New Possible Physical Evidence of the Homogeneous Electromagnetic Vector Potential for Quantum Theory.
Idea of a Test Based on a G. P. Thomson-like Arrangement

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It is suggested herein a test able to reveal the physical evidence of the homogeneous electromagnetic vector potential field in relation to quantum theory. We take into consideration three reliable entities as main pieces of the test: (i) influence of a potential vector of the de Broglie wavelength (ii) a G. P. Thomson-like experimental arrangement and (iii) a special coil designed to create a homogeneous vector potential. The alluded evidence is not connected with magnetic fluxes surrounded by the vector potential field lines, rather it depends on the fluxes which are outside of the respective lines. Also the same evidence shows that the tested vector potential field is a uniquely defined physical quantity, free of any adjusting gauge. So the phenomenology of the suggested quantum test differs from that of the macroscopic theory where the vector potential is not uniquely defined and allows a gauge adjustment. Of course, we contend that this proposal has to be subsequently subjected to adequate experimental validation.

1 Introduction

The physical evidence of the vector potential $\vec{A}$ field, distinctly of electric and/or magnetic local actions, is known as Aharonov-Bohm-effect (A-B-eff). It aroused scientific discussions for more than half a century (see [1–8] and references). As a rule in the A-B-eff context, the vector potential is curl-free field, but it is non-homogeneous (**n-h**) i.e. spatially non-uniform. In the same context, the alluded evidence is connected quantitatively with magnetic fluxes surrounded by the lines of $\vec{A}$ field. In the present paper we try to suggest a test intended to reveal the possible physical evidence of a homogeneous (**h**) $\vec{A}$ field. Note that in both **n-h** and **h** cases herein, we take into consideration only fields which are constant in time.

The announced test has as constitutive pieces three reliable entities (**E**) namely:

E_1: The fact that a vector potential $\vec{A}$ field changes the values of the de Broglie wavelength λ^{dB} for electrons. ■

E_2: An experimental arrangement of the G. P. Thomson type, able to monitor the mentioned λ^{dB} values. ■

E_3: A feasible special coil designed so as to create a **h**-$\vec{A}$ field. ■

Accordingly, on the whole, the test has to put together the mentioned entities and, consequently, to synthesize a clear verdict regarding the alluded evidence of a **h**-$\vec{A}$ field.

Experimental setup of the suggested test is detailed in the next Section 2. Essential theoretical considerations concerning the action of a **h**-$\vec{A}$ field are given in Section 3. The above-noted considerations are fortified in Section 4 by a set of numerical estimations for the quantities aimed to be measured through the test. Some concluding thoughts regarding a possible positive result of the suggested test close the main body of the paper in Section 5. Constructive and computational details regarding the special coil designed to generate a **h**-$\vec{A}$ field are presented in the Appendix.

2 Setup details of the experimental arrangement

The setup of the suggested experimental test is pictured and detailed below in Fig. 1. It consists primarily of a G. P. Thomson-like arrangement partially located in an area with a **h**-$\vec{A}$ field. The alluded arrangement is inspired by some illustrative images [9, 10] about G. P. Thomson's original experiment and it disposes in a straight line of the following elements: electron source, electron beam, crystalline grating, and detecting screen. An area with a **h**-$\vec{A}$ field can be obtained through a certain special coil whose constructive and computational details are given in the above-mentioned Appendix at the end of this paper.

The following notes have to be added to the explanatory records accompanying Fig. 1.

Note 1: If in Fig. 1 the elements 7 and 8 are omitted (i.e. the sections in special coil and the lines of **h**-$\vec{A}$ field) one obtains a G. P. Thomson-like arrangement as it is illustrated in the said references [9, 10]. ■

Note 2: Surely the above mentioned G. P. Thomson-like arrangement is so designed and constructed that it can be placed inside of a vacuum glass container. The respective container is not shown in Fig. 1 and it will leave out the special coil. ■

Note 3: When incident on the crystalline foil, the electron beam must ensure a coherent and plane front of de Broglie waves. Similar ensuring is required [11] for

