Dimension of Physical Space

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Each vector of state has its own corresponing element of the CayleyDickson algebra. Properties of a state vector require that this algebra was a normalized division algebra. By the Hurwitz and Frobenius theorems maximal dimension of such algebra is 8. Consequently, a dimension of corresponding complex state vectors is 4, and a dimension of the Clifford set elements is 4×4. Such set contains 5 matrices — among them — 3-diagonal. Hence, a dimension of the dot events space is equal to 3+1.

Further I use CayleyDickson algebras [1, 2]:

Let

$$1, \mathrm{i}, \mathrm{j}, \mathrm{k}, \mathrm{E}, \mathrm{I}, \mathrm{J}, \mathrm{K}$$

be basis elements of a 8-dimensional algebra Cayley (*the octavians algebra*) [1, 2]. A product of this algebra is defined the following way [1]:

1) For every basic element e:

$$\mathrm{ee} = -1;$$

2) If u_1, u_2, v_1, v_2 are real number then

$$(u_1 + u_2\mathrm{i})(v_1 + v_2\mathrm{i}) = (u_1v_1 - v_2u_2) + (v_2u_1 + u_2v_1)\,\mathrm{i}.$$

3) If u_1, u_2, v_1, v_2 are numbers of shape $w = w_1 + w_2\mathrm{i}$ (w_s, and $s \in \{1, 2\}$ are real numbers, and $\overline{w} = w_1 - w_2\mathrm{i}$) then

$$(u_1 + u_2\mathrm{j})(v_1 + v_2\mathrm{j}) = (u_1v_1 - \overline{v}_2u_2) + (v_2u_1 + u_2\overline{v}_1)\,\mathrm{j} \quad (1)$$

and $\mathrm{ij} = \mathrm{k}$.

4) If u_1, u_2, v_1, v_2 are number of shape $w = w_1 + w_2\mathrm{i} + w_3\mathrm{j} + w_4\mathrm{k}$ (w_s, and $s \in \{1, 2, 3, 4\}$ are real numbers, and $\overline{w} = w_1 - w_2\mathrm{i} - w_3\mathrm{j} - w_4\mathrm{k}$) then

$$(u_1 + u_2\mathrm{E})(v_1 + v_2\mathrm{E}) = (u_1v_1 - \overline{v}_2u_2) + (v_2u_1 + u_2\overline{v}_1)\,\mathrm{E} \quad (2)$$

and

$$\mathrm{iE} = \mathrm{I},$$
$$\mathrm{jE} = \mathrm{J},$$
$$\mathrm{kE} = \mathrm{K}.$$

Therefore, in accordance with point 2) the real numbers field (**R**) is extended to the complex numbers field (**R**), and in accordance with point 3) the complex numbers field is expanded to the quaternions field (**K**), and point 4) expands the quaternions fields to the octavians field (**O**). This method of expanding of fields is called *a Dickson doubling procedure* [1].

If

$$u = a + b\mathrm{i} + c\mathrm{j} + d\mathrm{k} + A\mathrm{E} + B\mathrm{I} + C\mathrm{J} + \mathrm{K}$$

with real a, b, c, d, A, B, C, D then a real number

$$\|u\| := \sqrt{u\overline{u}} = \left(a^2 + b^2 + c^2 + d^2 + A^2 + B^2 + C^2 + D^2\right)^{0.5}$$

is called *a norm* of octavian u [1].

For each octavians u and v:

$$\|uv\| = \|u\|\,\|v\|\,. \quad (3)$$

Algebras with this conditions are called *normalized algebras* [1, 2].

Any 3+1-vector of a probability density can be represented by the following equations in matrix form [4, 5]

$$\rho = \varphi^{\dagger}\varphi\,,$$
$$j_k = \varphi^{\dagger}\beta^{[k]}\varphi$$

with $k \in \{1, 2, 3\}$.

