Space-Time Uncertainty and Cosmology: a Proposed Quantum Model of the Universe

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The paper introduces a cosmological model of the quantum universe. The aim of the model is (i) to identify the possible mechanism that governs the matter/antimatter ratio existing in the universe and concurrently to propose (ii) a reasonable growth mechanism of the universe and (iii) a possible explanation of the dark energy. The concept of time-space uncertainty, on which is based the present quantum approach, has been proven able to bridge quantum mechanics and relativity.

1 Introduction

Physical cosmology is the science of the most fundamental questions about past, present and future of the universe. Born in the modern form with the early Einstein general relativity (1916), it involves today all branches of the theoretical physics. The conceptual basis of cosmology relies not only on the theories of gravity field, but also on the fundamental interactions between elementary particles. Likely the first attempt of extending the achievements of general relativity to propose a model of universe based on a physical theory was made by Einstein himself with the introduction of the cosmological constant Λ. At that time the quantum theory was at its very early beginning, while the gravitational interaction seemed the most general physical law governing the dynamics of celestial bodies; so the relativity, with or without Λ, soon appeared as the most valuable resource to proceed beyond the Newton physics.

The first milestone of the modern cosmology is due to Friedmann (1922) and (1924); the hypothesis of universe homogeneous and isotropic allowed inferring the equations that describe shape and expansion/contraction propensity of the universe depending on the value of the density parameter Ω. After these early contributions, have been proposed several models of universe, e.g. by Lemaitre (1929) and Eddington (1930).

The first experimental milestone of cosmology is due to Hubble, who measured the Doppler shift of light emitted by far galaxies (1929): the experimental data revealed the recession velocity law of galaxies with respect to earth. Since then, any model of universe should allow for this experimental evidence. The second experimental landmark was the discovery of the cosmic microwave background radiation (Penzias and Wilson, 1965).

An essential added value to the theoretical cosmology came from the almost simultaneous development of quantum mechanics. Without this physical background and the recent Standard Model, the modern cosmology would be inconceivable. The cosmic abundance of elements has been investigated by Weizsacker (1938) and then by Gamow et al (1948); Chandrasekhar (1942) and more recently Fowler et al [1] pointed out several processes in the stars that concurrently account for the formation of heavy elements in the universe.

On the one hand, the understanding of the nuclear processes explains the existence of stars and other objects (quasars, white dwarf and so on); on the other hand, however, is the general relativity that explains the existence and features of the black holes. The crucial point of the modern physics and cosmology is the difficulty of merging relativistic and quantum theories. Several papers have been published on quantum gravity, e.g. [2,3]. Today the string theory is deemed to be a step towards the unification of both theories [4,5]; unavoidably the string theory has been also implemented by cosmologists to investigate problems of mere quantum nature, like for instance the vacuum energy and the dark energy [6], and the cosmological constant as well [7,8,9]. However, the mathematical difficulties of these theories are daunting, and their previsions hardly testable.

Yet to shed light on fundamental issues of cosmology are also useful plain models that exploiting simple assumptions allow reliable order of magnitude estimates; simplified models are functional to focus essential but even so significant information.

The present paper aims to infer the order of magnitude estimates starting from a quantum standpoint. The input values implemented in this paper are the literature estimates of the universe diameter $d_u = 8.7\times10^{26}$ m and age $t_u = 4.3\times10^{17}$ s. The total mass of the universe reported in the literature is estimated to be about $m_u = 3 \times 10^{52}$ kg, counting however the stars only. Thus it is reasonable to expect that the effective value M_u of total mass should actually be considerably greater than m_u. Indeed this latter does not include contributions like the dark mass or the total mass of all black holes possible existing in our universe, which instead should be also taken into account when correlating these three main features of the universe; this reasonably suggests $M_u > m_u$. The fourth key value to be introduced is the expansion rate on the universe, usually expressed through the Hubble constant $H_0 = 2.3 \times 10^{-18}$ s^{-1}; this number, which presumably averages the value of a true function of time, has been object of great debate because of its importance in cosmology.

2 Quantum background

Physicists believe unsatisfactory a theory based on the wave function ψ without direct physical meaning, e.g. [10]; indeed $\psi^*\psi$ only has the statistical meaning of probability density and contains the maximum information obtainable about a physical system. Moreover also the Wigner function [11], although providing significant information about the quantum states, presents conceptual difficulties: it cannot be really regarded as a probability distribution in the classical sense, it is a quasi-probability that can take negative values; moreover it can represent the average value of an observable but not, in general, also its higher power moments. These difficulties, both inherent the wave formalism, are overcome in a model that exploits directly the statistical formulation of the quantum uncertainty, which becomes itself a fundamental assumption of the model and reads in one space dimension

$$\Delta x\, \Delta p_x = n\hbar = \Delta\varepsilon\, \Delta t. \qquad (2,1)$$

The second equality is formally obtained from the former rewritten $(\Delta x/\mathrm{v}_x)(\mathrm{v}_x\Delta p_x) = n\hbar$ with the same number n of states and defining $\mathrm{v}_x = \Delta x/\Delta t$ and $\Delta\varepsilon = \mathrm{v}_x\Delta p_x$; these definition hold because n and the uncertainty ranges are arbitrary. (2,1) compel the positions

$$x \to \Delta x; \qquad p_x \to \Delta p_x; \qquad t \to \Delta t; \qquad \varepsilon \to \Delta\varepsilon. \qquad (2,2)$$

No further hypothesis is necessary besides that of waiving the random local values of the dynamical variables, considered random, unknown and unpredictable. To clarify the kind of quantum approach required by the positions (2,2) and highlight why (2,1) have prospective interest also in cosmology, are useful two examples shortly sketched below. The quantum properties are inferred implementing directly the physical definitions of the observable of interest, without solving the pertinent wave equations; note however that the operator formalism of wave mechanics is also obtained as a corollary of these equations [12], which explains why the results are anyway the same.

The first example concerns the angular momentum $\mathbf{M} = \mathbf{r}\times\mathbf{p}$ whose component along the arbitrary unit vector $\mathbf{w}$ is $M_w = \mathbf{r}\times\mathbf{p}\cdot\mathbf{w}$; the vectors are defined in a reference system R. The positions (2,2) compel $\mathbf{r} \to \Delta\mathbf{r}$ and $\mathbf{p} \to \Delta\mathbf{p}$ to calculate the number l of states consistent with the ranges $\Delta\mathbf{r}$ and $\Delta\mathbf{p}$ physically allowed to the particle. Thus $M_w = (\Delta\mathbf{r}\times\Delta\mathbf{p})\cdot\mathbf{w} = (\mathbf{w}\times\Delta\mathbf{r})\cdot\Delta\mathbf{p}$ yields $M_w = \Delta\mathbf{W}\cdot\Delta\mathbf{p}$, where $\Delta\mathbf{W} = \mathbf{w}\times\Delta\mathbf{r}$. So $M_w = 0$ if $\Delta\mathbf{p}$ and $\Delta\mathbf{W}$ are orthogonal; else, rewriting $\Delta\mathbf{W}\cdot\Delta\mathbf{p} = (\Delta\mathbf{p}\cdot\Delta\mathbf{W}/|\Delta\mathbf{W}|)|\Delta\mathbf{W}|$ one finds $\pm\Delta p_W = \Delta\mathbf{p}\cdot\Delta\mathbf{W}/|\Delta\mathbf{W}|$ and thus $M_w = \pm\Delta W\Delta p_W$, i.e. $M_w = \pm l\hbar$ according to (2,1). One component of $\mathbf{M}$ only is knowable; repeating the same approach for another component trivially means changing $\mathbf{w}$. Therefore the average values $< M_x^2 >$, $< M_y^2 >$ and $< M_z^2 >$ calculated in the same way should be equal. The components are averaged over the possible states summing $(l\hbar)^2$ from $-L$ to $+L$, where L is an arbitrary maximum value of l; so $< M_i^2 > = \sum_{l_i=-L}^{l_i=L}(\hbar l)^2/(2L+1)$ i.e. $M^2 = \sum_{i=1}^{3} < M_i^2 > = L(L+1)\hbar^2$. The mere physical definition of angular momentum is enough to find quantum results completely analogous to that of the wave mechanics without any hypothesis on the angular motion. The same holds for the energy levels of hydrogenlike atoms. The concerned definitions are now the energy $\varepsilon = p^2/2m - Ze^2/r$, being m the electron mass, and the momentum $p^2 = p_r^2 + M^2/r^2$. The positions (2,2) $p_r \to \Delta p_r$ and $r \to \Delta r$ yield $\Delta\varepsilon = \Delta p_r^2/2m + M^2/2m\Delta r^2 - Ze^2/\Delta r$. Two numbers of states are expected because of the radial and angular uncertainties. The positions (2,2) and the previous result yield $\Delta\varepsilon = n^2\hbar^2/2m\Delta r^2 + l(l+1)\hbar^2/2m\Delta r^2 - Ze^2/\Delta r$ that reads also $\Delta\varepsilon = \varepsilon_o + l(l+1)\hbar^2/2m\Delta r^2 - E_{el}$ with $E_{el} = Z^2e^4m/2n\hbar^2$ and $\varepsilon_o = (n\hbar/\Delta r - Ze^2m/n\hbar)^2/2m$. Minimizing $\Delta\varepsilon$ with $\varepsilon_o = 0$ yields $\Delta r = n^2\hbar^2/Ze^2m$; so $l \le n-1$ in order to get $\varepsilon < 0$, i.e. a bound state; $\varepsilon_{rot} = l(l+1)E_o/n^4$ yields the rotational energy of the atom as a whole. Also here appears that the range sizes do not play any role in determining the energy levels. The physical meaning of Δr, the early Bohr radius, appears noting that actually $E_{el} = -Ze^2/2\Delta r$, i.e. E_{el} is the energy of two charges of opposite sign delocalized within a diametric distance $2\Delta r$ apart. It appears now that the quantum numbers of the eigenvalues are actually numbers of allowed states of quantum systems.

The key point of this introduction is not the chance of having found well known results, but the fact of having extended this kind of approach to the special and general relativity [13,14]; selected results of interest for the purposes of the present paper are reported in the appendix. In this respect, some relevant features of this approach will be exploited later and thus deserve attention.

- Both time and space coordinates are by definition inherent any model based on (2,1).
- Any uncertainty range is defined by two boundary values, e.g. $\Delta x = x_1 - x_0$; either of them is necessarily defined with respect to the origin of a reference system, the other one controls the range size. Since both x_0 and x_1 are arbitrary, unknown and unknowable by assumption, neither size nor reference system are specified or specifiable. Any result obtained from $\mathbf{M} = \mathbf{r}\times\mathbf{p}$ depends on the particular R where are defined $\mathbf{r}$ and $\mathbf{p}$. Yet, once having introduced the positions (2,2), any reference to the initial R is lost, whereas the eigenvalues are correctly inferred from $\Delta\mathbf{r}$ and $\Delta\mathbf{p}$ only; indeed $\Delta\mathbf{M} = \Delta\mathbf{r}\times\Delta\mathbf{p}$ yields a range $\Delta\mathbf{M}$ of angular momenta corresponding to all values of the arbitrary number n of states concurrently introduced via (2,1). Otherwise stated, the previous examples have shown that the boundary values r_0 and r_1 of each i-th component Δr_i are unnecessary and do not play any role to find the eigenvalues; so, since the same holds also for the momentum range, once disregarding both coordinates

neither the range sizes nor the reference system are in fact specifiable. Hence, in general, privileged reference systems are inherently excluded by the agnostic form of space-time uncertainty of (2,1), i.e. the results hold in any four dimensional reference system.

- These examples emphasize that both boundary coordinates could even be time dependent without changing approach or result: once ignoring the local dynamical variables, conceptually and not to simplify or approximate some calculation, no information on the ranges is actually required.
- The positions (2,2) skip the necessity of solving the pertinent wave equations and allow working directly on the physical definitions of the observables; (2,1) extract the allowed quantum information from the analytical form itself of the equation defining the observable.
- The concept of delocalization resulting from (2,1) has more agnostic meaning than that of the wave formalism: here is waived even the concept of probability density.
- (2,1) and the positions (2,2) rule out the classical concept of distance, because the local coordinates that define the distance are disregarded themselves "a priori"; this means that comoving and proper distances cannot in fact be calculated, while saving however their conceptual physical meaning.

Two questions arise at this point: are (2,1) usefully applicable also in cosmology? If they really do, why not think that even the physical dimensions of G could be regarded like that of the angular momentum previously sketched? Nothing excludes "a priori" positive answers, which however imply clearly that the universe is understandable like a quantum object. In fact is just this the crucial point that justifies the present model. These quantum examples have been shortly introduced to highlight the strategy of the present paper, i.e. to emphasize the role of the space-time quantum uncertainty in cosmology. The same kind of approach will be extended to the physics of the universe exploiting both (2,1) to implement G via its physical dimensions: the idea is to regard the physical definition of G likewise as done with the angular momentum. Accordingly the gravity constant is not a mere numerical value, but a physical amount defined by its dimensional factors. In effect, at least in principle, nothing prevents regarding the numerical value of G as that resulting from a combination of mass and time and space uncertainties; so these factors can be replaced by the respective time-space ranges that characterize the properties of the universe and handled exactly as done previously. Three examples useful in the following are highlighted below.

Write $G = \Delta r^3 m^{-1} \Delta t^{-2}$ and calculate

$$\delta G = (dG/d\Delta r)_0\, \delta\Delta r + (dG/d\Delta t)_0\, \delta\Delta t + (dG/dm)_0\, \delta m$$

in an arbitrary reference system R; the subscript emphasizes that the derivatives are calculated at arbitrary Δr_0, m_0 and Δt_0. Apparently a well defined value of gravity constant seems inconsistent with the arbitrariness of Δt, Δr and m inherent its physical dimensions and required by the positions (2,2). Yet the chance of compelling $\delta G = 0$ establishes a constrain on the variability of the constituent factors that makes the definition of G compatible even with a constant value; moreover this constrain is ensured at any age of the universe just because of the arbitrary values of Δr_0 and m_0 that represent its size and total mass at any age Δt_0. So the problem is not the constancy of G, but that of demonstrating a sensible physical meaning of the constrain itself. Divide both sides of the previous expression by $\Delta r_0^3/(m_0 \Delta t_0^2)$ and put $\delta G = 0$; this is not necessarily true because some theories regard G as time dependent function [15, 16], yet let us implement for simplicity this usual position. Here $\delta m \neq 0$ because some models of universe, the so called self-creation cosmology models [17], introduce mass production as a function of time. One finds thus $3\delta\Delta r/\Delta r_0 - \delta m/m_0 - 2\delta\Delta t/\Delta t_0 = 0$. Exploit the fact that the range sizes are arbitrary and that the increments $\delta\Delta r$, δm and $\delta\Delta t$ are arbitrary as well and of course defined independently of Δr_0, m_0 and Δt_0; then regard

$$\left(\frac{3}{2} - \frac{\Delta r_0}{2m_0}\frac{\delta m}{\delta\Delta r}\right)\delta\Delta r = \frac{\Delta r_0}{\Delta t_0}\,\delta\Delta t$$

in order that this equation has in particular a physical meaning of specific interest for the present model. So let us write

$$a(t) = \frac{3}{2} - \frac{\Delta r_0}{2m_0}\frac{\delta m}{\delta\Delta r}; \qquad c = \frac{\Delta r_0}{\Delta t_0}; \qquad \delta\Delta r = \frac{c}{a(t)}\,\delta\Delta t$$

where $a(t)$ is a dimensionless arbitrary function of time. Consider now the particular case of very small range size increments via the positions $\delta\Delta r \rightarrow dr$ and $\delta\Delta t \rightarrow dt$, possible just because of their arbitrariness, and integrate both sides of the former equation between two arbitrary r_1 and r_2 to which correspond the respective times t_1 and t_2 necessary for a photon to travel the space range $\chi = r_2 - r_1$. Of course the integration reads $\chi = \int_{t_1}^{t_2} a(t)^{-1} c dt$. Therefore with these integration limits and this definition of the constant ratio $\Delta r_0/\Delta t_0$, the resulting equation has the well known physical meaning of particle horizon distance and introduces the concept of scale function $a(t)$.

