ΔI=1 Signature Splitting in Signature Partners of Odd Mass Superdeformed Nuclei

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The spins, transition energies, rotational frequencies, kinematic and dynamic moment of inertia of rotational bands of signature partners pairs of odd–A superdeformed bands in A∼190 region were calculated by proposing a simple model based on collective rotational model. Simulated search program was written to determine the model parameters. The calculated results agree with experimental data for fourteen signature partner pairs in Hg/Tl/Pb/Bi/nuclei. We investigated the ΔI=1 signature splitting by extracted the difference between the average transitions I+2 ⟶ I and I ⟶ I-2 energies in one band and the transition I+1 ⟶ I-1 energies in its signature partner. Most of the signature partners in this region show large amplitude staggering. The signature splitting has the effect of increasing dynamical moment of inertia J^2 for favored band and decreasing J^2 for the unfavored band.

1 Introduction

Since the first observation of superdeformation in ^{152}Dy [1] and in ^{191}Hg [2] more than 350 settled SD bands in more than 100 nuclei have been will established in several mass regions of nuclear chart A∼190, 150, 130 [3–6]. With the aid of large γ-ray detectors arrays, new regions of SD nuclei have been discovered encircle mass A∼80, 60, 70, 90 regions. The A∼190 mass region is of special interest, more than 85 SD bands have now been observed in this mass region alone in Au, Hg, Tl, Pb, Bi and Po nuclei. The SD states in A∼190 mass region have been observed down to quite low spin also many SD bands in the A∼190 show the same smooth rise in the dynamical moment of inertia as rotational frequency increase, which is associated [7–9] with the successive gradual alignments of a pair of nucleons occupying specific high-N intruder orbitals in the presence of pairing correlations.

Spin assignment is one of the most difficult and still unsolved problems in the study of nuclear superdeformation, because spins have not been determined experimentally in SD nuclei. This is due to the difficulty of establishing the excitation of a SD band into known yrast states. Several related approaches to assign the spins of SD bands in terms of there observed γ-ray energies were proposed [10–28]. For all approaches an extrapolation fitting procedure was used.

The development of large γ-ray arrays has allowed experimentalists to find new phenomena at high angular momenta. For example some SD nuclear bands in mass regions A∼150 and A∼190 show an unexpected regular staggering effects in the transition energies E_γ (a zigzag behavior as a function of rotational frequency or spin). At high rotational frequencies a ΔI=2 staggering was observed [29–39]. It has attracted much attention and interest, and has thus become one of the most frequently debated subjects.

The ΔI=2 rotational bands are perturbed and two ΔI=4 rotational sequences emerge with an energy splitting of about some hundred eV. This is commonly called ΔI=4 bifurcation or as C_4 oscillation, because the SD-energy levels are consequently separated into two spin sequences with spin values I_0, I_0+4, I_0+8,... and I_0+2, I_0+6, I_0+10,. . . respectively.

Many ΔI=1 signature splitting have been observed in ND nuclei for different bands, like odd-even staggering (OES) in the gamma vibrational band at low spin [40], the beat odd-even ΔI=1 staggering patterns observed in the octuple bands [41] and the ΔI=1 odd-even staggering structure of alternating parity bands in even-even nuclei [42, 43].

There is another kind of staggering happens in SD odd-A nuclei, the ΔI=1 signature splitting in signature partner pairs. It was seen that most of SD rotational bands in odd-A nuclei in the A∼190 region are signature partners [44–53]. Most of these signature partners show large amplitude signature splitting and the bandhead moments of inertia of each pair are almost identical.

2 Sketch of the Model

In the model used, the excitation energy of a SD State E(I) and spin I is expressed as:

$$\hat{I}^2 = I\,(I+1) = \sum_n b_n\, E^n(I). \tag{1}$$

With $\hat{I}$ $[I\,(I+1)]^{1/2}$. If we restrict to three terms only, then

$$I(I+1) = b_0 + b_1 E(I) + b_2 E^2(I). \tag{2}$$

Solving for E(I) we get the two-parameters formula for E(I)

$$E(I) = E_0 + a\left([1 + bI(I+1)]^{1/2}\right) \tag{3}$$

with a,b and E_0 simply expressed by b_0, b_1 and b_2

$$a = \frac{1}{2b_2}[b_1^2 - 4b_0b_2]^{1/2} \tag{4}$$

$$b = \frac{4b_2}{b_1^2 - 4b_0 b_2} \tag{5}$$

$$E_0 = a - \frac{b_1}{2b_2} \tag{6}$$

where b characterizes the nuclear softness.