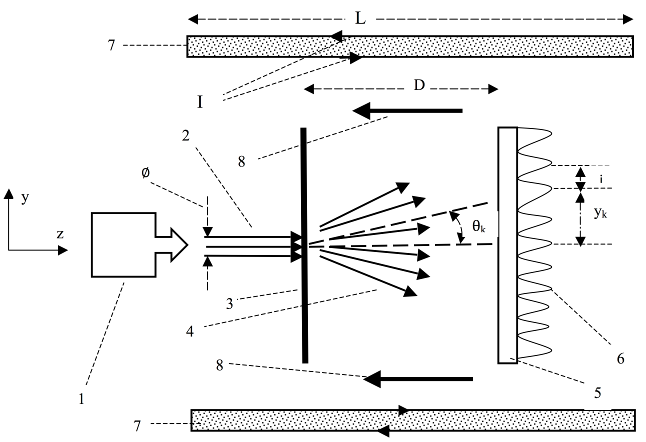

Fig. 1: Plane section in the image of suggested experimental setup, accompanied by the following explanatory records. 1 – Source for a beam of mono-energetic and parallel moving electrons; 2 –Beam of electrons in parallel movements; 3 – Thin crystalline foil as diffraction grating; 4 – Diffracted electrons; 5 – Detecting screen; 6 – Fringes in the plane section of the diffraction pattern; 7 – Sections in the special coil able to create a **h**-$\vec{A}$ field; 8 – **h**-$\vec{A}$ field ; ϕ = the width of the electron beam with $\phi \gg a$ (a = interatomic spacing in the crystal lattice of the foil -3); θ_k = diffraction angle for the k-th order fringe ($k = 0, 1, 2, 3, \ldots$); y_k = displacement from the center line of the k-th order fringe; i = interfringe width = $y_{k+1} - y_k$; D = distance between crystalline foil and screen ($D \gg \phi$); L = length of the special coil ($L \gg D$) ; I = intensity of current in wires of the coil.

optical diffracting waves at a classical diffraction grating. ■

Note 4: In Fig. 1 the detail 6 displays only the linear projections of the fringes from the diffraction pattern. On the whole, the respective pattern consists in a set of concentric circular fringes (diffraction rings). ■

3 Theoretical considerations concerning action of a h-$\vec{A}$ field

The leading idea of the above-suggested test is to search for possible changes caused by a **h**-$\vec{A}$ field in the diffraction of quantum (de Broglie) electronic waves. That is why we begin by recalling some quantitative characteristics of the diffraction phenomenon.

The most known scientific domain wherein the respective phenomenon is studied regards optical light waves [11]. In the respective domain, one uses as the main element the so-called *diffraction grating* i.e. a piece with a periodic structure having slits separated each by a distance a and which diffracts the light into beams in different directions. For a light normally incident on such an element, the grating equation (condition for intensity maximums) has the form: $a \cdot \sin\theta_k = k\lambda$, where $k = 0, 1, 2, \ldots$ In the respective equation, λ denotes the light's wavelength and θ_k is the angle at which the diffracted light has the k-th order maximum. If the diffraction pattern is received on a detecting screen, the k-th order maximum appears on the screen in the position y_k given by the relation $\tan\theta_k = (y_k/D)$, where D denotes the distance between the screen and the grating. For the distant screen assumption, when $D \gg y_k$, the following relation holds: $sin\theta_k \approx \tan\theta_k \approx (y_k/D)$. Then, with regard to the mentioned assumption, one observes that the diffraction pattern on the screen is characterized by an interfringe distance $i = y_{k+1} - y_k$ given through the relation

$$i = \lambda \frac{D}{a} . \qquad (1)$$

Note the fact that the above quantitative aspects of diffraction have a generic character, i.e. they are valid for all kinds of waves including de Broglie ones. The respective fact is presumed as a main element of the experimental test suggested in the previous section. Another main element of the alluded test is the largely agreed upon idea [1–8] that the de Broglie electronic wavelength λ^{dB} is influenced by the presence of a $\vec{A}$ field. Based on the two afore-mentioned main elements the considered test can be detailed as follows.

