Diffusion Equations, Quantum Fields and Fundamental Interactions

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The paper concerns an "ab initio" theoretical model based on the space-time quantum uncertainty and aimed to identify the conceptual root common to all four fundamental interactions known in nature. The essential information that identifies unambiguously each kind of interaction is inferred in a straightforward way via simple considerations involving the diffusion laws. The conceptual frame of the model is still that introduced in previous papers, where the basic statements of the relativity and wave mechanics have been contextually obtained as corollaries of the quantum uncertainty.

1 Introduction

Understanding the fundamental interactions of nature is certainly one among the most challenging topics of the modern physics; a unified theory able to account for the fundamental forces is a dream of the physicists since a long time [1, 2]. The science of the fundamental interactions progressed with the advancement of the physics of the elementary particles [3], whose properties could be tested by examining their way of interacting with other particles. The theoretical models bridging quantum and relativistic theories [4, 5] progressed along with the merging of the physics of the elementary particles and quantum fields [6] with that of the fundamental interactions. All this culminated with the formulation of the standard model [7] and with the superstring theory [8]. The way the particles interact involves significantly even the cosmology [9, 10]. The GU theories [11, 12] share some general concepts about the four fundamental interactions, their basic idea to model the force between quantum particles is in principle simple: to exchange appropriate elementary particles that transfer momentum and energy between the interacting partners. The vector bosons are acknowledged to mediate the forces between particles according to their characteristic features of lifetime and action range [13]. These messenger particles, quanta of the respective fields, are said to mediate the interaction that propagates with finite velocity and perturbs the space-time properties. This way of thinking suggests reasonably the key role of the displacement mechanism of the particles that propagate the interaction, e.g. the different transport rates of massive or massless messengers; this means, in particular, that the space in between a set of interacting particles is filled with the vector bosons mutually exchanged. As clouds of these latter flow throughout the space-time, it is reasonable to expect that the global properties of the resulting interaction should depend on the ability of the messengers to spread around the respective partners. Eventually, since the mutual positions of each particle in the set are in general functions of time, even random local density gradients of these messengers are expectedly allowed to form throughout the space-time.

These preliminary considerations feed the idea of implementing a model of fundamental interactions based on a appropriate mechanism of transport of matter/energy, sufficiently general to be suitably extended from sub-nuclear to infinite range interactions. Among the possible transport mechanisms deserves attention the particle diffusion, driven by a gradient law originated by a non-equilibrium situation; as it has been shown in a previous paper [14], this law is strictly connected with the global entropy increase of an isolated thermodynamic system, the diffusion medium plus the diffusing species both tending to the equilibrium configuration in the state of maximum disorder. So the driving force of the diffusion process is actually the second principle of thermodynamics, i.e. a law so general to hold at the nano-micro-macro scales of interest in the present context. As a matter of fact, it has been found that this law allows describing not only the concentration gradient driven mass transport but also other important laws of physics: for instance Ohm's electric conductivity or Fourier's heat conductivity or Poiseuille pressure laws [14]. So, in agreement with the quantum character of the approach therein introduced, appears stimulating in principle the idea of testing via the diffusion laws even the exchange of vector bosons to describe the fundamental interactions. This hint leads in a natural way to the idea of dynamical flux of messenger particles, by consequence of which are exchanged momentum and energy of the interacting partners. This assumption merely requires that the messengers of the forces are exchanged as clusters of particles randomly flowing through the space-time and thus characterized in general by local concentration gradients. The physics of the four fundamental interactions has been already concerned in a dedicated paper [15]; in that paper the interactions have been described starting directly from the concept of space-time uncertainty. Here this problem is reformulated via the diffusion laws only in a surprisingly simple way. This paper aims to show that the key features of the fundamental forces are obtained by elaborating purposely the diffusion laws; it will be emphasized that these laws provide interesting hints also for relativistic and thermodynamic considerations. Of course the purpose of the paper is not that of providing an exhaustive description of the fundamental interactions, which would require a much longer review of the huge amount of literature existing about each one of them; the paper intends instead to emphasize an even more crucial aspect of this topic, i.e. how to infer the essential features of all known interactions from a unique fundamental principle; in other words, the aim is to focus on a unique conceptual root from which follow contextually as corollaries all fundamental interactions. The paper introduces an "ab initio" model via considerations limited to the minimum necessary to infer the distinctive features of the various forms of interaction that identify unambiguously each one of them. Despite this topic is usually tackled via heavy computational ways, the present theoretical model is conceptual only but surprisingly straightforward. While the idea of interactions due to a diffusion-like flux of vector bosons has been early introduced [16], in the present paper this hint is further implemented. The model concerned in this paper exploits first the quantum origin of the diffusion laws, shortly reported for completeness of exposition, to infer next the interactions directly via the diffusion laws. Some concepts already published [14, 15, 16] are enriched here with further considerations in order to make this paper as self-contained as possible. It is clear the organization of the paper: the section 2 introduces the quantum background of the model and both Fick diffusion laws, plus ancillary information useful in the remainder sections; the section 3 introduces some thermodynamic considerations; the section 4 concerns the fundamental interactions, whereas the section 5 concerns a few additional remarks on the gravity force.

2 Physical background

The statistical formulation of the quantum uncertainty reads in one dimension

$$\Delta x \Delta p_x = n\hbar = \Delta\varepsilon\Delta t, \qquad \Delta\varepsilon = \mathrm{v}_x \Delta p_x, \qquad \mathrm{v}_x = \Delta x/\Delta t. \quad (1)$$

The subscript indicates the component of momentum range along an arbitrary x-axis. The second equality is actually consequence of the former merely rewritten as $(\Delta x/\mathrm{v}_x)(\Delta p_x \mathrm{v}_x)$, being Δt the delocalization time lapse necessary for the particle to travel throughout Δx; so this definition leaves unchanged the number n of quantum states allowed to the concerned system. Since the local coordinates are waived "a priori", i.e. conceptually and not as a sort of approximation aimed to simplify some calculation, these equations focus the physical interest on the region of the phase space accessible to the particle rather than on the particle itself. As these equations link the space range Δx to the time range Δt via n, any approach based on these equations is inherently four-dimensio-nal by definition. The sizes of the uncertainty ranges are arbitrary, unknown and unknowable; it has been shown that they do not play any role in determining the eigenvalues of the physical observables [17], as in effect it is known from the operator formalism of the wave mechanics. Actually it is possible to show that the wave formalism can be inferred as a corollary of the Eqs. (1) [17], coherently with the fact that n plays just the role of the quantum number in the eigenvalues inferable via these equations only [18, 19]. The Eqs. (1), early introduced in these papers to provide a possible way to describe the quantum systems in alternative to the solution of the pertinent wave equations, have been subsequently extended to the special and general relativity [20]. It has been shown for instance that a straightforward consequence of the space time uncertainty is

$$c^2 \Delta p_x = \mathrm{v}_x \Delta\varepsilon. \quad (2)$$

The demonstration is so short and simple to deserve of being mentioned here for completeness: this equation and the next Eq. (3) are enough for the purposes of the present paper. Consider a free particle delocalized in Δx. If this particle is a photon in the vacuum, then $\Delta x/\Delta t = c$; i.e. the time range Δt is necessary by definition for the photon to travel Δx. Yet, trusting to the generality of the concept of uncertainty, the Eqs. (1) must be able to describe even the delocalization of a massive particle moving at slower rate $\mathrm{v}_x = \Delta x/\Delta t < c$. Let us examine now this problem according to the Eqs. (1), i.e. starting from $\Delta x \Delta p_x = \Delta\varepsilon\Delta t$ to infer $\Delta\varepsilon/\Delta p_x = \Delta x/\Delta t$; as c represents the maximum velocity allowed to any particle, it must be true that $\Delta x/\Delta t \leq c$, whence $\Delta\varepsilon/\Delta p_x \geq c$. The inequality therefore constrains the ratio of the range sizes $\Delta\varepsilon$ and Δp_x depending on whether the delocalized particles are massive or not. Anyway both chances are considered writing $\Delta\varepsilon/\Delta p_x = (c/\mathrm{v}_x)c$. One finds thus the sought Eq. (2), which implies the local functional dependence $c^2 p_x = \mathrm{v}_x \varepsilon$ between energy and momentum and velocity components of the massive particles. Also note that the Eq. (2) implies the concept of mass simply introducing the limit

$$\lim_{\mathrm{v}_x \to 0} \frac{\Delta p_x}{\mathrm{v}_x} = \frac{\Delta\varepsilon_{rest}}{c^2} = m. \quad (3)$$