There $\beta^{[k]}$ are complex 2-diagonal 4×4-matrices of Clifford's set of rank 4, and φ is matrix columns with four complex components. The light and colored pentads of Clifford's set of such rank contain in threes 2-diagonal matrices, corresponding to 3 space coordinates in accordance with Dirac's equation. Hence, a space of these events is 3-dimensional.

Let $\rho(t, \mathbf{x})$ be a probability density of event $A\,(t, \mathbf{x})$, and

$$\rho_c(t, \mathbf{x}|t_0, \mathbf{x}_0)$$

be a probability density of event $A\,(t, \mathbf{x})$ on condition that event $B\,(t_0, \mathbf{x}_0)$.

In that case if function $q(t, \mathbf{x}|t_0, \mathbf{x}_0)$ is fulfilled to condition:

$$\rho_c(t, \mathbf{x}|t_0, \mathbf{x}_0) = q(t, \mathbf{x}|t_0, \mathbf{x}_0)\rho(t, \mathbf{x}), \quad (4)$$

then one is called *a disturbance function B* to *A*.

If $q = 1$ then B does not disturbance to A.

A conditional probability density of event $A\,(t, \mathbf{x})$ on condition that event $B\,(t_0, \mathbf{x}_0)$ is presented as:

$$\rho_c = \varphi_c^{\dagger}\varphi_c$$

like to a probability density of event $A\,(t, \mathbf{x})$.

Let

$$\varphi = \begin{bmatrix} \varphi_{1,1} + \mathrm{i}\varphi_{1,2} \\ \varphi_{2,1} + \mathrm{i}\varphi_{2,2} \\ \varphi_{3,1} + \mathrm{i}\varphi_{3,2} \\ \varphi_{4,1} + \mathrm{i}\varphi_{4,2} \end{bmatrix}$$

and

$$\varphi_c = \begin{bmatrix} \varphi_{c,1,1} + i\varphi_{c,1,2} \\ \varphi_{c,2,1} + i\varphi_{c,2,2} \\ \varphi_{c,3,1} + i\varphi_{c,3,2} \\ \varphi_{c,4,1} + i\varphi_{c,4,2} \end{bmatrix}$$

(all $\varphi_{r,s}$ and $\varphi_{c,r,s}$ are real numbers).

In that case octavian

$$u = \varphi_{1,1} + \varphi_{1,2}\mathrm{i} + \varphi_{2,1}\mathrm{j} + \varphi_{2,2}\mathrm{k} + \varphi_{3,1}\mathrm{E} + \\ + \varphi_{3,2}\mathrm{I} + \varphi_{4,1}\mathrm{J} + \varphi_{4,2}\mathrm{K}$$

is called *a Caylean* of φ. Therefore, octavian

$$u_c = \varphi_{c,1,1} + \varphi_{c,1,2}\mathrm{i} + \varphi_{c,2,1}\mathrm{j} + \varphi_{c,2,2}\mathrm{k} + \varphi_{c,3,1}\mathrm{E} + \\ + \varphi_{c,3,2}\mathrm{I} + \varphi_{c,4,1}\mathrm{J} + \varphi_{c,4,2}\mathrm{K}$$

is Caylean of φ_c.

In accordance with the octavian norm definition:

$$\|u_c\|^2 = \rho_c\,, \\ \|u\|^2 = \rho\,. \tag{5}$$

Because the octavian algebra is a division algebra [1, 2] then for each octavians u and u_c there exists an octavian w such that

$$u_c = wu.$$

Because the octavians algebra is normalized then

$$\|u_c\|^2 = \|w\|^2\,\|u\|^2\,.$$

Hence, from (4) and (5):

$$q = \|w\|^2\,.$$

Therefore, in a 3+1-dimensional space-time there exists an octavian-Caylean for a disturbance function of any event to any event.

