Motion-to-Motion Gauge for the Electroweak Interaction of Leptons

Felix Tselnik

Ben-Gurion University of the Negev, P.O.B. 653 Beer-Sheva 84105, Israel. E-mail: tselnik@ee.bgu.ac.il

Comprised of rods and clocks, a reference system is a mere intermediary between the motion that is of interest in the problem and the motions of auxiliary test bodies the reference system is to be gauged with. However, a theory based on such reference systems might hide some features of this actual motion-to-motion correspondence, thus leaving these features incomprehensible. It is therefore desirable to consider this correspondence explicitly, if only to substantiate a particular scheme. To this end, the very existence of a (local) top-speed signal is shown to be sufficient to explain some peculiarities of the weak interaction using symmetrical configurations of auxiliary trajectories as a means for the gauge. In particular, the unification of the electromagnetic and weak interactions, parity violation, $SU(2)_L \times U(1)$ group structure with the values of its coupling constants, and the intermediate vector boson are found to be a direct consequence of this gauge procedure.

1 Introduction

We shall apply a direct motion-to-motion gauge to the electroweak interactions. In so doing, our sole tool is the counting of the numbers of the top-speed signal oscillations in order to arrange test particles in special configurations of their trajectories possessing a particular symmetry. First we shortly review the basics of the motion-to-motion measurements (Sec. 1). Second we introduce compact symmetric configurations suitable for the gauge (Secs. 2, 3). Third we apply this gauge to construct a regular lattice suitable to unambiguously transport the (integer) value of the electric charge unit over the space-time and find that parity violating weak interaction is a necessary component of this (Sec. 4). In the Sec. 5, we describe some other applications of the gauge. The burden of the argument is as follows. The cube-star arrangement of electron and positron trajectories allows for the construction of a regular gauging lattice only under some conditions. In particular, it turns out that the particle charges must be altered, so as to let them leave the gauging cell intact notwithstanding the residual uncertainty pertinent to the gauge. Aiming at the finest lattice, we have found its minimal cell size required for the gauge. This size defines the range which is free to introduce an additional ("weak") interaction having no effect on the gauge itself. We can use this additional interaction to realize the necessary charge conversion (the electrons into the neutrinos). However, the top-speed signal oscillation numbers define not a single but two trajectories, and we have to provide the weak interaction with the property to select one of them. This interaction must depend on spin and contains parity violation as a necessary ingredient of electric charge gauge and transport.

2 Motion gauged with auxiliary motion

Ultimately, mechanics is based on comparing a trajectory of the body which is of interest in a problem to the trajectories of test bodies that are measuring force in the related points. Applications of the scheme also require a means to follow motions of the body. Otherwise, one could never be sure (in the absence of instant communication) that at a later moment it is the same body rather than a similar one. To this end, a top speed signal must exist in the scheme for not to loose the object upon its possible accelerations. In the conventional version, the required comparison is being carried out via an intermediate reference system comprised of rods and clocks. However, finally the real devices designed to measure the trajectories of bodies are to be gauged with the use of some standard motions. Thus, narrow light rays or free fall are used to determine whether or not the rod is rectilinear, and clock readings are to agree with astronomical and/or atomic processes. So, a reference system comprised of rods and clocks is just an intermediary in the comparison of one motion to another. One could guess that this intermediary might either add some features of its own to the gauge or, on the contrary, hide some important information in cases when the standard procedure is used beyond its traditional scope. It is therefore desirable to dispense with any intermediary so as to gauge motions directly, if only to obtain a criterion of suitability of the intermediary. To this end, many authors attempted to define the structure of space-time solely in terms of trajectories. In particular, natural topologies have been proposed to conform to the special role of the time axis [1-5]. A drawback of some of these approaches is the premise of a four-dimensional differentiable manifold for the space-time a topology to be introduced in. (However, the very idea to construct open subsets out of all trajectories, rather than of only free ones, and to deduce space-time properties, e.g., its dimension, out of their intersections was already considered [2].) Furthermore, topology is a too general structure, and practice requires more details. Thus, in order to define metrics based on a subset of trajectories, it was proposed to eliminate rigid rods (see, e.g., [6] and references therein); still clocks, at least in the form of affine parameters, seemed unavoidable.

But then, trajectories cannot be taken as primary entities in a theory to be developed from scratch, even though they might be considered directly observed (contrary to empty space-time!). It must be explained why just the trajectories of bodies are of particular interest rather than arbitrary changes in nature. Already in the Einstein's picture of the space-time, the event is defined as the intersection point of world lines of particles or of light pulses. This approach was further developed by Marzke and Wheeler [7]. In actual fact, primary definitions must be substantiated by the intended application of the theory, and therefore they must arise directly from the desirable statement of the problem, that is, to be axioms rather than hypotheses. Of course, a general uncertain concept of event cannot be basic for technical use that aims at a method to provide predictions, and therefore mechanics offers a particular kind of event, namely, contact (collision). The idea is to leave aside the question as to what results from the contact, assuming instead that nothing will happen, provided the contact does not occur. Whereas the notion "material point", i.e. "infinitesimal body", requires a preliminary concept of metric, the concept of contact is self-contained: the contact either exists or not. Only such problems are allowed for the analysis in mechanics. To this end, we define trajectories merely as a means to predict whether or not the contact of interest will occur in a particular problem upon detecting only some auxiliary contacts to be appropriately selected. Each trajectory possesses its own linear order, since it is introduced just for step-to-step predictions. This order introduces the topology of a simple arc on the trajectory to provide the basis for emerging structures. For this to be possible, we have to prepare a set of auxiliary (standard) trajectories in order to encode final, initial and intermediary states (contacts) solely in terms of these. Yet the choice of standard trajectories needs an explanation of its own. Can we dispense with geodesics? What properties of these are actually necessary for the scheme? Might these properties be deduced from meager information?

