Motion-to-Motion Gauge Entails the Flavor Families

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Charge and mass gauging procedure is carried out by means of counting the oscillation numbers of an auxiliary top-speed signal ("photons") between the appropriately ordered electrons and positrons, moving under their interaction along the diagonals of the cube toward its center (the "cube star"). Regular lattices composed of such stars transport the values of charge and mass over space-time regions. The gauge consists in detection of the cube symmetry in each star. However, the detected symmetry can also be observed, even if some particles of the basic electron/positron star are replaced with heavy mesons. These become an unavoidable byproduct of the gauge procedure. Two possible sub-symmetries of the cube realizing such replacement correspond to two mesons, but the regularity of the whole lattice holds only for some particular values of their masses. Numerical solutions to the non-linear ODE systems describing this situation yield these masses in terms of electron mass, which are close to those of the μ- and τ-mesons.

1 Introduction

The existence of the three flavor families remains a mystery, and it appears rather artificial in the otherwise self-contained structure of the standard model of particle physics (see, for instance, [1]). As in all basic structures of physics, theories must agree with experimental facts, and, in turn, the performance of experiments depends on existing theoretical conceptions. The design of measuring devices includes their gauge, which is an intermediary between the measurement of interest and some standard test measurements. In order to eliminate clocks and rods in the gauge, which might hide some features of the desired correspondence, we suggested a direct motion-to-motion gauge [2, 3]. We shall show that the flavor families naturally arise from the particular way this gauge could be carried out. Since all related experiments are ultimately based on the observation of the trajectories of charged particles in external electromagnetic fields, the gauge of electric charges and masses of particles is at the heart of any measurement. A relevant gauge procedure could use a regular lattice comprised of elementary cells ("stars"), each one being a standard configuration of the trajectories of test particles that are identical, apart from the sign of their charges [2, 3]. Starting with the stars that are primary for the gauge lattice, the whole lattice is constructed in such a way that the primary stars completely define secondary ones. The resulting relay races make it possible to transport the initial values of charge and mass over a chosen space-time region. In an appropriate construction of the lattice, each star could be connected to a previous star along various sequences of intermediate stars. The preservation of charge and mass over such transports might be detected, provided various paths connecting a pair of stars reveal the same symmetry at both ends according to the dynamics of involved particles.

In order to realize this program one needs a method to construct standard stars unambiguously. For this purpose, it was proposed to count top signal oscillations between the particles of the star [2]. No rods or clocks are then needed, provided the elementary stars possess some symmetry belonging to the Platonic solids. In this communication, we confine ourselves to the lepton sector of elementary particles, corresponding to the cube subsystem of the full dodecahedron structure. To this end, consider electrons and positrons moving along the diagonals of the cube toward its center under mutual attraction — the "cube star". The cube consists of two interlaced tetrahedrons — one for electrons, another for positrons, and the star is thus electrically neutral as a whole. Charge is being gauged by means of detecting the cube symmetry as being seen in the equality of the related numbers of photon oscillations, so that the detection of even one extra oscillation is sufficient to find this symmetry broken. (It is convenient to replace formally the counting of inter-particle oscillations with that between the particle and an imaginary central body; the translation is straightforward.) Of particular interest is the limiting case of the finest lattice, in which only one photon oscillation is sufficient to detect the symmetry of the star. Just this finest star will be considered in what follows.

The regular lattice comprises the stars as elementary cells to form a whole charge gauging structure. For this to be possible, the electrons/positrons are bound to turn into neutrinos at the center: Otherwise, the exit potential together with the radiation reaction force would prevent their leaving the star, so destroying the lattice forming connections. We regard neutrinos massless (or having a mass that is negligible as compared to that of other involved particles), hence moving practically with the velocity of light independently of their kinetic energy.

Only the simplest case of cube star symmetry breaking was considered in the charge gauging procedure [2, 3], i.e. that in which asymmetry may occur only between the two opposite-charge tetrahedrons of the cube. The breaking of cube symmetry in this case consists in this that particles belonging to different tetrahedrons have dissimilar masses m and/or absolute values of charge e, while these parameters remain identical within each tetrahedron. Perfect symmetry will be observed, provided all the involved particles have the same values of *both* m and $|e|$. In this case, the asymmetry to be detected is, in a sense, the weakest, and we assign it to the first flavor family, i.e. to that of the electron. We regard this — electron/positron — star as the basic one and ask whether or not our photon oscillations counting procedure might detect the symmetry as observed, even if some electrons/positrons in the star are being replaced with different particles. Detection of a perfect star with our method requires both charges and masses of the involved particles to be identical. Upon assuming the charges to remain equal, let us consider the lattice, in which some particles have a different mass. While electrons must turn into neutrinos in each star, these foreign particles ("mesons") are able to pass the center intact, since the exit barrier decreases there. They can then take part in the secondary stars. For this to be possible, they must satisfy three following requirements:

i. Preserve proper charge distribution in each star;
ii. Pass successfully the symmetry detection in the stars as carried out by counting photon oscillations;
iii. Yield the definite output velocity (e.g., equal to the input velocity) to be suitable over a long line of successive stars.

To fulfill these requirements, we have only two parameters at our disposal to be controlled over the whole lattice, that is, the mass and the velocity of the meson at the star entrance. We guess only much heavier mesons to be met with. Since the lattice is a ready structure and the slower mesons are just "impurities" in it, they will enter the next star with some time lag.

Besides the basic star, there exist only two configurations having weaker sub-symmetries. Depending on the mass found for the related foreign particle, one of the sub-symmetries will be ascribed to the τ-meson, and another to the μ-meson.