As there is no compelling reason to expect a vanishing $\Delta\varepsilon_{rest}$ for $\mathrm{v}_x \to 0$, one concludes that the left hand side is in general finite and corresponds to the definition of mass. Both signs are allowed in principle to v_x and thus to Δp_x; yet squaring $c^4 \Delta p_x^2 = \mathrm{v}_x^2 \Delta\varepsilon^2$ and implementing again $\mathrm{v}_x < c$, one finds $c^2 \Delta p_x^2 < \Delta\varepsilon^2$ i.e. $\Delta\varepsilon^2 = c^2 \Delta p_x^2 + \Delta\varepsilon_o^2$; thus the local functional dependence $\varepsilon^2 = c^2 p_x^2 + \varepsilon_o^2$, well known, combined with the Eq. (3) yields $\varepsilon_o = mc^2$ and also the explicit expressions of ε and p_x compliant with the respective Lorentz transformations.

2.1 Quantum basis of the diffusion laws

This subsection assumes that the diffusion medium is an isotropic body of solid, liquid or gas matter at constant and uniform temperature. The following considerations shortly summarize the reasoning introduced in [14]. Let us divide both sides of the Eq. (2) by $\mathrm{v}_o V$, being v_o an arbitrary velocity and V an arbitrary volume. So one finds

$$\mathrm{v}_x C = \Delta J_x, \qquad C = \frac{\Delta p_x}{\mathrm{v}_o V}, \qquad \Delta J_x = \frac{\Delta\varepsilon/c^2}{V} \mathrm{v}_x. \quad (4)$$

As C has physical dimensions $mass/volume$, it represents the average concentration of a mass m in the volume V, whereas ΔJ_x is the net change of the flux of particles moving at average rate v_x through V. So ΔJ_x, whose physical dimensions are $mass/(time \times surface)$, describes the net flux of matter entering in and leaving out two opposite surfaces delimiting V; the first Eq. (4) also implies that the functional dependence of any J_x within its uncertainty range ΔJ_x upon the corresponding local flux of m fits the classical definition $J_x = C\text{v}_x$. Assuming that $\Delta\varepsilon/c^2$ is the energy equivalent of mass, the last equation inferred with the help of the Eq. (2) extends the definition of flux of the first equation to the change of energy density inside V. Write now $V = \Delta x^3$, which is certainly possible regardless of the particular geometric shape because both V and Δx are arbitrary; so any shape factor, e.g. $4\pi/3$ for spherical V, is inessential because it would still yield $V = \Delta x'^3$ once included in $\Delta x'$. Since $\Delta x^{-3} = -\partial\Delta x^{-2}/2\partial\Delta x$, one finds

$$\Delta J_x = \frac{\Delta p_x}{\Delta x^3} = -\frac{\Delta p_x}{2}\frac{\partial\Delta x^{-2}}{\partial\Delta x}.$$

Moreover $\Delta x^{-2} = \Delta p_x^2/(n\hbar)^2$, so that

$$\Delta J_x = -\frac{\Delta p_x^2}{(n\hbar)^2}\frac{\partial\Delta p_x}{\partial\Delta x} = -\frac{1}{3(n\hbar)^2}\frac{\partial\Delta p_x^3}{\partial\Delta x}$$

which yields in turn

$$\Delta J_x = -\frac{n\hbar}{3}\frac{\partial(1/\Delta x^3)}{\partial\Delta x} = -\frac{n\hbar}{3m}\frac{\partial(m/\Delta x^3)}{\partial\Delta x}. \tag{5}$$

The last equality holds under the reasonable assumption of constant mass m in the volume Δx^3: as both V and m are arbitrary, the former can be conveniently chosen in order to fulfil the requirement that the latter is simply redistributed within Δx^3 during an assigned diffusion time Δt related to ΔJ_x. Indeed the fact of having defined C as the average concentration of a constant amount of diffusing mass does not exclude the existence of a concentration gradient within V; in effect ΔJ_x results in the Eq. (5) as the concentration gradient driven mass flux at the boundary surfaces of V. Also note that $\hbar/m$ has the same physical dimensions, $length^2/time$, of a diffusion coefficient D; so, as shown in [14], it is possible to write $D = qn\hbar/m$ being q an appropriate numerical coefficient able to fit the experimental value of D of any species moving in any diffusion medium. Owing to the generality of the Eqs. (1), no specific hypothesis is necessary about whether the concerned diffusion process occurs in gas or liquid or solid phase or even in the vacuum; also, this holds at any temperature and value of C. So the last equation (5) reads

$$\Delta J_x = -D\frac{\partial C}{\partial\Delta X}, \quad C = \frac{m}{\Delta x^3}, \quad \Delta X = \frac{\Delta x}{q}, \quad D = \frac{qn\hbar}{m}. \tag{6}$$

Of course the inessential factor 3 has been included into q. Here C is related to the given amount of mass m redistributed within V; so it depends not only on m itself, but on the space extent through which this redistribution was allowed to occur. This result is nothing else but the well known first Fick gradient law, now straightforward consequence of the fundamental Eqs. (1). So far, for simplicity has been concerned the one-dimensional case, symbolized by the subscript x denoting the actual vector components of momentum and displacement velocity of m along an arbitrary x-axis. Yet it is useful to account explicitly for the vector nature of the equations above summarizing the Eqs. (4) and (6) as follows:

$$\Delta\mathbf{J} = C\mathbf{v} = -D\nabla C. \tag{7}$$

For the following purposes, it is interesting to extend these first results. Given an arbitrary function $f(x,t)$ of coordinate and time, express its null variation $\delta f(x,t) = 0$ as $(\partial f/\partial x)\delta x + (\partial f/\partial t)\delta t = 0$ that reads $\text{v}_x(\partial f/\partial x) + (\partial f/\partial t) = 0$ i.e. $\mathbf{v}\cdot\nabla f + \partial f/\partial t = 0$; this yields $\nabla\cdot(f\mathbf{v}) - f\nabla\cdot\mathbf{v} = -\partial f/\partial t$. It is convenient in the present context to specify this result putting $f = C$, in which case $f\mathbf{v} = \mathbf{J}$; thus

$$\nabla\cdot\Delta\mathbf{J} = -\nabla\cdot(D\nabla C) = -\frac{\partial C}{\partial t} + C\nabla\cdot\mathbf{v} \qquad C = C(x,y,z,t). \tag{8}$$

In the particular case where $\mathbf{v}$ is such that the second addend vanishes, one obtains a well known result, the second Fick equation subjected to the continuity boundary condition required by $\delta f = 0$ i.e.

$$\nabla\cdot\Delta\mathbf{J} = -\frac{\partial C}{\partial t} \qquad \nabla\cdot\mathbf{v} = 0. \tag{9}$$

The condition on $\mathbf{v}$ is satisfied if in particular:
(i) $\mathbf{v} = \mathbf{i}\text{v}_1(y,z,t) + \mathbf{j}\text{v}_2(x,z,t) + \mathbf{k}\text{v}_3(x,y,t)$ or (ii) $\mathbf{v} = \mathbf{v}(t)$ or (iii) $\mathbf{v} = const$.