A reliable concept to begin with is the communication of bodies with a top speed signal (which is necessary anyway to follow motions of the body, while ensuring its unambiguous identification). Top speed signal can be defined independently of any general concept of speed. Consider two bodies A and B, the problem being stated of whether or not they will come into contact. Let A contact with an auxiliary body X which then contacts with B. Among these X's we look for the first to reach B, whatever ways they go. Only the order of these contacts matters, e.g., an X might put a mark on B, so that all the X's, except the first, find B already marked. It is this top-speed body that will serve as the signal. Let further there be a triple contact (B,X,Y), such that Y is, in turn, the first to meet A afterwards. If the contact (A,B) occurs, the number of these oscillations (multiple "echo") is infinite, corresponding to the so-called Zeno sequence. Otherwise, the last oscillation would occur before (A,B), and then this last oscillating body would not be a top-speed one. We could reverse this argument, suggesting that tending the number of oscillations to infinity could be used to predict the occurrence of (A,B), if in the absence of this contact the number of oscillations were not infinite as well. In conventional notions this implies infinite time of oscillations, but we cannot introduce space-time terms a priori aiming at a solution solely in terms of contacts. For this purpose, let us provide in our scheme some auxiliary X, such that (B,X) does occur. Then we can state that (A,B) occurs, provided the ratio of the (infinite) numbers of oscillations between A and B to that between B and X tends to a finite limit. For this to be actually used, one begins to count oscillation numbers at a moment, and the value of the ratio is determined as its limit when the number of oscillations as measured for the contacts of the signal with, e.g., A tends to infinity. This limit does not depend on the moment it starts from or on the reciprocal positions of the signals coming to A from B and X within one oscillation cycle [8]. We emphasize, that only local existence of the top speed signals is important (no cm/sec and no free trajectories to appear from the outset!). The counting of such ratios will be our sole tool in the sequel; however in some cases also finite oscillation numbers are suitable. (Finite numbers of top-speed signal oscillations were already used to compare distances [6, 7].)

We *define* space-time R not as something pre-existing but rather as an envelope of combination of all possible trajectories, the occurrence of contacts between which can be determined by means of top speed signal. In fact a (single!) reference system does exist in this approach, comprising an appropriate subset of trajectories – X's – chosen under the following conditions: i. Any pair of them either have no common contact or have only one (at least locally – with respect to their own topology); ii. If some trajectory A has a contact with some other trajectory B, there exists some X with the triple contact (A,B,X); iii. Although X's might have multiple contacts with trajectories not belonging to the subset, any pair of such contacts could be separated to insert a sequence of the top-speed signal oscillations for each of them. Moreover, just multiple contacts determine dynamics in terms of X's upon using an additional subset of "charged" test bodies, the trajectories of which are also encoded via X's. Under these conditions, infinite oscillation numbers provide the space-time with differential topology as a means to clearly separate possible contacts. Moreover, space-like hyper-surfaces S and the projections of trajectories thereon ("paths") might also be defined in these terms. The condition for a so defined contact scheme to represent any finite arrangement of the projections with their mutual intersections, while excluding any unnecessary for this purpose subsets, defines the topology of S. In the framework of traditional topology [1-5], dim R=ind R=1, but S is *not* a sub-space of R, though the set of its neighborhoods can be induced by trajectories from R: Each neighborhood of a point of the trajectory defines the corresponding neighborhood of this point in S, consisting of all its points connected with top-speed signal to this neighborhood of the trajectory. However, S is not a topological image of R, and its dimension is to be defined independently of R.

Unlike the trajectory itself, its projection on S need not be a simple arc, and it might have various self-intersections. However, according to the Nöbeling-Pontryagin embedding theorem, any n-dimensional metrizable topological space with a countable base of open subsets can be embedded into the (2n+1)-dimensional Euclidean space. In fact, the theorem states that in this space its n-dimensional subspaces are free to intersect or not, while a space of a lower dimension might be too "dense" forcing some of them to intersect necessarily, and a space of a higher dimension would add nothing to this freedom. This is particularly clear for n=1: In only two dimensions a line cannot pass a closed boundary line without crossing it, whereas in three dimensions this is always possible (traffic interchange, say), while the fourth dimension would be redundant. For a finite (and even for a countable) array of trajectories its map in S has n=1, so dim S = 3. It follows that each contact might be encoded with only three X's. This fact could never be understood unless the space-time is defined as a union of actual trajectories [2] rather than being accepted in advance. In this version, the extension of bodies should itself be regarded just as their property to obstruct some trajectories or their paths.

Upon focusing in this presentation only on some features of motion-to-motion measurements relevant for weak interactions, further analysis of geometrical properties of the space-time that arise from this approach is left for discussion elsewhere.

3 Compact arrangements of trajectories

Consider a set of trajectories with their common contact. We can choose some triple of them to provide a basis, so that any other member of the set can be specified with its oscillation numbers ratios with those of the basis. *However, there exists the twin to any so defined trajectory.* Indeed, let us consider for the sake of visualization such decomposition in the rest frame of one of bodies of the basis. Then the other two define a surface, e.g., a plane, and the dual to a trajectory is its mirror image with respect to this plane. In order to specify the trajectory uniquely, we have to add some internal degree of freedom, a doublet, in close analogy to the spin variable.

Among all such sets we select a particular subset – *spheres* – that is defined as follows. It is convenient to introduce an additional body for the center of the sphere. The sphere is comprised of a finite number of trajectories with equal oscillation numbers with respect to the center body, that is, their ratio equals 1 for each pair of the sphere members. The sphere might be viewed as a compact arrangement of trajectories which are specified solely by their mutual angles. While the ratios of the oscillations numbers between the members themselves to those between them and the center are in general different, we can define for each trajectory its neighbors as those for which this ratio is maximal. The spheres might be used to specify a condition for forces that are permitted to act on bodies over their trajectories. If we accept that everything in sight must be described in terms of signals, we have to define forces in these terms as well. Such a rule must be independent of the space-time point, i.e. to require the force not to alter oscillation number ratios.

Let us take the sphere consisting of A, B, C and use conventional variables in order to reveal the familiar forces that satisfy this condition. The ratio ${}_A\Gamma_{BC}$ of the oscillation numbers between the bodies A and B to that of A and C is [8]:

$$ {}_A\Gamma_{BC} = \lim_{n_{AB}\to\infty} \frac{n_{AB}}{n_{AC}} = \frac{\ln\left(u_{Ai}u_{Bi} + \sqrt{(u_{Ai}u_{Bi})^2 - 1}\right)}{\ln\left(u_{Ai}u_{Ci} + \sqrt{(u_{Ai}u_{Ci})^2 - 1}\right)} \tag{1} $$

where u_{Ai} and others are the four-velocities of the bodies, and summation over i is implied. Evidently the ratio ${}_A\Gamma_{BC}$ will not change under a force if the scalar products of the four-velocities do not.

