Further Problems with Integral Spin Charged Particles

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The structure of the Lagrangian density of quantum theories of electrically charged particles is analyzed. It is pointed out that a well known and self-consistent expression exists for the electromagnetic interactions of a spin-1/2 Dirac particle. On the other hand, using the Noether theorem, it is shown that no such expression exists for the spin-0 Klein-Gordon charged particle as well as for the $W^\pm$ spin-1 particle. It is also explained why effective expressions used in practical analysis of collider data cannot be a part of a self-consistent theory. The results cast doubt on the validity of the electroweak theory.

1 Introduction

Since its very beginning, quantum theory has provided expressions describing electromagnetic interactions. In particular, the Dirac equation of spin-1/2 charged particle takes a covariant form [1, see pp. 16–24]. As is well known, electromagnetic interactions of a Dirac particle have an extraordinary experimental support. Later, a quantum theory of a spin-0 Klein-Gordon (KG) charged particle was published [2, see pp. 188–205]. In the electroweak theory which was constructed several decades later, the $W^\pm$ spin-1 charged boson plays a cardinal role. The discussion presented in this work examines the Lagrangian density of quantum theories. As is well known, the electromagnetic interaction term of these theories depends on a contraction of the charged particle's 4-current and the external 4-potential $j_\mu A^\mu$. Thus, the Noether theorem is used for deriving expressions for the charged particle's 4-current. In this way the analysis proves that electromagnetic theories of spin-0 and spin-1 particles contain inherent contradictions.

Units where $\hbar = c = 1$ are used in this work. Hence, only one dimension is required and it is the length, denoted by [L]. For example, mass, energy and momentum have the dimension [L^{-1}], etc. Greek indices run from 0 to 3 and the diagonal metric used is $g_{\mu\nu} = (1, -1, -1, -1)$. The symbol $_{,\nu}$ denotes the partial differentiation with respect to x^ν. The summation convention is used for Greek indices. The second section presents theoretical elements that are used in the discussion. The third section contains a proof showing that electromagnetic interactions cannot be a part of a self-consistent theory of spin-0 and of spin-1 quantum particles. Concluding remarks can be found in the last section.

2 The theoretical basis of the analysis

The following discussion examines the structure of a quantum theory of an electrically charged particle and its interaction with electromagnetic fields. The need for a Lagrangian density as basis for a relativistic quantum theory has become a common practice. This issue can be derived from the fact that the phase is an argument of an exponent. Thus, the power series expansion of the argument proves that the phase must be a dimensionless Lorentz scalar. This requirement is satisfied if the action (divided by $\hbar$) is used for the phase and the Lagrangian density is a Lorentz scalar whose dimension is [L^{-4}]. Indeed, in this case, the action

$$S = \int \mathcal{L}\, d^4x \tag{1}$$

is a dimensionless Lorentz scalar.

The form of the required Lagrangian density is

$$\mathcal{L}(\Phi^\dagger, \Phi^\dagger_{,\mu}, \Phi, \Phi_{,\mu}, A^\mu, F^{\mu\nu}), \tag{2}$$

where Φ denotes the function of the charged quantum particle and $A^\mu, F^{\mu\nu}$ denote the electromagnetic 4-potential and its fields, respectively. In the discussion presented herein the quantum function Φ represents either scalar, spinor or vector particle. In specific cases the notation ϕ represents a KG charged particle, ψ denotes a Dirac particle and W^μ denotes the $W^\pm$ particles. Evidently, (1) and (2) prove that the function Φ has dimension.

Maxwellian electrodynamics is derived from the following Lagrangian density [3, see pp. 71–81]

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} - j_\mu A^\mu, \tag{3}$$

where j^μ denotes the charge's 4-current and the last term of (3) represents the electromagnetic interaction.

This expression demonstrates the crucial role of the 4-current in a self-consistent theory of an electrically charged particle. As is well known, the charge 4-current must satisfy the continuity equation

$$j^\mu_{,\mu} = 0. \tag{4}$$

The standard method used for constructing such a 4-current relies on Noether's theorem [4, see p. 20]. Thus, in the present case, the expression for the 4-current boils down to the following form

$$j^\mu = i\frac{\partial \mathcal{L}}{\partial \Phi^\dagger_{,\mu}}\Phi^\dagger - i\frac{\partial \mathcal{L}}{\partial \Phi_{,\mu}}\Phi. \tag{5}$$

(Note that due to the opposite phase sign of $\Phi^\dagger$ and Φ, corresponding terms derived from these functions have opposite sign.) Thus, in the case of a charged particle, the Noether 4-current (5) is multiplied by the electric charge e. Relying on (5), one concludes that the 4-current is derived from terms of the Lagrangian density that contain a derivative of the field function with respect to the coordinates x^μ. The 0-component of (5) represents the particle's density. Hence, the dimension of j^μ is [L^{-3}].

A standard method used for the introduction of electromagnetic interaction is to substitute the following transformation in the free Lagrangian density of the particle (see e.g. [1, p. 10])

$$-i\frac{\partial}{\partial x^\mu} \to -i\frac{\partial}{\partial x^\mu} - eA_\mu(x^\nu). \tag{6}$$

Later, this substitution is called the standard form of electromagnetic interaction. This form as well as other forms of electromagnetic interactions are discussed in the next section.