In the first sub-symmetry, only one pair of opposite electron and positron is replaced by the meson/anti-meson pair. Their diagonal is the natural axis of the star symmetry, since under the interaction in the star the mesons keep moving along this axis. The trajectories of the remaining three electrons and three positrons are curvilinear, though confined pair-wise to three planes (the members of each pair don't belong to a common diagonal of the cube). Then the absolute values of the Cartesian coordinates of all six electrons/positrons, both along and transverse the axis, will be the same. We refer to this case as (6:2) sub-symmetry. (In this notation, the electron/positron star is (8:0) sub-symmetry.) Contrary to the basic (8:0) case, magnetic part of the interaction is no longer cancelled on the curved trajectories in stars possessing only sub-symmetries, though the total resulting interaction still leaves the particles on the same planes they would move under the electric force alone.

In the second sub-symmetry, two identical meson/anti-meson pairs replace electron-positron pairs. Now all eight trajectories are curved though confined to the two mutually orthogonal planes, one of which carries only electrons and positrons, while another — only mesons and anti-mesons. Within each of these planes, the absolute values of the appropriately chosen Cartesian coordinates of its particles will be the same. We refer to this case as (4:4) sub-symmetry. Following the previous argument [2], we ignore the terms with retarded interaction in the equations describing the motion of the particles in the star, but radiation reaction of the accelerated particles may be important. However, even rough estimation of this multiple soft photons radiation will be sufficient to distinguish flavor families, provided the mesons are much heavier than the electron, and the mesons related to the two possible sub-symmetries strongly differ in their masses. As was found [2], the radius of the star is much smaller than the classical electron radius, still the smallest radius down to which the photon oscillations are being counted might be of the order or even larger than the classical radius r_0 of the meson. Therefore, the effect of radiation on the motion of the star particles should be estimated for the electrons and the mesons differently though the very motion of the center of the electron wave packet, which only matters in the photon oscillations counting procedure, might be described classically in virtue of the Ehrenfest theorem. In so complicated systems as the stars containing several interacting particles, accurate calculation of radiation would be rather complicated, and, moreover, it is well known [4] that r_0 is the limit of validity of the electrodynamics, while the trajectories for the finest star lie well deeper this value. Therefore, QED is needed to determine single photon radiation of electrons, pair production etc. in the very strong (even vacuum violating) electric field [5]. However, the motion of the electrons is of interest here only inasmuch as it influences that of the mesons, and we need not go into fine details for the electron component of the star. We thus choose to model radiation of the electrons with an appropriate functional factor S that tempers the energy increase of the accelerated electrons. This factor will depend on a parameter q, varying which one can match a solution for the mesons according to the threshold where the quantum single photon radiation reaction exceeds the driving force in the star. We assume that S depends only on the kinetic energy of the electron via the relativistic factor γ_e: $S = \exp[-(\gamma_e^{-2} - \gamma_{ei}^{-2})/q^2]$, where γ_{ei} is the initial value γ_e of in the star. So, $S = 1$ at the initial moment, while for appropriate solutions the value of q must be so chosen that its final value $\gamma_{e,f} \cong 5$, in accordance with the charge gauge [2] in the basic (8:0) star unperturbed by mesons.

The mesons are expected to move unchanged over many successive stars. Their motion should be analyzed in respect of the possibility to sustain a regular lattice, that is, of the

$$
\left.
\begin{aligned}
S^{-1}\,\frac{d\beta_{eu}}{d\chi} &= -4u_e\gamma_e^{-5}\left[4u_e^2+v_e^2-(2u_e\beta_{ev}+v_e\beta_{eu})^2\right]^{-\frac{3}{2}}-\frac{1}{4}\,u_e\gamma_e^{-5}\left[u_e^2+v_e^2-(u_e\beta_{ev}-v_e\beta_{eu})^2\right]^{-\frac{3}{2}}+\\
&+\left[(u_M-u_e)\left(1-\beta_{eu}^2\right)-v_e\beta_{ev}\left(\beta_M-\beta_{eu}\right)\right]\gamma_M^{-2}\gamma_e^{-1}\left[(u_M-u_e)^2+v_e^2\gamma_M^{-2}\right]^{-\frac{3}{2}}+\\
&+\left[(u_M+u_e)\left(1-\beta_{eu}^2\right)+v_e\beta_{ev}\left(\beta_M-\beta_{eu}\right)\right]\gamma_M^{-2}\gamma_e^{-1}\left[(u_M+u_e)^2+v_e^2\gamma_M^{-2}\right]^{-\frac{3}{2}}\\
S^{-1}\,\frac{d\beta_{ev}}{d\chi} &= -v_e\gamma_e^{-5}\left[4u_e^2+v_e^2-(2u_e\beta_{ev}+v_e\beta_{eu})^2\right]^{-\frac{3}{2}}-\frac{1}{4}\,v_e\gamma_e^{-5}\left[u_e^2+v_e^2-(u_e\beta_{ev}-v_e\beta_{eu})^2\right]^{-\frac{3}{2}}+\\
&+3^{-\frac{1}{2}}v_e^{-2}\gamma_e^{-5}\left(1-\beta_{eu}^2-\frac{1}{4}\beta_{ev}^2\right)^{-\frac{3}{2}}-\left[(u_M-u_e)\beta_{eu}\beta_{ev}+v_e\left(1-\beta_M\beta_{eu}-\beta_{ev}^2\right)\right]\gamma_M^{-2}\gamma_e^{-1}\times\\
&\times\left[(u_M-u_e)^2+v_e^2\gamma_M^{-2}\right]^{-\frac{3}{2}}-\left[(u_M+u_e)\beta_{eu}\beta_{ev}-v_e\left(1-\beta_M\beta_{eu}-\beta_{ev}^2\right)\right]\gamma_M^{-2}\gamma_e^{-1}\left[(u_M+u_e)^2+v_e^2\gamma_M^{-2}\right]^{-\frac{3}{2}}\\
\eta^{-1}\,\frac{d\beta_M}{d\chi} &= -3\left(u_M-u_e\right)\gamma_M^{-3}\gamma_e^{-2}\left[(u_M-u_e)^2+v_e^2\gamma_e^{-2}\right]^{-\frac{3}{2}}+\\
&+3\left(u_M+u_e\right)\gamma_M^{-3}\gamma_e^{-2}\left[(u_M+u_e)^2+v_e^2\gamma_e^{-2}\right]^{-\frac{3}{2}}-\frac{1}{4}\,u_M^{-2}\gamma_M^{-5}
\end{aligned}
\right\}. \qquad (1)
$$