Consider the electromagnetic force, F_{ik}. Then for velocity of light c, the charges and masses of the bodies e and m:

$$ du_{Ai} = \frac{e}{mc} F_{ik} u_{Ak} ds_A \,. \tag{2} $$

Hence:

$$ d(u_{Ai}u_{Bi}) = \frac{e_A}{m_A c} F_{ik} u_{Ak} u_{Bi} ds_A + \frac{e_B}{m_B c} F_{ki} u_{Bi} u_{Ak} ds_B \,. \tag{3} $$

But $ds_A = ds_B$ since A and B are the members of a sphere. Then, $d(u_{Ai}u_{Bi}) = 0$, if $F_{ik} = -F_{ki}$ and also $e_A/m_A = e_B/m_B$. Apart from electromagnetic field, anti-symmetry of which can be expressed, in the connected space-time manifold, via potentials as $F_{ik} = \partial A_i/\partial x_k - \partial A_k/\partial x_i$, a field might also include commutators $[A_i, A_k]$ if the components of the potential do not commute. (Quanta of these fields must be bosons, whereas fermions would require only anti-commutators.) We can then reverse the argument to state that only fields preserving the ratios of the oscillation numbers can appear in the theory as bosons. Moreover, propagation of the fields can also be expressed via appropriate contact schemes by means of Green functions [9]. To complete the method, we stay in need of a condition, in terms of contacts, for the constancy of charge and mass in (3) everywhere, and in order to find this condition we need a means to translate these values over the whole space- time. For this purpose consider a particular subset of spheres, in which the oscillation number ratios with its neighbors are the same for each member of the sphere. Such a sphere will be called a star. In three-dimensional space only five stars are possible. These are known as Platonic solids. Note that the definition of star doesn't imply that its bodies move uniformly.

4 Star-based gauge of electric charge

Eventually, all that is actually measured in experiments relates to motion under electromagnetic force of, e.g., particles, products of their interactions etc. It is therefore this force proportional to the electric charge of the particle it acts upon that must be gauged in the first place. The value of this charge is commonly accepted to be the same everywhere. Still a method is needed to detect this identity in terms of contacts. We want to use the stars for gauging electric charge without any intermediary. To this end, we have to specify the charge not only locally but also to develop some motion-to-motion gauge for its translation to any point the body of interest might occupy along its trajectory. This should be based on the symmetries of stars, which can easily be expressed in terms of equality of some oscillation numbers.

Suppose all the members of a star (in the gauge procedure we will call them particles) are electrically charged with equal e/m values and move only under mutual electromagnetic interaction. In any star comprised of identical particles they move along straight lines repelling each other, and the particles cannot reach the center. Moreover, the trajectories might become curved, provided some of the particles differ from others in charges or masses, and for this reason they miss the center as well. But in a symmetry-based charge gauging procedure, it is the disparity of charges and masses that is detected as a star symmetry breaking. If the particles miss the star center anyway, we cannot be sure that the symmetry is not broken just at the closer vicinity of the center, still being observed far from it. We must therefore use for the gauge only neutral as a whole stars with equal numbers of positive and negative particles. Of all Platonic solids, the center is reached only in the cube with opposite signs of the charges between the tetrahedrons the cube is comprised of. Although in the cube star the particles keep moving along straight lines (even if the absolute values of their charges differ between its two tetrahedrons, while being identical within each of them), the symmetry will be broken because the tetrahedrons are being differently accelerated by mutual attraction.

Starting the counting of the oscillation numbers between the particle and an introduced, for the sake of simplicity, imaginary central particle at a moment before the contact, we detect the symmetry breaking if these numbers, as measured at the center, differ at least by one oscillation. In the limit of the smallest star size, defining the highest precision of electromagnetic gauge, one tetrahedron nears the center over only one oscillation while another — over two oscillations. At a smaller initial radius the second oscillation has no time to occur. Since we detect only integer numbers of signal oscillations, the values of charge to be detected must be discrete. Indeed, suppose that the charges differ by some infinitesimal value. However close to the center the symmetry was detected, we cannot be sure that asymmetry could still be detected upon continuing the counting, since nothing is being registered in between the neighboring contacts. So, we are able to detect with our method only discrete values of charge (and/or mass), hence a minimum value of charge e can be registered, the next value being $2e$. Now, whereas in a given external field acceleration depends on e/m, for a case of interacting particles it depends on e^2/m, and in order to observe the symmetry of a star the masses and the absolute values of the charges of its particles are to be equal.

The particles of the tetrahedron having the charge $2e$ experience smaller acceleration as compared to the tetrahedron having the charge e. The related symmetry breaking gauge condition — one extra oscillation — is reached at some final radius r_{min}. Smaller radii are not involved in the gauge procedure, leaving this region free to introduce a new interaction under our general trend to regard possible in mechanics everything described with the motion-to-motion schemes. In the next section, we will find such an interaction to be necessary for the gauge itself.

5 Application to electroweak interactions

With the basic cube star at hand we proceed to develop the whole regular lattice, along which the value of the electric charge can be transported to a point of the trajectory in question. Along our general lines, the regular lattice must comprise elementary cube-star cells. For this purpose, we use some particles of one star, after they pass its center, as a seed for the next star. According to Sec. 2, just three stars are sufficient to completely define their next star. As a matter of fact, this simple picture cannot be trusted, because deviation of the charge at radii that are smaller than those involved in the gauge for the finest lattice might either prevent electrons and positrons from escaping the star against the exit potential barrier formed by the attraction of the other members of the star or to have a final energy differing from what is needed as the input energy of the next star. Even small charge deviation are important, since the energy near the minimal radius is typically much higher than the energies of the particles at the star entrance, and momentum conservation would yield large final fluctuations there; moreover, the deviation might be collected over a sequence of stars. In particular, even low level radiation that has a small effect on the matching of incoming to outgoing energies in a single star might cause large deviations over long sequences.

