Identical Bands and ΔI = 2 Staggering in Superdeformed Nuclei in A ∼ 150 Mass Region Using Three Parameters Rotational Model

A.M. Khalaf*, M.M. Taha†, and M. Kotb‡

*Physics Department, Faculty of Science, Al-Azhar University, Cairo, Egypt. E-mail: Profkhalaf@gmail.com
†Mathematics and Theoretical Physics Department, NRC, Atomic Energy Authority, P.No. 13759, Cairo, Egypt. E-mail: mahmoudmt@hotmail.com
‡Physics Department, Faculty of Science, Al-Azhar University, Cairo, Egypt. E-mail: mahmoudkottb@gmail.com

By using a computer simulated search program, the experimental gamma transition energies for superdeformed rotational bands (SDRB's) in $A \sim 150$ region are fitted to proposed three-parameters model. The model parameters and the spin of the bandhead were obtained for the selected ten SDRB's namely: ^{150}Gd (yrast and excited SD bands), ^{151}Tb (yrast and excited SD bands), ^{152}Dy (yrast SD bands), ^{148}Gd (SD-1,SD-6), ^{149}Gd (SD-1), ^{153}Dy (SD-1) and ^{148}Eu (SD-1). The Kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are studied as a function of the rotational frequency $\hbar\omega$. From the calculated results, we notic that the excited SD bands have identical energies to their Z+1 neighbours for the twinned SD bands in N=86 nuclei. Also the analysis done allows us to confirm $\Delta I = 2$ staggering in the yrast SD bands of ^{148}Gd, ^{149}Gd, ^{153}Dy, and ^{148}Eu and in the excited SD bands of ^{148}Gd, by performing a staggering parameter analysis. For each band, we calculated the deviation of the gamma ray energies from smooth reference representing the finite difference approximation to the fourth derivative of the gamma ray transition energies at a given spin.

1 Introduction

The superdeformed (SD) nuclei is one of the most interesting topics of nuclear structure studies. Over the past two decades, many superdeformed rotational bands (SDRB's) have been observed in several region of nuclear chart [1]. At present although a general understanding of these SDRB's have been achieved, there are still many open problems. For example the spin, parity and excitation energy relative to the ground state of the SD bands have not yet been measured. The difficulty lies with observing the very weak discrete transitions which link SD levels with normal deformed (ND) levels. Until now, only several SD bands have been identified to exist the transition from SD levels to ND levels. Many theoretical approaches to predict the spins of these SD bands have been proposed [2–11].

Several SDRB's in the $A \sim 150$ region exhibit a rather surprising feature of a $\Delta I = 2$ staggering [12–25] in its transition energies, *i.e.* sequences of states differing by four units of angular momentum are displaced relative to each other. The phenomenon of $\Delta I = 2$ staggering has attached much attention and interest, and has thus become one of the most frequently considerable subjects. Within a short period, a considerable a mounts of effort has been spent on understanding its physical implication based on various theoretical ideas [9, 26–41]. Despite such efforts, definite conclusions have not yet been reached until present time.

The discovery of the phenomenon of identical bands (IB's) [42, 43] at high spin in SD states in even-even and odd-A nuclei aroused a considerable interest. It was found that the transition energies and moments of inertia in neighboring nuclei much close than expected. This has created much theoretical interest [44, 45]. The first interpretation [46] to IB's was done within the framework of the strong coupling limit of the particle-rotor model, in which one or more particles are coupled to a rotating deformed core and follow the rotation adiabatically. Investigation also suggest that the phenomena of IB's may result from a cancelation of contributions to the moment of inertia occurring in mean field method [47].

In the present paper we suggest a three-particle model to predict the spins of the rotational bands and to study the properties of the SDRB's and to investigate the existence of $\Delta I = 2$ staggering and also investigate the presence of IB's observed in the $A \sim 150$ mass region.