To complete this analysis on the physical dimensions of G, put $\delta m \rightarrow dm$ consistently with dr and dt and consider that the equation of $a(t)$ takes the form $dm = \alpha(3/2 - a(t))dr$, where $\alpha = 2m_0/\Delta r_0$; having defined $dr = cdt/a(t)$, one finds $dm/\alpha = 3ca(t)^{-1}dt/2 - cdt$. The integral of this equation between the fixed times t_1 and t_2 arbitrarily defined and the corresponding m_1 and m_2 yields $(m_2 - m_1)/\alpha = 3(r_2 - r_1)/2 - c(t_2 - t_1)$. In general an equation having the form $\alpha^{-1}\delta m = 3\delta r/2 - c\delta t$ does not have specific physical meaning, because the quantities at right hand side are arbitrary; for instance

$\delta m = 0$ if in particular $\delta r = 2c\delta t/3$, whereas any other value of $\delta m \neq 0$ would be in principle allowed as well. This simply emphasizes that the physical meaning of $a(t)$ is not hampered by constrains on the values of δm or $\delta\Delta t$ or χ. Yet it is also possible to split the equation into $m_2/\alpha - 3r_2/2 + ct_2 = r_0$ and $m_1/\alpha - 3r_1/2 + ct_1 = r_0$, with r_0 arbitrary, which read thus $m_2/\alpha = \delta r_2^*$ and $m_1/\alpha = \delta r_1^*$ with $\delta r_2^* = r_0 + 3r_2/2 - ct_2$ and $\delta r_1^* = r_0 + 3r_1/2 - ct_1$. These equations have in effect a well defined physical meaning, because they read $m_2/\delta r_2^* = m_1/\delta r_1^* = const$. The chance of having inferred from G an equation having the form $m/\delta r^* = const$ is important because it links uniquely any mass m to a corresponding range δr^* via a proportionality factor *const*; as this link must necessarily involve G via a constant term, one expects by dimensional reasons that necessarily $const \propto G/c^2$. Before concerning this point, note that these results have been obtained simply defining $G = \Delta r^3 m^{-1} \Delta t^{-2}$, rather than by implementing additional hypotheses; thus this way of regarding G contains inherently concepts essential to describe an expanding universe.

To better understand the last result, let us consider a further way to exploit the physical dimensions of G via (2,1). Rewrite $G = \Delta r^3/(m\Delta t^2)$ as $\Delta r = Gm/\mathrm{v}^2$ with $\mathrm{v} = \pm\Delta r/\Delta t$; so v is the average velocity necessary for a particle to travel Δr during a time range Δt in any R, as stressed before. The maximum value allowed to v, defined along one coordinate axis for simplicity, introduces a minimum range size Δr_0 of Δr given by $\Delta r_0 = Gm/c^2$. By definition Δr_0 is the distance traveled by a photon starting from an arbitrary point, defined without loss of generality as the origin of R. Since the photon can move around the origin towards the negative or positive side of the reference axis with equal probability, as indeed either sign of v is identically admissible, Δr_0 is one half of a total uncertainty range Δr_s where the photon is certainly enclosed; so $\Delta r_s = 2\Delta r_0$ yields

$$\Delta r_s = 2Gm/c^2 \tag{2,3}$$

that defines therefore the boundary of the space range outside which the photon cannot escape. This range size has a general physical meaning characterized by the ratio $m/\Delta r_s$ only; also, the same holds of course for a massive particle having $\mathrm{v} < c$. This equation, already inferred in a more general way still via (2,1) only [18], has the same form just found examining $a(t)$: here we simply acknowledge that $const = 2G/c^2$.

Consider eventually that (2,1) read $\Delta x = (\Delta\varepsilon/\Delta p_x)\Delta t$; moreover it is shown in the appendix that $\Delta p_x = \mathrm{v}_x\Delta\varepsilon/c^2$, so that $\Delta x^3 = (c^2/\mathrm{v}_x)^3\Delta t^3$. Dividing both sides of this equation by $m\Delta t^2$ one finds $\Delta x^3/(m\Delta t^2) = (c^2/\mathrm{v}_x)^3\Delta t/m$. Hence

$$\frac{\Delta x^3}{m\Delta t^2} = \frac{c^3}{\xi^3}\frac{\Delta t}{m}; \qquad \mathrm{v_x} = \frac{\mathrm{c^2\Delta t}}{\Delta \mathrm{x}}; \qquad \xi = \frac{\mathrm{v_x}}{\mathrm{c}}; \qquad \xi < 1. \tag{2,4}$$

Define $\xi = \xi_G\,\xi_c$, so that the right hand side of the first (2,4) reads $(c/\xi_c)^3\Delta t/m$ and the left hand side $\xi_G^3\Delta x^3/(m\Delta t^2)$. Moreover regard in particular $\Delta t \equiv \Delta t_u$ and $\Delta x \equiv \Delta r_u$; this is certainly possible because all range sizes of (2,1) are arbitrary, so they can be regarded with reference to any specific case of interest. It is also possible to define ξ_G in order that the left hand side term corresponds to the value of G with the known values of Δr_u and Δt_u, so that (2,4) yields also the value of ξ_c; in other words (2,4) splits as follows

$$G = \xi_G^3\frac{\Delta r_u^3}{m\Delta t_u^2}; \qquad G = \frac{c^3}{\xi_c^3}\frac{\Delta t_u}{m}; \qquad \xi = \xi_G\,\xi_c < 1. \tag{2,5}$$

The previous considerations have evidenced that both expressions are compatible with a constant value of G. The problem is to show that in this way ξ effectively verifies the required inequality. The numerical results for $m \equiv m_u$ yield $\xi_G = 0.17$ and $\xi_c = 1.79$, i.e. $\xi = 0.3$. According to (2,4) ξ does not depend directly on m, whereas (2,5) show that ξ_G and ξ_c do. For instance, repeating the calculation with $m \equiv 10m_u$ at the same Δt_u one would find $\xi_G = 0.36$ and $\xi_c = 0.84$, of course still consistent with the same ξ. In both cases ξ_G and ξ_c have reasonable values, as in general a proportionality constant between two correlated quantities is expected to be of the order of unity; if not, then some physical reason hidden in the concerned correlation should account for its actual order of magnitude. Actually the factor ten just introduced is not accidental, although it appears at the moment arbitrary and unjustified; its physical meaning will be highlighted in the next section. So are of interest the following values

$$M_u = 10m_u; \quad \xi_G = 0.36; \quad \xi_c = 0.84; \quad \mathrm{v}_u = 0.3c. \tag{2,6}$$

These estimates imply that v_x of (2,4) takes the meaning of recession velocity v_u of today's universe boundary, being specifically calculated via Δr_u at our current time Δt_u. Yet there is no reason to think that the ratio $\Delta r/\Delta t$ is necessarily constant; so (2,4) prospects in general a variable expansion rate controlled by this ratio at different ages of the universe. Moreover, since v_u should reasonably depend also on the amount of mass within the universe, one expects a link between Δr_u and m_u or more likely M_u; in effect this conclusion will be confirmed in the next section.

At this point, therefore, the first target of the present model is to highlight how $\mathrm{v_u}$ is related to M_u via Δr_u, see in particular the next equation (3,3) that is the key together with (2,5) to link Δr_u and Δt_u to M_u. The model is described implementing first these today data, useful to assess the results, then it is also extended to past times when necessary. For reasons that will be clear soon, it is useful to begin with the matter era. The starting points of the present paper are not the general relativity and the Friedmann equations, but the quantum equations (2,1). The paper aims to check the effectiveness of this approach to formulate a possible model of universe. The worth of the present approach relies in particular on the fact that just (2,1) have been proven suitable to link the roots of the quantum mechanics to that of the special and general relativity [13,14].

3 Physical background of a possible model of the universe

According to (2,1) and positions (2,2), the key quantities of the present paper are not r_u and t_u, but the ranges $\Delta r_u = r_u - r_0$ and $\Delta t_u = t_u - t_0$. Let r_u be the current coordinate of the boundary of the universe at the time t_u, respectively defined with respect to an arbitrary initial value r_0 at the arbitrary time t_0. As previously emphasized, these latter coordinates are in turn fixed in an arbitrary space-time reference system R. Once accepting the quantum approach shortly introduced in section 2 to describe the universe as a quantum system, however, both r_0 and t_0 are deemed unknown and unnecessary to infer the eigenvalues of the physical observables, described instead by Δr_u and Δt_u only; moreover no particular R is specifiable, in agreement with one of the basic hypotheses of the relativity according which all reference systems are equivalent to describe the physical systems. If the uncertainty ranges only have physical meaning to define the quantum eigenvalues describing the observables, as shortly sketched in section 2, then this kind of universe has no defined center; this latter should be determined with respect to the origin of R, which however is undefined and indefinable itself like r_0 and t_0. Hence the physical universe is a space-time shell between the radii r_0 and r_u that define Δr_u. As the same holds for the time, the beginning of time defining the cosmological space-time is conceptually unidentifiable; it could be $t = 0$ or $t = t_0$ or any intermediate time. Strictly speaking, Δr_u and Δt_u only characterize the actual physical features of today's quantum universe. It means that r_0 and t_0, and in an analogous way ε_0 and p_0 of the respective ranges, characterize a pre-universe only; i.e. they are precursors of the space-time quantum ranges of (2,1) to which are actually related the physical observables of the universe. In fact, the following considerations will confirm the idea that trying to determine the initial values r_0 and t_0 is in fact inessential. The starting point of the present model is introduced as follows. Consider $\Delta p_r = n\hbar/\Delta r$ putting $\Delta p_r = h/\lambda_r - p_0$: coherently with Δr, also Δp_r defines an allowed range of local radial momenta falling between h/λ_r and p_0, both arbitrary. This equation yields in particular, specifying $\Delta r = \Delta r_u$,

$$n\lambda_u = 2\pi\Delta r_u; \quad \lambda_u = \lambda_r\lambda_0/(\lambda_0 - \lambda_r); \quad \lambda_0 = h/p_0. \qquad (3,1)$$

Whatever λ_0 might be, λ_r introduces a new wavelength λ_u; this result has in principle general valence because of the fundamental character of (2,1). For instance (3,1) imply a condition well known in quantum mechanics: an integer number n of wavelengths λ_u around a circumference corresponds to steady electron waves around a nucleus, in agreement with the quantization here introduced just by n. As λ_u has been defined without specifying the nature of the wave it characterizes, let us concern the particular case of a steady electromagnetic wave of wavelength λ_u traveling on the surface of a sphere. The assumption $r_0 \ll r_u$ brings thus to mind a hyperspherical four dimensional closed universe of radius Δr_u surrounded by a light wave running around any diametric circumference. This preliminary standpoint suggests in turn a possible hypothesis about its hypervolume and hypersurface

$$V_u = (4\pi/3)\,\Delta r_u^3; \qquad A_u = 4\pi\Delta r_u^2 \qquad (3,2)$$

filled with an amount of matter such to fulfill both (2,3) and (3,1). This also suggests regarding the universe consistent with the condition of "maximum growth efficiency", i.e. like a supermassive black hole; in effect, the previous considerations show that this conclusion is compatible with the analysis of the physical dimensions of G. Usually a black hole is allowed to form when any system, e.g. a star of sufficient mass at the end of its life cycle, collapses down to a critical radius fulfilling (2,3); so is seemingly surprising an expanding universe regarded as a supermassive black hole. Yet there is no physical reason to think that in general the shrinking process is the distinctive condition allowing a black hole; this usual idea implemented to explain observable events occurring inside the universe cannot be extrapolated to the behavior of the whole universe itself. Indeed Δr_s has been inferred via the physical definition of G simply exploiting (2,1), regardless of any specific reference to collapse events. Actually the present hypothesis seems reasonable for a growing universe, whose main requirement is to prevent mass and radiation energy losses outside it that could avert its possible evolution. According to the Hawking mechanism based on the vacuum polarization in the presence of a strong gravity field, a black hole inside the universe is able to split a couple of virtual particles generated by vacuum quantum fluctuation; it captures one of them, while releasing the other that thus appears as an ordinary particle. Outside the universe however this mechanism does not hold, as the concept of vacuum is replaced by that of "nothing". So no energy can escape outside Δr_u. The universe is thus a closed box unobservable from an external observer possibly existing. This point of view is assessed preliminarily by introducing the Schwarzschild range (2,3) and identifying $\Delta r_s \equiv \Delta r_u$ and $m \equiv m_u$; this position yields $\Delta r_s = 4.5 \times 10^{25}$ m, which is not very far from the estimated literature radius of the universe. Considering however that m_u quoted above is surely underestimated, as already emphasized, it is not surprising a value of Δr_s smaller than the expected Δr_u consistent with (2,3). Trust thus to the size of Δr_u and try to replace m_u with a value $M_u > m_u$ defined by

$$\Delta r_u = 2M_uG/c^2; \qquad (3,3)$$

one finds

$$M_u = 3 \times 10^{53}\ \text{kg}; \qquad M_u = m_u + m_? \approx 10m_u \qquad (3,4)$$

i.e. a total mass higher than the literature estimate of the visible m_u, as anticipated in section 2. This equation includes both the visible mass m_u plus a further contribution $m_?$ to be explained next. Actually nothing excludes in principle the hypothesis (3,3), which in fact can be checked in several ways. So in the following M_u only, and not m_u, will be implemented. Estimate with the help of (3,2) and (3,3) the average density of the universe

$$\rho_u = \frac{3c^6}{32\pi M_u^2 G^3} = \frac{3}{8\pi G}\left(\frac{c}{\Delta r_u}\right)^2 \qquad (3,5)$$

which justifies why this paper starts just from the so called matter controlled era.

The most direct consequence of (3,3) is the Hawking entropy. Define first the circular frequencies of a light wave trapped by gravity around the border of the universe as

$$\omega_n = n\omega_u; \qquad \omega_u = c/(2\pi\Delta r_u)$$

in agreement with (3,1); so the boundary layer of the universe is marked out by the allowed frequencies of the electromagnetic field surrounding the total mass M_u, whose energy ε_ω is given by

$$\omega_n = 1.1n \times 10^{-19}\ \mathrm{s}^{-1}$$
$$\varepsilon_\omega = \frac{n\hbar c}{2\pi\Delta r_u} = 1.2n \times 10^{-53}\ \mathrm{J}. \qquad (3,6)$$

Then let us concern also the total energy $\varepsilon_u = M_u c^2$ due to the whole amount of mass present in the universe. Since one expects that bulk energy ε_u and surface energy ε_ω should be someway correlated, the simplest hypothesis is to introduce a dimensionless proportionality factor σ_H such that $\varepsilon_u = \sigma_\mathrm{H}\varepsilon_\omega$. To infer the physical meaning of σ_H, calculate the mean values of this equation, which reads $<\varepsilon_u> = \sigma_\mathrm{H} <\varepsilon_\omega>$. Clearly $<\varepsilon_u> \equiv \varepsilon_u$. The standard way to calculate $<n\hbar\omega_u>$ via the partition function is well known; noting that $\hbar\omega_u \ll k_B T$ is verified for T down to values of the order of 10^{-28} K, one finds $<n\hbar\omega_u> \approx k_B T$. So $k_B\sigma_\mathrm{H}$ defined by an energy over a temperature can be nothing else but entropy. With the help of the Plank length $l_P = \sqrt{\hbar G/c^3}$, one finds indeed thanks to (3,2) and (3,3)

$$\sigma_\mathrm{H} = \frac{<\varepsilon_u>}{<n\hbar\omega_u>} = \frac{A_u}{4l_P^2}; \quad \hbar\omega_u = \frac{\hbar c}{2\pi\Delta r_u}; \quad \varepsilon_u = \frac{c^4}{G}\frac{\Delta r_u}{2}.$$

In effect, σ_H coincides just with the well known Hawking surface entropy in Boltzmann's units.