The rigid rotor limit corresponds to b⟶0 and a, E_0 keeping finite. The value of the parameter a increases slowly with I. It is expected that a better expression may be obtained if a weak I dependence of the parameter a is taken into account. So equation (3) is tentatively modified as follows:

$$E(I) = a\Big([1 + b(I)(I+1)]^{1/2} - 1\Big) + cI(I+1) \tag{7}$$

with an additional parameter c. Leading to a form for the gamma transition energies

$$\begin{aligned} E_\gamma(I) \;=\; & a\Big([1{+}bI(I{+}1)]^{1/2} - [1{+}b(I{-}2)(I{-}1)]^{1/2}\Big) \\ & +2c(2I{+}1). \end{aligned} \tag{8}$$

The kinematic J^1 and dynamic J^2 moment of inertia associated with the a, b, c, formula are:

$$J^1 = ab[1 + bI(I+1)]^{1/2} + \frac{1}{2c} \tag{9}$$

$$J^2 = ab[1 + bI(I+1)]^{3/2} + \frac{1}{2c}. \tag{10}$$

The bandhead moment of inertia is

$$J_0 = \frac{\hbar^2}{ab + 2c}.$$

Each SD nucleus is described by three adjustable parameters a, b and c which are determined by fitting procedure of all known levels.

For the SD bands, one can extract the rotational frequency, dynamic and kinematic moment of inertia by using the experimental intra band E_2 transition energies as follows:

$$\hbar\omega = \frac{1}{4}[E_\gamma(I+2) + E_\gamma(I)] \tag{11}$$

$$J^2(I) = \frac{4\,\hbar^2}{\triangle E_\gamma} \tag{12}$$

$$J^1(I-1) = \frac{\hbar^2(2I-1)}{E_\gamma} \tag{13}$$

where

$$E_\gamma = E(I) - E(I-2),$$

$$\triangle E_\gamma = E_\gamma(I+2) - E_\gamma(I).$$

It is seen that whereas the extracted J^1 depends on I proposition, J^2 does not.

3 Analysis of ΔI=1 signature splitting in SD signature partner

To investigate the ΔI=1 staggering in signature partner pairs of odd SD band, one must extract the differences between the average transition I+2⟶I and I⟶I-2 energies in one band the transition I+1⟶I and I⟶I-1 energies in the signature partner

$$\begin{aligned} \triangle^2 E_\gamma(I) \;=\; & \tfrac{1}{2}\Big[E_\gamma(I{+}2 \longrightarrow I) + E_\gamma(I \longrightarrow I{-}2) \\ & -2E_\gamma(I{+}1 \longrightarrow I{-}1)\Big] \end{aligned}$$

where E_γ(I) is proposed in equation (8).

4 Numerical Calculation and Discussions

Our selected data set includes fourteen signature partner pairs in ten odd SD nuclei in the A∼ 190 mass region, namely:

^{191}Hg (SD2, SD3) ^{193}Hg (SD1, SD2) ^{193}Hg(SD3, SD4)
^{193}Hg (SD3, SD4) ^{195}Hg (SD3, SD4) ^{191}Tl(SD1, SD2)
^{193}Tl (SD1, SD2) ^{195}Tl (SD1, SD2) ^{193}Pb(SD3, SD4)
^{193}Pb (SD5, SD6) ^{195}Pb (SD1, SD2) ^{195}Pb(SD3, SD4)
^{197}Pb (SD1, SD2) ^{197}Bi (SD2, SD3)

The experimental transition energies are taken from reference [3]. To parameterize the spins of the SD bands, we assumed various values for the bandhead spin I_0 for each SD band and the model parameters a, b and c are adjusted by using a computer simulated search program in order to obtain a minimum root mean square deviation

$$\chi = \left[\frac{1}{N}\sum_{i=1}^{N} \frac{E_\gamma^{exp}(I) - E_\gamma^{Theor}(I)}{\triangle E_\gamma^{exp}(I)}\right]^{1/2}.$$

Of the calculated energies E_γ^{cal} from the observed energies E_γ^{exp}, where N is the number of data points considered and $\triangle E_\gamma^{exp}$ is the uncertainty of the γ-transition energies. The fitting procedure was repeated with spin I_0 fixed at the nearest half integer.

