

Atomic Weights Confirm Bipolar Model of Oscillations in a Chain System

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We apply the bipolar model of oscillations in a chain system to the data set of standard atomic weights. 90% of these masses could be reproduced by this model and were expressed in continued fraction form, where all numerators are Euler's number and the sum of the free link and all partial denominators yields zero. All outliers were either radioactive or polynuclidic elements whose isotopic compositions as found in samples on Earth might not be fully representative for the mean values when considering samples from all parts of the universe.

1 Introduction

In several previous papers we applied the model of oscillations in a chain system to various systems such as the solar system [1], excited electronic states of atoms [2], the electron density in the Hydrogen atom [3], and more recently to the mass distribution of elementary particles [4].

Initially, this model was founded by Müller in three fundamental articles [5–7] and assumes that all protons in the universe are oscillators, coupled through the physical vacuum. As a consequence we can consider (in the most simplest case) a chain of equal harmonic proton oscillators with an associated logarithmic spectrum of eigenfrequencies which can be expressed through continued fractions. In that way, every mass is interpreted as a proton resonance state and expressed in continued fraction form.

Recently, a bipolar version of this model was proposed for the description of elementary particles [4], because the traditional version could not reproduce their masses in a fully satisfactory way. The idea of bipolarity postulates that the fundamental spectrum of proton resonances has an opposite, an anti-oscillation or inverted oscillation spectrum, and this is the spectrum of electron resonances.

Mathematically, two opposite oscillation states are characterized through equal continued fraction representations, but with the difference that in one case all denominators, the free link and the phase shift have been multiplied by (-1). From the analysis of elementary particle masses it was suggested to express the electron mass as a proton resonance and the proton mass as an electron resonance through the following equations (e is Euler's number):

$$\ln \frac{m_{electron}}{m_{proton}} = p_p + (-6) + \frac{e}{12 + \frac{e}{-6}}$$

and

$$\ln \frac{m_{proton}}{m_{electron}} = p_e + 6 + \frac{e}{(-12) + \frac{e}{6}}$$

Numerically, p_p was found to be ≈ -1.75 [4]; for these phase shifts must hold $p_p = -p_e$.

In this article we show that the relative atomic masses can be reproduced by almost the same the bipolar model. The only parameter that must be adjusted is the phase shift (from $|p| \approx 1.75$ to $|p| \approx 1.79$) and this is a very minor change.

2 Data sources and computational details

The standard atomic weights, including the proton and electron reference masses were taken from the web-site of the National Institute of Standards (NIST) and were expressed in the atomic mass unit u . The following abbreviations and conventions for the numerical analysis hold:

The atomic masses are transformed into a continued fraction according to the equations

$$\ln \frac{m}{m_{electron}} = p_e + S, \quad \ln \frac{m}{m_{proton}} = p_p + S,$$

where p is the phase shift (it must hold $p_p = -p_e$) and S is the continued fraction (e is Euler's number)

$$S = n_0 + \frac{e}{n_1 + \frac{e}{n_2 + \frac{e}{n_3 + \dots}}} \quad (1)$$

The numerical value of the phase shift p is initially unknown and must be adjusted in such a way that the largest possible amount of atomic weights can be expressed through a continued fraction.

The continued fraction representation $p+S$ is abbreviated as $[p; n_0 | n_1, n_2, n_3, \dots]$, where the free link n_0 is allowed to be $0, \pm 3, \pm 6, \pm 9, \dots$ and all partial denominators n_i can take the values $e+1, -e-1, \pm 6, \pm 9, \pm 12, \dots$. In the tables these abbreviations were marked with P or E, in order to indicate proton or electron resonance states.

The absolute value of the difference between the atomic weight given by NIST and the atomic weight calculated from the associated continued fraction representation is defined as numerical error and listed in the tables.

An atomic weight is considered as an outlier when the corresponding continued fraction representation provides a mass value outside the interval "atomic mass \pm standard deviation".