Anyway, whatever the general analytical form of $\mathbf{v}$ might be, this condition means that the vector $\mathbf{v}$ is solenoidal, which classically excludes sinks or sources of matter in the volume Δx^3 enclosing m. Note however that since the boundaries of any uncertainty range are arbitrary and unknown, introducing the range $\Delta\mathbf{J} = \mathbf{J} - \mathbf{J}_0$ means implementing the actual $\mathbf{J}$ as change of the flux in progress with respect to a reference flux $\mathbf{J}_0$ appropriately defined. For instance $\mathbf{J}_0$ could be a constant initial value at an initial time t_0 of $\Delta t = t - t_0$ where the diffusion process begins, in which case $\mathbf{J}_0$ can be put equal to zero by definition; this means determining the initial boundary condition $\mathbf{J}_0 = 0$ at $t_0 = 0$. Yet more in general is remarkable the fact that, according to the Eq. (8), the usual classical form $\mathbf{J} = C\mathbf{v}$ is also obtained if $\mathbf{J}_0$ is regarded as a reference flux as a function of which is defined $\mathbf{J}$ that fulfils the condition

$$\nabla\cdot\mathbf{J} = -\frac{\partial C}{\partial t} \qquad \nabla\cdot\mathbf{J}_0 = -C\nabla\cdot\mathbf{v}. \tag{10}$$

The quantum chance of expressing the diffusion equations considering $\Delta\mathbf{J}$ instead of $\mathbf{J}$ emphasizes that the classical view point is a particular case of, and in fact compatible with, the Eqs. (1).

This section has shown that the usual Fick equation (8) written as a function of $\mathbf{J}$ and C does not hold necessarily in the absence of sinks or sources of matter only, it includes also the chance $\nabla \cdot \mathbf{v} \neq 0$ provided that the boundary condition about the reference flux gradient $\nabla \cdot \mathbf{J}_0$ is properly implemented. In this subsection it has been also shown that all this has a general quantum basis.

2.2 Diffusion and relativistic velocity addition rule

Let us consider the Eq. (7) $\Delta\mathbf{J} = C\mathbf{v}$ and express the change $\delta\Delta\mathbf{J}$ of $\Delta\mathbf{J}$ as a function of the variations of $\delta\mathbf{v}$ and δC

$$\delta\Delta\mathbf{J} = \mathbf{v}\delta C + C\delta\mathbf{v} \qquad \mathbf{v} = \mathbf{v}_x + \mathbf{v}_y + \mathbf{v}_z \qquad \mathbf{v} = \mathbf{v}(\Delta t) \tag{11}$$

to calculate the scalar product of $\delta\Delta\mathbf{J}$ by one component of $\mathbf{v}$, e.g. $\mathbf{v}_x$:

$$\mathbf{v}_x \cdot \delta\Delta\mathbf{J} = \mathbf{v}_x \cdot \mathbf{v}\delta C + C\mathbf{v}_x \cdot \delta\mathbf{v}. \tag{12}$$

It is interesting to define in particular $\delta\mathbf{v}$ orthogonal to this component $\mathbf{v}_x$ for reasons clarified below; hence

$$\mathbf{v}_x \cdot \delta\mathbf{v} = 0, \qquad \mathbf{v}_x = \delta\mathbf{v} - (\delta\mathbf{v})^2 \frac{\mathbf{v}_o}{\mathbf{v}_o \cdot \delta\mathbf{v}}. \tag{13}$$

The second equation shows the form of $\mathbf{v}_x$ that satisfies the former condition whatever the ancillary vector $\mathbf{v}_o$ might be. So, owing to the Eqs. (7) and (12), one finds

$$\mathbf{v}_x \cdot \delta\Delta\mathbf{J} = \mathbf{v}_x \cdot \mathbf{v}\delta C = \delta\Delta\mathbf{J} \cdot \delta\mathbf{v} - (\delta\mathbf{v})^2 \frac{\mathbf{v}_o \cdot \delta\Delta\mathbf{J}}{\mathbf{v}_o \cdot \delta\mathbf{v}}. \tag{14}$$

As concerns the second equality, eliminating $(\delta\mathbf{v})^2$ between the Eqs. (14) and (13) one finds

$$\mathbf{v}_x = \delta\mathbf{v} - \frac{(\delta\mathbf{v} - \mathbf{v}_x) \cdot \delta\Delta\mathbf{J}}{\mathbf{v}_o \cdot \delta\Delta\mathbf{J}} \mathbf{v}_o. \tag{15}$$

As concerns the first equality (14), it is possible to write

$$\mathbf{v}_x \cdot \delta\Delta\mathbf{J} = \pm \mathrm{v}_x \delta\Delta J_x,$$

$$\delta\Delta J_x = \pm \frac{\mathbf{v} \cdot \mathbf{v}_x \delta x}{\mathrm{v}_x} \frac{\delta C}{\delta x} = \pm(\mathrm{v}_x \delta x) \frac{\delta C}{\delta x}, \tag{16}$$

being $\delta\Delta J_x$ the modulus of the component of $\delta\Delta\mathbf{J}$ along $\mathbf{v}_x$. Note that $\mathbf{v} \cdot \mathbf{v}_x \delta x/\mathrm{v}_x = \mathbf{v} \cdot \mathbf{u}_x \delta x$, where $\mathbf{u}_x$ is a unit vector oriented along $\mathbf{v}_x$, has the physical dimensions of a diffusion coefficient D; so, being $|\mathbf{v}_x|$ arbitrary, the Eq. (16) reads

$$\delta\Delta J_x = \pm D \frac{\delta C}{\delta X}, \qquad D = q\mathrm{v}_x \delta x, \qquad \delta X = q\delta x, \tag{17}$$

with q again proportionality coefficient, as previously introduced. With the minus sign, the first equation fits the quantum result (6); this sign therefore is that to be retained. Also, this agreement supports the usefulness of the condition (13) and introduces a further result in the quantum frame of the present approach. Put $\mathbf{v}_x = \xi\mathbf{v}_o + \mathbf{v}_1$, being ξ an arbitrary constant and $\mathbf{v}_1$ another arbitrary vector; in this way $\mathbf{v}_x$ has been simply redefined through a linear combination of two vectors, as it is certainly possible. So the second Eq. (13) reads

$$\mathbf{v}_o = \frac{\delta\mathbf{v} - \mathbf{v}_1}{\xi - \dfrac{\xi(\delta\mathrm{v})^2}{\mathbf{v}_1 \cdot \delta\mathbf{v}}}.$$

Multiplying both sides of this equation by the unit vector $\mathbf{u}_z$ one finds

$$\mathrm{v}_{oz} = \frac{\delta\mathrm{v}_z}{\xi - \dfrac{\xi(\delta\mathrm{v})^2}{\mathbf{v}_1 \cdot \delta\mathbf{v}}}, \quad \mathrm{v}_{oz} = \mathbf{v}_o \cdot \mathbf{u}_z, \quad \delta\mathrm{v}_z = (\delta\mathbf{v} - \mathbf{v}_1) \cdot \mathbf{u}_z. \tag{18}$$

It is natural at this point to express the terms with physical dimensions of velocity and square velocity appearing in the last result as follows

$$\delta\mathrm{v}_z/\xi = u_a - u_b, \qquad (\delta\mathrm{v})^2 = u_a u_b, \qquad \mathbf{v}_1 \cdot \delta\mathbf{v} = c^2,$$

being u_a and u_b two arbitrary velocities; then one obtains

$$\mathrm{v}_{oz} = \frac{u_a - u_b}{1 - \dfrac{u_a u_b}{c^2}}. \tag{19}$$

The physical meaning of this result is acknowledged by reasoning "a posteriori", i.e. by assessing its implications. Trivial considerations show that, whatever the actual numerical value of c might be, if $u_a = u_b = c$ then $\mathrm{v}_{oz} = c$; also, the right hand side never exceeds c. Knowing that c is the upper value of velocity accessible to any particle [16], and so just for this reason invariant in different inertial reference systems in reciprocal motion [17], the Eq. (19) must have the physical meaning of addition velocity rule; the appropriate notation should be therefore $\mathrm{v}_{oz} = u'_a$ with u'_a corresponding to u_a in another reference system, which is possible because $\mathbf{v}_o$ has not been specifically defined. Also this conclusion is a corollary of the quantum principle of uncertainty, Eqs. (1), from which started the present reasoning.