In order to increase a space dimensionality the octavian algebra can be expanded by a Dickson doubling procedure:

Another 8 elements should be added to basic octavians:

$$\mathrm{z}_1,\ \mathrm{z}_2, \mathrm{z}_3,\ \mathrm{z}_4,\ \mathrm{z}_5,\ \mathrm{z}_6,\ \mathrm{z}_7,\ \mathrm{z}_8$$

such that:

$$\begin{aligned} \mathrm{z}_2 &= \mathrm{i}\mathrm{z}_1, \\ \mathrm{z}_3 &= \mathrm{j}\mathrm{z}_1, \\ \mathrm{z}_4 &= \mathrm{k}\mathrm{z}_1, \\ \mathrm{z}_5 &= \mathrm{E}\mathrm{z}_1, \\ \mathrm{z}_6 &= \mathrm{I}\mathrm{z}_1, \\ \mathrm{z}_7 &= \mathrm{J}\mathrm{z}_1, \\ \mathrm{z}_8 &= \mathrm{K}\mathrm{z}_1, \end{aligned}$$

and for every octavians u_1, u_2, v_1, v_2:

$$(u_1 + u_2\mathrm{z}_1)\,(v_1 + v_2\mathrm{z}_1) = (u_1v_1 - \overline{v}_2u_2) + (v_2u_1 + u_2\overline{v}_1)\,\mathrm{z}_1$$

(here: if $w = w_1 + w_2\mathrm{i} + w_3\mathrm{j} + w_4\mathrm{k} + w_5\mathrm{E} + w_6\mathrm{I} + w_7\mathrm{J} + w_8\mathrm{K}$ with real w_s then $\overline{w} = w_1 - w_2\mathrm{i} - w_3\mathrm{j} - w_4\mathrm{k} - w_5\mathrm{E} - w_6\mathrm{I} - w_7\mathrm{J} - w_8\mathrm{K}$).

It is a 16-dimensional Cayley-Dickson algebra.

In accordance with [3], for any natural number z there exists a Clifford set of rank 2^z. In considering case for $z = 3$ there is Clifford's seven:

$$\underline{\beta}^{[1]} = \begin{bmatrix} \beta^{[1]} & 0_4 \\ 0_4 & -\beta^{[1]} \end{bmatrix},\ \underline{\beta}^{[2]} = \begin{bmatrix} \beta^{[2]} & 0_4 \\ 0_4 & -\beta^{[2]} \end{bmatrix}, \tag{6}$$

$$\underline{\beta}^{[3]} = \begin{bmatrix} \beta^{[3]} & 0_4 \\ 0_4 & -\beta^{[3]} \end{bmatrix},\ \underline{\beta}^{[4]} = \begin{bmatrix} \beta^{[4]} & 0_4 \\ 0_4 & -\beta^{[4]} \end{bmatrix}, \tag{7}$$

$$\underline{\beta}^{[5]} = \begin{bmatrix} \gamma^{[0]} & 0_4 \\ 0_4 & -\gamma^{[0]} \end{bmatrix}, \tag{8}$$

$$\underline{\beta}^{[6]} = \begin{bmatrix} 0_4 & 1_4 \\ 1_4 & 0_4 \end{bmatrix},\ \underline{\beta}^{[7]} = \mathrm{i}\begin{bmatrix} 0_4 & -1_4 \\ 1_4 & 0_4 \end{bmatrix}. \tag{9}$$

Therefore, in this seven five 4-diagonal matrices (7) define a 5-dimensional space of events, and two 4-antidiagonal matrices (9) defined a 2-dimensional space for the electroweak transformations.

It is evident that such procedure of dimensions building up can be continued endlessly. But in accordance with the Hurwitz theorem* and with the generalized Frobenius theorem† a more than 8-dimensional Cayley-Dickson algebra does not a division algebra. Hence, there in a more than 3-dimensional space exist events such that a disturbance function between these events does not hold a Caylean. I call such disturbance *supernatural*.

Therefore, supernatural disturbance do not exist in a 3-dimensional space, but in a more than 3-dimensional space such supernatural disturbance act.

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References

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*Every normalized algebra with unit is isomorphous to one of the following: the real numbers algebra **R**, the complex numbers algebra **C**, the quaternions algebra **K**, the octavians algebra **O** [1].

†A division algebra can be only either 1 or 2 or 4 or 8-dimensional [2].