Table 1: Continued fraction representations of the 20 most accurately determined atomic weights (Helium and the set of 19 mononuclidic non-radioactive elements), $x = 1.7918229$ is the phase shift, SD = standard deviation.

Element symbol	Mass \pm SD [u]	Continued fraction representation(s)	Numerical error [u]
He	$4.002602 \pm 2.0 \times 10^{-6}$	P [-x; 3 15, e+1, -6, e+1, 33, (6, -e-1, -e-1, -51)]	1.2×10^{-9}
Be	$9.0121822 \pm 4.0 \times 10^{-7}$	P [-x; 3 e+1, -e-1, e+1, -e-1, -e-1, e+1, -6, -6, 12, (-9, 6)]	4.3×10^{-8}
F	$18.99840322 \pm 7.0 \times 10^{-8}$	E [x; 9 -9, e+1, -e-1, e+1, -e-1, -1680, (1680)]	2.6×10^{-9}
Na	$22.9897692809 \pm 2.9 \times 10^{-9}$	E [x; 9 -18, -9, -9, -e-1, e+1, -33, 12, (48)]	7.9×10^{-10}
Al	$26.98153863 \pm 1.2 \times 10^{-7}$	P [0; 3 9, 6, 18, -6, -6, e+1, 9, (-e-1, 33)]	1.2×10^{-9}
P	$30.97376163 \pm 2.0 \times 10^{-7}$	P [-x; 6 -e-1, 12, -e-1, 6, -15, e+1, 6, (-15, e+1)] E [x; 9 18, 15, -18, 18, (-42)]	1.6×10^{-7} 1.1×10^{-7}
Sc	$44.9559119 \pm 9.0 \times 10^{-7}$	P [0; 3 e+1, -9, 9, -e-1, e+1, -21, e+1, (-e-1, 18, -e-1)] E [0; 12 -e-1, -12, e+1, -6, 126, e+1, -e-1, -120]	4.5×10^{-7} 9.1×10^{-8}
Mn	$54.9380451 \pm 7.0 \times 10^{-7}$	P [0; 3 e+1, -e-1, e+1, -e-1, e+1, -e-1, 6, 18, e+1, -e-1, e+1, -e-1, (-27)] P [-x; 6 -12, -e-1, e+1, -e-1, e+1, -6, 111, (-99)] E [0; 12 -6, 6, e+1, -24, e+1, -e-1, e+1, -6, (18, -e-1, -e-1)] E [x; 9 e+1, 63, -e-1, 6, -9, -18, (-51)]	2.2×10^{-7} 4.5×10^{-8} 1.2×10^{-8} 3.6×10^{-10}
Co	$58.9331950 \pm 7.0 \times 10^{-7}$	E [0; 12 -6, -6, e+1, -e-1, e+1, -e-1, e+1, -e-1, 162, (-162)] E [x; 9 e+1, -9, -6, -e-1, e+1, -24, -9, (-e-1, 39)]	5.7×10^{-8} 4.4×10^{-9}
As	$74.9215965 \pm 2.0 \times 10^{-6}$	P [-x; 6 27, -30, -27, (24)]	2.3×10^{-7}
Y	$88.9058483 \pm 2.7 \times 10^{-6}$	P [-x; 6 9, e+1, -e-1, e+1, -e-1, -102, -e-1, (87, e+1)]	5.1×10^{-7}
Nb	$92.9063781 \pm 2.6 \times 10^{-6}$	P [-x; 6 9, -6, -e-1, 21, e+1, -e-1, (27, e+1, -57)] E [0; 12 69, -e-1, -6, 18, -e-1, (-93, e+1, e+1)]	2.2×10^{-8} 6.9×10^{-7}
Rh	$102.905504 \pm 3.0 \times 10^{-6}$	P [-x; 6 6, 6, -6, e+1, -e-1, -6, -6, -e-1, (e+1)]	9.1×10^{-7}
I	$126.904473 \pm 4.0 \times 10^{-6}$	P [-x; 6 e+1, e+1, e+1, -93, 6, -e-1, e+1, (-e-1, 81, -e-1, -e-1)]	3.6×10^{-7}
Cs	$132.905451933 \pm 2.4 \times 10^{-8}$	E [0; 12 6, e+1, -6, 99, e+1, 6, -6, (-111, -e-1, -e-1)]	1.7×10^{-8}
Pr	$140.9076528 \pm 2.6 \times 10^{-6}$	P [-x; 6 e+1, -330, -e-1, 6, -e-1, e+1, (-12, 330)] E [0; 12 6, -63, -e-1, e+1, 9, -12, (48)]	4.4×10^{-8} 2.5×10^{-7}
Tb	$158.9253468 \pm 2.7 \times 10^{-6}$	P [-x; 6 e+1, -6, e+1, -e-1, -6, 525, (-519, -e-1)]	5.8×10^{-8}
Ho	$164.9303221 \pm 2.7 \times 10^{-6}$	P [0; 6 -e-1, e+1, 18, e+1, -6, 6, 75, (-99, -e-1)]	5.2×10^{-9}
Tm	$168.9342133 \pm 2.7 \times 10^{-6}$	P [0; 6 -e-1, e+1, e+1, 6, 6, 6, 12, (15, -e-1, -51)]	3.2×10^{-9}
Au	$196.9665687 \pm 6.0 \times 10^{-7}$	P [0; 6 -e-1, -78, e+1, e+1, 6, -e-1, e+1, (15, -e-1, 51)] E [0; 12 e+1, -9, -e-1, -e-1, -15, e+1, 6, -e-1, e+1, -e-1, (e+1, 6)]	7.1×10^{-9} 4.6×10^{-7}