Before discussing further evidences to support the idea of black hole-like universe, as concerns in particular the value of M_u hypothesized here, let us implement the right hand side of (2,1): one finds $\Delta\varepsilon_u = \hbar/\Delta t_u$, whose physical meaning is clearly that of energy uncertainty range within which is defined the energy ε_u of the universe. Moreover, multiplying both sides by M_u, one finds

$$\Delta\varepsilon_u = \frac{\hbar}{\Delta t_u} = 2.4 \times 10^{-52}\ \mathrm{J}; \quad \Delta p_u = \sqrt{M_u\Delta\varepsilon_u} = 9\ \mathrm{kg\ m/s}.$$

So the uncertainty range of the momentum p_u of the universe has size of the order of the Planck momentum. The fact that the size of $\Delta\varepsilon_u$ is very narrow means of course that ε_u, whatever its value might be, is defined almost exactly. It is interesting to implement this result via the definition of G. Replace m with M_u and $\Delta t_u = \hbar/\Delta\varepsilon_u$ in the second (2,5); one finds thus $\Delta\varepsilon_u = \hbar c^3/(\xi_c^3 G M_u) = 1.4\xi_c^{-3} \times 10^{-52}$ J. Therefore $\Delta\varepsilon_u$ here calculated with $\xi_c = 0.84$, i.e. with the same value of (2,6), agrees with that obtained here directly from (2,1) via the age of the universe only. So this result on the one hand supports the value of M_u previously found, on the other hand it also confirms that the physical dimensions of G actually summarize the quantum features of the universe.

Owing to (3,3), the second (2,4) reads

$$\mathrm{v}_u = c^2\frac{\Delta t_u}{\Delta r_u} = \frac{c^4}{G}\frac{\Delta t_u}{2M_u} \qquad (3,7)$$

whose numerical value coincides of course with that of (2,6). According to (2,5), an increasing ratio $\Delta t_u/M_u$ means a smaller mass at Δt_u and thus a greater $\mathrm{v_u}$, as it is natural to expect.

To implement further these considerations, note that $\sqrt{\rho G}$ yields a frequency; so, replacing ρ with ρ_u of (3,5), one finds

$$\sqrt{\rho_u G} = 2.4 \times 10^{-19}\ \mathrm{s}^{-1}. \qquad (3,8)$$

This value is nicely twice the ground value of (3,6), even though calculated via G only and regardless of the condition (3,1); i.e. it requires $n = 2$. This result has a remarkable physical meaning that will be highlighted later. After having examined the physical meaning of the ratio $\hbar\omega_n/\varepsilon_u$ let us consider now the ratio $\hbar\omega_n/\Delta\varepsilon_u$: we emphasize that the deviation of M_u from the visible mass m_u is controlled by the constrain between (3,6) and (2,4), i.e. between the surface energy $\hbar\omega_{n=2} = \hbar c/(\pi\Delta r_u)$ of the electromagnetic wave surrounding the universe and the uncertainty energy range $\Delta\varepsilon_u = \hbar c^3/(\xi_c^3 G M_u) = \hbar/\Delta t_u$ of the bulk universe; indeed with the help of (3,3) and (3,6) we obtain

$$\omega_n\Delta t_u = \frac{nc\Delta t_u}{2\pi\Delta r_u} = \frac{n\mathrm{v}_u}{2\pi c}; \qquad \frac{\hbar\omega_n}{\hbar/\Delta t_u} = \frac{n}{2}\frac{\xi_c^3}{2\pi} \approx 0.05n \quad (3,9)$$

according to the values (2,6), which yields $\hbar\omega_{n=2}/(\hbar/\Delta t_u) \approx 0.1 = m_u/M_u$. This result is crucial to understand the physical meaning of $m_?$, as highlighted in section 4.

Consider now that the ratio $c/\Delta r_u$ of (3,5) has physical dimensions time^{-1}; thus it is definable in general as $\dot{a}/a$, being a a function of coordinate and time. It is known that Δr_u^{-1} describes the local curvature of a surface; so $c/\Delta r_u$ must be actually expressed as $(\dot{a}+b)/a$ via an additive constant b, without which the curvature of the universe would tend to zero merely for a tending to a constant. Instead it seems more sensible to think that even for constant Δr_u the curvature becomes constant itself, but not necessarily equal to zero. So (3,5) reads

$$\frac{8\pi\rho_u G}{3} = \left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{b}{a}\right)^2 + \frac{2\dot{a}b}{a^2}$$
$$\frac{c}{\Delta r_u} = \frac{\dot{a}}{a}\left(1+\frac{b}{\dot{a}}\right) \tag{3,10}$$

i.e., more expressively,

$$\frac{8\pi\rho_u G}{3H^2} = \frac{\rho_u}{\rho_c} = 1 + \frac{b}{\dot{a}}\left(2+\frac{b}{\dot{a}}\right); \quad \rho_c = \frac{3H^2}{8\pi G}; \quad H = \frac{\dot{a}}{a}.$$

Despite the quantum approach has been carried out regardless of the general relativity, the conclusion is that $b/\dot{a} < 0$ or $b/\dot{a} = 0$ or $b/\dot{a} > 0$ depending on the ratio ρ_u/ρ_c; either sign of $b/\dot{a}$ depends on that of $\dot{a}$ and b controlling the curvature according to (3,10). Calling $b = \pm c$ and $\Lambda = \mp 6H/(ac)$ the right hand side reads $H^2 + (c/a)^2 - \Lambda c^2/3$, i.e. this equation reduces to the Friedmann equation; H is the Hubble parameter and Λ the cosmological constant. The implications of the Friedmann equation, as concerns in particular the parameter k, are so well known that a detailed discussion of (3,10) is superfluous. We emphasize the crucial role of (3,3) to obtain directly from (3,5) this result, which however compels automatically accepting here $\rho_u/\rho_c > 1$ once having hypothesized since the beginning a closed universe with hyperspherical geometry. If this inequality is such that $\rho_u/\rho_c \gtrsim 1$, then the previous considerations are consistent with an almost Euclidean closed universe, in which case

$$\frac{b}{\dot{a}}\left(2+\frac{b}{\dot{a}}\right) \gtrsim 0. \tag{3,11}$$

This is verified by $0 < b \ll \dot{a}$ and $b/\dot{a} \gtrsim -2$. Now, after having preliminarily verified the hypothesis (3,3) suggested by (3,1), let us check also the self-consistency of the considerations hitherto exposed examining once more $c/\Delta r_u$.

It is reasonable to think Δr_u proportional to the age Δt_u of the universe; so it is possible to write a series expansion defining Δr_u as $\Delta r_u = \sum_{j=1} a_j (cf)^j$, where $f = f(\Delta t)$ is an appropriate function of time to be defined and a_j are constant coefficients of the series. Rewriting more conveniently this series as $\Delta r_u = a_1 c f \varphi$, where $\varphi = 1 + a_2 c f/a_1 + a_3(cf)^2/a_1 + \cdots$, one expects that a_1 of the first order term should be close to the unity for the aforesaid reasons. Implement once again the physical dimensions of G similarly as done before and put in particular $f(\Delta t) \equiv \Delta t_u$; if this position is correct, then $\Delta r_u = a_1 c\varphi\Delta t_u$ with $\varphi \approx 1$ yields $a_1 \approx 2c/\xi_c^3$. On the other hand ξ_c of (2,6) has been calculated in order to fit the numerical value of $G = c^3\Delta t_u/(\xi_c^3 M_u)$ of (2,5), which results also in agreement with that of (3,9); as this equation of G reads $\Delta r_u = (2c/\xi_c^3)\Delta t_u$ with the help of (3,3), one finds at the first order $a_1 \approx 2c/\xi_c^3$ and thus $\Delta r_u \approx (2c/\xi_c^3)\varphi\Delta t_u$. Also this result agrees with the previous estimate of ξ_c defining $\Delta r_u/\Delta t_u$: in effect from (2,4) and (2,6), $\Delta r_u = (c/\xi)\Delta t_u$ compares well with $\Delta r_u = (2c/\xi_c^3)\Delta t_u$ because the values (2,6) verify $\xi^{-1} = 2/\xi_c^{-3}$. This confirms that effectively $\varphi \approx 1$. Hence defining

$$H_0 = \frac{1}{\varphi\Delta t_u}$$

one finds with $a_1 \approx 2c/\xi_c^3$ and once more the given value of ξ_c

$$H_0 = \frac{2c}{\xi_c^3\Delta r_u} = 2.4\times10^{-18}\ \mathrm{s}^{-1}.$$

So at the first order H_0 coincides with Δt_u^{-1}; moreover the second (3,10) yields $H(1+b/\dot{a}) = \xi/(\varphi\Delta t_u)$, i.e. $1+b/\dot{a} \approx \xi H_0/H$ and thus $1 + b/\dot{a} \approx 1$ in agreement with (3,11). The present estimate of H_0 fits well the average value of the Hubble constant, which according to recent measurements falls in the range $(2.2 \div 2.6)\times10^{-18}\ \mathrm{s}^{-1}$.

These results justify the advantage of introducing the present quantum model with the matter era; once having estimated H_0 and inferred the Friedmann equation, it is easy to describe also the radiation controlled era as shown below.

It is worth emphasizing the strategy of the present approach. The standard way to infer cosmological information is to find the solution of the gravity field equations and next to implement the Friedmann solutions: these equations provide information about the open or closed geometry of the universe. Here a different approach has been followed. The quantum equations (2,1) have been implemented since the beginning to introduce the wavelength λ_u and formulate by consequence the concurrent hypothesis (3,3) about a possible geometry of closed universe; thereafter this preliminary idea has been checked to infer (i) the Hawking entropy, (ii) the link between mass density and curvature radius of the universe, (iii) to obtain a Friedmann-like equation and (iv) to estimate the Hubble constant. Moreover, exploiting the same approach outlined in section 2 for the angular momentum, the factors that define the physical dimensions of G allowed to correlate correctly size, age and mass of the universe. The remainders of this paper aim to implement these preliminary ideas to show that further reasonable results are inferred hereafter.

3.1 The matter era

Let us estimate the average mass and energy densities ρ_u and $\eta_{in} = \rho_u c^2$ of the universe, which result to be with the help of (3,2) and (3,5) of the order of

$$\rho_u = \frac{M_u}{V_u} = \frac{3c^2}{2A_u G} = 8.7\times10^{-28}\ \mathrm{kg/m^3}$$
$$\eta_{in} = \frac{M_u c^2}{V_u} = \frac{3c^4}{2A_u G} = 7.8\times10^{-11}\ \mathrm{J/m^3}. \tag{3,12}$$

These values reasonably agree with that calculated in a very different way in [18]; the corresponding "non-visible" energy density is instead of the order of

$$\eta_? = 3m_? c^2/(4\pi\Delta r_u^3) = 7\times10^{-11}\ \mathrm{J/m^3}; \qquad m_? \approx 9m_u.$$

The ordinary visible mass of the universe is about 10% of the total mass only, whereas the remainder mass $m_?$ accounts for the 90% gravitational effect responsible of the black hole-like behavior of the whole universe. The average density ρ_u hides the physical nature of the actual total mass. Moreover, besides m_u of visible stars, M_u consists of a preponderant contribution $m_?$ of different physical nature: for instance all black holes possibly existing in the universe, or interstellar gas and dust, or free elementary particles, and so on including also the so called dark mass. A complex system of particles contributes to M_u, whose actual nature is however not explicitly concerned in neither of (3,12). According to some theories the elements were formed inside the stars by neutron bombardment of light nuclei and subsequent β decay, e.g. [19], other authors believed instead that elements were formed during the early stages after the big bang, e.g. [20]; more recently other authors returned to their formation inside the star by virtue of several nuclear processes [1]. Despite (3,12) waive specific information about the actual composition of M_u, the assumption of large scale homogeneity and isotropy of the universe supports the effective physical meaning of average ρ_u. Moreover the concept of quantum delocalization introduced by (2,1) stimulates itself the idea of average mass spreading uniformly throughout the universe likewise as the energy field of light radiation. This idea is useful to link the matter era to the earlier radiation era. It will be emphasized in the next subsection 3.2 that the radiation field, almost mono-chromatic at the beginning of the radiation era, turned into a more complex spectrum of wavelengths because of the concurrent expansion of the universe; so quantum fluctuations and possible events of constructive interference, statistically allowed to occur anywhere in the radiation field, promoted favorable conditions to form local couples of virtual particles uniformly distributed in the available volume of the early universe. It is known indeed that proton and antiproton virtual couples are formed by vacuum fluctuations and high order two-photon interactions during photon fluctuations able to generate fermion-antifermion pairs [21]. So it seems reasonable to guess that this mechanism triggered the evolution of the early radiation field to couples of virtual particles continuously annihilating and re-materializing up to the later formation of colder real matter. Some considerations on this point will be shortly sketched in the appendix. For the purposes of the present paper, however, it is enough to acknowledge that today's ρ_u corresponds on average to about one half proton mass per cubic meter of universe and that (3,12) hold identically while considering the mass of antiprotons. Despite this idea is mere statistical abstraction, (3,12) are useful for the purposes of the present model; they implement the assumed homogeneity and isotropy of the universe in its strongest form possible. Even with such information only, i.e. whatever the actual abundances of the j-th elements of mass m_j might be today within each unit volume of universe, it is possible to introduce: (i) an elementary volume V_0 physically located anywhere and defined as that containing on average one proton or one antiproton and (ii) a linear combination $m_p = \sum_j a_{ij} m_j$ that accounts via the local coefficients $a_{ij} = a_{ij}(x_i, y_i, z_i, t, m_j)$ for the actual composition of real matter progressively formed everywhere after the radiation era. These coefficients weight the time profile of the local effective abundances, e.g.: they are null if the pertinent coordinates of a_{ij} correspond to an empty volume of universe where $m_j = 0$, moreover all a_{ij} were equal to zero during the early radiation era, and so on. Since the local coordinates are conceptually disregarded by (2,1) and positions (2,2), however, let the indexes i and j number respectively the N_{in} elementary volumes V_{0i} of the universe and the various elements therein formed a time range Δt after its birth. The abundances are subjected to the boundary condition of the first (3,12); for instance, at today's Δt_u this point of view is summarized by the sums

$$\rho_u = \frac{1}{V_u} \sum_{i,j} a_{ij} m_j$$
$$a_{ij} = a_{ij}(V_{0i}, \Delta t, m_j) \qquad (3,13)$$
$$\sum_{i,j} a_{ij} m_j = N_{in} m_p.$$

The first two equations emphasize the local composition of ρ_u, the last one fits in particular the condition of today's average density. In fact (3,13) regard the universe as a lattice, whose elementary cells are the volumes V_{0i} uniformly occupied by one proton or one antiproton of every virtual couple with equal probability. Each cell is therefore a possible allowed state for either of them, i.e. the universe is statistically described by a total number $N_{in} = V_u/V_0 \approx 1.7 \times 10^{80}$ of degenerate states corresponding to η_{in}; also, since by definition each V_0 contains on average one proton mass, $m_p N_{in} = M_u$. So according to (3,12) the energy levels ε_{V_0} of one proton or one antiproton in the respective V_0 states are $m_p c^2/2$ and $m_{\bar{p}} c^2/2$, i.e.

$$\varepsilon_{V_0} = 7.8 \times 10^{-11}\ \text{J}; \qquad V_0 \approx 2\ \text{m}^3 \qquad (3,14)$$

in order that effectively $M_u/V_u = m_p/V_0$, in agreement with (3,12). Of course ε_{V_0} includes also the interaction energy between charges in different cells, e.g. that of couples of all virtual particles possibly generated together with energetic protons and antiprotons; this is possible because $M_u c^2$ involves the visible mass energy $m_u c^2$ plus the contribution of $m_? c^2$. Note eventually that despite $M_u c^2$ results statistically equivalent to the sums

$$\sum_i (\varepsilon_{V_{0\,prot}} + \varepsilon_{V_{0\,antiprot}}) = N_{in} m_p c^2/2 + N_{in} m_{\bar{p}} c^2/2 \quad (3,15)$$

over all the elementary volumes V_0, it will be shown later that an effective entropy driven mechanism in fact marked the transition from the radiation era to the matter era; so the sum of (3,15) reads actually

$$\sum_i (\varepsilon_{V_0\,prot} + \varepsilon_{V_0\,antiprot}) = N_{in} m_p c^2. \qquad (3,16)$$

Before describing this mechanism, the results so far obtained are summarized as follows: (i) each cell is in fact an allowed state for one proton or one antiproton; (ii) (3,14) represents the excitation energy necessary to remove either of them from its own V_0 and leave behind an empty cell; (iii) the latter represents a vacuum state, whereas either particle present in V_0 defines an occupied state.

