Black Hole Structure in Schwarzschild Coordinates

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In the analysis of the interior region of both stationary and rotating black holes, it is customary to switch to a set of in-falling coordinates to avoid problems posed by the coordinate singularity at the event horizon. I take the view here that to understand the physics of black holes, we need to restrict ourselves to bookkeeper or Schwarzschild coordinates of a distant observer if we are to derive measurable properties. I show that one can derive interesting properties of black holes that might explain some of the observational evidence available without the necessity of introducing further ad hoc conjectures.

1 The Schwarzschild black hole

Birkhoff's theorem [1] assures us that for any non-rotating spherically symmetric distribution of matter, the gravitational effect on any test mass is solely due to whatever mass lies closer to the center of symmetry. This allows us to infer what happens inside the event horizon, by comparing a hypothetical distribution of matter that is identical but with all mass outside the point of interest removed, with that of (say) a collapsing star. Making no further assumptions, let the density at any point inside the event horizon be $\rho_{initial}(r)$ where r is the reduced distance from the center of symmetry. Now consider a test mass m at a distance r_p from the center of a black hole, but inside an event horizon of radius r_{eh}. Now compare this in a thought experiment with a similar test mass m with an identical distribution of mass but with all mass at a distance greater than r_p set to zero. Clearly, our test mass in both cases will head towards the origin, but so too will every other particle that makes up the mass distribution $\rho_{initial}$ but is not yet at the origin. In our thought experiment, the spherical mass distribution will become increasingly compressed with our test particle riding on the collapsing surface. A point in time will be reached in our thought experiment where the mass enclosed by the collapsing surface becomes a black hole in its own right. To a distant observer, the test mass can then never in a finite time cross the event horizon formed by this newly created black hole. This will be true in our thought experiment, and thus must be equally true in the original black hole. At this point in time, to have formed a black hole, we must have

$$r' = \frac{2Gm'}{c^2},$$

where m' is the total mass enveloped by a surface with a radius of r'. As the test mass was at an arbitrary distance from the origin, this will become equally true for every point within the event horizon of the original black hole. As a consequence, the eventual distribution of mass must be such that for all r less than r_{eh}

$$r = \frac{2G}{c^2}\int_0^r 4\pi r^2 \rho(r)\,dr$$

with $\rho(r)$ being the eventual mass distribution function. This relation is satisfied by

$$\rho(r) = \frac{c^2}{8\pi G r^2}.$$

The black hole has a density inside its outer event horizon that is inversely proportional to the square of the (reduced) distance from the origin.

2 The Kerr black hole

In Boyer-Lindquist coordinates [2], there is a spherical inner event horizon for a Kerr black hole [3]; also in the limit of zero rotation, these coordinates, not surprisingly, reduce to Schwarzschild coordinates. The curvature tensors at the surfaces of the (inner) event horizons seem very different but are in fact identical. To understand this, see section 3, below. Therefore, in Boyer-Lindquist coordinates, both the Kerr black hole and the Schwarzschild black hole, have identical gravitational fields at their respective event horizons and therefore identical internal structure as a consequence of the holographic principal [4]. Let us clarify this: they are identical in Boyer-Lindquist coordinates but not from a viewing platform here on earth. From here, the spinning black hole will have an event horizon that appears as an oblate spheroid.

3 Comparing infinities

Consider two men with infinite piles of money, but with one having additional small piles of money. Which is the richer? Clearly they are equal. This was an example using scalar quantities, but let us extend this to vectors. Two vectors each have an infinite component but one of them has additional non-zero components at right angles. Which is the larger? Convert to polar coordinates to see that again they are equal. The same is true for tensors. Consider first two tensors each with one large and equal (but not infinite) component, but one tensor having small non-zero additional components (the other having all other components at zero). Now scale all components to the size of the largest by dividing through by the largest component. Then let the largest component increase without limit. The largest component remains at unity whilst all other components approach zero. Thus we are left with two identical tensors.

4 Consequences

With this solution, every point inside a black hole is sitting on a local event horizon, where, to a distant observer, time stands still, and so no two points inside a black hole will ever move closer together. Consequently, the black hole must be truly rigid in a way that no other physical object can be; it then follows directly from consideration of the Ehrenfest paradox [5] that the angular velocity of a black hole can never increase — it is fixed at birth. When a black hole increases in mass, it must also increase in angular momentum in order to keep the angular velocity constant up to the maximum speed of rotation set by the periphery being unable to exceed the speed of light, which thus limits the ultimate size a black hole can grow to. We thus formulate a new fifth law of black hole dynamics: **it is never possible to change the angular velocity of a black hole**. Rigidity means that black holes cannot be deformed by any outside processes, so it is difficult to comprehend a process that will allow black holes to coalesce. Ignoring this problem, it can be seen that the limitations of the laws of black hole dynamics severely restrict the possible outcomes whenever two black holes meet.

5 Observational justification

No definitive experimental evidence to confirm these results is produced at this time, but observe that with stellar black holes we would expect that at creation they would have to have a typical mass range of 3–30 solar masses. One would also expect them to be created with high spin due to the conservation of the angular momentum of the collapsing (spinning) star. This limits the maximum mass that a stellar mass black hole could ever grow to. This may apparently be justified by current observations but leaves the unanswered question of how supermassive black holes are ever formed. I suggest that although black holes may never merge, neutron stars can, and with counter-rotating neutron stars, this can give rise to a stellar mass black hole with exceptionally low spin. These black holes are not so limited in growth as normal stellar mass black holes and could grow to become supermassive. All measurements to date suggest that the spin rates for supermassive black holes are extremely high; that is they are approaching the end of their growth phase.

6 Counterarguments

In general relativity, any convenient system of coordinates can be used and is valid [6]. I suggest that as far as observational data goes, Schwarzschild coordinates are the most appropriate as these alone can correlate with observations. Two different coordinate systems — Schwarzschild and infalling coordinates — give very different results in the vicinity of a black hole horizon and yet we know that they must describe the same reality for different observers. Understanding the relation between these two results is therefore crucial to accepting the validity of this result. Consider twins, one of whom descends towards the event horizon of a black hole. We accept that one, the traveler, will appear to be slowing down due the gravitational effect on the passage of time. However, the traveler sees the opposite: time for the stay at home twin seems to speed up. There is nothing fictitious or illusory about this — if the traveler returns home, he will certainly be younger than his twin. Depending upon how close to the event horizon he travels, he could be many days or years younger. In principal, he could be 100,000 years younger and still not have crossed the event horizon. (Apart from the technical difficulties, we are assuming eternal life.) So when does the traveler cross the event horizon. By his own watch, it may be just a few hours but for the stay at home twin it will be eternity. So the traveler does arrive at the real singularity at the center, but for the stay at home twin, this is after the Universe has ceased to exist. Both are real but only one produces a measurable outcome.

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