repeatability of their initial and final velocities either in each star or, at least, for a long sequence of successive stars. For each of the sub-symmetries this possibility depends on the mass of the related meson. Interaction of the electrons and the mesons results in that that the motion of the electrons depends on the meson mass as well, hence the ratio of electron to meson masses might be obtained from our condition of the whole lattice regularity. Motion of heavy mesons might be described classically.

Strictly speaking, one has to include explicitly the meson radiation reaction term in the equation of motion. It would be convenient however to use, wherever possible, perturbation methods to determine the radiation reaction, provided it is much less than the driving force: The equation of motion could be solved for the driving force alone, and then the radiated energy is found using this solution. The final kinetic energy of the meson is determined by subtracting the radiation loss from its value as obtained before (see, e.g., [4]). This estimation is certainly valid for a large enough mass, since the radiation cross section contains inverse square of the mass value. For this reason, we may use classical, that is, multiple soft photon emission value for the radiation of heavy mesons.

2 (6:2) sub-symmetry

In this case, the meson/anti-meson pair still moves along a straight line, whereas the curved trajectories of the three electron family pairs confine to three planes intersecting over the meson axis with the relative angles $\frac{2\pi}{3}$. It is convenient therefore to measure the z coordinate along the meson axis, and to choose the second coordinate ρ at each electron plane as the distance from this axis. Then the values of ρ for each particle of the electron family (each one measured in its own plane) are equal, and the absolute values of z are the same for all electrons. In dimensionless variables:

$$
\chi = \frac{ct}{r_0}\,,\quad u_e = \frac{z_e}{r_0}\,,\quad v_e = \frac{\rho}{r_0}\,,\quad \beta_{eu} = \frac{du_e}{d\chi}\,,
$$

$$
\beta_{ev} = \frac{du_v}{d\chi}\,,\quad u_M = \frac{z_M}{r_0}\,,\quad \beta_M = \frac{du_M}{d\chi}\,,\quad \eta = \frac{m_e}{m_M}\,,
$$

$$
\gamma_e = \left(1-\beta_{eu}^2-\beta_{ev}^2\right)^{-\frac{1}{2}}\,,\quad \gamma_M = \left(1-\beta_M^2\right)^{-\frac{1}{2}}\,,
$$

where the subscript e marks electrons, M means mesons, c is the speed of light. The system of three ODEs describes the motion of the electrons and the mesons in the star under their interaction. Using the well-known expression for the field of a fast moving charge [4], this system can be written as shown in Eqs. 1 on top of this Page 101.

This system should be numerically solved under the initial conditions taken from the solution for the basic electron family [2]: the initial radius of the electrons $r_{e,i} = 0.24r_0$, and $\gamma_{e,i} = 3.2$. In our variables, these correspond to:

$$
\left.
\begin{aligned}
&\chi_i = 0\,,\quad u_{e,i} = \frac{r_i}{3r_0}\,,\quad v_{e,i} = \frac{2\sqrt{2}r_i}{3r_0}\\
&\beta_{eu,i} = \frac{1}{3}\beta_{e,i}\,,\beta_{ev,i} = \frac{2\sqrt{2}}{3}\beta_{e,i}\\
&\beta_{e,i} = \left(1-\gamma_{e,i}^{-2}\right)^{\frac{1}{2}}\,,\quad \beta_{M,i} = \left(1-\gamma_{M,i}^{-2}\right)^{\frac{1}{2}}
\end{aligned}
\right\}. \qquad (2)
$$

In the perturbation approach, the value of $\gamma_{M,i}$ for the regular lattice should be equal to the final $\gamma_{M,f}$ at the exit of the preceding star (or a group of stars) as obtained by subtracting the radiation term $\gamma_{M,rad}$ and the term of the exit potential barrier $\gamma_{M,ex}$ from the final value of the solution to the system (1). These terms are:

$$
\gamma_{M,rad} = \frac{2}{3}\,\eta\int_0^{\chi_f} d\chi\left(\frac{d\beta_M}{d\chi}\right)^2\gamma_M^3\,, \qquad (3)
$$

$$\gamma_{M,ex} \cong 12^{-1}\eta u_{M,f}^{-1}\gamma_{M,i}^{-2}. \qquad (4)$$

It is assumed in (4) that $\gamma_{M,ex} \ll \gamma_{M,f} - \gamma_{M,i}$, and $u_{M,f} \ll u_{M,i}$. (The first inequality holds since the deceleration from the opposite meson is at least an order of magnitude less than the acceleration from the electrons because of the relativistic anisotropy of the electric field of fast moving charges.)