To highlight the physical meaning of these points, consider an arbitrary mass m at the boundary of the universe. The shell theorem shows that the gravity force acting on m is that due to M_u regarded in the ideal center of a spherical body; so is accordingly calculated for a radius Δr_u its energy $\varepsilon = GM_u m/\Delta r_u$ that, exploiting once again (3,3), reads also $\varepsilon = mc^2/2$. If for instance m represents the mass of one proton or one electron, $m_p = 1.7 \times 10^{-27}$ kg and $m_e = 9.1 \times 10^{-31}$ kg, then one finds

$$\begin{aligned} \varepsilon_p &= G\,\frac{M_u m_p}{\Delta r_u} = \frac{m_p c^2}{2} = 1.0 \times 10^{-10}\ \text{J} \\ \varepsilon_e &= G\,\frac{M_u m_e}{\Delta r_u} = \frac{m_e c^2}{2} = 5.4 \times 10^{-14}\ \text{J}. \end{aligned} \qquad (3,17)$$

The second (3,17) emphasizes that if the volume V_0 would be occupied by one electron with its own energy level $m_e c^2/2$, then V_0 would represent a possible state for this electron. To clarify where anyway does m come from, note that at today's Δt_u the proton energy level ε_{V_0} inside any state V_0 of the bulk universe, (3,14), is equal to the energy ε_p, (3,17), of one proton at the boundary of the universe. So

$$\varepsilon_p = \varepsilon_{V_0}. \qquad (3,18)$$

This equation in fact reads $c^2/2 = M_u G/\Delta r_u$, which is nothing else but (3,3). Thus (3,18) and the first (3,17) do not depend on the proton mass, and hold whatever else m_p might represent. Moreover neither the analytical form of ρ_u nor that of η_{in} introduce explicitly m_p. Rather, the latter introduces the mere Planck force c^4/G acting on the total surface A_u of the universe. There are two reasons why the average values defined by (3,12) and (3,13) have importance for the following discussion: on the one hand, the right side of (3,12) links correctly energy density and pressure; on the other hand, being known that the pressure of a perfect gas is 2/3 of its energy density, the second (3,12) suggests regarding η_{in} in each volume V_0 as due to a proton/antiproton gas occupying uniformly all bulk states of the universe. As this average pressure appears to be a physical property of all elementary volumes V_0, then the internal pressure that characterizes the whole universe results to be, again via (3,3),

$$P_{in} = \frac{2}{3}\,\frac{M_u c^2}{V_u} = \frac{c^4}{A_u G} = 5.6 \times 10^{-11}\ \text{Pa}. \qquad (3,19)$$

The fact that even P_{in} does not depend explicitly on m_p suggests that (3,12) have actual physical meaning. The factor 2/3, numerically irrelevant in the frame of the order of magnitude estimates proposed here, is however conceptually significant to check the physical meaning of (3,12). Taking into account (3,16), (3,19) reads

$$P_{in} V_u = \frac{2}{3}\,E; \qquad M_u c^2 = E = N_{in} m_p c^2.$$

The surprising fact is that the mere definition of energy density, without any additional hypothesis, portrays the whole universe as a container full of quantum or classical gas, whose mass M_u exerts Planck force against its inner boundary; indeed the first equation holds for Boltzmann, Bose and Fermi statistics, which confirms that effectively any kind of quantum or classical particle, thus why not the proton, is compatible with M_u without affecting the validity of (3,19). Furthermore this picture holds at any time, because the surface A_u can be replaced by any A likewise related to the pertinent M/V whatever the numerical value of the ratio might be. Formally this is justified by the second equation, where E resulting from $M_u c^2$ is also associated to a number N_{in} of proton masses fulfilling the global energy conservation. Yet the simple equivalence matter/energy does not seem enough to explain why chunks of matter like asteroids or stars or cosmic powder could mimic the pressure of a proton gas of equivalent total mass filling uniformly the universe. This is however a classical way to think the universe. More stimulating appears in this respect the quantum character of the present model. First of all, the couples proton/antiprotons have been guessed as mere numerical hint due to the average value of the mass resulting in (3,12); but in fact any gas could be consistent with (3,19), which indeed does not make explicit reference to m_p. The chance that any gas mixture could contribute to E is a step towards introducing the actual existence of chemical abundances symbolized by various m_j; the first (3,13) merely means that the degenerate proton or antiproton energy levels $m_p c^2/2$ split into a complex system of non-degenerate energy levels describing the local bound states of cosmic matter. From this point of view, the energy conservation between two different systems of quantum energy levels appears more pertinent: since in principle one level could split into several non-degenerate levels in an infinite number of ways, the energy conservation appears as essential boundary condition to calculate the latter from the former, rather than a mere statistical abstraction. More significant is however the dual wave/corpuscle behaviour of matter. A body of real matter is superposition of waves to form a group in principle spreading from minus infinity to infinity but with a maximum probability of being somewhere: the amplitude of the wave packet rapidly decreases at the edge of a region that determines the most probable position and the finite extent of the body, whose possible motion is nothing else but the group velocity of the wave packet. It is known that the electromagnetic waves exert a pressure, whence the photon gas physics: why not to think the same about delocalized matter waves, according to (2,1)? If so, then the matter era began when matter waves started to appear in the pre-existing field of electromagnetic radiation according to the mechanism [21]. The appendix gives some more hints on this topic.

On the one hand these considerations are interesting because P_{in} controls the expansion of the universe, as it will be shown below; on the other hand the idea of V_0 bulk states allowed to protons and antiprotons, although suggested by the numerical values of (3,12) only, is attracting because it links radiation era and matter era, at the beginning of which couples of matter/antimatter particles were in fact formed. Anyway the significant conclusion is that (3,17) to (3,19) skip m_p and thus can be further implemented in the following regardless of whether the volumes V_0 are really occupied by protons or any other mass.

Exploit (3,19) to infer the average temperature T related to P_{in} in V_0. Here $T \approx E/(N_{in}k_B) = m_pc^2/k_B$ helps to estimate the average temperature in each elementary volume V_0; one finds $T \approx 10^{13}$ K. This estimate fulfills the usual statistical meaning of temperature, as the proton here concerned has a statistical meaning itself. To better assess this result consider the pressure P of an ideal gas of molecular weight M_{mol} and average density ρ in the volume V_0, so that $\rho = PM_{mol}/RT$. Exploiting (3,12) and (3,19) at the time Δt_u to express $\rho \equiv \rho_u$ and $P \equiv P_{in}$, one finds $M_u/V_u = (2M_uc^2/3V_u)M_{mol}/RT$, i.e. $T = 2c^2M_{mol}/3R$. Hence T is explicitly related to the specific M_{mol} only, regardless of the time Δt_u and related universe volume V_u. A uniform distribution of hydrogen in each V_0, i.e. $M_{mol} = 10^{-3}$ kg, estimates again $T \approx 10^{13}$ K, in agreement with that inferred directly from m_pc^2/k_B. Even the formation of hydrogen will be justified in the subsection 3.4 as a consequence of the step from (3,15) to (3,16). This large value is enough for protons to form further couples of virtual photons and fermions/antifer-mions; this supports the idea that effectively the protons early formed trigger the successive energy balance in V_0 qualitatively indicated in (3,13).

The previous ways to estimate T refer to the time where early hadrons began to form everywhere in the radiation field of such universe and indicate a temperature corresponding to a uniform distribution of virtual couples occupying the available states at the end of the radiation era. The same equations could in principle estimate the local T even during the subsequent matter era, when the bombardment with energetic neutrons allowed forming heavy elements; yet the concurrent clustering of matter determined a structure of the universe locally inhomogeneous, so at that later time a unique average T does no longer make sense. Actually both time and volume of the universe determine the value of M_{mol}. In particular, the expansion of the universe is crucial to determine the time profile of T after the radiation era: the hypothesis (3,3) requires $M/\Delta r = const$, which also compels that $M/\Delta r^3$ is a decreasing function of time for increasing Δr. So an increasing fraction of empty zones of the universe corresponds in principle to a global decreasing value of T; the calculation of the respective temperatures is not as immediate and straightforward as in the previous case, characterized by a uniform distribution of a unique kind of early particles. In this case both local coefficients a_{ij} and atomic weights of the elements m_j must be known: the sums of (3,13) are related to the abundances within the various volumes V_{0i} of cosmic objects, characterized by the different kinds of elements and local coefficients a_{ij}, and to empty parts of the universe.

A question arises now: did (3,3) and (3,18) hold even in the past? In fact there is no reason to suspect that this condition is an exclusive feature of the today space-time coordinates Δr_u and Δt_u, which indeed have nothing special with respect to any past or future Δr and Δt. The only necessary hypothesis to answer affirmatively is that the current V_0 grows together with the size of the universe, which is possible if its sizes are comoving distances. Otherwise stated, let V_0' be the past value of V_0 at any $\Delta r < \Delta r_u$ and $\Delta t < \Delta t_u$; we require $m_pc^2/2 = Mm_pG/\Delta r$, being M the past total mass. This requirement emphasizes the previous remarks: the actual nature of proton mass m_p is irrelevant as concerns (3,18), which holds thus whatever m_p stands for, i.e. whatever the relative element abundance of (3,11) in V_0 might have been at Δt. On the one hand $c^2/2G = M_u/\Delta r_u$ requires $M_u/\Delta r_u = M/\Delta r$ and thus $M = c^2\Delta r/2G$, i.e. the black hole condition held also in the past. On the other hand one expects that V_0' scales with $\propto \Delta r^3$, in order that it be definable even for the smaller universe sizes of the early matter era; so $V_0' = (\Delta r/\Delta r_u)^3 V_0$, i.e. V_0' was reasonably much smaller than today's V_0. In this way multiplying both sides by N_{in} one finds $N_{in}V_0' = (\Delta r/\Delta r_u)^3 N_{in}V_0$; since by definition $N_{in}V_0 = V_u$, (3,2) yield $N_{in}V_0' = (4\pi/3)\Delta r^3$, i.e. in the early hypersphere volume defined by Δr the number of elementary volumes and thus of states allowed to the new born matter was the same as today's N_{in}. In summary

$$M = \frac{\Delta r}{\Delta r_u} M_u; \quad V_0' = \left(\frac{\Delta r}{\Delta r_u}\right)^3 V_0; \quad N_{in} = const. \qquad (3,20)$$

What is important for the following discussion is that under reasonable assumptions the condition (3,18) could hold also in the past and that N_{in} was since the beginning fingerprint of our universe. (3,20) help to guess the size of the universe at the beginning of the matter era. It is instructive to proceed stepwise calculating Δr and V_0' by trial and error, i.e. assessing these quantities as a function of sensible values of M. If M would be the mass of one couple proton/antiproton only, then $\Delta r \approx 4.9 \times 10^{-54}$ m, which would mean a volume $V_0' \approx 2.9 \times 10^{-240}$ m^3, unrealistically smaller than the expected order of magnitude of Planck volume. This value of V_0' suggests an early number of virtual couples much higher than this. More reasonable results are obtained putting $V_0' \approx 4.2 \times 10^{-105}$ m^3 to estimate via the second equation the order of magnitude of Δr, which results $\Delta r = 5.5 \times 10^{-9}$ m; with this range the first equation yields $M = 3.8 \times 10^{18}$ kg corresponding to about 2.2×10^{45} protons, i.e. about 10^{45} virtual couples proton/antiproton at the beginning of the matter era. Note that $Mc^2 = 3.4 \times 10^{35}$ J corresponds to an average fluctuation energy $\varepsilon_{fl} = 3.4 \times 10^{-10}$ J, i.e. 2.1 GeV, per virtual couple of matter particles newly created: this is the fluctuation energy of the radiation field able to create matter. It is interesting the fact that with the given choice of V'_0 this result fits well the energy of a couple of protons, despite it has been calculated implementing M_u and Δr_u via (3,20) only; this supports the interpretation of (3,12). Supposing that on average each couple of photons generates one virtual couple of matter/antimatter, the fluctuation extra energy of radiation field increases the early Planck frequency of each couple of photons by about $\delta\omega = 3.4 \times 10^{-10}/\hbar = 3 \times 10^{24}$ s^{-1} to produce matter. The obvious conclusion of this section is to admit that before the time of mass production there was an earlier massless era, i.e. the radiation era.

3.2 The radiation era

Consider the density ρ corresponding to M and Δr of (3,20) by replacing M with $h/(\lambda c)$; in this way the total mass of the universe is expressed via the momentum h/λ of an electromagnetic wave propagating with velocity c. For simplicity we have assumed that the refractive index of the medium where the wave propagates is 1, although in principle this is an approximation only; the aforesaid gamma-gamma physics [21] predicts photon fluctuations resulting in charged fermion -antifermion pairs, leptons or quarks, which couple with the photons themselves. In the presence of electron-positron and proton-antiproton couples of particles that typically also form as a consequence of this kind of interaction, a refraction index equal to 1 is certainly an approximation; yet this is acceptable for the following reasoning and order of magnitude estimates. So the late $\rho_u = 3M_u/(4\pi\Delta r_u^3)$ of matter era reads $\rho^r = 3h/(4\pi\lambda c\Delta r^3)$ at the time Δt. A boundary condition for λ comes from the fact that the early electromagnetic radiation waves bounced between diametric distances $2\Delta r$ inside a sphere, i.e. still $\lambda = 2\Delta r/n$ with n integer according to eq (3,1); in this way steady waves were allowed to fill the universe at any time Δt. The internal bouncing of radiation is justified even admitting that the early stages of growth were allowed in non-equilibrium condition, owing to the rapid growth of the universe size, and without radiation energy loss unfavorable for the subsequent growth and evolution of the new-born universe. So λ was a function of time like Δr, i.e. the number n of allowed frequencies increased along with Δr; it seems reasonable to guess that an initial field almost monochromatic evolved towards a complex spectrum of steady wavelengths. Anyway the density of the universe in the radiation era reads

$$\rho^r = \frac{3nh}{8\pi c\Delta r^4} = \frac{3n\hbar}{4c\Delta r^4}.$$

while (3,3) reads $\Delta r = 2hG/(\lambda c^3)$; so the condition $\lambda = 2\Delta r/n$ yields $\Delta r = \sqrt{nhG/c^3}$. Hence increasing n means increasing Δr and the number of states allowed for the radiation field. So radiation density, radiation energy density and pressure during the radiation era read

$$\rho^r = \frac{3c^5}{4n\hbar G^2}; \qquad \eta^r_{in} = \frac{3c^7}{4n\hbar G^2}; \qquad P^r_{in} = \frac{c^7}{4n\hbar G^2}.$$

At the beginning of the radiation era, therefore, $\Delta r = \sqrt{\hbar G/c^3}$ with $\lambda = \Delta r$ and $n = 1$ has the expected order of the Planck length with which in effect has been calculated the Planck volume V'_0. Moreover estimating hc/λ with λ of the order of the Planck length, $\approx 10^{-35}$ m, yields a temperature $T \approx hc/k_B\lambda$ of the order of 10^{33} K. The fact that this characteristic temperature is much higher than that estimated for the proton in today's V_0, confirms that actually the radiation era precedes the matter era. Putting Δr of the order of the Planck length, with $n = 1$ one finds $\rho^r \approx 4 \times 10^{96}$ kg/m^3 and $P^r_{in} \approx 10^{113}$ Pa and $\eta^r_{in} = 3.5 \times 10^{113}$ J/m^3; at this stage of evolution of the universe the energy $\varepsilon^r_{in} = (4\pi/3)\Delta r^3 P^r_{in}$ results about $\varepsilon^r_{in} \approx 1.7 \times 10^9$ J, to which corresponds a temperature of the order of $\varepsilon_{in}/k_B \approx 10^{32}$ K in agreement with that already estimated. Estimating an energy $k_B T \approx 1.3 \times 10^9$ J of the radiation field corresponding to this temperature, one finds $\omega^r = 1.6 \times 10^9/\hbar \approx 1.6 \times 10^{43}$ s^{-1} i.e. a radiation field with Planck frequency. These values correspond well therefore to the Planck pressure, energy, frequency and temperature.