In the experimental setup depicted in Fig. 1 the crystalline foil 3 having interatomic spacing a plays the role of a diffraction grating. In the same experiment, on the detecting screen 5 it is expected to appear a diffraction pattern of the electrons. The respective pattern would be characterized by an interfringe distance i^{dB} definable through the formula $i^{dB} = \lambda^{dB} \cdot (D/a)$. In that formula, D denotes the distance between the crystalline foil and the screen, supposed to satisfy the condition $D \gg \phi$), where ϕ represents the width of the incident electron beam. In the absence of a **h**-$\vec{A}$ field, the λ^{dB} of a non-relativistic electron is known to satisfy the following expression:

$$\lambda^{dB} = \frac{h}{p_{kin}} = \frac{h}{mv} = \frac{h}{\sqrt{2m\mathcal{E}}} . \qquad (2)$$

In the above expression, h is Planck's constant while p_{kin}, m, v and $\mathcal{E}$ denote respectively the kinetic momentum, mass, velocity, and kinetic energy of the electron. If the alluded energy is obtained in the source of the electron beam (i.e. piece 1 in Fig. 1) under the influence of an accelerating voltage U, one can write $\mathcal{E} = e \cdot U$ and $p_{kin} = mv = \sqrt{2meU}$.

Now, in connection with the situation depicted in Fig. 1, let us look for the expression of the electrons' characteristic λ^{dB} and respectively of $i^{dB} = \lambda^{dB} \cdot (D/a)$ in the presence of a **h**-$\vec{A}$ field. Firstly, we note the known fact [6] that a particle with the electric charge q and the kinetic momentum $\vec{p}_{kin} = m\vec{v}$ in a potential vector $\vec{A}$ field acquires an additional (add) momentum, $\vec{p}_{add} = q\vec{A}$, so that its *effective* (eff) momentum is $\vec{P}_{eff} = \vec{p}_{kin} + \vec{p}_{add} = m\vec{v} + q\vec{A}$. Then for the electrons (with $q = -e$) supposed to be implied in the experiment depicted in Fig. 1, one obtains the effective (eff) quantities

$$\lambda^{dB}_{eff}(A) = \frac{h}{mv + eA} ; \quad i^{dB}_{eff}(A) = \frac{hD}{a\,(mv + eA)} . \qquad (3)$$

Further on, we have to take into account the fact that the **h**-$\vec{A}$ field acting in the experiment presented before is generated by a special coil whose plane section is depicted by the elements 7 from Fig. 1. Then from the relation (10) established in the Appendix, we have $A = \mathcal{K} \cdot I$, where $\mathcal{K} = \frac{\mu_0 N}{2\pi} \cdot \ln\left(\frac{R_2}{R_1}\right)$. Add here the fact that in this experiment $mv = \sqrt{2meU}$. Then for the effective interfringe distance i_{eff}^{dB} of the diffracted electrons, one finds

$$i_{eff}^{dB}(A) = i_{eff}^{dB}(U, I) = \frac{hD}{a\left(\sqrt{2meU} + e\mathcal{K}I\right)}, \quad (4)$$

respectively

$$\frac{1}{i_{eff}^{dB}(U, I)} = f(U, I) = \frac{a\sqrt{2me}}{hD}\sqrt{U} + \frac{ae\mathcal{K}}{hD} I. \quad (5)$$

4 A set of numerical estimations

The verisimilitude of the above-suggested test can be fortified to some extent by transposing several of the previous formulas into their corresponding numerical values. For such a transposing, we firstly will appeal to numerical values known from G. P. Thomson-like experiments. So, as regarding the elements from Fig. 1, we quote the values $a = 2.55 \times 10^{-10}$ m (for a crystalline foil of copper) and $D = 0.1$ m. As regarding U, we take the often quoted value: $U = 30$ kV. Then the kinetic momentum of the electrons will be $p_{kin} = mv = \sqrt{2meU} = 9.351 \times 10^{-23}$ kg m/s. The additional (add) momentum of the electron, induced by the special coil, is of the form $p_{add} = e\mathcal{K} \times I$ where $\mathcal{K} = \frac{\mu_0 N}{2\pi} \times \ln\left(\frac{R_2}{R_1}\right)$. In order to estimate the value of $\mathcal{K}$, we propose the following practically workable values: $R_1 = 0.1$ m, $R_2 = 0.12$ m, $N = 2\pi R_1 \times n$ with $n = 2 \times 10^3$ m^{-1} = number of wires (of 1 mm in diameter) per unit length, arranged into two layers. With the well known values for e and μ_0 one obtains $p_{add} = 7.331 \times 10^{-24}$(kg m C^{-1}) $\cdot I$ (with C = Coulomb).