Then the value of $u_{M,i}$ is: $u_{M,i} = \frac{r_i}{r_0}(2-\beta_{M,f})$, where $\beta_{M,f} = (1-\gamma_{M,f}^{-2})^{1/2}$.

The solution for (1) goes down to the final value $r_{e,2} = 0.002\,r_0$, that is, $(u_{e,2}^2 + v_{e,2}^2 = 0.002)^{1/2}$. This value of $r_{e,2}$ corresponds to the average value of the weak Yukawa-type potential (instead of $r_{e,2} = 0.003r_0$ found in the charge gauge procedure for (8:0) case [2]). We assume that the electrons and positrons disappear at $r < r_{e,2}$. The value of χ_f in (3) should be defined by the condition that the function $r_e(\chi_f) = r_{e,2}$ for the first time.

The solution must meet the requirement for the meson to be unrecognized with our method of symmetry detection, i.e., that the numbers of the photon oscillations remain equal for the electron and the meson. To this end, consider the photon emitted at $r_e(0) = r_i$ and reaching $r_{e,1} = r_e(\chi_{e,1})$ after being reflected at the star center. Then, $\chi_{e,1} = (u_{e,i}^2 + v_{e,i}^2)^{1/2} + (u_{e,1}^2 + v_{e,1}^2)^{1/2}$, where the last member should be taken from (1). Similarly for the meson: $\chi_{M,1} = u_{M,i} + u_{M,1}$. Neglecting the small (because $r_{M,1} \gg r_{M,2} \approx r_{M,i} - r_{e,i}$) difference in the initial positions, we write the condition for the second photon not to have enough time to oscillate between the electron and the center over the first oscillation of the meson as:

$$\left(u_{e,1}^2 + v_{e,1}^2\right)^{\frac{1}{2}} + 0.002 > u_{M,1}. \qquad (5)$$

This inequality ensures that the electron annihilates within the time of the first oscillation for the meson. Since the meson doesn't annihilate, the opposite inequality preventing the second photon oscillation for the meson within the time of the first photon oscillation for the electron is:

$$u_{M,1} > \left(u_{e,1}^2 + v_{e,1}^2\right)^{\frac{1}{2}}. \qquad (6)$$

Upon solving the system (1) with $q = 2$, it was found that only for $\eta = 0.0003$ there exists an "equilibrium cycle" that repeats itself over the series of the stars (possibly with small shift of $\gamma_{M,f}$ from a mean value in a star to be compensated with some opposite shift in the next star) under the conditions (5) and (6) for some particular value of $\gamma_{M,i}$. For $\gamma_{M,i} = 5.150408$, and $u_{M,i} = 0.244567$, the system (1) yields $\gamma_{M,f} = 5.248322 \neq \gamma_{M,i}$, but already in the next star with $u_{M,i}$, following from this $\gamma_{M,f}$: $u_{M,i} = 0.244397\,(\gamma_{M,i} = 5.248322)$, we obtain $\gamma_{M,f} = 5.150408$, and the solution for the whole trajectory of the meson repeats itself infinitely. For these two consecutive stars: $(u_{e,1}^2 + v_{e,1}^2)^{1/2} = 0.00437$ and 0.003782, $u_{M,1} = 0.004815$ and 0.003785 respectively, so both (5) and (6) are fulfilled for each of them; $u_{M,f} = u_{M,2} = 0.00214$, $(u_{e,1}^2 + v_{e,1}^2)^{1/2} = 0.002$, $\gamma_{e,f} = 5.280387$. According to (3) and (4), radiation decreases $\gamma_{M,f}$ by only $\gamma_{M,rad} \approx 10^{-4}$, and the exit potential by $\gamma_{M,ex} \approx 10^{-3}$. Both are small as compared to the variation in the energy of the meson along its trajectory: $\left|\gamma_{M,f} - \gamma_{M,i}\right| \approx 0.05$. Hence, our assumption for deceleration from the exit potential barrier to be negligible for (6:2) sub-symmetry is reasonable. No acceptable solutions exist for other values of η. Although at each η there is a value of $\gamma_{M,i}$, for which the electrons and the mesons meet at $(u_{e,2}^2 + v_{e,2}^2)^{1/2} \approx u_{M,2} \leqslant 0.002$, but $\gamma_{M,f} \neq \gamma_{M,i}$, tending to increase monotonously, when extended over the next stars. Eventually the electron radius $(u_{e,2}^2 + v_{e,2}^2)^{1/2}$ becomes larger than the weak interaction threshold 0.002 everywhere on the trajectory. E.g., for $\eta = 0.00015$ this happens at $\gamma_{M,i} \approx 5.46$, while for $\eta = 0.0004$ at $\gamma_{M,i} \approx 5.48$. (For $\eta = 0.0002$, only (6) is broken.) Then the annihilation of the electrons becomes impossible, and our lattice will be ruined.

This behavior of the solutions to (1) can be explained as follows. If in the immediate vicinity of the star center the positive, say, meson lags with respect to the three nearest electrons, it is accelerated, while the electrons are decelerated to be overtaken by the meson and vice versa. Which case is realized depends on η and on $\gamma_{M,i}$. The equilibrium along the whole series comes from balance in the interaction. If situation is far from the balance, the meson will move much ahead or behind the electrons. Then its attraction will not be able to compensate for the reciprocal repulsion of the electrons, resulting in the increase of $(u_{e,2}^2 + v_{e,2}^2)^{1/2}$, and this quantity becomes eventually larger than 0.002.

3 (4:4) sub-symmetry

In this case, electrons and mesons move in two orthogonal planes intersecting at some axis of the cube (z) that connects the centers of the pair of its opposite faces. In each of these planes, the absolute values of the two Cartesian coordinates of the particles are the same for its four particles — electrons or mesons — due to the (4:4) symmetry. It is convenient therefore to choose a coordinate frame with the (x) axis in the electron plane and the (y)axis in the meson plane.