So, trying to understand the physical meaning of these results beyond the numerical estimates, the radiation era was just after the very early time step of the creation of radiation just concerned; this initial step can be therefore nothing else but the Planck era. The huge internal pressure accounts for the rapid volume of the universe. Note that the value of ε^r_{in} is large, but not spectacularly high like P^r_{in} and η^r_{in}; these latter are due to the extremely small values of Planck volume. These ideas explain thus the subsequent beginning of the matter era, during which however the expansion mechanism of the universe was somehow different.

3.3 The universe expansion in the matter era

Comparing (3,17) and (3,14), it has been already noted the similarity between the gravitational energy ε_p of one proton at the boundary distance Δr_u and the energy ε_{V_0} existing within each V_0 just because of the presence of the proton itself. (3,20) have been accordingly inferred. If the proton, or whatever else its mass might actually represent, would be ideally removed from any volume V_0 internal to the universe and displaced to the boundary of the universe, the energy lost by V_0 is balanced by that transferred to the boundary; within the limits of the present order of magnitude estimates, there is no net gain or loss of energy in this ideal process. This suggests that creating a vacancy in the universe after ideally moving its average amount of matter per unit cell just to the external boundary of the universe occurs at zero energy cost. Strictly speaking ε_p should have been calculated in principle writing $M_u - m_p$, the numerical difference being however completely irrelevant for one proton only. Actually this reasoning is extensible to describe a relevant number of protons regarded at the boundary; as $M_u/m_p \approx 10^{80}$, for a large number n_p of protons such that $1 \ll n_p \ll M_u/m_p$ still holds (3,18) because $M_u \approx M_u - n_p m_p$. This means that large numbers of protons are expected to contribute to this ideal transfer process, i.e. large numbers of empty cells are to be expected in the universe. Of course the comparison between ε_{V_0} and ε_p has statistical meaning only, despite the actual structure of the visible mass in the universe and even regardless of the local element abundances in the universe, hidden within the global value of M_u and still undisclosed when reasoning about the mere average distribution of M_u. The following remarks are useful at this point.

- There is no actual flow of protons moving inwards or outwards throughout the universe; the uncertainty in the most agnostic form of (2,1) requires any quantum particle completely delocalized everywhere in the whole universe. The diameter $2\Delta r_u$ is a quantum delocalization range inside which no information is conceptually allowed about the local position and dynamical variables of any kind of particle, proton or else. So any particle could be in V_0 or at the boundary simply provided that there are available allowed states; (3,18) merely compares the energies of protons in two different places where they could in fact be, i.e. everywhere because V_0 could be itself everywhere in the universe.
- Two states of equal energy are allowed to the proton: the bulk state in V_0 and the boundary state at the rim of the universe. A proton at the boundary state leaves behind an empty cell V_0, i.e. a hole in one of the bulk allowed states. In general occupied and empty states are possible in the bulk and at the boundary of the universe. The global electroneutrality is ensured by the identical chance statistically allowed to antiprotons too.
- Both ideal chances are possible in principle despite the black hole character of the universe: the protons do not escape far from the boundary, they remain "glued" on the boundary like any electromagnetic radiation possibly arriving up there from the bulk of the universe. The Hawking entropy supports this idea.
- The chance of either alternative is consequence of the second law of thermodynamics; these bulk and boundary chances concurrently possible for the protons increase their number of allowed states and thus their configuration entropy. This crucial point, which will be further concerned later, agrees with the fact that (3,17) describes identically the total mass M_u at the ideal center of the universe and the mass m_p at the boundary Δr_u apart or, vice versa, the mass m_p at the ideal center of the universe and the total mass M_u concentrated on a point at the boundary Δr_u apart; indeed, according to the considerations of section 2, the local position of any particle is physically meaningless because of the quantum delocalization within an uncertainty range. Either extremal configuration, in principle possible for the universe, is however unlikely by entropy considerations.
- If V_0 scales as described by (3,20), which is admissible as no restraining hypothesis has been made on it, then (3,18) previously introduced for the proton at the time Δt_u is unchanged at any $\Delta t < \Delta t_u$; moreover the number of states N_{in} is expected constant, as in effect it has been found.

These ideas encourage regarding the proton in V_0 as a sort of template that symbolizes the average behavior of real matter in any bulk state and at the boundary state; as previously remarked, this is certainly the strongest form to affirm the large scale isotropy and homogeneity of the universe. Actually particles and antiparticles with the same m_p concurrently formed after the radiation era have statistically the same probability of being found in the boundary state; if so, the initial configuration of coexisting protons and antiprotons uniformly occupying all available bulk states generates subsequently a boundary halo of virtual couples plus possible annihilation photons along with corresponding vacuum states and matter states in the bulk universe. This configuration change increases the total entropy of the universe. In particular, the surface entropy at the boundary of the universe consists of the Hawking term σ_{H} plus a contribution related to the configuration of boundary states shared with that of the bulk universe. The entropy will be considered in some more detail in the next section. It will be shown that the way of thinking based on the degenerate quantum states of the universe rather than on the multiplicity of states describing its actual structure of matter, helps formulating a possible growth mechanism of the universe. Usually growth and expansion are synonyms; the next section emphasizes why actually it is not so in the present model, where growth does not merely mean swelling.

3.4 The universe growth in the matter era

Let the bulk universe at an arbitrary time after the big bang consist of a number N_{out} of V_0 empty cells and a corresponding number $N_{in} - N_{out}$ of filled V_0 cells; the external boundary is thus a layer formed by N_{out} glued protons and antiprotons missing in the bulk. So even this statistical picture of universe is consistent with the existence of an empty part of the real universe and its real matter structure: correspondingly to the further redistribution of $N_{out}V_0$ and $(N_{in} - N_{out})V_0$ volumes, in principle located randomly in the total volume $N_{in}V_0$ available, clusters of matter tend to coalesce together by gravitational interaction: the vacuum corresponds indeed to the N_{out} residual holes left in between. Anyway, if clusters of empty cells and clusters of occupied cells are numerous enough, then their random distribution within V_u is still consistent with the assumption of overall statistical homogeneity and isotropy. This seems indeed the case, as the number N_{in} of V_0 volumes has been estimated of the order of the Eddington number 10^{80}. The chance of introducing arbitrary numbers N_{out} and $N_{in} - N_{out}$ of cells brings the universe towards a situation of dynamical equilibrium between the former and the latter; yet this final configuration, somehow attained, could be imagined as the conclusion of a gradual process consisting of a first redistribution step $N_{in} - N'_{out}$ and N'_{out} of filled and empty cells, which in turn generates progressively a subsequent redistribution $N_{in} - N'_{out} - N''_{out}$ and $N'_{out} + N''_{out}$ of new filled and empty cells along with possible coalescence of cells still filled, and so on. This idea stimulates considering the dimensionless entropy of a current configuration, $\sigma_b = N_{in}!/(N_{out}!(N_{in} - N_{out})!)$, due to the fact that all transient configurations compatible with zero energy balance are equiprobable; the subscript b stands for "bulk". As σ_b has a maximum as function of N_{out}, the formation of bulk holes fulfills the second law until this maximum is reached. Let σ_b describe a transient configuration at a given time and $\sigma'_b = N_{in}!/(N'_{out}!(N_{in} - N'_{out})!)$ that at a later time; the latter is allowed if N_{out} and the subsequent N'_{out} fulfill $\sigma'_b > \sigma_b$. Hence, after an arbitrary numbers of steps, are formed as a function of time multiple clusters of matter aggregates subsequently attained and thus differently configured, together with a progressive modification of the empty space between them. At the dynamical equilibrium no net state exchange occurs. Of course σ_b and σ'_b neglect, for simplicity and brevity, the further contributions σ_{arr} and σ'_{arr} due to the ways to arrange the respective clusters of matter into actual universe structures; yet σ_b and σ'_b symbolize qualitatively the first conceptual step to understand the actual configuration of the universe. Clearly, by virtue of (3,13), the σ_{arr} driven final arrangements of filled cells are nothing else but stars or galaxies or flows of elementary particles or any other observable object. The existence of P_{in} related to the matter energy density agrees with and justifies the universe expansion, which however at this point still seems like a mere bubble blowing up by internal pressure effect. But just this point poses a further question: does the universe in the matter era expand freely or is it constrained by an external pressure P_{out} opposing to its expansion? In principle the expansion requires $P_{in} > P_{out}$, not necessarily $P_{out} = 0$: the force that pushes forwards the unit surface of universe boundary must simply overcome that possibly tending to pull it backwards, i.e. to squeeze the universe size towards a big crunch. If the former position is correct, then P_{out} tends to decrease the acceleration with which the universe expands. Yet, what does originate P_{out}? A possible answer relies just on the presence of protons and antiprotons at the boundary states of the universe previously introduced. The boundary here introduced is not mere spherical rim; in effect the plain idea of geometrical margin would be unphysical itself. More sensibly, the mobile contour of the universe is defined by a crowd of N_{out} virtual protons and antiprotons along with electromagnetic radiation trapped on a fading shell, recall the Hawking entropy. In fact the previous considerations propose in a natural way that the boundary should be a physical layer of finite volume and finite thickness; so the chance of defining an energy density η_{out} due to these particles seems the most straightforward way to define P_{out}. In this respect, the further chance of demonstrating that $P_{out} \neq P_{in}$ is important not only to infer information about the acceleration of the boundary of the universe, controlled by the net force $P_{in} - P_{out}$ per unit surface of boundary, but also to infer that the physical nature of the outer layer must be different from that inside the universe. Before assessing the importance of this conclusion as concerns the matter/antimatter ratio, let us examine two points: the expansion equation and the physical meaning of η_{out}, to which is related the pressure P_{out} equivalently as in (3,19). This external pressure could be likewise regarded as external force acting towards the center of the universe or resistance of the universe to increase the total surface of its boundary. The latter idea is more easily viable to introduce the existence of a boundary layer, whose thickness surrounds the universe and characterizes $\eta_{in} \neq \eta_{out}$; if the layer would have the same physical nature of the bulk vacuum, then the boundary should be at rest or steadily moving rather than accelerating. Let $\rho_u V_u c^2$ be the energy stored inside the universe; since today's universe expands, according to the first law its total energy E must also include a PV_u-like term. Let $\delta E = c^2\delta(\rho_u V_u) + P_{net}\delta V_u$ be the change δE of total energy during the time interval δt, where $P_{net} = P_{in} - P_{out}$ describes the net force pushing forwards the boundary. As no energy escapes outside of a black hole universe $\dot{E} = \dot{\rho} V_u c^2 + \rho \dot{V}_u c^2 + (P_{in} - P_{out})\dot{V}_u = 0$; so $\dot{\rho} + \rho\dot{V}_u/V_u + (P_{in} - P_{out})\dot{V}_u/(V_u c^2) = 0$. According to (3,20), the size of the elementary volume V_0 scales as Δr^3, i.e. like $V_u = N_{in}V_0$; then $\dot{V}_u/V_u = 3\dot{a}/a$, whence the well known result

$$\dot{\rho} + 3\,\frac{\dot{a}}{a}\left(\rho + \frac{P_{net}}{c^2}\right) = 0; \qquad P_{net} = P_{in} - P_{out} \tag{3,21}$$

The notation emphasizes that the time derivative of the radius defines the change rate of a co-moving length. The excess of internal pressure means that the layer outside the boundary is slightly different from the bulk. Note that also a negative pressure P_{out} counteracting P_{in} has been introduced in this reasoning.

Regard the boundary as if it would be a material layer characterized by a contractive energy per unit surface $\gamma = \varepsilon_\gamma/l^2$ that opposes to its stretching during the expansion; for instance, this effect can be guessed thinking to the opposite charges of the particles/antiparticles that crowd the boundary surface. Anyway the total contractive energy of a spherical bubble having internal radius Δr_u and volume V_u is $\varepsilon_\gamma = 4\pi\Delta r_u^2\gamma$. Moreover the Young-Laplace equation of such surface tension-like model of boundary reads $P_{in}-P_{out} = 2\gamma/\Delta r_u$ Suppose that $P_{out} = P_{in}/2$; then $P_{out} = \eta_{in}/3$, like that inside a universe with radiation only. This is equivalent to say that P_{in} is due to two contributions: one coming from its radiation density content and one due to the ability of the radiation to generate matter via quantum fluctuations. The former is counterbalanced by P_{out}, the latter is the active energy excess pushing outwards the boundary. Hence the expansion of the universe is controlled by the quantum contribution of radiation fluctuation extra energy that generates matter, without which the universe would still be a radiation volume. To check this idea note that (3,3) yields $M_u^2G/\Delta r_u = M_uc^2/2$, i.e. one half of the universe energy is equal to the first (3,17) with the proton mass replaced by that of the whole universe. The same holds for the energy density, obtained dividing both sides by V_u. So if $P_{in}/2 = \eta_{in}/3$, then $P_{out} = \eta_{in}/3$ requires $\eta_{out} = \eta_{in}/2$. Hence the right hand side yields

$$\eta_{out} = \frac{M_u^2G}{V_u\Delta r_u} \approx 4.2\times10^{-11}\mathrm{J/m^3} \qquad P_{out} = 2.8\times10^{-11}\mathrm{Pa} \tag{3,22}$$

as it reasonably appears comparing with η_{in} of (3,12). This result implies interesting consequences. The total contractive energy of a spherical bubble of radius Δr_u and volume V_u is $\varepsilon_\gamma = 4\pi\Delta r_u^2\gamma$. Moreover the Young-Laplace equation reads $P_{in} - P_{out} = P_{in}/2 = 2\gamma/\Delta r_u$, so that $\gamma = P_{in}\Delta r_u/4$ yields $\varepsilon_\gamma = \pi\Delta r_u^3P_{in} = M_uc^2/2$ thanks to (3,19). Hence the whole energy of the boundary layer generating its contractive surface tension is one half of the total bulk energy of the universe, i.e. that corresponding to the net pushing effect of the big-bang quantum fluctuation only. Also, this confirms that $P_{out} = P_{in}/2$ is an external pressure opposite to P_{in} and directed towards the universe center consistently with the curvature radius Δr_u. The numerical value of γ is $\approx 6\times10^{15}$ J/m^2, corresponding to $M_uc^2/2A_u$. It is interesting the fact that the boundary layer can be regarded as a real matter sheet curved by the pressure difference according to the Laplace equation. The initial black hole condition (3,3) is essential for this result. Note that it is possible to write

$$\eta_{out} = \frac{3}{16\pi\Delta r_u^2}\frac{c^4}{G} = \frac{3}{4}\frac{c^4}{A_uG}; \qquad \gamma = \frac{c^4\Delta r_u}{4A_uG} \tag{3,23}$$

i.e. the compression force at the boundary of the universe is of the order of the Planck force acting on its total surface. It is interesting to note that replacing $A_u = 4l_P^2\sigma_\mathrm{H}$, it is possible to express γ as a function of the Hawking entropy. Moreover, once knowing η_{out} it is easy to find the thickness of the boundary layer. This energy density is that stored in a layer surrounding the universe δr_u thick. i.e. the boundary protons and antiprotons are actually contained in a shell of volume $(4\pi/3)[(\Delta r_u + \delta r_u)^3 - \Delta r_u^3]$; so

$$V_{out} = \zeta V_u; \qquad \zeta = (1 + \delta r_u/\Delta r_u)^3 - 1 \tag{3,24}$$

which means that in fact the size of the universe is still described just by its radius Δr_u via a correction factor ζ. Having defined η_{out} at Δr_u, it is immediate to estimate also energy, mass and number of protons/antiprotons of the boundary layer through the following equations

$$\varepsilon_{out} = V_{out}\eta_{out} = \zeta M_u^2G/\Delta r_u = \zeta\,\frac{c^4}{4G}\,\Delta r_u$$

$$m_{out} \approx \varepsilon_{out}/c^2 = \zeta\,\frac{c^2}{4G}\,\Delta r_u$$

$$n_{out} \approx m_{out}/m_p = \zeta\,\frac{c^2}{4Gm_p}\,\Delta r_u.$$

If $\delta r_u \gg \Delta r_u$, then $\zeta \approx (\delta r_u/\Delta r_u)^3$; if instead $\delta r_u \approx \Delta r_u$, then $\zeta \approx 7$. Moreover, trusting to the idea that $\delta r_u \ll \Delta r_u$ at the today time Δt_u, one finds $\zeta \approx 3\delta r_u/\Delta r_u$ and then $V_{out} \approx 4\pi\Delta r_u^2\delta r_u$. Suppose that $\delta r_u \approx 10^{-15}$ m, which corresponds to the size of the proton; then $V_{out} \approx 2\times10^{39}$ m^3 yields $\varepsilon_{out} \approx 10^{29}$ J; i.e. the boundary layer consists of a total mass $m_{out} \approx 10^{12}$ kg, to which correspond about $n_{out} \approx 6\times10^{38}$ protons and antiprotons. It would be also easy with the help of (3,20) to repeat the estimates also a different past times. Going beyond the raw numerical estimates, one concludes: (i) the number density n_{out}/V_{out} is of the order of 1/3 proton per cubic meter, a figure similar to that found in V_0 of the bulk universe; (ii) the number of boundary protons results $\ll N_{in}$, as it must be according to the previous considerations; (iii) the fact that the size of the proton is of the order of one fm means that the boundary layer is actually formed by a monolayer of protons and antiprotons; also this result seems in effect quite reasonable. The connection of these conclusions with the previous (3,1), (3,6) and (3,8) will appear shortly.