Table (1) gives the optimized model parameters a, b, c, the bandhead spin proposition I_0 and the lowest transition energies E_γ (I_0+2 ⟶ I_0) for each SD band.

The systematic behavior of kinematic J^1 and dynamic J^2 moments of inertia are guideline for the spin prediction and to understand the properties of the SD bands. We studied the variation of $J^1$and J^2as a function of rotational frequency $\hbar\omega$. The value of $J^1$and J^2 approaches each other at the bandhead spin I_0 . The J^1 moment of inertia is found to be smaller than that of J^2 for all values of $\hbar\omega$. Both $J^1$and J^2 plots are concave upwards. In general the bandhead moments of inertia in our selected signature partners odd-A SD nuclei $J_0 \cong (94 \pm 4)\hbar$ MeV^{-1} are longer than that of the yrast SD bands in neighboring even-even nuclei. The best fitted parameters were used to calculate the theoretical transition energies extracted from our proposed model.

Table 1: The calculated best model parameters a, b, c and suggested bandhead spins I_0 for our selected signature partners in the odd SD nuclei in A $\simeq$ 190 region.

SD Bands	a	b	c	I_0	E_γ
	MeV	10^{-4} MeV	MeV	$\hbar$	keV
^{191}Hg(SD2)	19074.6639	3.0809	2.3765	10.5	252.4
^{191}Hg(SD3)	15810.8517	3.6987	2.4037	11.5	272
^{193}Hg(SD1)	1569.7883	23.4445	3.7662	9.5	233.2
^{193}Hg(SD2)	12654.6097	4.3858	2.6051	10.5	254
^{193}Hg(SD3)	12243.4329	4.4984	2.6289	9.5	233.5
^{193}Hg(SD4)	12654.6098	4.3858	2.6051	10.5	254
^{195}Hg(SD3)	72779.9405	1.8708	-0.9723	10.5	284.5
^{195}Hg(SD4)	22034.6647	2.4110	2.4673	15.5	341.9
^{191}Tl(SD1)	307519.2819	0.7272	-5.7903	11.5	276.77
^{191}Tl(SD2)	249002.6385	0.8532	-5.2350	12.5	296.75
^{193}Tl(SD1)	13573.6592	3.7666	2.6759	9.5	227.3
^{193}Tl(SD2)	6380.8736	5.3776	3.5196	8.5	206.6
^{195}Tl(SD1)	6380.8738	5.3776	3.5196	5.5	146.2
^{195}Tl(SD2)	33124.3911	2.4266	1.2551	6.5	167.5
^{193}Pb(SD3)	4702.3802	6.2778	3.8243	10.5	251.5
^{193}Pb(SD4)	16892.1756	3.5957	2.2986	11.5	273
^{193}Pb(SD5)	4337.5276	8.2523	3.6196	8.5	213.2
^{193}Pb(SD6)	3574.7877	9.4219	3.7378	9.5	234.6
^{195}Pb(SD1)	600.9413	13.4593	4.6737	7.5	162.58
^{195}Pb(SD2)	15864555.765	0.0139	-5.9659	6.5	182.13
^{195}Pb(SD3)	2362.3559	13.4225	3.9167	7.5	198.2
^{195}Pb(SD4)	18884.3711	3.5500	2.0732	8.5	213.6
^{197}Pb(SD1)	9713.0371	2.5870	3.8497	7.5	183.7
^{197}Pb(SD2)	724986.6813	0.1798	-1.3692	6.5	204.6
^{197}Bi (SD2)	6.09E+08	8.24E-07	-245.7806	8.5	166.2
^{197}Bi (SD3)	6.09E+08	8.24E-07	-245.7806	9.5	186.7