We guess in this case $\eta \gg 0.0003$, since the effect of four electrons on four mesons is smaller than that of six electrons on two mesons. Hence, radiation is expected to be important, since the meson must radiate much more energy with main contribution coming from the close neighborhood of the star center. This effect owes to the smaller meson mass as well as to the curvature of the trajectory, since, given force, transverse acceleration scales as γ^{-1} while longitudinal one only as γ^{-3}. Although it was long shown [4, 6] that, in the relativistic case, the energy radiated by the particle might be even larger than that received under external acceleration, we cannot use this result directly. In these references, the accelerating field was considered given in advance, i.e. independent of the particle's motion, whereas in our case back influence of radiation

$$
\left.
\begin{aligned}
S^{-1}\frac{d\beta_{eu}}{d\chi} = & -\frac{1}{4}\left(1-\beta_{ev}^2\right)^{-\frac{1}{2}}\left(1-\beta_{eu}^2\right)\gamma_e^{-3}u_e^{-2} - \frac{1}{4}\left(1-\beta_{eu}^2\right)^{-\frac{1}{2}}\beta_{eu}\beta_{ev}\gamma_e^{-3}v_e^{-2} - \\
& -\frac{1}{4}\left[u_e\left(\gamma_e^{-2}+\beta_{eu}^2\beta_{ev}^2+\beta_{eu}\beta_{ev}^3\right) - v_e\left(2\beta_{eu}\beta_{ev}-\beta_{eu}^3\beta_{ev}^2-\beta_{eu}^2\beta_{ev}^2\right)\right]\gamma_e^{-3}\left[u_e^2+v_e^2-\left(u_e\beta_{ev}-v_e\beta_{eu}\right)^2\right]^{-\frac{3}{2}} + \\
& +2\left[\left(u_M+u_e\right)\left(1-\beta_{ev}\beta_{Mu}\right)\left(1-\beta_{eu}^2\right) - v_e\left(1-\beta_{eu}\beta_{Mu}\right)\beta_{eu}\beta_{ev}\right]\gamma_e^{-1}\gamma_M^{-2} + \\
& +\left\{v_e^2\gamma_M^{-2}+w_M^2+\left(u_M+u_e\right)^2-\left[\left(u_M+u_e\right)\beta_{Mw}-w_M\beta_{Mu}\right]^2\right\}^{-\frac{3}{2}} + \\
& +2\left[\left(u_M-u_e\right)\left(1-\beta_{ev}\beta_{Mu}\right)\left(1-\beta_{eu}^2\right) - v_e\left(1-\beta_{eu}\beta_{Mu}\right)\beta_{eu}\beta_{ev}\right]\times \\
& \times\gamma_e^{-1}\gamma_M^{-2}\left\{v_e^2\gamma_M^{-2}+w_M^2+\left(u_M-u_e\right)^2-\left[\left(u_M-u_e\right)\beta_{Mw}-w_M\beta_{Mu}\right]^2\right\}^{-\frac{3}{2}} \\
S^{-1}\frac{d\beta_{ev}}{d\chi} = & -\frac{1}{4}\left\{u_e\beta_{ev}\left(\beta_{eu}+\beta_{ev}\right)\left(1-\beta_{ev}^2\right) + v_e\left[1-\beta_{ev}\left(\beta_{eu}+\beta_{ev}\right)\left(1-\beta_{eu}\beta_{ev}\right)\right]\right\}\gamma_e^{-3}\times \\
& \times\left[u_e^2+v_e^2-\left(u_e\beta_{ev}-v_e\beta_{eu}\right)^2\right]^{-\frac{3}{2}} + \frac{1}{4}\left(1-\beta_{eu}^2\right)^{\frac{1}{2}}\left(1-\beta_{ev}^2\right)\gamma_e^{-3}v_e^{-2} + \frac{1}{4}\left(1-\beta_{eu}^2\right)^{-\frac{1}{2}}\beta_{eu}\beta_{ev}\gamma_e^{-1}u_e^{-2} + \\
& +2\left[\left(u_M+u_e\right)\left(1-\beta_{ev}\beta_{Mu}\right)\beta_{eu}\beta_{ev} - v_e\left(1-\beta_{eu}\beta_{Mu}\right)\left(1-\beta_{ev}^2\right)\right]\times \\
& \times\gamma_M^{-2}\gamma_e^{-1}\left\{v_e^2\gamma_M^{-2}+w_M^2+\left(u_M+u_e\right)^2-\left[\left(u_M+u_e\right)\beta_{Mw}-w_M\beta_{Mu}\right]^2\right\}^{-\frac{3}{2}} - \\
& -2\left[\left(u_M-u_e\right)\left(1-\beta_{ev}\beta_{Mu}\right)\beta_{eu}\beta_{ev} - v_e\left(1-\beta_{eu}\beta_{Mu}\right)\left(1-\beta_{ev}^2\right)\right]\gamma_M^{-2}\gamma_e^{-1}\times \\
& \times\left\{v_e^2\gamma_M^{-2}+w_M^2+\left(u_M-u_e\right)^2-\left[\left(u_M-u_e\right)\beta_{Mw}-w_M\beta_{Mu}\right]^2\right\}^{-\frac{3}{2}} \\
\frac{d^2\beta_{Mu}}{d\chi^2} = & \frac{3}{2}\left(\eta^{-1}\frac{d\beta_{Mu}}{d\chi}-U\right)\gamma_M^{-1} - 2\left(\beta_{Mu}\frac{d\beta_{Mu}}{d\chi}+\beta_{Mw}\frac{d\beta_{Mw}}{d\chi}\right)\times \\
& \times\left[\frac{d\beta_{Mu}}{d\chi}-\gamma_M^{-2}\beta_{Mw}\left(\beta_{Mu}\frac{d\beta_{Mw}}{d\chi}-\beta_{Mw}\frac{d\beta_{Mu}}{d\chi}\right)\right] - 2\gamma_M^{-2}\beta_{Mu}\left(\beta_{Mu}\frac{d\beta_{Mu}}{d\chi}+\beta_{Mw}\frac{d\beta_{Mw}}{d\chi}\right)^2 \\
\frac{d^2\beta_{Mw}}{d\chi^2} = & \frac{3}{2}\left(\eta^{-1}\frac{d\beta_{Mw}}{d\chi}-W\right)\gamma_M^{-1} - 2\left(\beta_{Mu}\frac{d\beta_{Mu}}{d\chi}+\beta_{Mw}\frac{d\beta_{Mw}}{d\chi}\right)\times \\
& \times\left[\frac{d\beta_{Mw}}{d\chi}+\gamma_M^{-2}\beta_{Mu}\left(\beta_{Mu}\frac{d\beta_{Mw}}{d\chi}-\beta_{Mw}\frac{d\beta_{Mu}}{d\chi}\right)\right] - 2\gamma_M^{-2}\beta_{Mu}\left(\beta_{Mu}\frac{d\beta_{Mu}}{d\chi}+\beta_{Mw}\frac{d\beta_{Mw}}{d\chi}\right)^2
\end{aligned}
\right\}, \quad (7)
$$