3 Results and discussion

It can be easily verified that the standard Müller model with the phase shifts $p = 0$ and $p = 1.5$ does not apply at all to the relative atomic weights, while the bipolar model with phase shifts of approximately ± 1.75 (as used in a previous study [4]) produces around 30% outliers. When working with the complete data set, varying the phase shift does not lead to a clear result. In that case we obtain a wealth of slightly different phase shifts, all providing a quite similar number of outliers and a similar sum of squared residuals (sum of squared numerical errors).

In order to arrive at a conclusion, the data set of 84 atomic masses was divided into two parts. The first part is composed of the element Helium (two stable isotopes, but still very low standard deviation) and the set of 19 non-radioactive mononuclidic elements. Here, the maximum measurement error is 2.7×10^{-6} u. The second part consists of the remaining el-

ements; their standard deviations vary from $\sim 10^{-5}$ to 0.1 u (Pb) due to isotopic variations found in samples taken at different locations on Earth.

It is fact that the “mean atomic mass” of a mononuclidic element is everywhere in the universe exactly the same, while we would expect some variations in the atomic masses of polynuclidic elements when analyzing rock samples obtained from different galaxies. It is reasonable to assume that the conditions during the formation of the chemical elements were subjected to variations throughout the universe.

Therefore we give priority to the atomic masses of the mononuclidic chemical elements and only the first part of the data set has been analyzed thoroughly. The phase shift was adjusted in such a way that (a) the number of outliers, and (b) the sum of squared residuals are minimized. This leads to a phase shift of ≈ 1.79 (exact value is 1.7918229) which is close to $\ln(6)$. Table 1 lists these atomic masses together with

Table 2: Continued fraction representations of the atomic weights of polynuclidic elements, from H to Kr (except He), $x = 1.7918229$ is the phase shift, SD = standard deviation.