3 Quantum charged particles

The Dirac Lagrangian density of a free spin-1/2 particle is [4, see p. 54]

$$\mathcal{L} = \bar{\psi}\,[\gamma^\mu i\partial_\mu - m]\,\psi. \tag{7}$$

This expression is linear in the mass. Hence, the dimension [L^{-4}] of the Lagrangian density means that the dimension of the Dirac function ψ is [$L^{-3/2}$]. An application of the Noether relation (5) for a construction of the 4-current yields the well known Dirac expression [1, see pp. 23–24] which is written below in the standard notation

$$j^\mu = e\,\bar{\psi}\gamma^\mu\psi. \tag{8}$$

The dimension [$L^{-3/2}$] of the Dirac function ψ shows that (8) has the required dimension.

The case of the KG and of the W Lagrangian density is different. Here the mass term takes the form (see [4, p. 26] and [5, p. 309], respectively)

$$-m^2\Phi^\dagger\Phi. \tag{9}$$

Different numerical factors of (9) are not mentioned and the same is true for the contraction of the 4 components of the W function. Relationship (9) means that the dimension of the KG and of the W functions is [L^{-1}]. Thus, in order to satisfy the [L^{-4}] dimension of the Lagrangian density of these particle, it must contain terms that are *bilinear* in derivatives with respect to the space-time coordinates x^μ. Applying the Noether relation for the 4-current (5), one finds that *the 4-current of the KG and of the W particles contains a derivative with respect to* x^μ. This property means that utilizing of the standard form of the introduction of electromagnetic interactions (6), one finds that *the 4-current of the KG and of the W particles depends linearly on the 4-potential of the electromagnetic fields.* (This is certainly inconsistent with gauge invariance, because here a gauge transformation alters charge density and the associated field values as well. However, this matter is not discussed in the present work.) The dependence of the charged KG 4-current on the external electromagnetic 4-potential has already been shown a long time ago [2, see p. 199].

Let us turn to the electromagnetic fields. The interaction term of the Maxwellian Lagrangian density (3) is $j_\mu A^\mu$. Now, if the 4-current j^μ of the KG and of the W particles depends linearly on the 4-potential of electromagnetic fields then *there is a quadratic term of the 4-potential in the expression for the interaction term in the Maxwellian Lagrangian density (3).* This is a contradiction because in Maxwellian electrodynamics the interaction term must be linear in the 4-potential [3, see pp. 78–79].

The foregoing discussion proves that there is no theoretically valid expression for the electromagnetic interaction of a KG particle and of the W boson as well. Thus, in the case of the W boson people resort to a phenomenological expression that goes by the name *effective Lagrangian density* [6, 7]. Using standard notation for the W field, one of the nonvanishing electromagnetic interaction terms of the effective Lagrangian density is

$$\mathcal{L}_{int} = -ie\,(W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu W^{\mu\nu}A_\nu). \tag{10}$$

The articles [6,7] have been cited many times and (10) is still used in a collider data analysis [8, see eq. (1)] [9, see eq. (3)].

The following argument proves that (10) is indeed an effective expression which cannot be justified theoretically. Let us assume that (10) is a term in a theoretically justifiable Lagrangian density. In this case the following expression

$$j^\nu = -ie\,(W^{\dagger\nu}_\mu W^\mu - W^\dagger_\mu W^{\mu\nu}) \tag{11}$$

represents the electric 4-current of the W boson. But (11) contains the factors $W^{\dagger\mu\nu}$ and $W^{\mu\nu}$, and by the definition $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, each of which is a derivative with respect to x^μ. Therefore, due to the Noether theorem (5), the interaction term (10) alters the 4-current of the W boson and adds to it a troublesome term that is proportional the the external electromagnetic 4-potential A^μ. Hence, *contrary to the assumption examined herein, (11) does not represent the 4-current of the W boson.* This contradiction substantiates the proof.

A second electromagnetic term which is introduced into the effective Lagrangian density of the W is [6–9]

$$\mathcal{L}_{int} = ie\,W^\dagger_\mu W_\nu F^{\mu\nu}. \tag{12}$$

This term is certainly inconsistent with electromagnetic interactions because these interactions are proportional to the 4-current of the charged particle and the dimension of the 4-current is [L^{-3}]. On the other hand, it is proved above that the dimension of the W function is [L^{-1}] and that of $W^\dagger_\mu W_\nu$ is [L^{-2}]. Therefore, (12) cannot represent a consistent electromagnetic interaction.

4 Conclusions

The solid mathematical structure of the spin-1/2 Dirac equation and its successful experimental status are pointed out above. Here a self-consistent relativistically covariant electromagnetic interaction exists. Thus, nobody finds the need to resort to "effective Lagrangian density".

A different situation holds for the cases of spin-0 and spin-1 elementary particles. It is proved in this work that for these particles the standard methods used for constructing electromagnetic interactions fail. Furthermore, it is proved above that the authors of [6, 7] are right in their description of the W boson electromagnetic interaction (10) as an effective expression. However, a proof presented in the previous section shows that (10) cannot be a part of a theoretically self-consistent Lagrangian density. This outcome means that *the W boson cannot carry an electric charge.* Now, the W boson takes a vital part in the unification of electrodynamics with weak interaction which is called electroweak theory. Therefore, the results cast doubt on the validity of the electroweak theory.

Another result of the discussion presented above is that the experimentally detected W boson cannot be an elementary particle described by a field function that takes the form $W^{\pm\mu}(x^\nu)$. Indeed, a dependence on a single set of space-time coordinates x^μ is a property of a structureless pointlike elementary particle like the electron etc. Thus, the actual $W^\pm$ particles must be composite particles and it looks plausible to regard them as a combination of mesons of the top quark and either of the d, s, b antiquarks or vice versa. It turns out that the conclusions of this work provide an independent support to similar conclusions that have been published earlier [10]. It should also be noted that the results of this work are consistent with Dirac's lifelong objection to the KG equation [11].

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