Let us summarize the results achieved in this subsection. The Eqs. (6) and (7) introduce the laws of physics where the gradient of some non-equilibrium property, e.g. the non-uniform concentration of matter or charges and even temperature or pressure field gradients, generates the respective mass or charge or heat flows and related driving forces; this expresses the tendency of nature towards an equilibrium configuration corresponding to the maximum entropy [14]. Next the Eq. (12) enabled to infer the x-component of $\delta\Delta\mathbf{J}$ corresponding to that of the Eq. (6), thus emphasizing the connection of the present analysis with the straightforward quantum result. Eventually the orthogonality position of the Eq. (13) was also necessary to ensure that $\delta\mathbf{v}$ associated to $\delta\Delta\mathbf{J}$ does not imply the change of $\mathbf{v}_x$ to which is related D of the Eq. (17); so the Eq. (19) results pertinent to the Eq. (6) although obtained via $\delta\mathbf{v}$. This last result, Eq. (19), is a well known relativistic equation: the addition of the velocities, here expressed through one velocity component along an arbitrary axis identified by $\mathbf{u}_z$, cannot overcome the limit speed c despite u_a or u_b or both are themselves equal to c. All of these results have been obtained via the first equation (13) only, which is straightforward consequence itself of the Eqs. (1). Besides the concrete importance of these results, however, the question arises at this point: what is the physical connection between the gradient laws of physics and the relativistic composition of the velocities? Otherwise stated: if the gradient law describes the tendency of physical systems towards the equilibrium state, why this result has been inferred contextually to the velocity addition rule of the special relativity? This question can be further extended also considering the dimensional properties of the flux of matter of the Eq. (7), whose time derivative obtained differentiating the Eq. (7) yields

$$\frac{\delta\Delta\mathbf{J}}{\delta\Delta t} = C\dot{\mathbf{v}} + \mathbf{v}\dot{C}, \qquad \dot{\mathbf{v}} = \frac{\delta\mathbf{v}}{\delta\Delta t}, \qquad \dot{C} = \frac{\delta C}{\delta\Delta t}; \quad (20)$$

as explained in [17], the derivatives are defined in the present model via the Eqs. (1) only, i.e. as ratios of the uncertainty ranges therein introduced. In the present context the ratio regards the change $\delta\Delta\mathbf{J}$ during $\delta\Delta t$. Being $C = mass/volume$ and noting that $\Delta\dot{\mathbf{J}}$ is $force/volume$, one infers that $\mathbf{F} \approx m\mathbf{a}$ in the case where $\mathbf{v}\dot{C}$ can be neglected with respect to the former addend. As it is known, force and acceleration are parallel vectors in the non-relativistic approximation only; since both C and $\mathbf{v}$ are arbitrary, in general they are expected to contribute at increasing $\mathbf{v}$ to the relativistic limit $|\mathbf{v}| \to c$ where reasonably the second addend becomes important. In effect is sensible the fact that $\mathbf{v}\dot{C}$ someway surrogates the relativistic consequences of the space-time deformation, recalling that $C = m/V$; writing $V = \Delta x^3$ and regarding the time derivative as that due to the change of V pertinent to a fixed amount m of mass, in agreement with the Eq. (5), one infers $\dot{C} = -3C\Delta\dot{x}/\Delta x$. In fact $\Delta\dot{x}/\Delta x$ is a deformation of the space-time uncertainty range Δx, being by definition $\Delta\dot{x} = \delta\Delta x/\delta\Delta t$; so, at least in principle, the involvement of relativistic concepts like the deformation of the space-time in the presence of the mass is understandable. In effect, is not accidental the fact that just this space-time deformation is the relativistic contribution to the Newtonian term $m\dot{\mathbf{v}}$.

In conclusion, the actual quantum origin of the diffusion equations stimulates the question about why relativistic implications, apparently dissimilar, have been contextually obtained without any "ad hoc" hypothesis. The only possible answer is that the mere context of the quantum uncertainty contains itself the intimate connection that underlies fundamental laws even of apparently different nature. All considerations have been carried out by elaborating the Eqs. (1), which are thus the common root of these results: so this conclusion is not surprising because, as shown in [17], even the basic statements of quantum mechanics and special and general relativity are obtained as corollaries of the Eqs. (1). Therefore further considerations are expectedly hidden in this kind of approach, even as concerns the field gradient driven forces.

2.3 Diffusion and driving forces

The second equality (7) reads $\mathbf{v} = -D\nabla\log(C)$ and suggests a reasonable link with the known expression of the chemical potential $\mu = k_BT\log(C)$; this hint yields

$$\mathbf{v} = -\frac{D}{k_BT}\nabla k_BT\log(C) \qquad \mathbf{F} = -\nabla k_BT\log(C); \quad (21)$$

then merging the thermodynamic definitions of μ and mobility β, i.e. $\mathbf{v} = \beta\mathbf{F}$, one finds contextually the force $\mathbf{F} = -\nabla\mu$ acting on the diffusing species and the Einstein equation $D = \beta k_BT$ linking mobility and diffusion coefficient. Note however that it is convenient to define μ as

$$\mu = k_BT\log(C/C_j) \qquad C_j = C_j(t) \quad (22)$$

which leaves unaffected $\mathbf{F}$ and $\mathbf{v}$ and is still consistent with the asymptotic limits $\mathbf{F} \to 0$ and $\mathbf{v} \to 0$ for $C \to const$: i.e. the driving force of the diffusion process vanishes when C evolves as a function of time to reach any constant concentration. This limit implies a gradient free distribution of matter attained for $C \to C_j$ evolving as well e.g. to fit the limit value of C. Further information is also inferred with the help of the Eq. (2); dividing both sides by Δt, this equation reads in vector form $\mathbf{F} = \Delta\mathbf{p}/\Delta t = (\Delta\varepsilon/c^2\Delta t)\mathbf{v}$, which yields with the help of the Eqs. (1)

$$\mathbf{F} = \frac{n\hbar(c\Delta t)\mathbf{v}}{(c\Delta t)^3} = \frac{n\hbar}{\delta x^3}\mathbf{v}\delta x, \quad \delta x = c\Delta t, \quad \beta = \frac{c^2\Delta t}{\Delta\varepsilon} = \frac{(c\Delta t)^2}{n\hbar}.$$

Calculate the component of $\mathbf{F}$ along the arbitrary direction of a unit vector $\mathbf{u}$; owing to the Eq. (17) the scalar $\mathbf{v}\cdot\mathbf{u}\delta x$ at right hand side defines the diffusion coefficient D, so

$$\frac{F_u}{D} = \frac{n\hbar}{V}, \qquad V = \delta x^3, \qquad D = \mathbf{v}\cdot\mathbf{u}\delta x. \quad (23)$$

Merging the last equation with the Eq. (6), one finds $\mathbf{v}\cdot\mathbf{u}\delta x = qn\hbar/m$, which reads $mv_u\delta X = n\hbar$ and thus is just nothing else but the first equality (1). Implementing again the idea of expressing D via $n\hbar/m$ by dimensional reasons, see the Eqs. (6), the Eq. (23) reads

$$F_u = \frac{(n\hbar)^2}{mV}; \quad (24)$$

this step of the reasoning introduces diffusing mass and volume in the expression of the driving force of the macroscopic process whose diffusion coefficient is D. Interesting evidence about the importance of this result has been already emphasized in [16]; this point is so simple that it is worth being shortly summarized here for completeness.