For wires of 1 mm in diameter, by changing the polarity of the voltage powering the coil, the current I can be adjusted in the range $I \in (-10\ to\ +10)A$. Then the effective momentum $\vec{P}_{eff} = \vec{p}_{kin} + \vec{p}_{add}$ of the electrons shall have the values within the interval (2.040 to 16.662) $\times 10^{-23}$ kg m/s. Consequently, due to the above mentioned values of a and D, the effective interfringe distance i_{eff}^{dB} defined in (4) changes in the range (1.558 to 12.725) mm, respectively its inverse from (5) has values within the interval (78.58 to 641.84) m^{-1}. Then it results that in this test the **h**-$\vec{A}$ field takes its magnitude within the interval $A \in (-4.5\ ,\ +4.5) \times 10^{-4}$ kg m C^{-1}, (C = Coulomb).

Now note that in the absence of the **h**-$\vec{A}$ field (i.e. when $I = 0$) the interfange distance i^{dB} specific to a simple G. P. Thomson experiment has the value $i^{dB} = \frac{hD}{a\sqrt{2meU}} =$ 2.776 mm. Such a value is within the range of values of i_{eff}^{dB} characterizing the presence of the **h**-$\vec{A}$ field. This means that the quantitative evaluation of the mutual relationship of i_{eff}^{dB} versus I, and therefore the testing evidence of a **h**-$\vec{A}$ field can be done with techniques and accuracies similar to those of the G. P. Thomson experiment.

5 Some concluding remarks

The aim of the experimental test suggested above is to verify a possible physical evidence for the **h**-$\vec{A}$ field. Such a test can be done by comparative measurements of the interfringe distance i_{eff}^{dB} and of the current I. Additionally it must examine whether the results of the mentioned measurements verify the relations (4) and (5) (particularly according to (5) the quantity $(i_{eff}^{dB})^{-1}$ is expected to show a linear dependence of I). If the above outcomes are positive, one can notice the fact that a **h**-$\vec{A}$ field has its own characteristics of physical evidence. Such a fact leads in one way or another to the following remarks (**R**):

R$_1$: The physical evidence of the **h**-$\vec{A}$ field differs from the one of the **n-h-** $\vec{A}$ field which appears in the A-B-eff. This happens because, by comparison to the illustrations from [12], one can see that: *(i)* by changing the values of **n-h-** $\vec{A}$, the diffraction pattern undergoes a simple translation on the screen, without any modification of interfringe distance, while *(ii)* according to the relations (4) and (5) a change of **h**-$\vec{A}$ (by means of current I) does not translate the diffraction pattern but varies the value of associated interfringe distance i_{eff}^{dB}. The mentioned variation is similar to that induced [12] by changing (through accelerating the voltage U) the values of kinetic momentum $\vec{p}_{kin} = m\vec{v}$ for electrons. ■

R$_2$: There is a difference between the physical evidence (objectification) of the **h**-$\vec{A}$ and the **n-h**-$\vec{A}$ fields in relation with the magnetic fluxes surrounded or not by the field lines. The difference is pointed out by the following subsequent aspects:

(i) On the one hand, as it is known from the A-B-eff, in case of the **n-h**-$\vec{A}$ field, the corresponding evidence depends directly on magnetic fluxes surrounded by the $\vec{A}$ field lines.

(ii) On the other hand, the physical evidence of the **h**-$\vec{A}$ field is not connected to magnetic fluxes surrounded by the field lines. But note that due to the relations (4) and (5), the respective evidence appears to be dependent (through the current I) on magnetic fluxes not surrounded by the field lines of the **h**-$\vec{A}$. ■