2 Nuclear SDRB's in framework of three parameters rotational model

In the present work, the energies of the SD nuclear RB's $E(I)$ as a function of the unknown spin I are expressed as:

$$E(I) = E_0 + a[[1 + b\hat{I}^2]^{1/2} - 1] + c\hat{I}^2 \qquad (1)$$

with $\hat{I}^2 = I(I + 1)$, where a, b and c are the parameters of the model. The rotational frequency $\hbar\omega$ is defined as the derivative of the energy E with respect to the angular momentum $\hat{I}$

$$\begin{aligned}\hbar\omega &= \frac{dE}{d\hat{I}} \\ &= [2c + ab[1 + bI(I + 1)]^{1/2}](I(I + 1))^{-1/2}.\end{aligned} \qquad (2)$$

Two possible types of nuclear moments of inertia have been suggested which reflect two different aspects of nuclear dynamics. The kinematic moment of inertia $J^{(1)}$, which is

Table 1: The adopted best parameters a, b, c of the model and the band-head spin assignment I_0 of our ten SDRB's. The rms deviations are also shown.

SD Band	$E\gamma(I+2 \to I)$ (keV)	I_0 ($\hbar$)	a (keV)	b (keV)	c (keV)	χ
^{148}Gd (SD-1)	699.9	31	-0.313446E+07	0.163069E-04	0.311027E+02	7.387009E-01
(SD-6)	802.2	39	-0.106162E+06	0.107495E-03	0.105003E+02	2.104025E-01
^{150}Gd (SD-1)	815.0	47	-0.148586E+06	-0.517219E-04	0.954401E-01	5.250988E-01
(SD-2)	727.9	31	-0.617154E+06	-0.134929E-04	0.163288E+01	1.734822E+00
^{152}Dy (SD-1)	602.4	26	-0.144369E+06	0.207972E-04	0.733270E+01	5.217181E-01
^{149}Gd (SD-1)	617.8	27.5	-0.825976E+05	-0.698261E-04	0.285641E+01	4.559227E-01
^{148}Eu (SD-1)	747.7	29	-0.131028E+06	0.432608E-04	0.928191E+01	7.010767E-01
^{151}Tb (SD-1)	726.5	30.5	-0.852833E+06	-0.546382E-05	0.364770E+01	2.023767E+00
(SD-2)	602.1	26.5	-0.136986E+07	-0.431179E-05	0.289128E+01	6.644767E-01
^{153}Dy (SD-1)	721.4	30.5	-0.671437E+06	-0.386442E-05	0.464507E+01	2.171267E+00

equal to the inverse of the slope of the curve of energy E versus $\hat{I}$:

$$\begin{aligned} J^{(1)} &= \hbar^2 \hat{I} \left(\frac{dE}{d\hat{I}}\right)^{-1} \\ &= \frac{\hbar^2}{ab}[1 + bI(I+1)]^{1/2} + \frac{1}{2c} \end{aligned} \tag{3}$$

and the dynamic moment of inertia $J^{(2)}$, which is related to the curvature in the curve of E versus $\hat{I}$:

$$\begin{aligned} J^{(2)} &= \hbar^2 \left(\frac{d^2E}{d\hat{I}^2}\right)^{-1} \\ &= \frac{\hbar^2}{ab}[1 + bI(I+1)]^{3/2} + \frac{1}{2c}. \end{aligned} \tag{4}$$

For the SD bands, one can extract the rotational frequency, dynamic and kinematic moment of inertia by using the experimental interband E2 transition energies as follows:

$$\hbar\omega = \frac{1}{4}[E_\gamma(I+2) + E_\gamma(I)], \tag{5}$$

$$J^{(2)}(I) = \frac{4\hbar^2}{\Delta E_\gamma}, \tag{6}$$

$$J^{(1)}(I-1) = \frac{\hbar^2(2I-1)}{E_\gamma}, \tag{7}$$

where

$$\begin{aligned} E_\gamma &= E(I) - E(I-2), \\ \Delta E_\gamma &= E_\gamma(I+2) - E_\gamma(I). \end{aligned}$$

It is seen that whereas the extracted $J^{(1)}$ depends on I proposition, $J^{(2)}$ does not.