Element symbol	Mass \pm SD [u]	Continued fraction representation(s)	Numerical error [u]
H	$1.00794 \pm 7 \times 10^{-5}$	P [0; 0 4128, (-4128)] E [x; 6 -9, e+1, 6, -e-1, e+1, -e-1, e+1, (-e-1, 9, -12)]	2.5×10^{-8} 5.9×10^{-5}
Li	$6.941 \pm 2 \times 10^{-3}$	P [-x; 3 e+1, 57, (e+1, -e-1, -e-1, -60)] E [0; 9 6, 27, (-42)]	5.5×10^{-4} 4.9×10^{-4}
B	$10.811 \pm 7 \times 10^{-3}$	P [0; 3 -e-1, -e-1, -e-1, -9, (e+1, 6, e+1, e+1)] E [0; 9 e+1, -e-1, -6, -e-1, (21, -24, e+1)] E [x; 9 -e-1, e+1, 24, (-33)]	2.0×10^{-4} 5.5×10^{-5} 3.5×10^{-3}
C	$12.0107 \pm 8 \times 10^{-4}$	E [0; 9 e+1, -e-1, e+1, -e-1, e+1, e+1, -6, (e+1, -9, 6, -e-1, -e-1, -e-1)] E [x; 9 -e-1, 9, -9, -e-1, e+1, (-9, e+1)]	1.4×10^{-6} 6.8×10^{-4}
N	$14.0067 \pm 2 \times 10^{-4}$	E [x; 9 -e-1, -6, e+1, e+1, -e-1, 9, (-12)]	6.2×10^{-5}
O	$15.9994 \pm 3 \times 10^{-4}$	P [0; 3 -12, 6, 6, -e-1, (-e-1, 6, -9, e+1, e+1)]	9.0×10^{-7}
Ne	$20.1797 \pm 6 \times 10^{-4}$	P [0; 3 -1056, (1053)] E [x; 9 -9, -e-1, 42, (-42, e+1)]	2.3×10^{-5} 1.6×10^{-5}
Mg	$24.3050 \pm 6 \times 10^{-4}$	P [0; 3 15, -15, (-138, 135)] E [x; 9 -30, e+1, -18, (-e-1, 39)]	8.4×10^{-9} 8.7×10^{-5}
Si	$28.0855 \pm 3 \times 10^{-4}$	P [0; 3 9, -e-1, -27, -e-1, e+1, (15, e+1)] P [-x; 6 -e-1, e+1, e+1, e+1, -6, 18, (-e-1, -18, -e-1)]	2.7×10^{-4} 2.1×10^{-5}
S	$32.065 \pm 5 \times 10^{-3}$	P [0; 3 6, -27, -e-1, (e+1, 18)] P [-x; 6 -e-1, 33, (-33, -6, e+1)] E [x; 9 15, -12, (6, -18)]	3.1×10^{-4} 1.4×10^{-5} 2.7×10^{-4}
Cl	$35.453 \pm 2 \times 10^{-3}$	E [0; 12 -e-1, e+1, -12, -e-1, 6, (-6, e+1)]	9.1×10^{-5}
Ar	$39.948 \pm 1 \times 10^{-3}$	P [0; 3 e+1, 9, e+1, -9, e+1, (-e-1, 6, -e-1, -e-1, -9)] P [-x; 6 -6, e+1, -6, e+1, -12, (18, -e-1, -e-1)] E [0; 12 -e-1, 9, -e-1, e+1, -e-1, (-21, e+1, e+1)] E [x; 9 6, e+1, 348, (-e-1, -363)]	4.3×10^{-7} 6.8×10^{-4} 3.9×10^{-4} 2.0×10^{-5}