To investigate the ΔI=1 signature splitting, the difference between the averaged transitions I+2⟶I and I⟶I-2 energies in one band and the transition I+1⟶I-1 energies in its signature partner $\triangle^2 E_\gamma$(I) are determined and its value as a function of spin I for each signature partner pairs are plotted in figure (1). Most of there signature partners show large amplitude staggering with the exception of ^{193}Hg (SD1, SD2), ^{193}Pb (SD5, SD6) and ^{195}Pb (SD3, SD4).

A clear out amplification of $\triangle^2$E(I) is seen in ^{193}Pb (SD3, SD4). For most cases one finds that $\triangle^2$E(I) is very small at lower spins, increasing faster and faster as the spin I increase. The of $\triangle^2$E(I) in ^{193}Tl (SD1, SD2) and ^{195}Tl (SD1, SD2) are remarkable similar.

5 Conclusion

The nuclear superdeformed rotational bands of signature partners of odd-mass number in the A∼190 region have been studied in the framework of a simple formula based on collective rotational model containing three parameters. The formula connected directly the unknown spin and the energy of the level the spins of the observed levels were extracted by assuming various values to the lowest spin of the bandhead at the nearest half integer. The optimized three parameters have been deduced by using a computer simulated search program in order to obtain a minimum root mean square deviation of the calculated transition energies from the measured energies.

The calculated transition energies, level, spins, rotational frequencies, kinematic and dynamic moments of inertia are examined for fourteen signature partner pairs. To investigate the ΔI=1 signature splitting for each signature partner pair, we calculated the difference between the average transitions I+2 ⟶I and I⟶I-2 energies in one band and the transition I+1⟶I-1 energies in its signature partner. Most of the signature partners in this region show large amplitude ΔI=1 staggering.

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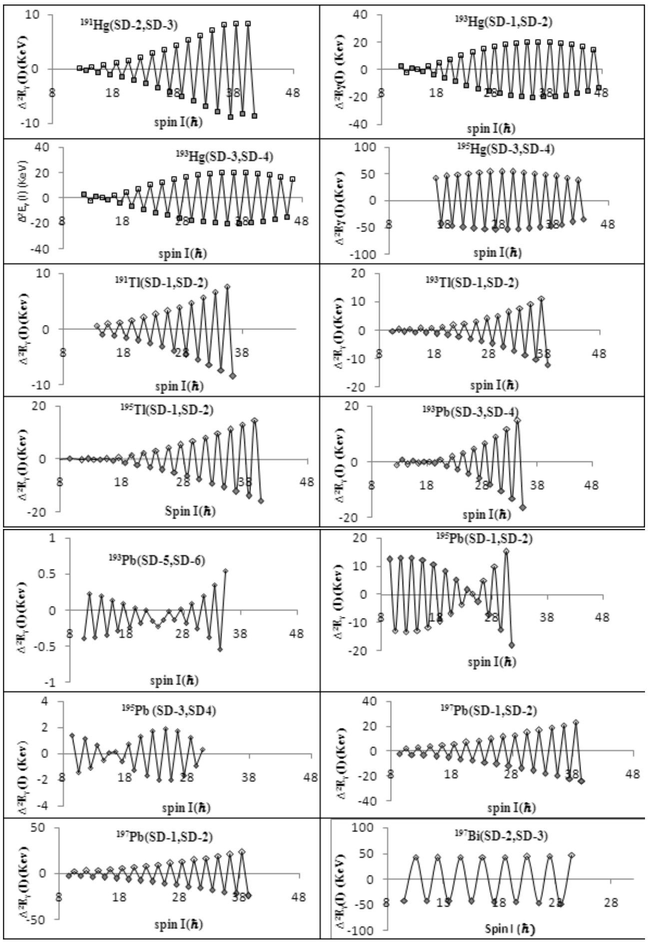

Fig. 1: The ΔI=1 signature splitting in some signature partners of odd-A superdeformed nuclei.

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