Radiation is negligible in stars comprising large bodies, and our gauge is quite feasible in this case. Long sequences might then be directly arranged, in which the outgoing bodies are directly used in the next star, since their velocity return to the initial values being decelerated after passing the star center. This is impossible in the limit of elementary particles. If, however, a new — "weak" — interaction converts the charge of the particles to zero at the smallest radius of the symmetry detection, the gauge becomes independent of radiation. Being constrained to radii that are smaller than those involved in the electric charge gauge, such a conversion doesn't obstruct the gauge. Over a larger scale, one could consider stars consisting, e.g., of ions, which can change their charge via charge exchange or stripping. We, however consider the limiting case of the finest lattice comprised of stars having the smallest possible size, still allowing the motion-to-motion charge gauge. Then only elementary particles might participate in the lattice, and an elementary neutral particle must complete the collection. It is just in this extreme case the weak interaction with its parity violation appears.

In order to form the lattice, this newborn particle, the "neutrino", has then to be converted back into the electron of the next star. This can happen under the same weak interaction, provided the neutrino collides with the anti-neutrino to create the electron-positron pair. Though never observed in practice, such a limiting process, as well as the charged star itself — with its eight particles' simultaneous collision, should be considered a feature of our formal language to question nature, providing as concepts for theories so also rules for experiments. We need therefore to introduce intermediate cube stars made up of only neutrinos and anti-neutrinos and positioned at the corners of the charged cube. These neutral stars are "blind" in the sense that their symmetry cannot be detected electromagnetically. Still, suitability of the whole regular lattice might be detected, provided the following charged star is found to repeat the original symmetry. So, we need a doublet consisting of electrons and neutrinos to prepare a regular lattice. The doublet corresponds to two charge states that convert one into another at the vertices, suggesting the SU(2) group for transformations of the inner (charge) space in the gauge field theory, but now it appears as an indispensable mechanism to realize the regular lattice by means of the motion-to-motion gauge.

Our next step is to define all the members of the next star starting with the trajectories that are continuations of its three preceding stars. For any star, it is sufficient to take a basis of three trajectories to determine all the others. In order to visualize this construction, it is convenient to proceed using the conventional picture, that is, to imagine the cube star in its center-of-mass (CM) reference system as eight particles at its vertices moving toward the center with equal velocities v ($\beta = v/c$). Let us take, for example, the trajectory A and its neighbors B and C as the basis for the star to be constructed and choose the line of A for the x-axis, the line through the cube center parallel to the line between the vertices B and C for the y-axis, and the z-axis as orthogonal to these two. We have to find D as the third neighbor of A. In so chosen coordinates, the decomposition coefficients of the basis are:

$$\left.\begin{aligned} &\beta_{Ax} = \beta, \quad \beta_{Ay} = \beta_{Az} = 0 \\ &\beta_{Bx} = \beta_{Cx} = \frac{\beta}{3}, \quad \beta_{By} = -\beta_{Cy} = \beta\sqrt{\frac{2}{3}} \\ &\beta_{Bz} = \beta_{Cz} = \beta\,\frac{\sqrt{2}}{3} \end{aligned}\right\}, \tag{4}$$

and those of D:

$$\beta_{Dx} = \frac{\beta}{3}, \quad \beta_{Dy} = 0, \quad \beta_{Dz} = -\beta\,\frac{2\sqrt{2}}{3}. \tag{5}$$

But we know from Sec. 2 that via the oscillation number ratios counting — our sole means — the basis A, B, C defines actually two trajectories, that is, there exists another trajectory E besides D with the same ratios of the oscillations. In order to determine the coordinates of E, we transform (4) and (5) to the reference system, in which A is at rest, to find E there as the mirror image of D, and then to return to the CM system to find the coordinates of E there. From the relativistic transformation formulae for velocities, we find:

$$\left.\begin{aligned} &\beta'_{Ax} = \beta'_{Ay} = \beta'_{Az} = 0 \\ &\beta'_{Bx} = \beta'_{Cx} = -\beta\,\frac{2}{3-\beta^2} \\ &\beta'_{By} = -\beta'_{Cy} = \beta\,\frac{\sqrt{6(1-\beta^2)}}{3-\beta^2} \\ &\beta'_{Bz} = \beta'_{Cz} = \beta\,\frac{\sqrt{2(1-\beta^2)}}{3-\beta^2} \\ &\beta'_{Dx} = -\beta\,\frac{2}{3-\beta^2}, \quad \beta'_{Dy} = 0 \\ &\beta'_{Dz} = -\beta\,\frac{2\sqrt{2(1-\beta^2)}}{3-\beta^2} \end{aligned}\right\}. \tag{6}$$

Using (6), we obtain the mirror image E of D trajectory with respect to the plane defined by the transformed B and C velocities as:

$$\left.\begin{aligned} &\beta'_{Ex} = -\beta\,\frac{2(3-5\beta^2)}{(3-\beta^2)^2} \\ &\beta'_{D'y} = 0 \\ &\beta'_{D'z} = \beta\,\frac{2\sqrt{2(1-\beta^2)}\,(3+\beta^2)}{(3-\beta^2)^2} \end{aligned}\right\}. \tag{7}$$

Upon back transforming (7) to the laboratory reference frame, we find finally:

$$\left.\begin{aligned} \beta_{Ex} &= \frac{\beta\left((3-\beta^2)^2 - 2(3-5\beta^2)\right)}{(3-\beta^2)^2 - 2\beta^2(3-5\beta^2)} \\ \beta_{Ez} &= \frac{2\sqrt{2}\,\beta\,(1-\beta^2)(3+\beta^2)}{(3-\beta^2)^2 - 2\beta^2(3-5\beta^2)} \end{aligned}\right\}. \tag{8}$$

Though in our example (D placed between B and C) E moves in the same xz plane as D, (8) does not define a vertex of the cube. Even the absolute values of the D and E velocities differ already in the order of β, though their oscillations numbers with respect to the basis are the same. So, upon constructing the next star we must introduce some additional — internal — degree of freedom, helicity, to define just D but not E by means of choosing a particular order in the basis A, B, C. Mathematically, this is similar to the spin variable, the spin being directed either in the direction of the momentum of the particle or oppositely. So, parity violation turns out to be a necessary property of the motion-to-motion gauge, since only the projection of spin on the momentum direction conveys the necessary information to select the appropriate trajectory out of the two. In the electron/positron cube star, the opposite sense particles belong to different tetrahedrons, and of the two particles on each main diagonal of the cube one is the electron while another – the positron. Therefore the order of the basis for the electron is seen as reversed from its opposite positron, and the product of parity and charge conjugation is the same for both (CP conservation).