K [Outlier]	$39.0983 \pm 1 \times 10^{-4}$	P [0; 3 e+1, 6, e+1, 9, -e-1, e+1, -e-1, e+1, -e-1]	6.3×10^{-4}
Ca	$40.078 \pm 4 \times 10^{-3}$	P [-x; 6 -6, e+1, -6, -e-1, 6] E [0; 12 -e-1, 9, -e-1, -e-1, -e-1, (-21, e+1, e+1, e+1, e+1)] E [x; 9 6, e+1, 9, (-24, -e-1)]	1.1×10^{-3} 1.3×10^{-4} 8.0×10^{-4}
Ti	$40.078 \pm 1 \times 10^{-3}$	E [0; 12 -e-1, -e-1, -6, e+1, 45, (-51, e+1)] E [x; 9 e+1, e+1, -e-1, 9, (-23202, -e-1, 23184)]	1.2×10^{-5} 6.8×10^{-12}
V [Outlier]	$50.9415 \pm 1 \times 10^{-4}$	P [0; 3 e+1, -e-1, 12, e+1, -e-1, e+1, -e-1, e+1, -e-1]	4.7×10^{-4}
Cr	$51.9961 \pm 6 \times 10^{-4}$	P [0; 3 e+1, -e-1, 6, -9, -12, (12)] E [0; 12 -6, e+1, -e-1, e+1, -15, e+1, (-e-1, 9, -e-1)]	1.8×10^{-5} 3.4×10^{-5}
Fe	$55.845 \pm 2 \times 10^{-3}$	P [-x; 6 -15, e+1, -e-1, -30, (39)] E [0; 9 e+1, -69, 6, (54, -e-1)]	1.6×10^{-4} 2.4×10^{-4}
Ni	$58.6934 \pm 4 \times 10^{-4}$	P [-x; 6 -18, -e-1, e+1, -e-1, e+1, -e-1, (12, e+1)] E [0; 12 -6, -6, 9, -e-1, e+1, (-9)]	2.9×10^{-4} 3.9×10^{-4}
Cu	$63.546 \pm 3 \times 10^{-3}$	P [-x; 6 -42, -e-1, -15, (51, e+1)] E [x; 9 e+1, -e-1, -e-1, e+1, -75, (66)]	8.5×10^{-5} 7.5×10^{-5}
Zn	$65.38 \pm 2 \times 10^{-2}$	P [-x; 6 -78, e+1, (-e-1, 72)] E [0; 12 -9, 9, e+1, (-e-1, -12)] E [x; 9 e+1, -e-1, -12, e+1, (-21, 24, -e-1)]	3.4×10^{-3} 4.4×10^{-3} 9.8×10^{-5}
Ga	$69.723 \pm 1 \times 10^{-3}$	P [-x; 6 93, 6, e+1, (-105, -e-1)] E [x; 9 e+1, -e-1, e+1, 6, e+1, -e-1, e+1, -e-1, (-e-1, -15)]	3.3×10^{-5} 4.5×10^{-6}
Ge	$72.64 \pm 1 \times 10^{-2}$	P [-x; 6 39, -12, (-33)]	1.1×10^{-3}
Se	$78.96 \pm 3 \times 10^{-2}$	P [-x; 6 18, -9, (-15)]	2.1×10^{-3}
Br	$79.904 \pm 1 \times 10^{-3}$	E [0; 12 -24, -6, 6, (12)]	4.4×10^{-4}
Kr	$83.798 \pm 2 \times 10^{-3}$	P [-x; 6 12, e+1, -18, (6, -e-1, -6)] E [0; 12 -42, -e-1, e+1, e+1, (-e-1, 30)]	3.4×10^{-5} 2.3×10^{-4}