3 Analysis of the ΔI = 2 staggering effects

It has been found that some SD rotational bands in different mass region show an unexpected $\Delta I = 2$ staggering effects in the gamma ray energies [12–25]. The effect is best seen in long rotational sequences, where the expected regular behavior of the energy levels with respect to spin or to rotational frequency is perturbed. The result is that the rotational sequence is split into two parts with states separated by $\Delta I = 4$ (bifurcation) shifting up in energy and the intermediate states shifting down in energy. The curve found by smoothly interpolating the band energy of the spin sequence I, I+4 ,I+8... is somewhat displaced from the corresponding curve of the sequence I+2, I+6, I+10....

To explore more clearly the $\Delta I = 2$ staggering, for each band the deviation of the transition energies from a smooth reference ΔE_γ is determined by calculating the fourth derivative of the transition energies $E_\gamma(I)$ at a given spin I by

$$\begin{aligned} \Delta E_\gamma(I) = \tfrac{3}{8}\Big(&E_\gamma(I) - \tfrac{1}{6}[4E_\gamma(I-2) + 4E_\gamma(I+2) \\ &- E_\gamma(I-4) - E_\gamma(I+4)]\Big). \end{aligned} \tag{8}$$

This expression was previously used in [15] and is identical to the expression for $\Delta^4 E_\gamma(I)$ in Ref. [33]. We chose to the use the expression above in order to be able to follow higher order changes in the moments of inertia of the SD bands.

4 Superdeformed identical bands

A particularly striking feature of SD nuclei is the observation of numerous bands with nearly identical transition energies in nuclei differing by one or two mass unit [42–45]. To determine whether a pair of bands is identical or not, one must compare the dynamical moment of inertia or compare the E2 transition energies of the two bands.

5 Numerical calculations and discussions

Nine SDRB's observed in nuclei of mass number $A \sim 150$ have been analyzed in terms of our three parameter model. The experimental transition energies are taken from Ref. [1]. The studied SDRB's are namely:

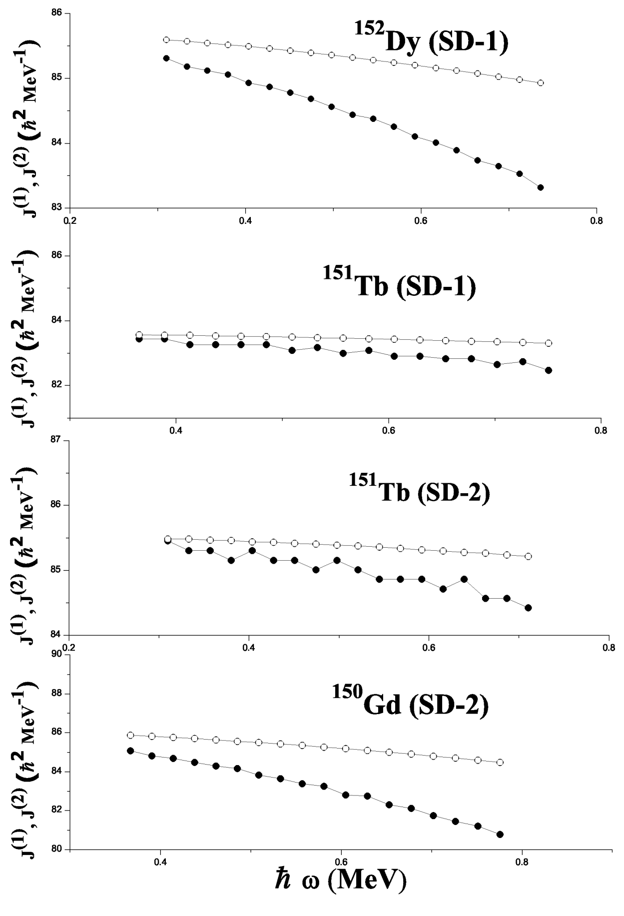

Fig. 1: Calculated Kinematic $J^{(1)}$ (open circles) and dynamic $J^{(2)}$ (closed circles) moments of inertia as a function of rotational frequency $\hbar\omega$ for the set of identical bands ^{151}Tb(SD-1), ^{152}Dy(SD-1), ^{150}Gd(SD-3) and ^{151}Tb(SD-2).