We are able now to use parity in the electric charge gauge as performed solely with photon oscillations counting. In the symmetric cube star magnetic field is zero on the trajectories, hence there is no orbital angular momentum, and only the spin of the particle defines its total angular momentum. Then our electric charge gauge fails to distinguish between particles with left and right orientations, letting both enter the weak interaction zone. In order to define the fourth trajectory, the neutrinos must be provided with a definite, e.g., left, helicity, and therefore the charged star must generate only these. To this end, the weak interaction must be spin-dependent to create only left-handed neutrinos (and right-handed antineutrinos) in the collision of the particles in the charged star. It is sufficient to consider only the electron and its neutrino, the argument being similar for their antiparticles. In the blind star the neutrino will turn into the electron with the same projection of its spin in virtue of the angular momentum conservation.

For the left-handed electrons in the charged star, the function of the weak interaction is dual. On the one hand, the weak interaction for the left-handed electrons possesses its own dynamics, since it should match the output and input energies in the sequence of charged stars over the whole lattice. On the other hand, its intensity defines charge conversion probability, scaling as $\gamma^2 = (1-\beta^2)^{-1}$ according to the general properties of all acceptable fields as satisfying the condition (1), and the same field should also accelerate the electrons to maximize the cross-section of charge conversion along with minimizing that of annihilation. (The latter scales as γ^{-2}; so the ratio of the related probabilities (however small) is proportional to γ^4.) This relationship of the dynamics and the charge conversion implies their common coupling constant. For the same reason charged particles created in the neutral star are to leave the weak interaction region avoiding annihilation.

When the left-handed electron passes the weak interaction region of the star, it has some probability either to turn into the neutrino or to annihilate or to cross this region intact. In the latter case this left-handed electron might be reflected by the exit potential to pass the star center in the opposite direction now as a right-handed one. Being reflected once again, this electron can turn into the neutrino becoming left-handed again, thus sharing the total neutrino flux. This cannot be allowed for the gauge, since the time moment of this electron would differ from that of the normally leaving star electron to result further on in the incorrect initial moment of the newborn electron in the next star. This unwanted process can be suppressed by annihilation of the electron-positron pair when the reflected particles flip their helicity. The related probabilities depend on the value of the weak coupling constant g_L, given the electromagnetic coupling constant e (the subscript L refers to the left-handed electron).

Let us first consider the energy matching dynamics ignoring radiation. In the charged star, the electron is being accelerated from γ_i at the radius r_i, as defined by the finest star cell still possible for the gauge of electron charge, up to some γ_f at $r_{min} \ll r_i$ [10]. As any field satisfying the general motion-to-motion condition (1), the weak field has to satisfy a wave equation [9]. In particular, the finite range weak interaction could be expressed via the Yukawa potential $gr^{-1}\exp(-r/r_{min})$ satisfying the wave equation with an additional "mass" term. For not to disturb the charge gauge, the weak potential should be at most of the order of the Coulomb potential e^2/r at the minimal gauge-defined radius r_{min}. Apart from the short range, parity violation and electric neutrality, the dynamical behavior of weak field should be quite similar to that of electric field, as prescribed by (1). For the estimations let us approximate the weak field Yukawa potential with its averaged factor g^2/r, analogous to the electromagnetic e^2/r, though defined only within the weak field range $r/r_{min} \sim 1$: For $r/r_{min} < 1$, the potential $gr^{-1}\exp(-r/r_{min}) \approx g/r - g/r_{min}$ constant second term being immaterial. We introduce therefore a combined radius r_L, $r_L = (e^2 + g_L^2)/mc^2$ to write the following equation for γ in the CM reference system:

$$\gamma^3 = \gamma_f^3 + 3Ar_L\left(\frac{1}{r} - \frac{1}{r_{min}}\right) \tag{9}$$

where $A \approx 10$ represents the force created by all the other particles of the cube star together [10]. In dimensionless variables $\eta_L = 3Ar_L/r_{min}$ and $x = r/r_{min}$ (8) reads:

$$\gamma^3 = \gamma_f^3 + \eta_L\left(x^{-1} - 1\right). \tag{10}$$

In the transition from one star to the next, the electron starting with $\gamma = \gamma_f$ is accelerated by both the electromagnetic and weak forces from r_{min} down to some smaller r', where it turns into the neutrino, which moves to some r'' on the opposite end of the weak region under the weak force only, then this neutrino moves freely to start being accelerated by the weak field of the neutral star at r_{min}, where it turns into the new electron at r'', which finally decelerates by both the electromagnetic and weak fields to become a member of the next star, now at its own r_i, where it must have $\gamma = \gamma_i$. In this oversimplified scenario the total contribution of the weak field over the whole path from the output of one charged star to the input of the next charged star is zero, and it is the sole electric field, which is active only over its parts, defines the final velocity. In order to obtain a non-zero result also for the weak field, we have to switch it on and off over some parts of the transition. A natural means to realize this switch is to include an intermediate particle with a different mass as its carrier. This is the typical situation for a random process (at least, for a local one [11]), e.g., for quantum mechanics: the described with the wave function particle can be found (with some probability) anywhere at the same moment, still remaining point-like. The required intermediate particle will then have some mass M, the value of which must be large, being defined only within the short weak field range $\Lambda = \hbar/Mc \sim r_{min}$, so describing the transition solely in terms of the charge gauge.

For the energies relevant in our gauge procedure such a massive particle can only be a virtual one, its sole role consisting in correctly transporting the momentum, charge and spin data. For this to be possible, this meson must possess its own charge and polarization, having the spin equal to 1 to preserve the total angular momentum in the charge conversion, since the two other particles — the electron and the neutrino — have spin 1/2. Similarly, transporting the value of momentum as encoded by means of the boson properties implies its motion. Then the moments of creation and decay of the boson must be separated by a time interval, however short due its small velocity for the large mass. The whole transition between the charged stars will now look as follows. In electron at r_{min} having $\gamma = \gamma_f$ is being accelerated to reach the energy $mc^2\gamma'$ at $r = r'$. Here the electron turns into the intermediate boson, non-relativistic because of its large mass, moving with the velocity $v = c(\gamma' 2m/M)^{1/2}$.