Now let us explain why the presence of the proton/antiproton couples at the boundary is important for the growth of the bulk universe. Assume that the empty V_0 cells of the universe, i.e. our core vacuum, actually includes couples of virtual particles and antiparticles that annihilate and then rematerialize: whatever their specific nature might be, a simple reasoning shows that the main effect of sharing these virtual couples between bulk states and boundary states is that of transferring to the aforesaid boundary layer the properties of the bulk universe. It is essential that both virtual particles and antiparticles have equal probability of being in either state, see the next section for more details; in this sense it is possible to regard them as a couple. These forerunner quantum couples are the precursors that generate a new boundary of the universe and activate its expansion. Indeed transferring the energy early contained in any V_0 towards the boundary means reproducing at the boundary the quantum states characterizing the bulk universe, i.e. not only that of protons and antiprotons but also the vacuum energy fluctuation generating them. This also means that the universe grows by replicating part of itself outside itself; the duplication concerns of course also the virtual couples of particles and antiparticles characterizing the core vacuum, which once more confirms why (3,12) and (3,14) have been calculated with M_u and not with m_u. So in the present model expansion does not mean merely swelling: the chance that these couples annihilate and rematerialize at the external boundary of the universe likewise as they did inside the universe, means that even the external boundary assumes the feature of the core cells V_0. In effect the previous figures recalculated with a value of δr_u slightly larger than one proton monolayer yield a proton/antiproton density comparable to that within V_0 of the bulk universe; this clearly indicates that increasing δr_u means increasing the number of boundary states allowed to protons and antiprotons. Yet proton and antiproton density in the boundary layer equal to that existing in V_0 means that the bulk of the universe has been in fact expanded by a supplementary layer $\gtrsim \delta r_u$, i.e. the actual boundary is located a step δr_u beyond the previous one, and so on by successive steps consistent with a growth rate presently given by v_u of (2,6). The driving force of this "onion growth" process is the entropy increase required by the second law: all protons and antiprotons filling the bulk universe only, anyhow distributed and arranged, would define a degree of order greater than that where some of them have the additional chance of being further arranged in the only region furthermore conceivable, i.e. that glued to the external boundary of the black hole universe. Yet the key concept is clearly the quantum uncertainty, in its most agnostic form of (2,1): being completely delocalized everywhere in the universe, the particles can preferentially be in fact wherever they ensure the most advantageous entropy and growth conditions.

However, the question that then arises immediately is: does this chance expel to the boundary exactly equal amounts of particles and antiparticles or is there preferential transfer of either kind of them? From a statistical standpoint the answer is indeed that reasonably couples of virtual particles only should share this growth mechanism: drawing randomly from a multitude of particles and antiparticles, the realistic chance is that equal numbers of either kind are involved in the quantum state change. Despite this statistical equivalence, however, the next section will emphasize why the overall effect of the entropy increment is that of increasing the matter/antimatter ratio in the bulk universe.

3.5 The problem of matter and antimatter

This section describes a mechanism really possible soon after the end of the radiation era; the couples proton/antiproton just formed from the very hot radiation field have actual physical meaning, instead of being mere statistical entities suggested by (3,12). Is useful here a reasoning similar to that of the Dirac sea, which in the present context seems physically even more appropriate than the original one: are inherent here neither infinite states occupied by electrons with negative energy nor the doubtful concept of "neutrality" conventionally defined by the presence of infinite electrons in negative energy occupied states; the Pauli principle is no longer necessary to avert a weird radiation of negative energy.

In the original Dirac idea, a photon of energy $\geq 2m_ec^2$ excites an electron in the negative state above the forbidden gap; as a result, the electron just removed appears as a standard electron that leaves behind a related positive hole, the positron. Today we know that in fact two photons of sufficient energy are able to create a couple particle/antiparticle while fulfilling the conservation laws. Let us implement here this standpoint, emphasizing however that the driving energy has now entropic character: the energetic photons necessary to modify the Dirac sea of negative energy electron states is here replaced by the entropy increase $T\delta S$ that results from the combined configuration option, bulk state and boundary state, allowed for each proton and each antiproton. The number of proton and antiproton quantum states is the large but finite N_{in}. It has been already estimated that just after the radiation era T was of the order of $10^{32} \div 10^{33}$ K; this range of values seems high enough to account for a Dirac-like process. Discuss separately what happens when one proton and one antiproton pass from their own bulk states in V_0 to their respective boundary states; two V_0 states are involved in the process, the probability that this happens is equal for both.

One proton in the first V_0 has the same energy as in the boundary state; with the proton in this latter state a hole is left behind in this V_0, i.e. a neutral vacuum state forms in the bulk universe. No constrain is necessary about the energy $T\delta S$ to allow the change from bulk to boundary state, either configuration is allowed at zero energy cost; now one V_0 state is chargeless, whereas one boundary state is positively charged.

The Dirac reasoning for an antiproton in the second V_0 sounds as follows. A proton in the negative energy state in this V_0 is excited concurrently and with the same statistical probability of the previous process; now a constrain about the excitation energy is required and reads $T\delta S \geq 2m_pc^2 + m_ec^2$. This proton is thus excited, leaves unoccupied its initial state, overcomes the forbidden gap at the right hand side and appears as an ordinary proton; a negative hole, i.e. one antiproton, results by consequence. This hole is to be regarded in the boundary state, previously raised to a positive charge state by the first proton, to ensure the local electric neutrality; the ordinary proton co-generated in the second bulk state V_0 remains inside the bulk universe together with the negative charge of one electron; this latter, necessary for the total spin conservation and for the overall bulk neutrality at the minimum energy cost, occupies the former empty vacuum state V_0 left behind from the first proton.

Clearly this mechanism requires that both a proton and an antiproton change contextually and with the same probability their bulk states, in which case we have: (i) two boundary states altogether neutral occupied by one proton and one negative antiproton, which can yield by annihilation the electromagnetic radiation trapped at the boundary of the universe and concerned since the beginning by (3,1), as confirmed by

(3,6) and (3,8); (ii) a neutral bulk state formed by one proton and one electron occupying the two volumes V_0 left vacant. Also the electroneutrality in the bulk universe is thus fulfilled thanks to the electron energy included in the energy balance of $T\delta S$. On the one hand, therefore, the equal probability of exciting statistically one proton or one antiproton is essential to ensure the neutrality of both bulk and boundary states; on the other hand, by consequence of this mechanism a couple proton/antiproton is formed in the boundary state, whereas in the bulk one proton has replaced the antiproton with the help of one electron. In fact this process removes antimatter from the bulk universe, which appears as electromagnetic radiation surrounding the universe via entropy driven process; the holes of negative energy states, i.e. protons, concurrently generated along with electrons appear as bulk matter. Moreover just the annihilation electromagnetic halo ensures the growth of the universe, which therefore does not simply swell but replicates itself far at the boundary via annihilation energy. The separation boundary-antimatter from bulk-matter was likely allowed to occur just at the beginning of the matter era, when the matter started being generated from the extremely hot radiation field consistent with its $T\delta S$. It is reasonable to think that without this separation the bulk universe would have remained in the radiation era, because the two photon mechanism previously hypothesized would have continued to produce virtual matter that however endlessly annihilated with the virtual antimatter contextually generated. Since no energy escapes from the black hole universe, $T\delta S = \delta(TS) - S\delta T$ caused decrease of internal energy and cooling of the universe, until when the temperature decrease made impossible the radiation driven formation of virtual proton/antiproton couples and the consequent antimatter expulsion to the boundary along with the concurrent formation of low T matter. Begins just now the matter era. Of course all this is possible because of the total uncertainty of the quantum particles introduced in its most agnostic form of (2,1): these particles do not need any actual travel to go from bulk to boundary of the universe, being instead totally delocalized; they are simultaneously everywhere without any chance of specify their actual location. These ideas have been exploited to discuss the EPR paradox in the frame of a relativity model entirely based on the space-time uncertainty [12].

As concerns the point (i) above, (3,6) to (3,9) and related considerations about $\hbar\omega_{n=2}$ agree with the idea that both protons and antiprotons existing at the boundary of the universe contribute with their annihilation to form the halo of electromagnetic radiation surrounding the universe.

As concerns the point (ii), the presence of the electron is evidenced simply implementing the second (3,17): the electron energy ε_e early contributed by $T\delta S$ replaces ε_p in the empty V_0 left behind by the previous proton now occupying the boundary state, so the energy density in the bulk volume V_0 becomes ε_e/V_0. To confirm this mechanism, it is enough to estimate $T = (\varepsilon_e/V_0\mathrm{a})^{1/4}$ via the black body constant a $= 5.67 \times 10^{-16}$ J/m^{3}K^4; today's $V_0 \approx 2$m^3 yields $T \approx 2.63$ K. Of course in the past, when $V_0' \ll V_0$ according to (3,20), the energy density was higher and thus the background cosmic temperature accordingly higher; the low energy of the present cosmic radiation is due to the swelling of the early V_0', formerly of the order of the Planck volume, to the size of today's V_0 that decreases the electron energy density. This conclusion agrees with the condition $n\lambda = 2\Delta r$ previously introduced to describe the evolution of the radiation field as a function of the growing universe size during the radiation era. The mechanism that originates the CBMR dates back to the early beginning of the matter era when this mechanism took place, but is operating even presently: the today wavelength, due to the swelling of the early V_0' to the current V_0, is related to the virtual couples of particles/antiparticles that feed the growth of the universe keeping constant its black hole ratio $M_u/\Delta r_u$ according to (3,20) and the concept of vacuum. The small % discrepancy from the experimental value 2.72 K of today background cosmic radiation is due to having implemented the mere rest mass of the electron, whose kinetic energy instead is presumably not exactly zero; being the electron much lighter than the proton, a relativistic correction factor in the energy balance of $T\delta S$, corresponding to $\mathrm{v}_e \approx 0.5c$ and reasonably expected, increases slightly the energy density in V_0 and allows to fit exactly the experimental value. Yet this is not the main point: the most important aim of the model is to verify the sensibleness of estimated values with respect to the available experimental data and assess the conceptual consistency of the theoretical model with the current knowledge of the universe.

4 The dark mass

A crucial point that deserves a rational explanation, hitherto not yet concerned, regards the mass $m_?$. Some comments on this mass are here reported starting from (3,9) and (3,17) and comparing the energy $\hbar\omega_n$ with $\Delta\varepsilon_u = \hbar/\Delta t_u$. One finds

$$\frac{nc\Delta t_u}{2\pi\Delta r_u} \approx \frac{n}{20}. \tag{4,1}$$

In effect, with the help of (3,1) and (3,3) the ratio at left hand side is equal to about $n\xi/2\pi$ with $\xi = 0.3$ according to (2,5) and (2,6). In section 3.1 it has been highlighted that $n = 2$ means considering electromagnetic waves surrounding the universe whose energy corresponds to the annihilation of several protons with antiprotons; also, in agreement with (3,8), for $n = 2$ the right hand side of (4,1) becomes 10^{-1}. Recall now that just a factor ten has been already found in (3,4), when describing the ratio M_u/m_u. So it seems natural to introduce this ratio into (4,1) that becomes therefore

$$\frac{M_u}{m_u}\frac{c\Delta t_u}{\pi\Delta r_u} \approx 1. \tag{4,2}$$

Very large numbers that fit such a simple numerical value suggest a significant physical meaning hidden in the last equation: the fact that $M_u c\Delta t_u \approx \pi m_u \Delta r_u$ is interesting because it provides a new link between M_u and m_u, i.e. according to (3,4) $m_u c^2\hbar/\Delta t_u \approx m_u c^2\hbar\omega_{n=2} + m_? c^2\hbar\omega_{n=2}$ with $\omega_{n=2} = c/\pi\Delta r_u$. Going thus beyond the mere numerical result, let us generalize (4,2) to any Δt by replacing 1 with a number $q = q(\Delta t, \Delta r)$; so the subscript u characterizing today's quantities will be omitted, whereas different values are expected for $\omega_{n=2}$ and $m_?$. Multiply both sides of the resulting equation by c^2; recalling again (3,4), (4,2) turns into

$$qmc^2 \frac{\hbar}{\Delta t} = mc^2\hbar\omega_{n=2} + m_? c^2\hbar\omega_{n=2}; \quad \omega_{n=2} = \frac{c}{\pi\Delta r}. \quad (4,3)$$

This equation is interesting because its terms are cross linked: a couple of terms shares mc^2, another couple $\hbar\omega_{n=2}$. This shows that m and $m_?$ are correlated. Moreover the fact that this equation contains squared energy terms, brings to mind an important equation inferred in the appendix, i.e.

$$\varepsilon^2 = (pc)^2 + \varepsilon^2_{rest}. \quad (4,4)$$

Add $\zeta mc^2\hbar/\Delta t$ to both sides of (4,3); by comparison these equations suggest the following correspondences

$$\begin{aligned} &(q+\zeta)\, mc^2 \frac{\hbar}{\Delta t} = \varepsilon^2 \\ &mc^2\left(\hbar\omega_{n=2} + \zeta\frac{\hbar}{\Delta t}\right) = (pc)^2 \\ &m_? c^2\,\hbar\omega_{n=2} = \varepsilon^2_{rest} \end{aligned} \quad (4,5)$$

being $\zeta = \zeta(\Delta t, \Delta r)$ a function of Δr and Δt whose physical meaning will appear soon. In principle these correspondences, merely based on the one-to-one association between (4,3) and (4,4) having an analogous form, propose a possible explanation of the mass $m_?$.

