Planck's Radiation Law: Thermal Excitations of Vacuum Induced Fluctuations

Fernando Ogiba
E-mail: ogiba@cpovo.net

The second Planck's radiation law is derived considering that "resonators" induced by the vacuum absorb thermal excitations as additional fluctuations. The maximum energy transfer, as required by the maximum entropy equilibrium, occurs when the frequencies of these two kind of vibrations are equal. The motion resembles that of the coherent states of the quantum oscillator, as originally pointed by Schrödinger [1]. The resulting variance, due to random phases, coincides with that used by Einstein to reproduce the first Planck's radiation law from his thermal fluctuation equation [2].

1 Introduction

In 1901, Planck derived the spectral distribution of radiant heat, simply calculating entropy from the number of ways that thermal energy can be distributed among all blackbody resonators (maximum entropy). This forced him to interpret the possible energies of the resonators, for a given mode and temperature, as multiples of a fixed energy; the quantum of electromagnetic energy [3]. In such approach, the appearance of a collection of resonators — with all sort of frequencies — depends only on thermal excitations, that is, for $T = 0$ they do not exist. However, in 1912 Planck realized that *thermal equilibrium with radiation* would make sense only if the resonators remain even for $T = 0$ [4]. In this new approach the quantization of the first Law was preserved, but only in the emissions, that is, oscillators in equilibrium with radiation absorbs continuously until a certain $nh\nu$ is reached, and then they emit or continues absorbing. From this semi-classical derivation, one concludes that exists vibrations not induced by thermal excitations. In this way, arose the concept of zero-point energy (ZPE), which is a term of the second Planck's radiation law, i.e.

$$\langle E \rangle = \frac{1}{2}\,\hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}. \tag{1}$$

At the time, the ZPE was a controversial concept; at best, it was accepted as "virtual photons due to nearby matter". The concept of a radiation field permeating the vacuum, and then inducing "matter-oscillators" with an energy given by the first term of Eq. (1), only gained credibility after the predictions of the quantum field theory (quantum vacuum states) and the experimental proof of the Casimir's force [5]. In fact, around the middle of the last century they begin appear works that assume explicitly that the matter (elementary electrical charges or agglomerates of them) are in permanent interaction with a zero-point radiation field (ZPF); absorbing and emitting electromagnetic radiation in a conservative way, independently of temperature.

In accordance with the experimentally proved work of Casimir [6] and the proponents of the stochastic electrodynamics [7], the ZPF is a homogeneous and isotropic distribution of electromagnetic plane waves pervading all space; each one carrying energy proportional to its frequency (ranging from zero to infinite, or a big cutoff value), $\hbar\omega/2$. Moreover, its spectral energy density is proved to be a Lorentz invariant. As the phases of such waves are randomically distributed in the range $[0, 2\pi]$, then electrical charges (or any agglomerate of them) are permanently receiving unpredictable impulses with the following features: First, the ZPF isotropy ensures zero net momentum transfer. Second, the emitted radiation, due to non uniform acceleration, responds by the local energy conservation. Third, the symmetric distribution of emissions ensures zero net self-momentum (no liquid radiation reaction). Fourth, the permanent nature of the absorption-emission process imply a remnant random trembling motion, whose energy in the particle-bound reference frame, in the case of a free electron, is the well-known rest energy

$$m_0 c^2 = \frac{\hbar\omega_Z}{2}, \tag{2}$$

where ω_Z is the zitterbewegung frequency [8, 9].

This zitterbewegung, strongly correlated with the translational motion trough the de Broglie's periodicity, prevent such particles to follow predictable paths (quantum randomness). Even so, the overall motion obeys the dynamical principle founded on trajectories. Non relativistically, this obedience means that the center of mass of the particle's vibrations can be found — instantly — over any one of the trajectories dictated by the stochastic Hamilton-Jacobi-Bohm equation, which is implicit in the Schrödinger's equation [10].

What follows is a derivation of Planck distribution, which replaces the quantization a priori by the presence of the ZPF, which, therefore, is the responsible by "resident blackbody resonators". Nevertheless, quantization is implied. Indeed, the zero-point energy ε_0, besides being a fixed quantity for each mode, is indispensable to get a discrete Boltzmann's distribution from a continuous one [12].