Over the characteristic time Λ/c the boson moves a distance of the order $\Lambda(\gamma' 2m/M)^{1/2} \sim r_{min}(\gamma' 2m/M)^{1/2}$ (neglecting acceleration due to its large mass) to turn into the neutrino, moving with the same energy the distance $\sim r_i$ with velocity c to turn back into the boson at $r = r''$ (now measured from the center of the neutral star). Here the newborn electron is being decelerated, again by the electromagnetic and weak forces to reach $\gamma = \gamma_i$ at $r = r_i$ as measured from the center of the next charged star. In order to get in the course of the transition to the required γ_i given γ_f, we put $r'' = r' - r_{min}(\gamma' 2m/M)^{1/2}$ to obtain for the whole transition:

$$\gamma_i^3 = \gamma_f^3 + \eta_L \left(x'^{-1} - \frac{1}{x' - \sqrt{2m/M\gamma'}} \right). \tag{11}$$

This equation should be supported with the equation for $\gamma' = \gamma(x')$:

$$\gamma'^3 = \gamma_f^3 + \eta_L \left(x'^{-1} - 1 \right). \tag{12}$$

We eliminate x' from the system of (11) and (12) to obtain:

$$\begin{aligned} F(\gamma', \eta_L) = \;& \gamma_f^3 - \gamma_i^3 \\ & - \frac{\left(\eta_L + \gamma'^3 - \gamma_f^3\right)\sqrt{2m/M\,\gamma'}}{\eta_L - \left(\eta_L + \gamma'^3 - \gamma_f^3\right)\sqrt{2m/M\,\gamma'}} \\ = \;& 0. \end{aligned} \tag{13}$$

Still, the condition of reducing γ from γ_f to γ_i in the course of the whole transition doesn't define the points r' and r'' of the charge flips uniquely, unless the charge conversion is connected with the related dynamics (otherwise the flip might occur at any point within the weak interaction region), and we look for the maximum of γ' to achieve the maximal ratio (increasing as γ') of the charge conversion cross section to that of the dominating (two-photon) electron/positron annihilation.

The equation (13) implicitly defines $\gamma'(\eta)$ given γ_f and γ_i, and the condition for its maximum $d\gamma'(\eta_L)/d\eta_L = \partial F/\partial \eta_L = 0$ (provided $\partial F/\partial \gamma' \neq 0$ at $\eta_L = \eta_{L(max)}$) yields:

$$\eta_{max} = \left(\gamma'^3_{max} - \gamma_f^3\right) \frac{1 + \sqrt{2m/M\,\gamma'_{max}}}{1 - \sqrt{2m/M\,\gamma'_{max}}}\,. \tag{14}$$

Substituting (14) in (13), we obtain the equation for γ'_{max}, given γ_f and γ_i:

$$\gamma'^3_{max} - \left(\gamma_f^3 - \gamma_i^3\right) \frac{\left(1 - \sqrt{2m/M\,\gamma'_{max}}\right)^2}{4\sqrt{2m/M\,\gamma'_{max}}} - \gamma_f^3 = 0. \tag{15}$$

For the finest lattice as defined by the electron charge gauge, the equation for γ_f is similar to (9), in which, however, the electric force, introduced via $r_e = e/mc$, acts alone:

$$\gamma_f^3 = \gamma_i^3 + 3Ar_e \left(\frac{1}{r_{min}} - \frac{1}{r_i} \right). \tag{16}$$

In the gauge procedure, the value of γ_i is of great importance, because it is this lowest velocity that mainly contributes to the sensitivity of asymmetry detection in the stars: Since $r_i \gg r_{min}$, it will be: $\gamma_f \gg \gamma_i$ and the exact value of γ_f (since β_f is very close to 1) is but of minor importance in the integration of the disparity between the tetrahedrons [10]. However, γ_f is important in equations (9)-(15).

With resulting from the gauge condition [10] $\gamma_i \sim 3$ and $r_{min} \sim 3\times10^{-3} r_e$, we find from (16): $\gamma_f \sim 30$. Then from (15) and (14): $\gamma'_{max} \sim 50$ and $\eta_{L(max)} \sim 10^5$. This value of $\eta_{L(max)}$ corresponds to $g \sim 2e$, in agreement with the experimental data: $\sin\theta_w \sim 0.5$.

Until now we ignored radiation, and we have to consider its importance. In the gauge process itself, i.e. for $r_i > r > r_{min}$, radiation decreases the value of γ_f, and in the weak field regions, $r_{min} > r > r'$ and $r'' < r < r_{min}$, radiation is active as well. Both effects decrease the related γ's and therefore the probability of the charge conversions.

Whereas only the mean values of mechanical variables (behaving classically) are important in our gauge, as based solely on the top-speed signal oscillations, the analysis of the role of radiation requires the full quantum theory. Indeed, it was shown [10] that in the classical limit, corresponding to multiple soft-photons emission [11], radiation restricts the size of the star for the finest lattice down to the order of r_e. But it is well known that the classical field theory is no longer valid at these distances. Instead, we are bound to calculate only the cross sections for the emission of single photons.

Contrary to the classical limit, single photon radiation in QED occurs only with some probability, i.e. there is also a finite probability for the absence of emission. Only this case is relevant for our gauge, since radiation decreasing the related γ accordingly decreases the proportional to γ ratio of charge conversion cross section to that of annihilation. If the radiation cross section is not too close to unity, the charge conversion events which are not accompanied by radiation might be isolated as providing correct γ_f to γ_i transitions in accord with (11).