The universe as a whole is to be regarded like a free spinless neutral macro-particle moving at uniform speed, whose kinetic and total energy are respectively related to the terms $(pc)^2$ and ε^2; accordingly $m_?$ accounts for the rest energy of the macro-particle universe. It seems surprising that this link, suggested by mere numerical analysis of the values of $\hbar\omega_{n=2}$ and $\hbar/\Delta t_u$ of (4,2), is provided by a formula of special relativity and not of general relativity. The energies of (4,3) concern the universe as a whole and not the interaction of its parts, galaxies and stars and so on, whose gravitational dynamics is governed by the general relativity. In effect, (3,3) regards the black-hole universe as a global object, a spinless macro-particle, whose properties are due to its total mass and total size only, regardless of its complex internal structure, mass composition and mass distribution assumed homogeneous at least on large scale. A valid support to propose a rectilinear uniform motion of the whole universe comes from the fact that indeed this idea cannot be excluded by any experiment: since Galileo it is known that such an inertial motion cannot be detected by any observer inside the universe. Perhaps a harder implication of this idea could concern the hypothetical reference system R_u able to describe this motion; however also this dilemma is actually a false problem in the present model, once thinking the size of the universe as an uncertainty range $\Delta r = r_1 - r_0$ in principle similar to that introduced in section 2 to describe energy levels and angular momentum of the quantum particles. It has been emphasized: (i) that neither r_0 nor r_1 must be specified to describe the quantum properties; (ii) that in fact both coordinates are not specifiable; (iii) that this conceptual lack of information prevents specifying the reference system R_u where is defined r_0 and the actual size of Δr defined by r_1. So it is conceptually impossible, but also inessential, to specify such R_u as regards the quantum properties of a particle within the range Δr_u during the time range Δt_u: if the properties of the quantum macro-particle we call universe do not depend on r_0 or r_1 but on Δr only, then the difficulty of defining R_u, e.g. its origin, becomes marginal. Anyway, since (4,1) and (4,3) come directly from the experimental values of Δr_u and Δt_u, there is no reason to reject them; in effect (4,3) and its relativistic free particle interpretation explain why one addend concerns the mass $m_?$ and its energy $m_? c^2$ additional to the visible mass m_u of stars. Now is justified the function ζ knowing that $\varepsilon = mc^2/\sqrt{1-(\mathrm{v}/c)^2}$ and $p = m\mathrm{v}/\sqrt{1-(\mathrm{v}/c)^2}$; also these formulas are shown in the appendix in the frame of the present model. Let us rewrite the three terms of (4,4) that define the relativistic energy of the free macro-particle universe of (4,5) as a function of its displacement constant velocity v_{mp} and mass M_{mp}; this means replacing v and m with v_{mp} and M_{mp}. Hence

$$\begin{aligned} &(q+\zeta)\, mc^2 \frac{\hbar}{\Delta t} = \frac{M^2_{mp}c^4}{1-\mathrm{v}^2_{mp}/c^2} \\ &mc^2\left(\hbar\omega_{n=2} + \zeta\frac{\hbar}{\Delta t}\right) = \frac{M^2_{mp}\mathrm{v}^2_{mp}c^2}{1-\mathrm{v}^2_{mp}/c^2} \\ &m_? c^2\,\hbar\omega_{n=2} = M^2_{mp}c^4. \end{aligned} \quad (4,6)$$

Taking the ratio side by side of the first two equations one finds with the help of (3,9)

$$\frac{\mathrm{v}^2_{mp}}{c^2} = \frac{\omega_{n=2}\Delta t + \zeta}{q+\zeta}; \qquad \omega_{n=2}\Delta t = \frac{c}{\pi}\frac{\Delta t}{\Delta r} = \frac{\mathrm{v}}{\pi c} \quad (4,7)$$

where v is the average expansion rate of the universe at Δt. Now we impose that v_{mp} is constant via the function ζ; so

$$\mathrm{v}_{mp} = \pm c\sqrt{\zeta_0}; \quad \zeta = \frac{\omega_{n=2}\Delta t - \zeta_0 q}{\zeta_0 - 1}; \quad q = \frac{M}{m}\,\omega_{n=2}\Delta t \quad (4,8)$$

i.e. q generalizes (4,2). Note that M_{mp} does not appear in these equations; it is merely defined by the third (4,6) as a function of $m_?$, on which however no hypothesis has been made. So the definitions of ζ and q hold regardless of M_{mp}. An obvious condition is $0 < \zeta_0 < 1$; moreover $q + \zeta > 0$ and $\omega_{n=2}\Delta t + \zeta > 0$ are also evident because both sides of (4,6) and (4,7) are positive. The former condition reads $q + \zeta = (\omega_{n=2}\Delta t - q)/(\zeta_0 - 1) > 0$, the latter reads $\omega_{n=2}\Delta t + \zeta = \zeta_0(\omega_{n=2}\Delta t - q)/(\zeta_0 - 1)$; owing to the expression of q both reduce to the unique condition $1 - M/m < 0$, which is indeed true as it has been introduced since the beginning in the present quantum model. Impose also $m_? \omega_{n=2} = const$, which yields $m_? c/\pi\Delta r = const$: this equation extends (3,20) that reads $M/\Delta r = const = m/\Delta r + m_?/\Delta r$ according to (3,4). In this way M_{mp} becomes a constant. Note that owing to (3,8) this result reads $m_? c^2\hbar\sqrt{\rho G} = \varepsilon^2_{rest}$; being by definition $\rho = (m_? + m)/V$, one concludes that ε_{rest} is defined even during the early the radiation era when the visible mass was $m = 0$ and the universe volume of the order of the Planck volume V_P. So, with obvious meaning of symbols, $\varepsilon^2_{rest} = m^{(0)}_? c^2\hbar\sqrt{m^{(0)}_? G/V_P}$ and remained constant since then; hence the third (4,5) reads $m_? = m^{(0)}_? \sqrt{\rho^{(0)}/\rho}$, with $\rho^{(0)} = m^{(0)}_?/V_P$. Of course, as already noted, $1/\rho$ is an increasing function of $m_?$ because the black hole condition $M/\Delta r = const$ requires M/V decreasing function of Δr^3. In conclusion (4,5) are appropriate to describe a free macro-particle of constant mass M_{mp} moving at constant rate v_{mp}. Eventually, note that the square energies of (4,5) are actually products of two different energies, as if they would come from geometrical averages like for instance $< \varepsilon > = \pm((q + \zeta)mc^2\hbar/\Delta t)^{1/2}$. So the black hole we call universe has, as a whole, the average energy $< \varepsilon >$ of a free particle that moves with average constant momentum $< pc > = \pm(mc^2(\hbar\omega_{n=2} + \zeta\hbar/\Delta t))^{1/2}$, whereas $m_? c^2$ and $\hbar\omega_{n=2}$ defining $< \varepsilon_{rest} > = \pm(m_? c^2\hbar\omega_{n=2})^{1/2}$ appear to be the ingredients of its average rest energy. Otherwise stated, the well defined mass balance between $m_?$ and m_u proposed here appears rationally motivated: m_u is due to the capability of the universe to create ordinary visible mass after the radiation era exploiting the available big-bang fluctuation energy; the additional mass $m_?$ ensures the existence of an efficient black hole universe that does not waste uselessly its valuable energy content. So it follows also the necessity of a displacing universe. Are unavoidable at this point at least three questions: does actually the equation $< \varepsilon > = \pm\sqrt{< pc >^2 + < \varepsilon_{rest} >^2}$ admit the minus sign? could an anti-universe actually exist with a matter/antimatter mechanism equal and opposite to that described in the previous section? is our whole universe a wave/corpuscle subjected itself to the uncertainty principle?

5 Discussion

The cosmology is probably the most difficult among the physical sciences because of both its multidisciplinary conceptual basis and scarcity of experimental data, besides inferred in a limited domain of time and space consistent with the light speed: past, present and future of the whole universe must be guessed despite the space-time horizon gives us access to a limited window of observable objects only. Just for this reason the theoretical models have a special role in cosmology. Usually the experimental data validate a theoretical model; here instead seems true the exact contrary, i.e. a sound self-consistent model highlights the physical meaning of the available experimental data. In this particular context is crucial the role of quantum mechanics. The correspondence principle states that the classical physics is the limit of quantum physics for high quantum numbers, which implicitly means that just the quantum principles are the true essence of physics and thus of cosmology as well. This explains the attempt of the present model, mostly based on quantum considerations rather than on relativistic considerations. Two important experimental values, the Hubble constant and the cosmic background radiation temperature, have been estimated with accuracy enough to conclude that the physical approach of the present quantum model of the universe is basically correct. (2,1) enable the most important equations of quantum mechanics and relativity to be inferred [12,13,14,18]; their generality is also proven in particular by the ability of describing quantum fluctuations of a relativistic free particle. For instance the appendix shows how to find the well known equation $p = v\varepsilon/c^2$ via $\Delta p = v\Delta\varepsilon/c^2$, whose importance for the present model has been already emphasized, e.g. (2,4) and (3,7); however Δp and $\Delta\varepsilon$ are not classical ranges but quantum uncertainty ranges. So a quantum particle whose local momentum and energy are included within the respective ranges, recall the explicative results of section 2, is subjected to quantum fluctuations of p and ε that expectedly alter also its propagation rate. This fact prospects new chances for the known equations of special relativity, which here appear in fact as quantum equations subjected to the weirdness of the quantum world. Further considerations on this topic are outside the purposes of the present paper. Yet it is worth mentioning that the EPR paradox, according which particles billions of light years apart can instantaneously exchange information via the so called quantum entanglement, is explained according to the agnostic physical meaning of (2,1); the concept of distance becomes itself undetermined once disregarding the local coordinates. Renouncing even to the concept of probability density for any particle to be somewhere, replaced by the mere idea of delocalization within an uncertainty range, the concept of distance is no longer definable. So it is unphysical to expect a different quantum behavior for particles definable very close or very far apart only classically. Certainly this odd conclusion is not the only weirdness of the quantum world: as it is shown in section 2, this agnostic standpoint has unexpectedly heuristic physical meaning. One kind of weird phenomenon is the quantum fluctuation, according which any macroscopic object at rest could suddenly excited to a self-perturbed state because of a transient excess of energy, justifiable via the uncertainty principle only. The behavior of a relativistic quantum particle during a quantum fluctuation is quoted here because it is in effect pertinent to the purposes of the paper. The considerations proposed in the appendix usefully contribute to explain cosmological problems like the inflationary era. In the paper [13] it was shown that (2,1) only are enough to infer the following corollaries: (i) equivalence of all reference systems in describing the physical laws, (ii) existence of a maximum average displacement rate allowed for any particle in its delocalization range and (iii) invariance in all reference systems of such a maximum velocity. These corollaries are in fact the basic statements of special relativity. Moreover also the equivalence principle of general relativity and the coincidence of inertial and gravitational mass were also inferred [14] along with the concept of mass as corollaries of the space-time uncertainty.

6 Appendix

This appendix sketches shortly how the relativistic momentum and energy are obtained exploiting (2,1) only; it aims to make the present paper as self-contained as possible. Let the arbitrary delocalization ranges be defined in an arbitrary reference system R, where a photon travels at speed c through $\Delta x^{(c)}$; so ((2,1)) read $\Delta x^{(c)}\Delta p_x^{(c)} = n^{(c)}\hbar = \Delta t^{(c)}\Delta\varepsilon^{(c)}$. The superscripts emphasize that the ranges are sized to fulfil the delocalization condition during an appropriate time range $\Delta t^{(c)}$. Being by definition $\Delta x^{(c)}/\Delta t^{(c)} = c$, then $c\Delta p_x^{(c)} = \Delta\varepsilon^{(c)}$. To find how the momentum and energy ranges of a massive particle traveling at rate $\mathrm{v}_x < c$ through $\Delta x^{(c)}$ scale with respect to $\Delta p_x^{(c)}$ and $\Delta\varepsilon^{(c)}$, write $\Delta x^{(c)}\Delta p_x^{(\mathrm{v})} = n^{(\mathrm{v})}\hbar = \Delta t^{(\mathrm{v})}\Delta\varepsilon^{(\mathrm{v})}$. As neither v_x nor c appear explicitly in this equation, write $n^{(\mathrm{v})}\hbar = \Delta t^{(c)}\Delta\varepsilon^{(c)} = \Delta t^{(\mathrm{v})}\Delta\varepsilon^{(\mathrm{v})}$; this is true if $\Delta t^{(c)}$ and $\Delta\varepsilon^{(c)}$ scale respectively like $\Delta t^{(\mathrm{v})} = (c/\mathrm{v}_x)\Delta t^{(c)}$, as it is reasonable, and $\Delta\varepsilon^{(\mathrm{v})} = (\mathrm{v}_x/c)\Delta\varepsilon^{(c)}$, as a consequence. Replacing these positions in the former equation, $\Delta x^{(c)}\Delta p_x^{(\mathrm{v})} = \Delta t^{(c)}(\mathrm{v}_x/c)\Delta\varepsilon^{(c)}$ yields $c\Delta p_x^{(\mathrm{v})} = (\mathrm{v}_x/c)\Delta\varepsilon^{(c)}$. Actually the superscripts can be omitted because they have been introduced for clarity of exposition only, not to identify particular range sizes; both $\Delta p_x^{(\mathrm{v})}$ and $\Delta\varepsilon^{(c)}$ are indeed completely arbitrary like v_x itself; the superscripts are also irrelevant as concerns the functional relationship between the local values of the respective variables. Hence

$$p_x = \mathrm{v}_x\varepsilon/c^2; \qquad \Delta p_x = \mathrm{v}_x\Delta\varepsilon/c^2 \tag{A1}$$

regardless of how the respective uncertainty ranges are defined. Since an identical reasoning holds in any other reference system R', one concludes that $p'_x = \mathrm{v'}_x\varepsilon'/c^2$ is an invariant of special relativity. In principle the component of velocity defining the momentum component can be positive or negative; yet squaring this equation one surely handles positive terms. So write $\varepsilon^2(\mathrm{v_x}/c)^2 = (p_x c)^2$; since $\mathrm{v_x}/c < 1$ for a massive particle one finds $\varepsilon^2 > (p_x c)^2$, which compels writing $\varepsilon^2 = (p_x c)^2 + \varepsilon_o^2$. Calculate the limit $p_x/\mathrm{v_x}$ for $\mathrm{v} \to 0$; denoting this limit as

$$\lim_{\mathrm{v}\to 0}\frac{p_x}{\mathrm{v}_x} = m \tag{A2}$$

the concept of mass m is introduced as a consequence of the uncertainty, whereas (A1) yields $\lim_{\mathrm{v}\to 0}\varepsilon = \varepsilon_{rest} = mc^2$ in agreement with the idea that the limit must be finite; indeed no reason requires $\varepsilon \to 0$ for $\mathrm{v_x} \to 0$. Thus $p_x = m\mathrm{v}_x$ is the non-relativistic form of (A1). So the previous equation yields $mc^2 = \varepsilon_o^2$, i.e.