The Eqs. (1) and (6) yield $qF_u/D = n\hbar/V$ and thus $qF_u/D = \Delta\varepsilon/\nu V$ having defined $\nu = \Delta t^{-1}$; so the right hand side is an energy range per unit frequency and unit volume. Putting $\Delta\varepsilon = h\nu$ one finds thus $qF_u/D = nh/V$. Let now V be the volume of a cavity in a body filled with radiation in equilibrium with its internal walls, whose size is able to contain the longest wavelength $\lambda = c/\nu$ of the steady radiation field; of course λ is arbitrary. Then $V = (2c/\nu)^3$, where the factor 2 accounts for λ with nodes just at the boundaries of the cavity, whose size is thus one half wavelength. Hence $F_u/D = 8h(\nu/c)^3 n/q$. Is significant here the physical meaning of the ratio F_u/D, which has physical dimensions $h/volume$, regardless of the specific values of F_u and D separately; thus, being F_u/D the component of the vector $\mathbf{F}/D$ along the arbitrary direction defined by $\mathbf{u}$, regard this latter as a unit vector drawn outwards from the surface of the body at the centre of the cavity. As $\mathbf{u}$ represents any possible path of the radiation leaving the cavity, let q be defined in this case in agreement with $\int(F_u/D)d\Omega = \pi nh/V$. Actually F_u/D is taken out of the integral because it has no angular dependence, whereas the integral $\int d\Omega$ is carried out over the half plane above the surface of the cavity only, which yields 2π; a factor 1/2 is also necessary as this is the probability that one photon at the surface of the cavity really escapes outwards instead of being absorbed inwards within the cavity. So $\int(F_u/D)d\Omega = 8\pi h(\nu/c)^3 n$ yields the Planck black body formula once replacing the number n of states allowed to the radiation field with the factor $(\exp(h\nu/k_BT) - 1)^{-1}$ of the Bose distribution statistics of all oscillators: as an arbitrary number of particles is allowed in each state, n is also representative of any number of particles concerned by the statistical distribution.

Implement now the definition of mobility to write $\delta\mathbf{v} = \beta\delta\mathbf{F} + \mathbf{F}\delta\beta$; dividing both sides by $\delta\beta$ one finds $\delta\mathbf{v}/\delta\beta - \mathbf{F}' = \mathbf{F} = -\nabla\mu$, having put $\mathbf{F}' = \beta\delta\mathbf{F}/\delta\beta$. By analogy with $\mathbf{F}$, let us introduce the position $\mathbf{F}' = -\nabla Y$ with Y appropriate energy function related to $\delta\mathbf{F}$; thus the result is

$$\frac{\delta\mathbf{v}}{\delta\beta} = -\nabla(Y + \mu). \tag{25}$$

The physical meaning of this result is highlighted thinking that the physical dimensions of β are $time/mass$; considering in particular a volume V of matter where the mass is conserved and simply redistributed, exactly as assumed in the Eq. (5), $\delta\mathbf{v}/\delta\beta$ is proportional to $mass \times \delta\mathbf{v}/\delta t$, i.e. it is nothing else but the law of dynamics previously found via $\partial\mathbf{J}/\partial t$. The Eq. (25), which agrees with the additive character of the force vectors, could be also obtained via Euler's homogeneous function theorem. Here $\mathbf{F}'$ is regarded as if it would be a function of β, whereas it is usually implemented as a function of the position vector $\mathbf{r}$ defined in an appropriate reference system. To this purpose it is enough to put the modulus $r = a\beta$, being a a parameter that controls the local values of mobility as a function of r, to write $\mathbf{F}(a\beta) = a^k\mathbf{F}(\beta)$. So calculating $\partial\mathbf{F}(a\beta)/\partial a\beta = \beta\partial\mathbf{F}(a\beta)/\partial a = ka^{k-1}\mathbf{F}(\beta)$ and putting then in particular $a = 1$, as shown in standard textbooks, one finds $\beta\partial\mathbf{F}/\partial\beta = k\mathbf{F}(\beta)$; this is the essence of the Euler theorem. Eventually, once having inferred $\mathbf{F}' = \beta\delta\mathbf{F}/\delta\beta = a^k\mathbf{F}(\beta)$, similarly to $\mathbf{F} = -\nabla\mu$ one concludes $\mathbf{F}' = -\nabla Y$ too. An example to elucidate Y could be the familiar force $\nabla Y = -ze\nabla\phi$ to which is subjected an ion of charge ze under the electric potential gradient $\nabla\phi$, in which case $Y + \mu = ze\phi + \mu$ is the well known electro-chemical potential controlling the working conditions of a fuel cell. The result (25) is in fact possible because $\delta\mathbf{F} = \mathbf{F}_2 - \mathbf{F}_1$ is an arbitrary force; whatever $\mathbf{F}_2$ and $\mathbf{F}_1$ might be, their arbitrariness ensures the general physical meaning of $\mathbf{F}'$ and thus its ability to be specified according to some particular physical condition. Suppose known for instance C, solution of the Eq. (8) with or without the condition (9). This solution provides one with information about the momentum pertinent to the mass transfer involved by the diffusion process. Indeed $\Delta\mathbf{J}$ represents from the dimensional point of view the momentum change per unit volume related to the redistribution of the mass within V. Thus, collecting the Eqs. (2) and (7), one finds $\Delta\mathbf{J} = \Delta\mathbf{p}/V = \mathbf{v}\Delta\varepsilon/c^2V = C\mathbf{v}$ being $\Delta\varepsilon/c^2 = m$ and $mC = V$ by definition. Putting then $\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_o$, trivial manipulations with the help of the first Eq. (21) yield

$$\frac{\mathbf{p}}{m} = \frac{\mathbf{p}_o}{m} - D\nabla\log(C).$$

The ratios involve the velocities $\mathbf{v}$ and $\mathbf{v}_o$ in agreement with the Eqs. (21); for instance, the former is the rate with which occurs the redistribution of m in V, the latter is the initial velocity of the concerned species before the redistribution. In summary, this section has shown that the diffusion equations imply the transfer of matter, energy and momentum; moreover, the velocity addition rule shows that the particles responsible of the mass transfer move in agreement with the relativistic requirements under the condition (13). Eventually the fact of having inferred $\mathbf{F} \approx m\mathbf{a}$ without precluding, at least in principle, even its possible generalization to the relativity, suggests that the quantum basis of these preliminary results is appropriate to carry out further tasks to describe the fundamental interactions too.

3 Entropy and chemical potential

As concerns μ of the Eq. (22) it is known that [21]

$$\left(\frac{\partial\mu}{\partial T}\right)_{P,\mathrm{n}} = -\left(\frac{\partial S}{\partial \mathrm{n}}\right)_{T,P}, \tag{26}$$

being dS the entropy change calculated keeping constant the pressure and temperature during the time necessary to increase n by dn; here n is a dimensionless amount of the concerned substance, e.g. a number of atoms or molecules, whereas dn can be approximately treated as a differential for large n only. The following considerations aim to integrate the Eq. (26) with respect to dn with the help of the Eq. (22).

