x; 6 | e+1, -9, e+1, 6]	2.50×10^{-4}
Ω_c^0	$2.6952 \pm 1.7 \times 10^{-3}$	
	P [x; 3 | -15, e+1, -e-1, e+1]	6.84×10^{-4}
	E [0; 9 | -6, -9, e+1, (-12)]	3.15×10^{-6}
	E [-x; 6 | e+1, -6, e+1]	9.61×10^{-4}
$\Omega_c(2770)^0$	$2.7659 \pm 2.0 \times 10^{-3}$	
	E [0; 9 | -6, -e-1, (93)]	9.99×10^{-6}
	E [-x; 6 | e+1, -6, e+1, e+1]	3.47×10^{-4}

Table 11: Continued fraction representations of masses of the bottom baryons (x = -1.75083890054)

Particle	Mass ± SD [GeV] Continued fraction representation(s)	Numerical error [GeV]
Λ_b^0	$5.6202 \pm 1.6 \times 10^{-3}$	
	P [x; 3 | 6, e+1, -e-1, e+1, 9]	1.25×10^{-4}
	E [0; 9 | 9, -27]	3.49×10^{-4}
Σ_b^+	$5.8078 \pm 2.7 \times 10^{-3}$	
	E [0; 9 | 9, -e-1, e+1, -e-1, (-27)]	2.47×10^{-6}
Σ_b^-	$5.8152 \pm 2.0 \times 10^{-3}$	
	E [0; 9 | 9, -e-1, e+1, -e-1, e+1, (-e-1, 24)]	4.30×10^{-6}
$\Sigma_b{*}^+$, $\Sigma_b{*}^0$	$5.8290 \pm 3.4 \times 10^{-3}$	
	P [x; 3 | e+1, e+1, -e-1, e+1]	8.39×10^{-4}
$\Sigma_b{*}^-$	$5.8364 \pm 2.8 \times 10^{-3}$	
	P [x; 3 | e+1, e+1, -e-1, e+1, -6]	7.39×10^{-5}
$\Xi_b^{-,0}$	$5.7905 \pm 2.7 \times 10^{-3}$	
	E [0; 9 | 9, -e-1, e+1, 9]	2.20×10^{-4}

Table 12: List of the 10 possible continued fraction representations of the electron mass when considering the rules that denominators must be small and their sum including the free link equals zero, together with their associate phase shifts and the number of outliers when considering the following set of 18 particles: μ^-, K^0, B^+, B^0, $B^{*0,+}$, B_s^0, $B_s{*}^0$, $B_{s2}{*}(5840)^0$, $J/\psi(1S)^0$, $\chi_{c1}(1P)^0$, $h_c(1P)^0$, $\Lambda(1520)^0$, Σ^0, $\Sigma(1385)^+$, Ξ^-, Λ_c^+, $\Sigma_b{*}^{0,+}$ and $\Sigma_b{*}^-$

Continued fraction representation for ln $\frac{m_{electron}}{m_{proton}}$ = x + S	phase shift x	number of outliers
P [x; -9 | 15, -6]	1.29770965366	3
P [x; -9 | -6, 15]	1.95172884111	5
P [x; -9 | 18, -9]	1.33097940724	4
P [x; -9 | -9, 18]	1.79175802145	2 μ^-, Σ^0
P [x; -6 | -6, 12]	-1.04460536299	6
P [x; -6 | 12, -6]	-1.75083890054	1 $\chi_{c1}(1P)^0$
P [x; -6 | -9, 15]	-1.20718990898	6
P [x; -6 | 15, -9]	-1.70037040878	6
P [x; -6 | 18, -12]	-1.66836807753	3
P [x; -6 | -12, 18]	-1.2860171871	4

so one single outlier might not be sufficiently significant to make a clear decision. Here it is now worth looking at the outlier particles. In the first case, the two outliers are the muon and the Σ^0. The muon has a comparatively long mean lifetime of 2.2 μs. So it is fare more stable than the average elementary particle. Therefore it is reasonable to request that the muon mass is reproduced by the model, i.e. the muon must not be an outlier.

4 Conclusions

The here presented bipolar version of Müller's continued fraction model is so far the best description of elementary particle masses. It demonstrates two facts: first, electron and proton can be interpreted as a manifestation of the "Yin and Yang" principle in nature. They both can be interpreted as fundamental reference points in the model of a chain of harmonic oscillations. Second, the proton resonance idea alone is an incomplete concept and we have to recognize that electron resonances also play an important role in the universe.

These results can be obtained only when strictly considering the individual measurement errors of the particles and all similar future analyses should be based on the most accurate data available.

Until now, this bipolar version of Müller's model has reproduced only one data set. It is obvious that this alone cannot be considered as a full proof of correctness of this model variant and much more data should be analyzed.

Acknowledgments

The author greatly acknowledges the financial support from the Brazilian governmental funding agencies FACEPE and CNPq.

Submitted on September 6, 2012 / Accepted on September 11, 2012

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