$$\varepsilon^2 = (pc)^2 + (mc^2)^2 \tag{A3}$$

as it is well known. Hence (2,1) define themselves without additional hypotheses the concept of mass and the relativistic and non-relativistic form of the respective local variables included in the ranges Δp and $\Delta\varepsilon$. Note that merging together both equations one finds the well known expressions consistent with the Lorentz transformations. Also note that the local values of p_x and ε are exactly definable in relativity, which is substantially classical physics subjected to the covariancy principle in a four dimensional space-time context; here instead, as shown in section 2, coordinates, momentum and energy are dynamical variables random, unknown and unknowable within the respective uncertainty ranges. This is the conceptual key to understand the further considerations of this appendix. In classical physics momentum and energy of a free particle are constants; yet it is not so in the quantum world, where quantum fluctuations are allowed to occur. The crucial point is that (A1) and (A3) are quantum results, despite their form agrees of course with that of special relativity; yet, being the particles completely delocalized, the local p and ε must be intended as random values within the respective uncertainty ranges. So these equations can be accordingly handled. Let us admit that during a short time range δt even the energy of a free particle is allowed to fluctuate randomly by $\delta\varepsilon$. Since during the time transient the particle is expectedly allowed to move in a arbitrary way, (A1) is now exploited to highlight the link between $\delta\varepsilon$ and the related changes δp and $\delta\mathrm{v}$. Differentiating (A1) one finds $\delta\varepsilon = c^2\delta p/\mathrm{v} - p(c/\mathrm{v})^2\delta\mathrm{v}$: with given p and v, this result defines the functional dependence of $\delta\varepsilon$ upon arbitrary δp and $\delta\mathrm{v}$. Sum $\delta\varepsilon$ and (A1) to find $\varepsilon + \delta\varepsilon = c^2(p+\delta p)/\mathrm{v} - \varepsilon\delta\mathrm{v}/\mathrm{v}$. In general $\delta p\delta x = n\hbar$ reads $(\delta p)^2 = n\hbar\delta p/\delta x$, whereas in an analogous way $(\delta\varepsilon)^2 = n\hbar\delta\varepsilon/\delta t$. Regard just in this way $\varepsilon + \delta\varepsilon$ and $p + \delta p$; putting $\delta x = \mathrm{v}\delta t$ and replacing in the last expression to calculate $\delta(\varepsilon + \delta\varepsilon)/\delta t$, one finds

$$\begin{gathered}(n\hbar)^{-1}(\Delta\varepsilon)^2 = (n\hbar)^{-1}(\Delta pc)^2 - \varepsilon\delta\omega \\ \Delta\varepsilon = \varepsilon + \delta\varepsilon; \qquad \Delta p = p + \delta p.\end{gathered} \tag{A4}$$

The term $\varepsilon\delta\omega$ results because $\mathrm{v}/\delta x$ has physical dimensions of a frequency ω, so that $\delta\mathrm{v}/\delta x = \delta\omega$. As $n\hbar\omega\delta\varepsilon = \delta(\varepsilon n\hbar\omega) - \varepsilon\delta(n\hbar\omega)$, replacing this identity in the last equation one finds $(\Delta\varepsilon)^2 = (\Delta pc)^2 + n\hbar\omega\delta\varepsilon - \delta(\varepsilon n\hbar\omega)$. Let us specify this result via the position

$$n\hbar\omega = \delta\varepsilon$$

which yields also $(\Delta\varepsilon)^2 - (\Delta pc)^2 = (\delta\varepsilon)^2 - \delta(\varepsilon\delta\varepsilon)$. At left hand side appear terms containing the ranges $\varepsilon+\delta\varepsilon$ and $p+\delta p$ only, at right hand side the ranges $\delta\varepsilon$ and δp only. These latter are both arbitrary; moreover ε and p are arbitrary as well. So it is reasonable to expect that the last equation splits into two equations linked by a constant energy ε_o

$$(\Delta\varepsilon)^2 - (\Delta pc)^2 = \varepsilon_o^2 = (\delta\varepsilon)^2 - \delta(\varepsilon\delta\varepsilon). \tag{A5}$$

Indeed ε_o agrees with both of them just because it does not depend upon neither of them. Trivial manipulations show that the first equation yields

$$p = \pm\frac{\varepsilon_o \mathrm{v}/c^2}{\sqrt{r_\varepsilon^2 - r_p^2(\mathrm{v}/c)^2}}; \qquad \varepsilon = \pm\frac{\varepsilon_o}{\sqrt{r_\varepsilon^2 - r_p^2(\mathrm{v}/c)^2}}$$
$$r_p = 1 + \frac{\delta p}{p} \qquad\qquad r_\varepsilon = 1 + \frac{\delta\varepsilon}{\varepsilon}. \tag{A6}$$

(A5) is fulfilled even during the transient. The value of the constant ε_o is immediately found as a consequence of (A2): in agreement with (A5) $\varepsilon_o^2 = \varepsilon_{rest}^2$, because A6 hold during the time transient allowing $\delta\varepsilon$; before and after that transient one must put $\delta\varepsilon = 0$ and $\delta p = 0$ in order to have the "standard" Einstein momentum and energy of the free particle, here inferred from A1 to A3. So

$$\varepsilon_{Ein}^2 = c^2 p_{Ein}^2 + \varepsilon_{rest}^2$$

$$p_{Ein} = \pm\frac{m\mathrm{v}}{\sqrt{1-(\mathrm{v}/c)^2}}; \qquad \varepsilon_{Ein} = \pm\frac{mc^2}{\sqrt{1-(\mathrm{v}/c)^2}}.$$

It is easy now to calculate the energy and momentum gaps during the time transient δt as a function of $\delta p/p$ and $\delta\varepsilon/\varepsilon$ as follows

$$\delta l\left(\frac{m\mathrm{v}}{\sqrt{r_\varepsilon^2 - r_p^2(\mathrm{v}/c)^2}} - \frac{m\mathrm{v}}{\sqrt{1-(\mathrm{v}/c)^2}}\right) = n_{fl}\hbar$$
$$\delta t\left(\frac{mc^2}{\sqrt{r_\varepsilon^2 - r_p^2(\mathrm{v}/c)^2}} - \frac{mc^2}{\sqrt{1-(\mathrm{v}/c)^2}}\right) = n_{fl}\hbar \tag{A7}$$

where δt is the time length of the fluctuation, δl the path traveled by the particle during δt and n_{fl} the number of states allowed to the particle during the energy transient. These equations are in effect nothing else but the uncertainty equations of the fluctuation gaps $\delta p_{fl} = p_{fl} - p_{Ein}$ and $\delta\varepsilon_{fl} = \varepsilon_{fl} - \varepsilon_{Ein}$. Of course $\delta p \to 0$ and $\delta\varepsilon \to 0$ after the transient, so the amounts within parenthesis vanish, while $n_{fl} = 0$ too; i.e. the fluctuation states are no longer accessible to the particle. Taking the ratio of these expressions, one finds

$$\frac{\delta l}{\delta t} = c\,\frac{c}{\mathrm{v}}. \tag{A8}$$

According to (A8), during a quantum fluctuation of time length δt the uncertainty range δl allowed to any quantum particle corresponds to an average displacement rate $\delta l/\delta t = c^2/\mathrm{v} > c$, i.e. *as if* the particle would really propagate at superluminal rate. The reasoning to explain this result is similar to that explaining the recession motion of celestial objects mostly as a consequence of the expansion of the space-time itself. If the fluctuation modifies the size of the energy and momentum ranges, then according to (2,1) it must modify also the space and time range sizes. Yet the space range includes all local coordinates allowed to the particle: since this latter is anywhere in the space range because it is delocalized, and not because it really travels from point to point, modifying the space size means affecting the ability of the particle of being somewhere in the universe regardless of the velocity necessary to cover the path. This explains the apparent anomaly of superluminal velocity to figure out a fluctuation driven displacement. From a mathematical point of view, this is indeed possible provided that (A7) verify two inequalities: the first is $r_p\mathrm{v}/cr_\varepsilon < 1$, to avoid imaginary quantities, the second is $r_\varepsilon^2 - r_p^2(\mathrm{v}/c)^2 < 1 - (\mathrm{v}/c)^2$, in order that both left hand sides be positive. These inequalities merge into the unique $r_\varepsilon^2 - 1 < (r_p^2 - 1)(r_\varepsilon/r_p)^2$, which yields $1 - r_\varepsilon^{-2} < 1 - r_p^{-2}$ i.e. $r_\varepsilon^{-2} > r_p^{-2}$ and thus $r_\varepsilon^2 < r_p^2$. So, being $\delta p/p > \delta\varepsilon/\varepsilon$ according to (A6), $\varepsilon/p > \delta\varepsilon/\delta p$ reads thanks to (A1) and (2,1) $\mathrm{v}/c^2 > \delta t/\delta x$ and thus $\delta x/\delta t > c^2/\mathrm{v}$ even though $\mathrm{v} < c$. (A8) is confirmed noting that it could have been obtained more quickly and easily: rewrite (2,1) as $\Delta x/\Delta t = \Delta\varepsilon/\Delta p_x$ and recall (A1) $\Delta\varepsilon/\Delta p_x = c^2/\mathrm{v_x}$; replacing the latter into the former one finds $\Delta x/\Delta t = c^2/\mathrm{v}$. This result has the same form of (A8) and (3,7); without the steps (A4) to (A8) however the properties of the quantum fluctuation would not be evident. Owing to the arbitrariness of the range sizes, nothing in principle distinguishes Δx and Δt from δl and δt; yet (A7) emphasize the specific link between δl and δt and their conjugate momentum and energy just during the quantum fluctuation. For instance, (A7) admit $r_\varepsilon = 1$ and $r_p = 1$, i.e. $\delta\varepsilon = 0$ and $\delta p = 0$, in which case $n_{fl} = 0$ because of course there are no fluctuation states; instead $\Delta p_x = 0$ and $\Delta\varepsilon = 0$ are unphysical because they deny the concept of quantum uncertainty.

In conclusion the theoretical analysis describes the effect of the extra energy transient on the space-time uncertainty of the particle during the quantum fluctuation: a massive particle can displace more than allowed by its actual velocity. Transient displacement ranges $\delta l > c\delta t$ are possible for the boundary of the universe, even though forbidden in the early Einstein derivation of momentum and energy. Indeed the relativity is substantially classical physics; yet the beauty of the theory does not admit itself quantum phenomena like the fluctuations. These phenomena are instead allowed when deriving the Einstein formulas in the quantum frame of (2,1).

It is worth emphasizing however that in the particular case $\mathrm{v} = c$ even $\delta l/\delta t$ remains always and invariably equal to c.

It is clear now that also the universe expansion is interested by these results: the previous quantum considerations, unexpected in classical relativity, help to better understand and describe the so called "inflationary era". Regard the big bang as a vacuum fluctuation that begins at the arbitrary time t_0 and expands the primordial sphere of radius r_0 according to the concepts introduced at the beginning of section 3. During δt the displacement δl of the boundary of the universe could overcome $c\delta t$, in agreement with $\delta\varepsilon \neq 0$ and $\delta p \neq 0$. Inflation did occur when the radiation density was such that the photons were allowed moving in a medium with refractive index $n_r > 1$ and matter particles, virtual or not, were generated in the radiation field during the early beginning of the later matter era. This idea agrees with the presence at the boundary of the primordial universe of the virtual couples of particles and antiparticles generating locally via their annihilation the halo of electromagnetic radiation introduced in (3,1).

As in the present approach the ranges sizes are unknown and conceptually unknowable, it is impossible to know exactly how long lasts δt. Yet it is possible to say that after a certain time range, when $\delta\varepsilon = 0$ and $\delta p = 0$ i.e. after the end of the fluctuation, the universe expansion continued at rate compliant with the usual condition v < c.

Consider an arbitrary number of particles, assumed for simplicity non-interacting; holds for i-th of them $p_i = \mathrm{v}_i\varepsilon_i/c^2$. Let Δp and $\Delta\varepsilon$ be the momentum and energy ranges including all p_i and ε_i; being the range sizes arbitrary, it is possible to write $\Delta p = \mathrm{v}\Delta\varepsilon/c^2$ with v defined in agreement with (A1). Suppose that a quantum fluctuation starts at an arbitrary time and modifies momenta and energies of some of the particles, so that the respective ranges are modified as well; then $(c^2/\mathrm{v})\Delta p = \Delta\varepsilon$ yields $(c^2/\mathrm{v})\delta\Delta p - (c/\mathrm{v})^2\delta\mathrm{v}\Delta p = \delta\Delta\varepsilon$. Moreover (2,1), which read $\Delta\varepsilon/2\pi = n\hbar\omega$ with $\omega = 2\pi/\Delta t$ and $\Delta p/2\pi = n\hbar k$ with $k = 2\pi/\Delta x$, yield $\delta\Delta\varepsilon/2\pi = n\hbar\delta\omega$ and $\delta\Delta p/2\pi = n\hbar\delta k$. The former is the Planck equation expressed as a function of $\Delta\varepsilon/2\pi$ instead of $\Delta\varepsilon$, the latter is the De Broglie equation also expressed as a function of $\Delta p/2\pi$ instead of Δp; however being the range sizes arbitrary, unknowable and inessential as concerns the eigenvalues of the physical observables, as shown in section 2, the factor $(2\pi)^{-1}$ is trivially irrelevant. It is remarkable instead that Δx and Δt of (2,1) are regarded here as wavelength and frequency of a wave, which is in fact possible in agreement with the general character of (2,1). One finds concurrently

$$\frac{\delta\omega}{\delta k} = \frac{\omega'}{k}; \qquad \omega' = \omega - ku; \qquad u(\omega,k) = n_r^2\,\frac{\delta\mathrm{v}}{\delta\Delta p}\,\Delta p; \qquad n_r = \frac{c}{\mathrm{v}}. \tag{A9}$$

Being v and δv arbitrary, it is evident that these equations hold whatever n_r might be. This conclusion is interesting because in effect the physical meaning of these equations depends just on the features of v. Call $\mathrm{v}_p = \omega/k$ and $\mathrm{v}_g = \omega'/k$, being thus $\omega' = \omega'(k)$. For $\mathrm{v} \equiv c$ (A1) reads $\Delta\varepsilon = c\Delta p$ and describes a set of electromagnetic waves propagating in the vacuum, whence $u = 0$ i.e. $\omega'/k \equiv \omega/k \equiv c$. If $\mathrm{v} < c$ is again constant, then these equations still describe a set of light waves propagating at the same rate v_p in non-dispersive medium with refractive index n_r; yet they are also compatible with a set of massive free particles displacing at the same rate. The case where $\mathrm{v} < c$ depends on k is more interesting. The equations describe light waves propagating with different velocities in a dispersive medium dependent on n_r; the first (A9) defines the group velocity $\mathrm{v}_g \neq \mathrm{v}_p$ of the whole packet formed in the dispersive medium. Analogous conclusion holds also for the matter waves: now the displacement of matter wave packet at rate v_g is related to the maximum probability to find somewhere the set of particles; indeed the first (A9) is also obtained from $\delta(\omega'/k)/\delta k = 0$, which suggests that ω'/k corresponds to the rate with which moves the maximum of the packet defined by the dispersion curve ω'/k vs k. Both electromagnetic waves and matter particles, despite their different physical nature, are thus compatible with a unique kind of equation: their common feature is the dual wave/corpuscle nature strictly connected with the quantization condition of (2,1).

The changes $\delta\omega$ and δk have been introduced as a consequence of quantum fluctuation; in effect it would be also possible to infer from $\delta\omega = \omega\delta k/k - u\delta k$ the Einstein formula for the energy fluctuations of blackbody radiation. For brevity this point is waived here; yet, is significant the ability of the quantum fluctuation to generate packets of particle waves and packets of electromagnetic waves having similar behavior. This conclusion helps to figure out the formation of matter in the radiation field during the radiation era as superposition of electromagnetic radiation and matter wave packets, both propagating with their characteristic group velocities $\mathrm{v}_g^{(r)}$ and $\mathrm{v}_g^{(m)}$. This supports the idea of fermion/antifermion pairs formed via photon fluctuations at appropriate energy fulfilling momentum and angular momentum conservation rules. The matter waves extended to all space time available justify the presence of matter throughout the universe. Indeed it is possible to write $\delta\omega/\delta k = \delta\omega^{(r)}/\delta k^{(r)} + \delta\omega^{(m)}/\delta k^{(m)}$; then, the addends at right hand side read $\delta\omega^{(r)}/\delta k^{(r)} = \mathrm{v}_g^{(r)}$ and $\delta\omega^{(m)}/\delta k^{(m)} = \mathrm{v}_g^{(m)}$. So the extra energy transient of the fluctuation of the radiation field (term at left hand side, because $\delta\omega$ is proportional to $\delta\Delta\varepsilon$) has generated a matter wave propagating at rate in general different from that of further radiation (terms at right hand side); the quantum fluctuation of this latter could generate in turn further matter and further radiation and so on, until the available energy is sufficient to repeat the process. The matter particle propagates with a group velocity $\mathrm{v}_g^{(m)}$ having finite space length; in principle the matter wave packet can also represent a chunk of matter having finite size and given probability of being found somewhere and moving in the universe. This supports the physical meaning of (3,12) as discussed in section 3.1.

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