Let m consist of a cluster of n_m atoms or molecules randomly distributed over an arbitrary number of elementary volumes V_j forming V, i.e. such that $V = \sum_j V_j$: so the given amount m of mass in the actual volume V is in fact distributed into several elementary volumes $V_j = V_j(t)$. Regard thus each V_j as a possible state allowed to one or more particles among the n_m available: if for instance V_j would be all equal, then each ratio $V_j/V = 1/n_j$ would yield the probability $\Pi_j = 1/n_j$ of each state accessible to m, being by definition $\sum_j n_j^{-1} = 1$. Moreover the possible distributions of n_m objects into the various V_j are functions of time related to the corresponding number N_j of allowed quantum configurations: whatever N_j might be in general, depending on the kind of statistical distribution compliant with the possible spin of the n_m particles, V_j/V is in fact a parameter related to the degree of disorder characteristic of m in V. Hence integrating the Eq. (26) with respect to dn means summing over all of the probabilities n_j^{-1} consistent with all possible V_j compatible with V; this also means integrating over $d(V_j/V)$ while keeping constant the total number of particles n_m in V, as required at left hand side of the Eq. (26) and in agreement with the Eq. (5). Putting therefore $C_j = m/V_j$ by analogy with $C = m/V$, one infers $C/C_j = V_j/V$ and then

$$S_j = S_o - \int (\partial\mu_j/\partial T)_{P,n} dn = S_o - k_B \int \log(V_j/V) d(V_j/V) =$$

$$= S_o - k_B (V_j/V)\Big(\log(V_j/V) - 1\Big).$$

Clearly the reasoning about the j-th states in V can be repeated for the j'-th states pertinent to the ratios V'_j/V' concerning the volume V', which consists of related elementary volumes V'_j such that $\sum_{j'} V'_j/V' = 1$. The same holds also for a volume V'' defined as sum of elementary volumes V'' and so on; in this way it is possible to define the resulting extensive entropy collecting together all integrals on V_j/V plus that on V'_j/V' and V''_j/V'', with $V + V' + V'' + \cdot\cdot = V_{tot}$ and the respective masses $m + m' + m'' + \cdot\cdot = m_{tot}$ each one of which is that already concerned in the Eq. (5). Then since by definition $\sum_{j'} V'_j/V' = \sum_{j''} V''_j/V'' = 1$ and thus $\sum_j V_j/V + \sum_{j'} V'_j/V' + \sum_{j''} V''_j/V'' + \cdot\cdot = j_{tot}$, summing over all elementary volumes of which consist the total mass and volume of the body yields

$$S = (S_o + j_{tot} k_B) - k_B \sum_j \frac{V_j}{V} \log\left(\frac{V_j}{V}\right). \tag{27}$$

The first addend is clearly a constant. This result defines an extensive function that collects all possible configurations N_j corresponding to all distributions of the various m in the respective volumes V_j compatible with each V where holds the Eq. (5). In principle V is arbitrary; yet it must be sufficiently large to be subdivided into V_j whose n_j allow considering dn_j as differentials. Note that the Eq. (27) has been early obtained in [14] elaborating directly the Eqs. (5). Appears clear the link between diffusion, regarded as the way through which the nature drives a thermodynamic system towards the equilibrium state, and entropy, $-\Sigma_j \pi_j \log \pi_j$, which measures the tendency towards states of progressively increasing disorder: this link is the underlying chemical potential μ, strictly connected with the concentration gradient of the diffusing species on the one side and with the related entropy change on the other side. If in the Eq. (26) $d\mu = 0$, which corresponds to $\mathbf{F} = -\nabla\mu = 0$ for uniform distribution of C, then $dS = 0$ reveals that the concerned system is in the state of maximum disorder. The diffusion of matter and energy is thus the driving force that puts into action the second law.

4 Diffusion and fundamental interactions

This is the central section of the paper. The fact of having inferred the results of the previous section from the fundamental Eqs. (1) along with relativistic implications, suggests that additional outcomes should be obtainable elaborating further the concepts hitherto introduced. For the following considerations it is useful to remark that the physical dimensions of $\mathbf{J}$ imply $flux/velocity = density = \rho$ and $flux \times velocity = energy\ density = \eta$. The interactions are thus described by a flux $\mathbf{J}$ of messenger particles, the respective boson vectors, displacing at rate $\mathbf{v}$ and characterized by mass and energy densities ρ and η. The starting point of this section is again the initial Eq. (9) identically rewritten as

$$\nabla \cdot \Delta\mathbf{J} + \frac{\partial C}{\partial t} = +\nabla \cdot \nabla \times \mathbf{U}_+,$$

which holds whatever the arbitrary vector $\mathbf{U}_+$ might be; indeed the last addend is anyway null. Let us rewrite this equation with the help of the position $\nabla \cdot \mathbf{U}_- = C$, which in turn yields

$$\nabla \cdot \left(\Delta\mathbf{J} + \frac{\partial \mathbf{U}_-}{\partial t} - \nabla \times \mathbf{U}_+\right) = 0. \tag{28}$$

So the vector within parenthesis must be a constant or a function of time only; then in general

$$\Delta\mathbf{J} + \frac{\partial \mathbf{U}_-}{\partial t} - \nabla \times \mathbf{U}_+ = \mathbf{J}_w, \qquad \mathbf{J}_w = \mathbf{J}_w(t), \qquad \nabla \cdot \mathbf{U}_- = C. \tag{29}$$

The physical dimensions of $\mathbf{U}_-$ and $\mathbf{U}_+$ are $mass \times surface^{-1}$ and $mass \times time^{-1} \times length^{-1}$, whence $\mathbf{U}_+ = \mathbf{U}_- c$ from dimensional point of view; c is the pertinent constant velocity. The homogeneous differential equation obtained from the Eq. (29) is

$$\Delta\mathbf{J} + \frac{\partial \mathbf{U}_-}{\partial t} - \nabla \times \mathbf{U}_+ = 0, \qquad \mathbf{J}_w = 0. \tag{30}$$

Starting from this quantum groundwork, the next subsections aim to highlight the steps ahead toward the goal of inferring the four fundamental interactions of nature as contextual corollaries.

4.1 The Maxwell equations

This subsection summarizes the reasoning reported in [15]; it is emphasized in the next subsection 4.2 how to include also the weak interaction still in the frame of the same approach.