where the functions U and W are expressed as follows

$$
\begin{aligned}
U = & -\frac{1}{4}\left(1-\beta_{Mw}^2\right)^{-\frac{1}{2}}\left(1-\beta_{Mu}^2\right)\gamma_M^{-3}u_M^{-2} - \frac{1}{4}\left(1-\beta_{Mu}^2\right)^{-\frac{1}{2}}\beta_{Mu}\beta_{Mw}\gamma_{Mw}^{-3}w_M^{-2} - \\
& -\frac{1}{4}\left[u_M\left(\gamma_M^{-2}+\beta_{Mu}^2\beta_{Mw}^2+\beta_{Mu}\beta_{Mw}^3\right) - w_M\left(2\beta_{Mu}\beta_{Mw}-\beta_{Mu}^3\beta_{Mw}^2\right)\right]\gamma_M^{-3}\times \\
& \times\left[u_M^2+w_M^2-\left(u_M\beta_{Mw}-w_M\beta_{Mu}\right)^2\right]^{-\frac{3}{2}} + 2\left[\left(u_M+u_e\right)\left(1-\beta_{Mw}\beta_{eu}\right)\left(1-\beta_{Mu}^2\right) - w_M\left(1-\beta_{Mu}\beta_{eu}\right)\beta_{Mu}\beta_{Mw}\right]\times \\
& \times\gamma_M^{-1}\gamma_e^{-2}\left\{w_M^{-2}\gamma_e^{-2}+v_e^2+\left(u_M+u_e\right)^2-\left[\left(u_M+u_e\right)\beta_{ev}-v_e\beta_{eu}\right]^2\right\}^{-\frac{3}{2}} - \\
& -2\left[\left(u_M-u_e\right)\left(1-\beta_{Mw}\beta_{eu}\right)\left(1-\beta_{Mu}^2\right) - w_M\left(1-\beta_{Mu}\beta_{eu}\right)\beta_{Mu}\beta_{Mw}\right]\gamma_M^{-3}\times \\
& \times\left\{w_M^2\gamma_e^{-2}+v_e^2+\left(u_M-u_e\right)^2-\left[\left(u_e-u_M\right)\beta_{ev}-v_e\beta_{eu}\right]^2\right\}^{-\frac{3}{2}},
\end{aligned}
$$

$$
\begin{aligned}
W = & -\frac{1}{4}\left\{u_M\beta_{Mw}\left(\beta_{Mu}+\beta_{Mw}\right)\left(1-\beta_{Mw}^2\right)+w_M\left[1-\beta_{Mw}\left(\beta_{Mu}+\beta_{Mw}\right)\left(1-\beta_{Mu}\beta_{Mw}\right)\right]\right\}\times \\
& \times\ \gamma_M^{-3}\left[u_M^2+w_M^2-\left(u_M\beta_{Mw}-w_M\beta_{Mu}\right)^2\right]^{-\frac{3}{2}}+\frac{1}{4}\left(1-\beta_{Mu}^2\right)^{\frac{1}{2}}\left(1-\beta_{Mw}^2\right)\gamma_M^{-3}w_M^{-2}+ \\
& +\ \frac{1}{4}\left(1-\beta_{Mu}^2\right)^{-\frac{1}{2}}\beta_{Mu}\beta_{Mw}\gamma_M^{-1}u_M^{-2}+2\left[\left(u_M+u_e\right)\left(1-\beta_{Mw}\beta_{eu}\right)\beta_{Mu}\beta_{Mw}-w_M\left(1-\beta_{Mu}\beta_{eu}\right)\left(1-\beta_{Mw}^2\right)\right]\times \\
& \times\ \gamma_e^{-2}\gamma_M^{-1}\left\{w_M^2\gamma_e^{-2}+v_e^2+\left(u_M+u_e\right)^2-\left[\left(u_M+u_e\right)\beta_{ev}-v_e\beta_{eu}\right]^2\right\}^{-\frac{3}{2}}+ \\
& +\ 2\left[\left(u_M-u_e\right)\left(1-\beta_{Mw}\beta_{eu}\right)\beta_{Mu}\beta_{Mw}-w_M\left(1-\beta_{Mu}\beta_{eu}\right)\left(1-\beta_{Mw}^2\right)\right]\times \\
& \times\ \gamma_e^{-2}\gamma_M^{-1}\left\{w_M^2\gamma_e^{-2}+v_e^2+\left(u_M-u_e\right)^2-\left[\left(u_M-u_e\right)\beta_{ev}-v_e\beta_{eu}\right]^2\right\}^{-\frac{3}{2}}.
\end{aligned}
$$