R$_3$: A particular characteristic of the physical evidence forecasted above for the **h**-$\vec{A}$ regards the macroscopic versus quantum difference concerning the uniqueness (gauge freedom) of the vector potential field. As is known, in macroscopic situations [13, 14] the vector potential $\vec{A}$ field is not uniquely defined (i.e. it has a gauge freedom). In such situations, an arbitrary $\vec{A}$ field allows a gauge fixing (adjustment), without any alteration of macroscopic relevant variables/equations (particularly of those involving the magnetic field $\vec{B}$). So two distinct vector potential fields $\vec{A}$ and $\vec{A}^1$ have the same macroscopic actions (effects) if $\vec{A}^1 = \vec{A} + \nabla f$, where f is an arbitrary gauge functions. On the other hand, in a quantum context, a **h**-$\vec{A}$ has not any gauge freedom. This is because if this test has positive results, two fields like $\mathbf{h} - \vec{A} = A \cdot \vec{k}$ and $\mathbf{h} - \vec{A}^1 = \mathbf{h} - \vec{A} + \nabla f$ are completely distinct if $f = (-z \cdot A \cdot \vec{k})$, where $\vec{k}$ denotes the unit vector of the Oz axis. So we can conclude that, with respect to the **h**-$\vec{A}$ field, the quantum aspects differ fundamentally from those aspects originating in a macroscopic consideration. Surely, such a fact (difference) and its profound implications have to be approached in subsequently more elaborated studies. ■

Postscript

As presented above, the suggested test and its positive results appear as purely hypothetical things, despite the fact that they are based on essentially reliable entities (constitutive pieces) presented in the Introduction. Of course, we hold that a true confirmation of the alluded results can be done by the action of putting in practice the whole test. Unfortunately, at the moment I do not have access to material logistics able to allow me an effective practical approach of the test in question. Thus I warmly appeal to the concerned experimentalists and researchers who have adequate logistics to put in practice the suggested test and to verify its validity.

Appendix: Constructive and computational details for a special coil able to create a h-$\vec{A}$ field

The case of an ideal coil

An experimental area of macroscopic size with the **h**-$\vec{A}$ field can be realized with the aid of a special coil whose constructive and computational details are presented below. The announced details are improvements of the ideas promoted by us in an early preprint [15].

The basic element in designing the mentioned coil is the **h**-$\vec{A}$ field generated by a rectilinear infinite conductor carrying a direct current. If the conductor is located along the axis Oz and the current has the intensity I, the Cartesian components (written in SI units) of the mentioned **h**-$\vec{A}$ field are given [16] by the following formulas:

$$A_x(1) = 0\,, \qquad A_y(1) = 0\,, \qquad A_z(1) = -\mu_0 \frac{I}{2\pi} \ln r\,. \quad (6)$$

Here r denotes the distance from the conductor of the point where the hct-$\vec{A}$ is evaluated and where μ_0 is the vacuum permeability.

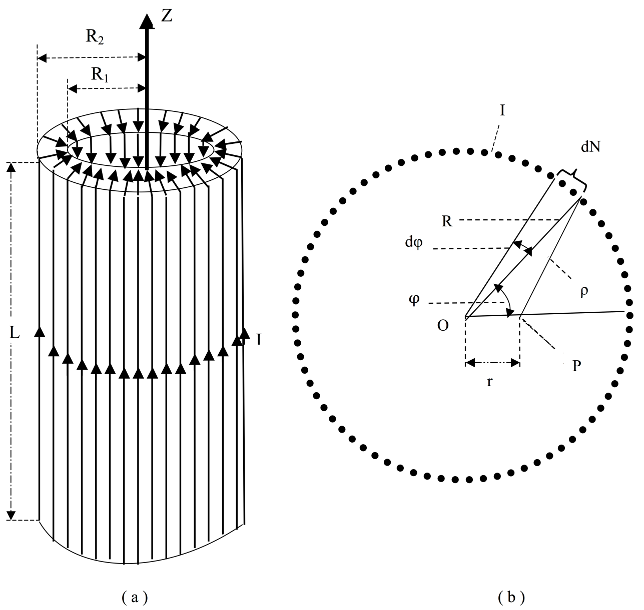

Fig. 2: Schemes for an annular special coil.

Note that formulas (6) are of ideal essence because they describe the **h**-$\vec{A}$ field generated by an infinite (ideal) rectilinear conductor. Further onwards, we firstly use the respective formulas in order to obtain the **h**-$\vec{A}$ field generated by an ideal annular coil. Later on we will specify the conditions in which the results obtained for the ideal coil can be used with fairly good approximation in the characterization of a real (non-ideal) coil of practical interest for the experimental test suggested and detailed in Sections 2,3 and 4.