2 Thermal excitations of vacuum induced fluctuations

The energy absorbed (emitted) from (to) the ZPF in order to form temperature independent primordial matter-oscillators (or "Blackbody resonators") is

$$\varepsilon_0(\omega) = \frac{1}{2}\,\hbar\omega. \tag{3}$$

When particles absorbs such vibrant energy, conservatively, it is expected that its coordinates fluctuates as

$$q_0(t,\phi) = \sqrt{\frac{2\varepsilon_0(\omega)}{m\omega^2}}\cos(\omega t + \phi), \tag{4}$$

which differs from a typical classical oscillation only by the presence of random phases ϕ (ZPF randomness), which imply that this equation does not describe the actual path followed by particles, but simply obedience to the dynamic principle at each occupied position. Indeed, this is the main feature of the Schrödinger's equation, as argued elsewhere.

Notice, now $\varepsilon_0(\omega)$ is the energy of the matter-oscillator (the zero-point energy), which, as can be seen by simple substitution of Eq. (4), obey the equality

$$\varepsilon_0(\omega) = \frac{1}{2\pi}\int_0^{2\pi}\left[(2)\frac{2\pi}{\omega}\int_0^{2\pi/\omega}\frac{1}{2}\,m\left(\frac{dq_0(t,\phi)}{dt}\right)^2 dt\right]d\phi, \tag{5}$$

where the factor (2) refers to equal contributions from kinetic and potential energies of the harmonic oscillator, ω is the angular frequency of the absorbed radiation, the integral in t is an average over the radiation period, and the integral in ϕ is an average over random phases.

Given the permanent nature of the interactions, the ZPE must be viewed as a remnant energy. It is indispensable to compose the ground state energy of quantum systems. The exact shape, as it should be, only appears in the case of the harmonic oscillator.

For $T \neq 0$, there are thermal excitations, which manifest as additional vibrations that increase the amplitude of existing fluctuations. In a sense, this can be inferred from the thermal dilatation of bodies. In other words, the center of mass of the matter-fluctuations, as expressed by Eq. (4), fluctuates due to thermal excitations. This implies the superposition

$$q_{\phi,\Phi}(t) = \sqrt{\frac{2\varepsilon_0(\omega)}{m\omega^2}}\cos(\omega t+\phi)+\sqrt{\frac{2\mathcal{E}_T(\Omega)}{m\Omega^2}}\cos(\Omega t+\Phi), \tag{6}$$

where $\mathcal{E}_T(\Omega)$ is the vibrational energy induced be thermal excitations at the temperature T, Φ is a random phase, and, for the sake of generality, Ω is an arbitrary frequency.

It is worth informing, the assumption of the last paragraph is in full agreement with what is inferred from the coherent states of the quantum harmonic oscillator (the perfect framework to derive the Planck's law); that is, the statistical Gaussian of the ground state (here, the primordial oscillator) is moved, as a whole, by classical oscillations [11, see p. 104].

Averaging the energy

$$(2)\times\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}\,m\left(\frac{dq_{\phi,\Phi}(t)}{dt}\right)^2 dt$$

over random phases, both ϕ and Φ, yields the energy absorbed (emitted) by this superposition of vibrations, i.e.

$$E(\omega,\Omega) = \varepsilon_0(\omega) + \mathcal{E}_T(\Omega), \tag{7}$$

where Ω still continues unknown.

Now, averaging the square deviation from $\varepsilon_0(\omega)$,

$$\left[(2)\times\frac{2\pi}{\omega}\int_0^{2\pi/\omega}\frac{1}{2}m\left(\frac{dq_{\phi,\Phi}(t)}{dt}\right)^2 dt - \varepsilon_0(\omega)\right]^2,$$

over both random phases, emerges the variance

$$\sigma^2_{\omega,\Omega} = \frac{2\hbar\omega^3\left(\omega^2+\Omega^2\right)\sin^2\left(\pi\Omega/\omega\right)E_T(\Omega)}{\pi^2\left(\omega^2-\Omega^2\right)} + \\ + \frac{\left[\omega^2 + 16\pi^2\Omega^2 - \omega^2\cos^2\left(4\pi\Omega/\omega\right)E_T(\Omega)\right]E_T(\Omega)}{16\pi^2\Omega^2} \tag{8}$$

which seems to diverges when $\Omega \to \omega$. In true, there is the maximum variance

$$\sigma^2 = \lim_{\Omega\to\omega}\sigma^2_{\omega,\Omega} = E_T^2(\omega) + \hbar\omega E_T(\omega), \tag{9}$$

which can also be obtained replacing Ω by ω in the starting Eq. (6), and then performing the indicated operations.

Maximum variance implies maximum entropy (or thermodynamical equilibrium). Indeed, calculating entropy form Gaussian or exponential distribution (like Boltzmann's distribution) one find that entropy is proportional to $[\ln(\sigma^2) + cte]$.