on the field-generating particles is important. We have thus to include the radiation reaction term explicitly in the equation of motion. But the value $\eta \approx 0.005$ is just at the boundary of self-contradiction of electrodynamics for the meson at the weak interaction threshold. Also quantum effects, however weaker than those for the electron, might alter radiation there. Moreover, deceleration of the meson at the exit potential barrier coming from other mesons as well as radiation accompanying this deceleration cannot be neglected now.

However, it would be inadequate merely to introduce a functional factor like that used above for the electron, because details of the meson trajectory are now in question. In order to trace the tendency, we shall instead try to approach the value $\eta = 0.005$ from below, i.e. from larger meson mass.

Again, in dimensionless variables

$$
\chi=\frac{ct}{r_0}\,,\quad u_e=\frac{z_e}{r_0}\,,\quad v_e=\frac{x_e}{r_0}\,,\quad \beta_{eu}=\frac{du_e}{d\chi}\,,\quad \beta_{ev}=\frac{dv_e}{d\chi}\,,
$$

$$
\gamma_e=\left(1-\beta_{eu}^2-\beta_{ev}^2\right)^{-\frac{1}{2}},
$$

$$
u_M=\frac{z_M}{r_0}\,,\quad w_M=\frac{y_M}{r_0}\,,\quad \beta_{Mu}=\frac{du_M}{d\chi}\,,\quad \beta_{Mw}=\frac{dw_M}{d\chi}\,,
$$

$$
\gamma_M=\left(1-\beta_{Mu}^2-\beta_{Mw}^2\right)^{-\frac{1}{2}},
$$

the system of four ODE equations — Eqs. 7 shown in the previous Page 103, with the functions U and W explained on the same Page Page 103 and on top of this Page 104 — describes the relativistic motion of electrons and mesons in the (4:4) cubic star under their interaction.

This system will be numerically solved under following initial conditions:

$$
u_{e,i}=u_{M,i}=\frac{r_i}{\sqrt{3}r_0}\,,\quad v_{e,i}=w_{M,i}=\sqrt{\frac{2}{3}}\,\frac{r_i}{r_0}\,,
$$

$$
\beta_{eu,i}=\frac{1}{\sqrt{3}}\,\beta_{e,i}\,,\quad \beta_{Mu,i}=\frac{1}{\sqrt{3}}\,\beta_{M,i}\,,\quad \beta_{ev,i}=\sqrt{\frac{2}{3}}\,\beta_{e,i}\,,
$$

$$
\beta_{Mw,i}=\sqrt{\frac{2}{3}}\beta_{M,i}\,,\quad \beta_{e,i}=\left(1-\gamma_{e,i}^{-2}\right)^{\frac{1}{2}},
$$

$$
\gamma_{e,i}=3.2,\quad \beta_{M,i}=\left(1-\gamma_{M,i}^{-2}\right)^{\frac{1}{2}}.
$$

At the star exit, the contribution of radiation coming from meson-meson interaction is expected to be rather low. It is thus convenient to follow the method used in the previous section in order to separate the radiation part in the total decrease of kinetic energy there. So, we solve first the equations of motion ignoring radiation, and then compute $\gamma_{M,rad}$ over the confined to a plane meson trajectory corresponding to this solution:

$$
\begin{aligned}
\gamma_{M,rad}=\frac{2}{3}\,\eta\int_0^{\chi_f}d\chi\Bigg[&\left(\frac{d\beta_{Mu}}{d\chi}\right)^2+\left(\frac{d\beta_{Mw}}{d\chi}\right)^2- \\
&-\left(\beta_{Mw}\frac{d\beta_{Mu}}{d\chi}-\beta_{Mu}\frac{d\beta_{Mw}}{d\chi}\right)^2\Bigg]\gamma_M^3\,.
\end{aligned}
\tag{8}
$$

The related ODE system is shown in Eqs. 9 on top of the next Page 104.

Since the lateral displacement of the heavy meson in a single star is expected to be small, the system (9) should be solved under the initial condition:

$$
u_{Mi}=\frac{r_{M2}}{\sqrt{3}}\,,\quad w_{Mi}=r_{M2}\sqrt{\frac{2}{3}}\,,\quad r_{M2}=\left(u_{M2}^2+w_{M2}^2\right)^{\frac{1}{2}},
\tag{10}
$$

where r_{M2} is the final radius of the meson in the accelerating phase of the star. It was found that the condition (6) holds only for $\eta \geqslant 0.005$. With $\eta = 0.005$, the equilibrium cycle looks as follows. (We have to choose $q = 1.3$ to agree with the charge gauge condition $\gamma_{e,f} \approx 5$ as in [2]). Unlike (6:2) case, in which the full cycle of returning to the initial state takes two neighboring stars, now it takes four.