In the close vicinity of the star center only some small central part of the wave packet can take a part in the interaction, which is the source of radiation. Therefore, only a small part of the infinite range Coulomb interaction is actually involved, behaving there like a short range interaction. A similar effect in scattering on (neutral) atoms is accounted for by means of "screening" the potential [11, 12]. When the particle interacts with atom, this screening appears as a form factor effectively reducing the range of Coulomb potential to the size of the atom. In the same way, the short range Yukawa potential could be regarded as a screened initially long range fictitious potential, and we consider also the electromagnetic interaction to be screened as well, because now the flux of incoming particles should be normalized for a wave packet of the relevant size rather than for a plane wave. We start with the ultra- relativistic case for the radiation cross section formula in the center-mass system [11]:

$$d\sigma_{rad} = 4\alpha r_e^2 \frac{df}{f}\left(1-\frac{2}{3}(1-f)+(1-f)^2\right) \times\left(\ln 4\gamma_0^2\left(\frac{1}{f}-1\right)-\frac{1}{2}\right), \tag{17}$$

where $\alpha = e^2/\hbar c \sim 1/137$ is the fine structure constant $f = \hbar\omega/\epsilon_0$ (ω is the frequency of the emitted photon, ϵ_0 is the energy of the incident electron in the CM system, $\gamma_0 = \epsilon_0/m$). Integrating (17), we find σ_{rad}. The integral diverges for small f. For a simple estimation let us replace $\ln(1/f-1)$ with its average value Q. Integrating f from some f_{min}, (to be determined later) to 1:

$$\sigma_{rad} = 4\alpha r_e^2\left(Q-\frac{1}{2}+2\ln 2\gamma_0\right) \times\left(\frac{5}{6}-\frac{4}{3}(\ln f_{min}-f_{min})-\frac{1}{2}f_{min}^2\right). \tag{18}$$

In the scattering matrix theory, the analysis is carried out over the infinite distances from the interaction region both for initial and final states of the system, so that the incoming and outgoing wave functions are plain waves over the whole continuum, and in the derivation of (17), the integral for the Fourier component of the infinite range Coulomb potential is taken from 0 to ∞. In our case, only radiation events within the star are important, e.g., for $r_i > r > r'$ in the charged star and for $r'' < r < r_{min}$ in the neutral one. We shall therefore accept a model, in which the wave functions outside the interaction regions are still plain waves though bounded laterally to the interaction radii. These functions are given in advance, not taking care of how they were actually prepared. Then we can replace r_e^2 with r_i^2 for the gauge region in (17) and (18), so normalizing the plane wave spinors in the S-matrix element with one particle in r_i^3 rather than in the unit volume, in accord with the flux density of one electron per r_i^2. Similarly, r_{min} will replace r_e for the weak field region. We have also to modify α to account for the weak potential: $\alpha_L = e(e+g_L)/\hbar c$.

It will then be possible to use the Feynman diagram technique to calculate the radiation cross sections. Considering the interactions as existing only in these regions, we calculate the related interaction potential in the momentum representation. In particular, for the pure Coulomb potential $eA_0(q)$ (the time component of the four-vector eA_i) in the gauge region ($r_i > r > r_{min}$) we write (see, e.g., [11]):

$$\begin{aligned} A_0(q) &= -4\pi e\int_{r_{min}}^{r_i} dr\ \exp(iqr) \\ &= \frac{4\pi e}{q^2}\left(\cos(qr_i)-\cos(qr_{min})\right), \end{aligned} \tag{19}$$

where we put the boundary radii instead of usual ∞ and 0. (If r_i were to tend to infinity, the exponential factor with a negative real power should be included in the integrand (to be set zero at the end in order to cancel the first term in the parenthesis, while and the second term becomes unity). In the derivation of (17) (see, e.g., [11, 12]), the argument q has to be set equal to the absolute value of the recoil momentum according to the total four-momentum conservation. In the ultra-relativistic case $q \approx mc/\hbar$, so for the gauge region ($r_i \sim r_e \gg r_{min}$), $qr_i \sim e^2/\hbar c = \alpha \ll 1$, and it follows from (19):

$$A_0(q) = -\frac{2\pi e}{q^2}\alpha^2. \tag{20}$$

Since the S-matrix element is proportional to (20), the radiation cross section (17), proportional to the S-matrix element squared, becomes modified by the additional factor $\alpha \sim 10^{-9}$. In order to obtain the total probability w_{rad} of emission in the interval ($r_i > r > r_{min}$) of a single photon with $f_{min} < f < 1$, the modified according to (20) cross section (18) is to be multiplied by the flux $j = 2v/V$ ($v \approx c$ is the velocity in the CM system, and $V \sim r_i^3$ is the gauge region volume) to obtain the probability for unit time, and then multiplying by r_i/v to find the probability for this region. With all these substitutions:

$$w_{rad} \approx 4\alpha^5 \left(Q - \frac{1}{2} + 2\ln 2\gamma_0 \right) \times \left(\frac{5}{6} - \frac{4}{3} \left(\ln f_{min} - f_{min} \right) - \frac{1}{2} f_{min}^2 \right). \tag{21}$$

Due to the factor α^5, this probability is very low, unless f_{min} is sufficiently small. For w_{rad} to be of the order of unity, it must be: $\ln(1/f_{min}) \sim \alpha^{-5}$, whatever all other factors in (21) might be. Evidently, such soft photons cannot bring about any changes in the value of γ_f in the gauge region. The same reasoning and with the same conclusion holds in the weak field region for γ'_{max} and η_{max}.

The factor α^4 in (21) suppresses radiation of the electron that does not pass the star center, the nearest vicinity of which provides main contribution to radiation. However, for the electron that passes the center without turning into the neutrino the full radiation cross section must be accounted for. As it follows from (18), the probability of emitting even rather high energy photons is of the order of unity, and it will be collected over a sequence of stars, since radiation can only decelerate the electron. Loosing even a small part of its final energy ($\geq mc^2\gamma_i$), this electron either reaches a lower value of γ_i than allowed for the next stars, or even fails to overcome the exit potential barrier of the last star of a short star sequence, so destroying the gauge lattice.

Although the right-handed electrons take no part in the charge conversion, they might ruin the charge gauge. Indeed, their helicity becomes opposite if they are reflected by the output electromagnetic barrier of the star, and the initially right-handed electron becomes a source of the left-handed neutrino as well. Such oppositely moving neutrinos would make uncertain the choice of the charge sign in the next star, being admixed to the proper antineutrinos generated by the positrons. The flux of these neutrinos could be somewhat suppressed by the electromagnetic electron-positron annihilation, provided the weak interaction acts against the electromagnetic acceleration. So, for the right-handed electron the weak interaction also receives some dynamical meaning.