From another point of view, the Eq. (9) also means that maximum energy transfer occurs when thermal vibrations are tuned with that induced by the ZPF, in full agreement with a well-known result of the theory of oscillations; that is, maximum energy transfer occurs at the natural frequency of the absorbing oscillator.

Therefore, from this tuned behavior — thermodynamical equilibrium — it follows that each possible energy, considering Eq. (7), obey

$$E = \frac{\hbar\omega}{2} + E_T(\omega), \tag{10}$$

and are distributed in such a way that the corresponding distribution has the variance σ^2.

It is crucial emphasizing, such ensemble of random energies is justified by a variance arising from random phases, ϕ and Φ. The first is a well-known feature of the ZPF (masterfully interpreted in the quantum mechanics framework), and the second is related to the myriad of ways that thermal excitations can move an elementary constituent of a body.

3 Thermal fluctuations and the Planck's radiation law

The variance expressed by Eq. (9) ensures that for each ω-mode at the remperature T there is a collection of random energies E, Eq. (10). From a thermodynamical point of view, the equilibrium involving such energy fluctuations must be treated in terms of the Boltzmann's statistics.

Deriving the moments of such distribution,

$$\langle E^r\rangle = \frac{\int_0^\infty dE E^r e^{-\beta E}}{\int_0^\infty dE e^{-\beta E}} = r!\langle E\rangle^r,$$

with respect to $\beta = 1/k_B T$, we obtain the Einstein's thermal fluctuation equation

$$\sigma_E^2 = k_B T^2 \frac{d\langle E\rangle}{dT}, \tag{11}$$

where, in the present calculations, $\langle E\rangle$ is the thermal average of the energies expressed by Eq. (10), i.e.

$$\langle E\rangle = \frac{\hbar\omega}{2} + \langle E_T\rangle, \tag{12}$$

and the thermal variance (thermal fluctuation) σ_E^2 is, therefore, the thermal average of Eq. (9):

$$\sigma_E^2 = \langle E_T\rangle^2 + \hbar\omega\langle E_T\rangle. \tag{13}$$

Combining the last three equations, we get the differential equation

$$k_B T^2 \frac{d\langle E_T\rangle}{dT} = \langle E_T\rangle^2 + \hbar\omega\langle E_T\rangle, \tag{14}$$

whose solution, considering $\langle E_T\rangle = 0$ for $T = 0$, is

$$\langle E_T\rangle = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}. \tag{15}$$

Therefore,

$$\langle E\rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}. \tag{16}$$

Submitted on February 10, 2015 / Accepted on February 14, 2015

References

1. Schrödinger E. Collected Papers on Wave Mechanics, Blackie Son Limited, 1929
2. Einstein A. Zum gegenwartgen Stand des Strahlungs-problems. *Physikalische Zeitschift*, 1900, v. 10, 185–193. Published in The Collected Papers of Albert Einstein,v. 2: The Swiss Years: Writings, 1900-1909 (English translation supplement), Princeton University Press, 1989, 357–369.
3. Planck M. The Theory of Heat Radiation, Dover, New York, 1959.
4. Milonni P. W. The Quantum Vacuum: An Introduction to Quantum Electrodynamics, Academic Press, 1994.
5. Casimir H. B. G. On the attraction between two perfectly conducting plates. *Proceedings of the Royal Netherlands Academy of Arts and Sciences*, 1948, v. B51, 793–795.
6. Lamoraux S. K. Demonstration of the Casimir Force in the 0.6 to 6m Range. *Physical Review Letters*, 1997, v. 78, n. 1, 5–8.
7. Boyer T. H. Random electrodynamics: The theory of classical electrodynamics with classical electromagnetic zero-point radiation. *Physical Review D*. 1975, v. 11, n. 4, 790–808.
8. Dirac P. A. M. The Principles of Quantum Mechanics, 4ed. Oxford University Press, 1999.
9. Sidharth B. G. Revisiting Zitterbewegung. *International Journal of Theoretical Physics*, 2009, v. 48, 497–506.
10. Ogiba F. Addendum to Phenomenological Derivation of the Schrödinger Equation. *Progress in Physics*, 2014, v. 1, 108–110.
11. Burkhardt C. E., Leventhal J. J. Foundations of Quantum Physics, Springer, 2008.
12. de la Peña L., Ceto A. M. The Quantum Dice: An Introduction to the Stochastic Electrodynamics, Kluwer Academic, Dordrecht, 1996