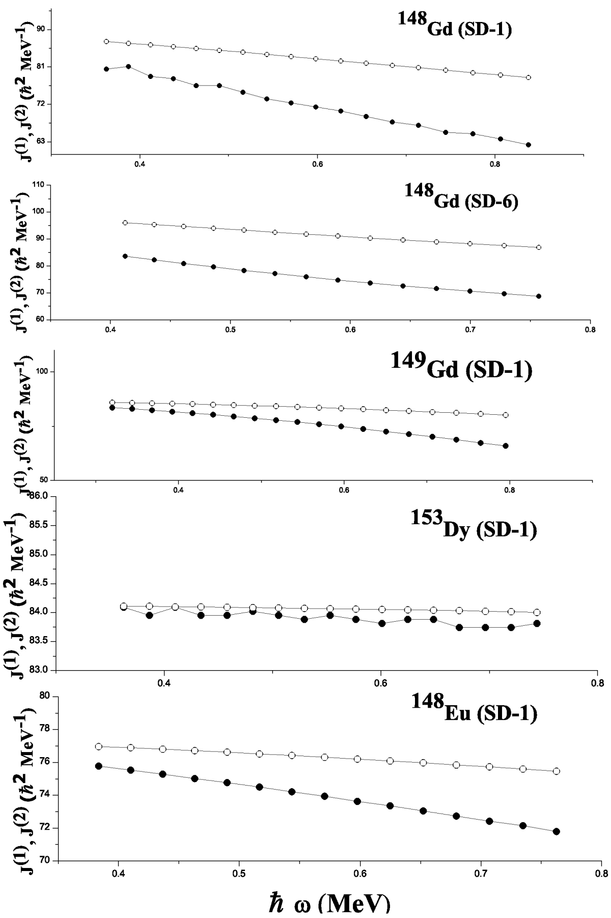

Fig. 2: Calculated Kinematic $J^{(1)}$ (open circles) and dynamic $J^{(2)}$ (closed circles) moments of inertia as a function of rotational frequency $\hbar\omega$ for the SDRB's ^{148}Gd(SD-1, SD-6), ^{149}Gd(SD-1), ^{153}Dy(SD-1) and ^{148}Eu(SD-1).

^{150}Gd(SD1, SD2), ^{151}Tb(SD1, SD2), ^{152}Dy(SD1), ^{148}Gd(SD1, SD6), ^{149}Gd(SD1), ^{153}Dy(SD1) and ^{148}Eu(SD1). The difference between the SD bands in various mass region are obviously evident through the behavior of the dynamical $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia seems to be very useful to the understanding of the properties of the SD bands. The bandhead moment of inertia J_0 at $J^{(2)} = J^{(1)}$ is a sensitive guideline parameter for the spin proposition.

A computer simulated search program has been used to get a minimum root mean square (rms) deviation between the experimental transition energies E_γ^{exp} and the calculated ones derived from our present three parameter model E_γ^{cal}:

$$\chi = \frac{1}{N}\left[\sum_{i=1}^{n}\left|\frac{E_\gamma^{cal}(I_i) - E_\gamma^{exp}(I_i)}{\delta E_\gamma^{exp}(I_i)}\right|^2\right]^{1/2} \tag{9}$$

where N is the number of data points enters in the fitting procedure and $\delta E_\gamma^{exp}(i)$ is the uncertainties in the $\gamma-$transitions. For each SD band the optimized best fitted four parameters a, b, c and the bandhead spin I_0 were obtained by the adopted fit procedure. The procedure is repeated for several sets of trail values a, b, c and I_0. The spin I_0 is taken as the nearest integer number, then another fit with only a, b and c as free parameters is made to determine their values. The lowest bandhead spin I_0 and the best parameters of the model a, b, c for each band is listed in Table(1). The SD bands are identified by the lowest gamma transition energies $E_\gamma(I_0 + 2 \rightarrow I_0)$ observed.