In order to determine the value of the corresponding coupling constant g_R in the Yukawa potential, we have to find the probability w_{an} of the two-photon electron-positron annihilation when they are decelerated from $\gamma = \gamma_f$ down to $\gamma = 0$ at the turning point. We start with the well-known Dirac's formula for the annihilation cross section in the CM system. In our case it looks:

$$\sigma_{an} = \frac{2\pi r_{min}^2}{\gamma^4 \sqrt{\gamma^2 - 1}} \left[\left(\gamma^4 + \gamma^2 - \frac{1}{2} \right) \ln \left(\gamma + \sqrt{\gamma^2 - 1} \right) - \frac{1}{2} \gamma \left(\gamma^2 + 1 \right) \sqrt{\gamma^2 - 1} \right]. \tag{22}$$

The probability of annihilation w_{an}, increasing with deceleration, depends on the function $\gamma(r)$, which, in turn, depends on r:

$$\gamma^3 = \gamma_f^3 - \eta_R \left(\frac{1}{x} - 1 \right) \tag{23}$$

where $\eta_R = 30 r_R / r_{min}$, $r_R = (g_R^2 - e^2)/mc^2$, $x = r/r_{min}$. Annihilation probability dw an over the interval dx is:

$$dw_{an} = \sigma_{an} \frac{2v}{r_{min}^2} \, dx. \tag{24}$$

From (22), (23) and (24) we obtain:

$$w_{an} = 12\pi\eta_R \int_1^{\gamma_f} d\gamma \times \frac{\left(\gamma^4 - \gamma^2 - \frac{1}{2}\right) \ln \left(\gamma + \sqrt{\gamma^2 - 1}\right) - \frac{1}{2}\gamma \left(\gamma^2 + 1\right) \sqrt{\gamma^2 - 1}}{\gamma^2 \sqrt{\gamma^2 - 1} \left(\gamma_f^3 + \eta_R - \gamma^3\right)^2}. \tag{25}$$

Given γ_f, this equation defines a function $w_{an}(\eta_R)$, which possesses a maximum. A simple numerical calculation with $\gamma_f \approx 30$ gives: $w_{an}(max) = 0.12$ for $\eta_R(max) \approx 2500$. This value of $\eta_R(max)$ corresponds to $g_R \approx 1.15e$, again in close correspondence with the experimental value of $\cos\theta_w$. In a standard probabilistic approach, this 12% difference is sufficient to reliably discern between particles and antiparticles.

6 Conclusion

In summary, our argument goes as follows:

i. A direct gauge of electric charge using motion-to-motion measurements might be based on the very existence of a (local) top-speed signal, no matter how high this speed is in any units whatsoever.

ii. Letting this signal oscillate between test particles and counting the ratios of the (infinite) numbers of these oscillations, we are able to detect the symmetry of the stars arranged as Platonic solids.

iii. Of the five Platonic solids, only the neutral as a whole cube-symmetrical star, consisting of the two tetrahedrons – one for the electron and another for the positron – is suitable for the electric charge gauge, since it is the only symmetry in which the particles move under electrical interaction along straight lines to cross at their common center.

iv. In order for the electron charge to be gauged as having the same value everywhere, the stars must be arranged in a lattice extended over the whole space-time, in which the initial star arrangement gives rise to its followings by means of the same signal oscillations counting.

v. For this to be possible, the method must uniquely define the transitions in the star sequences; however, the oscillation ratios counting method defines two trajectories rather than only one, and some internal degree of freedom (spin) should be given the particle to make the choice unique.

vi. With our gauge confined to integer charge values and sensitive to deviation from these, however small, beyond the gauge region, transitions between the stars in the lattice becomes uncertain; however, our charge gauge leaves free some vicinity of the star center, where an additional interaction not destroying the gauge might exist, and it could be used for charge conversion to make this uncertainty immaterial.

vii. The weak interaction realizes the necessary charge conversion with the neutrino that must also provide the necessary information to select a single trajectory out of the two in the next star, their spin projection onto the momentum direction being the sole source for this selection. The transition within the lattice also requires appropriate matching between the in and out energies of the electrons in the succeeding stars; this can be reached only with an intermediate vector boson.

viii. The design of the lattice requires only one conversion of the electric charge, so involving only two charge eigenstates (the SU(2) doublet).

ix. The charge gauge naturally combines the weak and electromagnetic interactions in a single interaction as pertaining to the common cube star, and the numerical relationships between the three coupling constants directly follow from this gauge.

It is fascinating that just the existence of top-speed signals is sufficient to predict the existence of the weak interactions with its range, parity violation and even the intermediate boson, basing solely on Platonic symmetries. The electroweak segment in the standard model suggesting $SU(2)_L \times U(1)$ group with adjusted coupling constants to account for the previously observed in experiments data including parity violation (while PC is still preserved for the leptons) provides good predictions as well. One should appreciate, however, the difference between a theory predicting these features from its own "first principles" and a developed ad hoc theory that only explains, however successfully, already known experimental results. Moreover in other applications, the existence of top-speed signal is sufficient to construct the nonsingular part of the Green function (the so-called Huygens' tail) in general relativity [9]. Also, motion-to-motion measurements are relevant in stochastic approach to quantum mechanics [13], in which random scattering on the measuring device, that is realized as a set of macroscopic bodies moving so as to correspond on average to that of the particle in question, leads to the Schrodinger equation: In the form of the Madelung's fluid with its "quantum potential" depending on the same wave function, the external force vector corresponds to the total average acceleration of the particle, that is, the "scattering medium" itself depends also on the own motion of the particle under measurement. One more application of the motion-to-motion gauge helps to explain the existence and masses of the heavy μ- and τ-mesons [14]: In the cube cell, the same gauge regular lattice might occur if one (for the τ) or two (for the μ) electron/positron pairs are being replaced by the heavy mesons. These two sub-symmetries of the cube star may form the whole regular lattice, provided these "foreign" entries move under the mutual acceleration in the cell nearly identically to other electrons and positrons. This situation was found to exist only for some particular values of the mesons' masses, found to be close to experimental data.

We deduce therefore that the pure motion-to-motion gauge eliminating all artificial ingredients (even free falling bodies) and basing only on the (local) existence of top-speed signals provides not only its own interpretation of observations, but it can predict experimental results, otherwise hidden. This is not surprising, since such a gauge is based solely on the very statement of practical problems, and the attached theoretical scheme merely prescribes appropriate notions to address nature. Experiments, as carried out along these lines, can give then nothing but what these notions already imply, in accord with the viewpoint of I. Kant [15] (see also H. Bergson [16]).

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