On the accelerating phase of the first star of the cycle: $r_{M,i} = 0.244912$; $r_{M,2} = 0.001923$; $\gamma_{M,i} = 4.927011$; $\gamma_{M,f} = 5.090523$; $\gamma_{e,f} = 5.353761$. On the decelerating phase: $\gamma_{M,f} = 4.925161$; $\gamma_{M,rad} = 0.014866$. Radiation energy decrease (8) is less than 0.1 of that from the exit potential barrier as found by subtraction the final energy for the deceleration phase (9) from that for the acceleration phase (7), the second being initial for the first. Hence, our approximation is appropriate. On the accelerating phase of the last star of the cycle: $r_{M,i} = 0.244921$; $r_{M,2} = 0.001934$; $\gamma_{M,i} = 4.926057$;

$$\left.\begin{aligned}
\eta^{-1}\,\frac{d\beta_{Mu}}{d\chi} &= \frac{1}{4}\left(1-\beta_{Mw}^2\right)^{-\frac{1}{2}}\left(1-\beta_{Mu}^2\right)\gamma_M^{-3}u_M^{-2} - \frac{1}{4}\left(1-\beta_{Mu}^2\right)^{-\frac{1}{2}}\beta_{Mu}\beta_{Mw}\gamma_M^{-3}w_M^{-2} - \\
&- \frac{1}{4}\left[u_M\left(\gamma_M^{-3}+\beta_{Mu}^2\beta_{Mw}^2+\beta_{Mu}\beta_{Mw}^3\right) - w_M\left(2\beta_{Mu}\beta_{Mw}-\beta_{Mu}^3\beta_{Mw}^2-\beta_{Mu}^2\beta_{Mw}^2\right)\right]\times \\
&\times\ \gamma_M^{-3}\left[u_M^2+w_M^2-\left(u_M\beta_{Mw}-w_M\beta_{Mu}\right)^2\right]^{-\frac{3}{2}} \\
\eta^{-1}\,\frac{d\beta_{Mw}}{d\chi} &= -\frac{1}{4}\left\{u_M\beta_{Mw}\left(\beta_{Mu}+\beta_{Mw}\right)\left(1-\beta_{Mw}^2\right) + w_M\left[1-\beta_{Mw}\left(\beta_{Mu}+\beta_{Mw}\right)\left(1-\beta_{Mu}\beta_{Mw}\right)\right]\gamma_M^{-3}\times\right. \\
&\times\left[u_M^2+w_M^2-\left(u_M\beta_{Mw}-w_M\beta_{Mu}\right)^2\right]^{-\frac{3}{2}} + \frac{1}{4}\left(1-\beta_{Mu}^2\right)^{\frac{1}{2}}\left(1-\beta_{Mw}^2\right)\gamma_M^{-3}w_M^{-2} + \frac{1}{4}\left(1-\beta_{Mu}^2\right)^{-\frac{1}{2}}\beta_{Mu}\beta_{Mw}\gamma_M^{-1}u_M^{-2}
\end{aligned}\right\}. \quad (9)$$

$\gamma_{M,f} = 5.089923$; $\gamma_{e,f} = 5.411567$. On its decelerating phase again: $\gamma_{M,f} = 4.927011$. The conditions (5) and (6) are satisfied in all four stars of the equilibrium cycle.

Contrary to the (6:2) case, both electron and meson energies have been found to increase in the close vicinity of the star center on the acceleration phase. Therefore, for (4:4) symmetry it is just meson radiation that dominates the mechanism to support equilibrium. An equilibrium cycle satisfying both (5) and (6) exists also for $\eta > 0.005$. Formal solution gives that only for $\eta > 0.02$ the condition (5) is broken. QED estimation with averaged Coulomb field [5] shows that for heavy meson ($\eta < 0.02$) quantum single photon corrections for radiation are small. However, classical electrodynamics is invalid for $\eta < 0.005$. Therefore $\eta = 0.005$ could only be accepted as the lowest value compatible with the above equations. This result by no means undermines the very fact of correspondence between the lepton families and the cube star sub-symmetries as detected with photon oscillation counting, which possesses its own meaning, independent of a particular theory to specify trajectories.

4 Concluding remarks

However imprecise, the obtained values for η strongly suggest the (6:2) and (4:4) sub-symmetries to be associated accordingly with the τ−meson ($\approx 1.5\,\mathrm{GeV}/c^2$, $\eta = 0.0003$) and the meson ($\approx\ 100\,\mathrm{MeV}/c^2$, $\eta\ =\ 0.005$). Our estimations are reliable because of sufficiently big differences in mass values between the leptons. In order to find precise values, more complicated calculations of bremsstrahlung [5] are required for the star involving many Feynman diagrams for the mesons, interacting between themselves and with the electrons. Another approximation relates to the assumed sharp cut-off in the electroweak interaction at $r_{e,2}$.

We point out that the similar analysis might be carried out for quarks, which correspond to the three subsets of the complementary to the cube 12-particle part of the dodecahedron star in the full gauge lattice [2].

Although being presented here in the conventional form, the motion-to-motion gauge is actually coordinate-less, basing solely on the existence of the top velocity signal and symmetrical patterns of particles' trajectories. The existence of the flavor families could never be comprehended, unless the direct motion-to-motion gauge of charge is used, because the intermediary involving reference systems comprised of clocks and rods hides some important features of actual measurements. Just the same situation comes about in the weak interaction [3], where the obstructive role of reference systems stimulates the appearance of auxiliary "principles" like gauge invariance with its artificial group structure that can only explain the already known results of experiments rather than predict them. As a matter of fact, the very statement of the basic problem in mechanics, i.e. the contact problem, must be sufficient to substantiate all principles, including Lorentz covariance, gauge invariance and so on [7].

Submitted on December 23, 2014 / Accepted on December 28, 2014

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