Table 3: Continued fraction representations of the atomic weights of polynuclidic elements, from Rb to Os, $x = 1.7918229$ is the phase shift, SD = standard deviation.

Element symbol	Mass \pm SD [u]	Continued fraction representation(s)	Numerical error [u]
Rb	$85.4678 \pm 3 \times 10^{-4}$	P [-x; 6 12, -9, 6, -6, -e-1, (-9, e+1)] E [0; 12 -63, e+1, -15, (66, -e-1)]	2.3×10^{-4} 2.1×10^{-4}
Sr	$87.62 \pm 1 \times 10^{-2}$	E [0; 12 -144, (132)]	5.8×10^{-3}
Zr	$91.224 \pm 2 \times 10^{-3}$	P [-x; 6 9, 21, e+1, (-36, -e-1)] E [0; 12 126, 6, (-144)]	2.8×10^{-4} 4.6×10^{-5}
Mo [Outlier]	$95.96 \pm 2 \times 10^{-2}$	E [0; 12 39, -e-1, e+1, -e-1, e+1]	5.7×10^{-2}
Ru	$101.07 \pm 2 \times 10^{-2}$	P [-x; 6 6, e+1, -9, -e-1, (6, -9)] E [0; 12 21, e+1, -e-1, (12, -45)]	3.5×10^{-3} 8.6×10^{-4}
Pd	$106.42 \pm 1 \times 10^{-2}$	P [-x; 6 6, 189, (6, -207)] E [0; 12 15, 6, -6, (-27)]	1.5×10^{-5} 8.2×10^{-3}
Ag [Outlier]	$107.8682 \pm 2 \times 10^{-4}$	E [0; 12 15, -e-1, -e-1, -6, e+1, -e-1, e+1, -e-1]	3.3×10^{-4}
Cd [Outlier]	$112.411 \pm 8 \times 10^{-3}$	E [0; 12 12, -15, e+1, -e-1, e+1, -e-1]	1.1×10^{-2}
In	$114.818 \pm 3 \times 10^{-3}$	P [-x; 6 6, -e-1, 6, -e-1, 6, e+1, (-24, e+1)]	2.5×10^{-3}
Sn	$118.710 \pm 7 \times 10^{-3}$	E [0; 12 9, 6, -e-1, e+1, -e-1, e+1, (-27)]	1.5×10^{-5}
Sb	$121.760 \pm 1 \times 10^{-3}$	P [-x; 6 e+1, e+1, -e-1, 30, 6, (-42, -e-1)] E [0; 12 9, -12, e+1, e+1, -e-1, e+1, (-e-1, -9, -e-1)]	8.6×10^{-5} 6.3×10^{-4}
Te [Outlier]	$127.60 \pm 3 \times 10^{-2}$	P [-x; 6 e+1, e+1, e+1, -e-1, e+1, -e-1, e+1]	7.2×10^{-2}
Xe [Outlier]	$131.293 \pm 6 \times 10^{-3}$	P [-x; 6 e+1, 6, e+1, -e-1, e+1, -e-1, e+1, -e-1]	1.1×10^{-2}
Ba	$137.327 \pm 7 \times 10^{-3}$	P [-x; 6 e+1, 21, 6, (-33, -e-1)] E [0; 12 6, 9, -9, e+1, (-18, -e-1)]	4.9×10^{-3} 8.9×10^{-4}
La	$138.90547 \pm 7 \times 10^{-5}$	P [-x; 6 e+1, 42, -e-1, -e-1, -6, (-42, e+1)]	3.1×10^{-5}