The mentioned special coil has the shape depicted in Fig. 2-(a) (i.e. it is a toroidal coil with a rectangular cross section). In the respective figure the finite quantities R_1 and R_2 represent the inside and outside finite radii of the coil while $L \to \infty$ is the length of the coil. For evaluation of the **h**-$\vec{A}$ generated inside of the mentioned coil let us now consider an array of infinite rectilinear conductors carrying direct currents of the same intensity I. The conductors are mutually parallel and uniformly disposed on the circular cylindrical surface with the radius R. The conductors are also parallel with Oz as the symmetry axis. In a cross section, the considered array is disposed on a circle of radius R as can be seen in Fig. 2b. On the respective circle, the azimuthal angle φ locates the infinitesimal arc element whose length is $Rd\varphi$. On the respective arc there was placed a set of conductors whose number is $dN = \left(\frac{N}{2\pi}\right) d\varphi$, where N represents the total number of conductors in the whole considered array. Let there be an observation point P situated at distances r and ρ from the center O of the circle respectively from the infinitesimal arc (see the Fig. 2b). Then, by taking into account (6), the z-component of the **h**-$\vec{A}$ field generated in P by the dN conductors is given by relation

$$A_z(dN) = A_z(1)\,dN = -\mu_0 \frac{NI}{4\pi^2} \ln \rho \cdot d\varphi\,, \quad (7)$$

where $\rho = \sqrt{(R^2 + r^2 - 2Rr\cos\varphi)}$. Then all N conductors will generate in the point P a $\mathbf{h}$-$\vec{A}$ field whose value A is

$$A = A_z(N) = -\mu_0 \frac{NI}{8\pi^2} \int_0^{2\pi} \ln\left(R^2 + r^2 - 2Rr\cos\varphi\right) \cdot d\varphi\,. \quad (8)$$

In calculating the above integral, the formula (4.224-14) from [17] can be used. So, one obtains

$$A = -\mu_0 \frac{NI}{2\pi} \ln R\,. \quad (9)$$

This relation shows that the value of A does not depend on r, i.e. on the position of P inside the circle of radius R. Accordingly this means that inside the respective circle, the potential vector is homogeneous. Then starting from (9), one obtains that the inside space of an ideal annular coil depicted in Fig. 2a is characterized by the $\mathbf{h}$-$\vec{A}$ field whose value is

$$A = \mu_0 \frac{NI}{2\pi} \ln\left(\frac{R_2}{R_1}\right). \quad (10)$$

From the ideal coil to a real one

The above-presented coil is of ideal essence because their characteristics were evaluated on the basis of an ideal formula (6). But in practical matters, such as the experimental test proposed in Sections 2 and 3, one requires a real coil which may be effectively constructed in a laboratory. That is why it is important to specify the main conditions in which the above ideal results can be used in real situations. The mentioned conditions are displayed here below.

On the geometrical sizes: In a laboratory, it is not possible to operate with objects of infinite size. Thus we must take into account the restrictive conditions so that the characteristics of the ideal coil discussed above to remain as good approximations for a real coil of similar geometric form. In the case of a finite coil having the form depicted in the Fig. 2a, the alluded restrictive conditions impose the relations $L \gg R_1$, $L \gg R_2$ and $L \gg (R_2 - R_1)$. If the respective coil is regarded as a piece in the test experiment from Fig. 1, indispensable are the relations $L \gg D$ and $L \gg \phi$.

About the marginal fragments: On the whole, the marginal fragments of coil (of width $(R_2 - R_1)$) can have disturbing effects on the Cartesian components of $\vec{A}$ inside the the space of practical interest. Note that, on the one hand, in the above-mentioned conditions $L \gg R_1$, $L \gg R_2$ and $L \gg (R_2 - R_1)$ the alluded effects can be neglected in general practical affairs. On the other hand, in the particular case of the proposed coil the alluded effects are also diminished by the symmetrical flows of currents in the respective marginal fragments.

As concerns the helicity: The discussed annular coil is supposed to be realized by winding a single piece of wire. The spirals of the respective wire are not strictly parallel to the symmetry axis of the coil (the Oz axis) but they have a certain helicity (corkscrew-like path). Of course, the alluded helicity has disturbing effects on the components of $\vec{A}$ inside the coils. Note that the mentioned helicity-effects can be diminished (and practically eliminated) by using an idea noted in another context in [18]. The respective idea proposes to arrange the spirals of the coil in an even number of layers, with the spirals from adjacent layers having equal helicity but of opposite sense.

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