The dynamical $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia using our proposed model at the assigned spin values are calculated as a function of rotational frequency $\hbar\omega$ and illustrated in Figs. (1,2). $J^{(2)}$ mostly decrease with a great deal of variation from nucleus to nucleus. The properties of the SD bands are mainly influenced by the number of the high-N intruder orbitals occupied. For example the large slopes of $J^{(2)}$ against $\hbar\omega$ in ^{150}Gd and ^{151}Tb are due to the occupation of $\pi 6_2$, $\nu 7_2$ orbitals, while in ^{152}Dy the $\pi 6_4$ level is also occupied and this leads to a more constant $J^{(2)}$ against $\hbar\omega$. A plot of

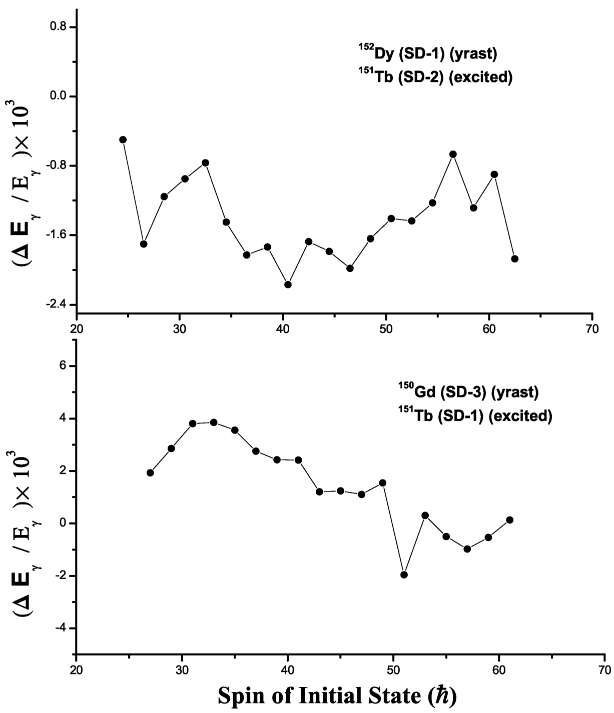

Fig. 3: Percentage differences $\Delta E_\gamma / E_\gamma$ in transition energies $E_\gamma = E(I) - E(I-2)$ as a function of spin I for the set of identical bands (^{151}Tb(SD-2), ^{152}Dy(SD-1)) and (^{150}Gd(SD-2), ^{151}Tb(SD-1)).

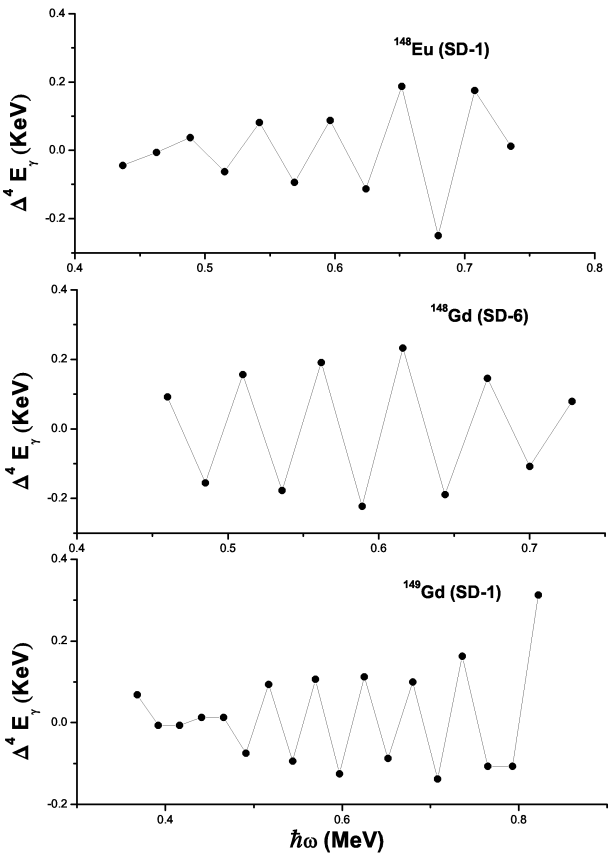

Fig. 4: The calculated $\Delta^4 E_\gamma$ staggering as a function of rotational frequency $\hbar\omega$ of the SDRB's ^{148}Eu(SD-1), ^{148}Gd(SD-6), ^{149}Gd(SD-1).