Ce	$140.116 \pm 1 \times 10^{-3}$	P [-x; 6 e+1, 132, 6, (-e-1, -144)] E [0; 12 6, 84, e+1, -e-1 (-102)]	5.8×10^{-4} 8.1×10^{-4}
Nd	$144.242 \pm 3 \times 10^{-3}$	E [0; 12 6, -9, e+1, -6, (9, -e-1, -12)]	8.9×10^{-4}
Sm	$150.36 \pm 2 \times 10^{-2}$	P [0; 6 -e-1, e+1, -e-1, e+1, -e-1, -e-1, (-18, 12, e+1, e+1)] P [-x; 6 e+1, -9, 12, (-e-1, -9)]	1.9×10^{-4} 2.8×10^{-3}
Eu	$151.964 \pm 1 \times 10^{-3}$	P [0; 6 -e-1, e+1, -e-1, e+1, e+1, -6, (-69, -e-1, 69)] E [0; 12 6, -e-1, e+1, 6, e+1, -24, (-e-1)]	8.3×10^{-6} 8.7×10^{-5}
Gd	$157.25 \pm 3 \times 10^{-2}$	P [0; 6 -e-1, e+1, -6, e+1, -e-1, e+1, (9, -e-1, -9)] P [-x; 6 e+1, -6, 6, -12, (-e-1, 6)]	9.7×10^{-4} 1.2×10^{-3}
Dy	$162.500 \pm 1 \times 10^{-3}$	P [-x; 6 e+1, -e-1, -e-1, -45, (e+1, 39)] E [0; 12 e+1, e+1, -6, -e-1, -9, -e-1, e+1, (-e-1, 9, -6)]	5.0×10^{-4} 7.4×10^{-4}
Er	$167.259 \pm 3 \times 10^{-3}$	P [0; 6 -e-1, e+1, 6, 6, e+1, (-18, -e-1)] P [-x; 6 e+1, -e-1, -27, 30, (-9)] E [0; 12 e+1, e+1, e+1, 63, (-e-1, -75, -e-1, -e-1)]	3.8×10^{-4} 1.3×10^{-4} 2.0×10^{-4}
Yb	$173.054 \pm 5 \times 10^{-3}$	P [0; 6 -e-1, 6, -e-1, e+1, (222, e+1, -234)] P [-x; 6 e+1, -e-1, 6, e+1, 9, e+1, (-e-1, -e-1, -21)] E [0; 12 e+1, 6, e+1, -e-1, e+1, -e-1, -e-1, (-18)]	1.5×10^{-5} 8.6×10^{-4} 1.0×10^{-3}
Lu	$174.9668 \pm 1 \times 10^{-4}$	P [0; 6 -e-1, 6, -6, 9, -e-1, e+1, e+1, (-15)] P [-x; 6 e+1, -e-1, 6, -e-1, e+1, -6, -e-1, e+1, -e-1, (-6, e+1)]	1.4×10^{-5} 7.3×10^{-5}
Hf	$178.49 \pm 2 \times 10^{-2}$	P [-x; 6 e+1, -e-1, e+1, -48, (-e-1, 42)]	1.9×10^{-3}
Ta	$180.94788 \pm 2 \times 10^{-5}$	E [0; 12 e+1, 21, -39, -6, (-e-1, 12)]	1.1×10^{-5}
W	$183.84 \pm 1 \times 10^{-2}$	P [0; 6 -e-1, 9, 9, -e-1, (e+1, -24, e+1)] P [-x; 6 e+1, -e-1, e+1, -e-1, e+1, -e-1, 15, (-21)] E [0; 12 e+1, 60, (-72, -e-1)]	3.4×10^{-3} 2.3×10^{-3} 1.8×10^{-3}
Re	$186.207 \pm 1 \times 10^{-3}$	P [0; 6 -e-1, 12, -6, -12, (e+1)] E [0; 12 e+1, -135, (123, -e-1)]	1.2×10^{-4} 5.8×10^{-4}
Os	$190.23 \pm 3 \times 10^{-2}$	E [0; 12 e+1, -21, -e-1, (9)]	3.6×10^{-3}