Consider first the homogeneous differential equation inferred from the Eq. (30)

$$\nabla \times \mathbf{U}_+ = \Delta \mathbf{J} + \frac{\partial \mathbf{U}_-}{\partial t}, \qquad \nabla \cdot \mathbf{U}_- = C. \tag{31}$$

The first equation (31) defines the vector $\mathbf{U}_+$ as a function of $\mathbf{U}_-$, the second one defines the vector $\mathbf{U}_-$ as a function of C. Putting $\Delta\mathbf{J} = \mathbf{J}_2 - \mathbf{J}_1$, it is reasonable to expect also $\mathbf{U}_- = \mathbf{U}_2 - \mathbf{U}_1$ and thus $C = C_2 - C_1$. Moreover, besides the dimensional link, appears now a preliminary reason to define $\mathbf{U}_+$ via the same vectors that implement $\mathbf{U}_-$: there is no compelling necessity to introduce further vectors additional to $\mathbf{U}_1$ and $\mathbf{U}_2$, about which specific hypotheses would be necessary to solve both Eqs. (31). This choice simply requires $\mathbf{U}_+ = (\mathbf{U}_2 + \mathbf{U}_1)\xi$, being ξ an appropriate proportionality factor. The vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ just introduced are arbitrary, likewise the respective C_1 and C_2; for this reason both $\mathbf{U}_+$ and $\mathbf{U}_-$ have been defined with coefficients of the linear combinations of $\mathbf{U}_1$ and $\mathbf{U}_2$ equal to 1 without loss of generality. Hence, combining these definitions with the dimensional requirements, one finds

$$\mathbf{U}_+ = c(\mathbf{U}_2 + \mathbf{U}_1), \qquad \mathbf{U}_- = \mathbf{U}_2 - \mathbf{U}_1, \tag{32}$$

$$\mathbf{U}_2, \mathbf{U}_1 = mass/surface,$$

so that the second Eq. (31) yields

$$\nabla \cdot \mathbf{U}_2 = C_2, \qquad \nabla \cdot \mathbf{U}_1 = C_1, \tag{33}$$

whereas the first Eq. (31) takes the form

$$c\nabla \times \mathbf{U}_2 + c\nabla \times \mathbf{U}_1 - \mathbf{J}_2 + \mathbf{J}_1 - \frac{\partial \mathbf{U}_2}{\partial t} + \frac{\partial \mathbf{U}_1}{\partial t} = 0. \tag{34}$$

Now the problem arises about how could be rearranged the terms appearing in this equation. For instance the chance

$$c\nabla \times \mathbf{U}_2 - \mathbf{J}_2 - \frac{\partial \mathbf{U}_2}{\partial t} = \mathbf{J}' = -c\nabla \times \mathbf{U}_1 - \mathbf{J}_1 - \frac{\partial \mathbf{U}_1}{\partial t} \tag{35}$$

separates the quantities with subscript "2" from those with subscript "1"; the ancillary arbitrary vector $\mathbf{J}'$ that satisfies both equalities (35) can be in general different from zero. If so, then one obtains two equations

$$c\nabla \times \mathbf{U}_2 - \mathbf{J}'_2 - \frac{\partial \mathbf{U}_2}{\partial t} = 0, \quad -c\nabla \times \mathbf{U}_1 - \mathbf{J}'_1 - \frac{\partial \mathbf{U}_1}{\partial t} = 0,$$

$$\mathbf{J}'_2 = \mathbf{J}_2 + \mathbf{J}', \qquad \mathbf{J}'_1 = \mathbf{J}_1 + \mathbf{J}'. \tag{36}$$

Note that it is possible to change the physical meaning of the mass concentrations C_1 and C_2 of the Eqs. (33) simply multiplying both sides by q_m/m and q_e/m respectively; q_e is the total amount of electric charge possibly owned by the mass m, the physical meaning of q_m will be explained later in analogy with that of q_e. The multiplicative factors convert the mass density C_2 into the q_e charge density C_2^*, whereas C_1 turns into the q_m density C_1^*; analogously $\mathbf{U}_1$ and $\mathbf{U}_2$ turn into $\mathbf{U}_1^*$ and $\mathbf{U}_2^*$ in the Eqs. (33), whereas the same holds for $\mathbf{J}'_2$ and $\mathbf{J}'_1$ that turn respectively into charge and q_m flows $\mathbf{J}_2^*$ and $\mathbf{J}_1^*$ in the Eqs. (36). This means having converted $\mathbf{U}_1$ and $\mathbf{U}_2$ into quantities corresponding to the respective $\mathbf{J}_1^*$ and $\mathbf{J}_2^*$. Indeed the Eqs. (33) and the last two equations read

$$\nabla \cdot \mathbf{U}_1^* = C_1^*, \qquad \nabla \cdot \mathbf{U}_2^* = C_2^*,$$

$$C_1^* = C_1 \frac{q_m}{m}, \qquad C_2^* = C_2 \frac{q_e}{m}, \tag{37}$$

whence

$$c\nabla\times\mathbf{U}_2^* - \mathbf{J}_2^* - \frac{\partial \mathbf{U}_2^*}{\partial t} = 0, \qquad \mathbf{U}_2^* = \mathbf{U}_2 \frac{q_e}{m}, \qquad \mathbf{J}_2^* = \mathbf{J}'_2 \frac{q_e}{m}, \tag{38}$$

and

$$-c\nabla \times \mathbf{U}_1^* - \mathbf{J}_1^* - \frac{\partial \mathbf{U}_1^*}{\partial t} = 0$$

$$\mathbf{U}_1^* = \mathbf{U}_1 \frac{q_m}{m}, \qquad \mathbf{J}_1^* = \mathbf{J}'_1 \frac{q_m}{m}. \tag{39}$$

The Eqs. (38) and (39) have physical meaning different from that of the respective Eqs. (36); subtracting side by side these latter one of course finds again the initial Eq. (34), whereas the same does not hold for the Eqs. (38) and (39) that have been multiplied by the respective factors implemented in the Eqs. (37).

Exploit now the fact that the Eqs. (38) and (39) can be still merged together because anyway $c\nabla \times \mathbf{U}_2^* - \mathbf{J}_2^* - \partial\mathbf{U}_2^*/\partial t = -c\nabla \times \mathbf{U}_1^* - \mathbf{J}_1^* - \partial\mathbf{U}_1^*/\partial t$. Note however that the vectors $\mathbf{U}_1^*(\mathbf{J}_1^*)$ and $\mathbf{U}_2^*(\mathbf{J}_2^*)$ obtained solving separately the Eqs. (38) and (39) have scarce physical interest, because the boundaries of the initial uncertainty range $\Delta\mathbf{J}$ are arbitrary; whatever their form might be, they provide two independent solutions that are functions of their own flux vectors only. More interesting seems instead a general solution like $\mathbf{U}_1^*(\mathbf{J}_1^*, \mathbf{J}_2^*)$ and $\mathbf{U}_2^*(\mathbf{J}_1^*, \mathbf{J}_2^*)$, in fact also prospected by the initial Eqs. (35) themselves: this hint appears sensible because $\mathbf{U}_+$ and $\mathbf{U}_-$ consist by definition of the same vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ in the Eq. (31). So rewrite the last result as

$$c\nabla \times \mathbf{U}_1^* - \mathbf{J}_2^* - \partial\mathbf{U}_2^*/\partial t = 0 = -c\nabla \times \mathbf{U}_2^* - \mathbf{J}_1^* - \partial\mathbf{U}_1^*/\partial t,$$

where we have simply exchanged the sides where appear the curl vectors. For simplicity of notation, but without loss of generality, has been omitted the new flux vector $\mathbf{J}''$ possibly shared by both equalities; indeed, as previously done with $\mathbf{J}'$ to infer the Eqs. (36) from the Eq. (35), $\mathbf{J}''$ would have been once more incorporated within $\mathbf{J}_2^*$ and $\mathbf{J}_1^*$. In conclusion one obtains from the Eqs. (37) to (39)

$$\nabla \cdot \mathbf{U}_1^* = C_1^*, \qquad \nabla \cdot \mathbf{U}_2^* = C_2^*, \tag{40}$$

$$c\nabla \times \mathbf{U}_1^* - \mathbf{J}_2^* - \frac{\partial \mathbf{U}_2^*}{\partial t} = 0, \qquad c\nabla \times \mathbf{U}_2^* + \mathbf{J}_1^* + \frac{\partial \mathbf{U}_1^*}{\partial t} = 0.$$