$J^{(2)}$ against $\hbar\omega$ for the excited SD band in ^{151}Tb gives a curve that is practically constant and which closely follows the $J^{(2)}$ curved traced out by the yrast SD band in ^{152}Dy but which is very different from the yrast SD band in ^{151}Tb. Similarly the ^{150}Gd excited SD band has $J^{(2)}$ values which resemble those observed in the ^{151}Tb yrast SD band. It is concluded that the N=86 isotones SD nuclei have identical supershell structures:

Nucleus	Yrast band	Excited band
$^{150}_{64}Gd$	$\pi(3)^{\bar{0}}[(4)^{10}(5)^{12}](i_{13/2})^2$	$\pi(3)^{\bar{1}}[(4)^{10}(5)^{12}](i_{13/2})^3$
$^{151}_{65}Tb$	$\pi(3)^{\bar{0}}[(4)^{10}(5)^{12}](i_{13/2})^3$	$\pi(3)^{\bar{1}}[(4)^{10}(5)^{12}](i_{13/2})^4$
$^{152}_{66}Dy$	$\pi(3)^{\bar{0}}[(4)^{10}(5)^{12}](i_{13/2})^4$	$\pi(3)^{\bar{1}}[(4)^{10}(5)^{12}](i_{13/2})^5$

6 Identical bands in the isotones nuclei N=86

A particularly striking feature of SD nuclei is the observation of a numerous bands with nearly identical transition energies in neighboring nuclei. Because of the large single particle SD gaps at Z=66 and N=86, the nucleus ^{152}Dy is expected to be a very good doubly magic SD core. The difference in $\gamma-$ray energies ΔE_γ between transition in the two pairs of N=86 isotones (excited ^{151}Tb (SD-2), yrast ^{152}Dy (SD-1)) and (excited ^{150}Gd (SD-2), yrast ^{151}Tb (SD-1)) were calculated.

The gamma transition energies of the excited band (SD-2) in ^{151}Tb are almost identical to that of the yrast band (SD-1) in ^{152}Dy. This twin band has been associated with a [301]1/2 hole in the ^{152}Dy core. The orbitals $\pi 6_2$ and $\nu 7_2$ are occupied in ^{151}Tb, while in ^{152}Dy the $\pi 6_4$ level is occupied and this leads to a more constant in dynamic moment of inertia $J^{(2)}$. Clearly the $J^{(2)}$ values for the excited SD bands are very similar to the yrast SD bands in their Z+1, N=86 isotones. The plot of percentage differences $\Delta E_\gamma / E_\gamma$ in transition energies versus spin for the two pairs (^{151}Tb(SD-2), ^{152}Dy(SD-1)) and (^{150}Gd(SD-2), ^{151}Tb(SD-1)) are illustrated in Fig. (3).

7 ΔI = 2 Staggering

Another result of the present work is the observation of a $\Delta I = 2$ staggering effects in the $\gamma-$ray energies, where the two sequences for spins $I = 4j, 4j+1$ (j=0,1,2,...) and $I = 4j+2$ (j=0,1,2,...) are bifurcated. For each band the deviation of the $\gamma-$ray energies from a smooth reference ΔE_γ is determined by calculating the fourth derivative of the $\gamma-$ray energies $\Delta E_\gamma(I)$ at a given spin $\Delta^4 E_\gamma$. The staggering in the $\gamma-$ray energies is indeed found for the SD bands in ^{148}Eu(SD-1), ^{148}Gd(SD-6) and ^{149}Gd(SD-1) in Fig. (4).

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