Table 4: Continued fraction representations of the atomic weights of polynuclidic and radioactive elements, from Ir to U, $x = 1.7918229$ is the phase shift, SD = standard deviation.

Element symbol	Mass \pm SD [u]	Continued fraction representation(s)	Numerical error [u]
Ir	$192.217 \pm 3 \times 10^{-3}$	E [0; 12 e+1, -15, -e-1, -9, -e-1, (e+1, 12)]	1.1×10^{-3}
Pt	$195.084 \pm 9 \times 10^{-3}$	P [0; 6 -e-1, 195, (-201, e+1)] E [0; 12 e+1, -12, e+1, 15, (-e-1, -e-1, -15)]	1.9×10^{-3} 4.2×10^{-4}
Hg	$200.59 \pm 2 \times 10^{-2}$	P [0; 6 -e-1, -21, 6, (e+1, 9)]	1.6×10^{-3}
Tl	$204.3833 \pm 2 \times 10^{-4}$	P [0; 6 -e-1, -12, 6, -e-1, e+1, e+1] E [0; 12 e+1, -6, -12, 6, e+1, -e-1, (-e-1)] E [x; 12 -e-1, e+1, -e-1, -12, -27, (27, e+1)]	1.9×10^{-4} 5.4×10^{-5} 7.9×10^{-5}
Pb	207.2 ± 0.1	P [0; 6 -e-1, -9, 6, (12, -15, e+1)] E [0; 12 e+1, -6, 6, (27, -39, -e-1)] E [x; 12 -e-1, e+1, -6, e+1, -e-1, e+1, -e-1 (-6)]	1.4×10^{-3} 8.4×10^{-5} 6.8×10^{-2}
Bi [Outlier]	$208.98040 \pm 1 \times 10^{-5}$	E [0; 12 e+1, -6, e+1, -9, -e-1, -e-1, 6, e+1, -e-1, e+1, -e-1]	1.8×10^{-5}
Pa	$231.03588 \pm 2 \times 10^{-5}$	E [x; 12 -e-1, 6, -6, -e-1, -e-1, -18, -6, (e+1, e+1, e+1, 12)]	1.2×10^{-6}
Th	$232.03806 \pm 2 \times 10^{-5}$	E [0; 12 e+1, -e-1, e+1, e+1, -9, e+1, e+1, -9, e+1, -e-1 (6, -e-1, -e-1, -e-1, -e-1)] E [x; 12 -e-1, 6, -12, -e-1, 6, 6, -e-1 (9, e+1, e+1, e+1, -27)]	1.7×10^{-5} 5.0×10^{-7}
U	$238.02891 \pm 3 \times 10^{-5}$	E [0; 12 e+1, -e-1, e+1, -6, e+1, e+1, -e-1, -15, (-e-1, -e-1, 9)]	1.8×10^{-5}

the corresponding continued fraction representations and the numerical errors. As it can be seen, no outlier is present.

Moreover, many continued fractions show the effect of successively canceling denominators. For instance, the continued fraction representation for Be, as calculated by the computer is: P [-x; 3 | e+1, -e-1, e+1, -e-1, -e-1, e+1, -6, -6, 12, (-6, -360, ...)]. The denominators in brackets are not required to obtain a mass value inside the interval "atomic mass \pm SD". Through a minimal manipulation, we obtain a zero sum of all denominators and the free link, without significantly changing the value of the fraction: P [-x; 3 | e+1, -e-1, e+1, -e-1, -e-1, e+1, -6, -6, 12, (-9, 6)]. As this procedure can be applied in a similar way to all elements, we demonstrate this and opted to express all continued fractions as a zero sum. Only redundant denominators (given in brackets) were manipulated to achieve the zero sums.

In a second step, the so-adjusted model was tested against the remaining 64 chemical elements. Only eight outliers were found (K, V, Mo, Ag, Cd, Te, Xe, Bi [radioactive]). Tables 2 to 4 show the results; for outliers, the best possible continued fraction is displayed (not as a zero sum), and it can be seen that in most cases the atomic mass is reproduced with a numerical error very little higher than the standard deviation.

4 Conclusions

The relative atomic masses are now the second data set that can be described by the bipolar model of oscillations in a chain system. In total, 10% outliers were found which might be attributed to the fact that the isotopic compositions of these outlier elements as found here on Earth are not good representatives for the true mean compositions when considering samples from distant parts of the universe.

Anyway, it is important to note that all mononuclidic elements can be described perfectly by this model.

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