Despite the notations, mere consequence of the fact that the starting point to attain the Eqs. (40) were the diffusion equations of the section 2, is evident the conceptual equivalence of these equations with the well known ones

$$\nabla \cdot \mathbf{H} = 0, \qquad \nabla \cdot \mathbf{E} = \rho_{ch}, \tag{41}$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{E}}{\partial t} - \mathbf{J}_{ch} = 0, \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t} = 0,$$

simply regarding $\mathbf{U}_2^* \equiv \mathbf{E}$ and $\mathbf{U}_1^* \equiv \mathbf{H}$ together with the charge density $C_2^* \equiv \rho_{ch}$ and $C_1^* = 0$. So, being $\mathbf{J}_2^*$ by definition identified with the charge current density $\mathbf{J}_{ch}$, the Eqs. (41) are nothing else but the Maxwell equations, usually written putting $C_1^* = \rho_{qm} = 0$ and $\mathbf{J}_1^* = \mathbf{J}_{qm} = 0$; these positions, due to $q_m = 0$, acknowledge the lack of experimental evidence of magnetic monopoles. Since these monopoles have not yet been observed experimentally, the correspondence has been emphasized as in the Eqs. (41), despite it would be very attracting and convincing to consider $q_m \neq 0$ too in the equations (41) by formal symmetry: it is worth emphasizing indeed that the reasoning hitherto carried out does not exclude at all the theoretical existence of the magnetic monopoles, rather this approach suggests explicitly them. The positions above that read now

$$\mathbf{U}_+^*/c = \mathbf{E} + \mathbf{H}, \qquad \mathbf{U}_-^* = \mathbf{E} - \mathbf{H},$$

entail four more reasons to validate the positions (32), according which $\mathbf{U}_-$ and $\mathbf{U}_+$ can be expressed through the same vectors they introduce:
(i) $\mathbf{U}_+^*/c + \mathbf{U}_-^* = 2\mathbf{H}$ and $\mathbf{U}_+^*/c - \mathbf{U}_-^* = 2\mathbf{E}$;
(ii) the same holds for the scalars $\mathbf{U}_+ \cdot \mathbf{U}_-/c = H^2 - E^2$ and $U_+^2/c^2 - U_-^2 = 4\mathbf{E} \cdot \mathbf{H}$;
(iii) $\mathbf{U}_- \times \mathbf{U}_+/c = 2\mathbf{E} \times \mathbf{H}$;
(iv) $U_+^2/c^2 + U_-^2 = 2(H^2 + E^2)$.

Once having specified in particular $\mathbf{H}$ and $\mathbf{E}$ as vectors proportional to magnetic and electric fields, then the proposed definitions of $\mathbf{U}_-$ and $\mathbf{U}_+$ entail the well known features: the scalars (ii) define two invariants with respect to Lorentz transformations, whereas the vector (iii) is proportional to the Poynting vector and defines the energy density flux; moreover the point (iv) defines a scalar proportional to the energy density of the electromagnetic field; finally, the integral $c^{-1} \int \mathbf{U}_+ \cdot \mathbf{U}_- dV$ over the volume previously introduced is proportional to the Lagrangian of a free field. As the only velocity that appears in these equations is c, one must conclude that the carriers of this kind of interaction are the photons. Despite these last considerations are well known, their mentioning here is not redundant: indeed these outcomes of the diffusion laws come from and complete the quantum frame of the Maxwell equations.

4.2 The weak interactions

The starting point of this subsection is the non-homogeneous Eq. (29) which concerns $\mathbf{J}_w \neq 0$. Of course even the results of the previous subsection hold when $\mathbf{J}_w \neq 0$ is negligible with respect to $\Delta\mathbf{J}$; so the content of this subsection is not to be regarded separately from the previous one, rather as its completion and generalization. Note that the Eq. (29) results formally similar to the Eqs. (35); the only difference is that $\mathbf{J}'$ is in general function of x, y, z, t, as no hypothesis has been necessary about it, whereas $\mathbf{J}_w$ is instead by definition function of time only in agreement with the Eq. (28). So this case can be formally handled as before, simply rewriting the Eq. (29) as

$$\Delta\mathbf{J}' + \frac{\partial \mathbf{U}_-}{\partial t} - \nabla\times\mathbf{U}_+ = 0, \quad \Delta\mathbf{J}' = \Delta\mathbf{J} - \mathbf{J}_w, \quad \mathbf{J}_w = \mathbf{J}_w(t). \tag{42}$$

Once replacing the previous change of flux $\Delta\mathbf{J} = \mathbf{J}_2 - \mathbf{J}_1$ with $\Delta\mathbf{J}' = \mathbf{J}_2 - \mathbf{J}_1 - \mathbf{J}_w$, is attracting the idea that in the present problem $\mathbf{J}_w$ describes a quantum time fluctuation of energy range $\Delta\varepsilon_w$ and time length Δt_w consistent with the uncertainty equations (1). To highlight the link between the flux modulus $J_w = |\mathbf{J}_w|$ and $\Delta\varepsilon_w$, let $\eta_w = \mathbf{v} \cdot \mathbf{J}_w$ be the energy density transient of time length $\Delta t_w = \hbar/\eta_w V$, being $V = \Delta x^3$ the volume within which is generated the mass density transient $\rho_w = m_w/V = J_w/\mathrm{v}$; of course $\mathrm{v} = |\mathbf{v}|$ is the modulus of the velocity with which the messenger particles propagate this kind of interaction, whereas $\Delta\varepsilon_w$ is the fluctuation energy change necessary to create messengers with lifetime Δt_w. It is possible to express the mass flux J_w of m_w as $\hbar/\Delta x_w^4$ by dimensional reasons; so $J_w = \xi\hbar/\Delta x_w^4$, being ξ a proportionality constant. Hence $\xi\hbar/\Delta x_w^4 = m_w\mathrm{v}/\Delta x_w^3$ yields

$$\zeta\frac{\hbar}{\Delta x_w} = m_w c, \qquad \mathrm{v} = \gamma c, \qquad \zeta = \frac{\xi}{\gamma};$$

so the range of this interaction force is $\Delta x_w = (\xi/\gamma)(\hbar c/m_w c^2)$. Let us estimate Δx_w putting preliminarily $\xi/\gamma \approx 1$, according to the reasonable idea that a proportionality constant correlating two quantities should be of the order of the unity; otherwise some further physical effect should be identified and implemented to justify $\xi/\gamma >> 1$. So one expects

$$\Delta x_w \approx \frac{\hbar c}{m_w c^2}, \quad \Delta x_w \approx 10^{-16}\mathrm{cm}, \quad m_w c^2 \approx 250\ \mathrm{GeV}. \tag{43}$$

The estimates have been guessed to exemplify the correlation between space range and energy scale; the figures are plausibly typical of the weak interactions. This preliminary estimate aimed merely to show that the positions $J_w \approx \hbar/\Delta x_w^4$ and $\rho_w \approx J_w/\mathrm{v}$ and mass m_w of the messenger particles are reasonable; this result must be however better assessed and more thoroughly justified.

The basic idea is that during the time transient described by $\mathbf{J}_w$, the range of the related interaction cannot be very wide; a long distance travel of messenger particles would require an extended time length, incompatible with the short-lasting transient Δt_w during which the classical energy conservation is temporarily replaced by the related quantum energy uncertainty $\Delta\varepsilon_w$. The next reasoning attempts to introduce a short range force mediated by massive particles created somewhere in the space-time by the energy fluctuation $\Delta\varepsilon_w$ and moving at rate $\mathrm{v}_w < c$: once having waived in the Eqs. (1) the local time and space coordinates, it is possible to say that at an arbitrary time t_0 the quantum fluctuation nucleates at the arbitrary point x_0, y_0, z_0 the total mass m_w that flows along with $\mathbf{J}_w$ within a volume V with average density ρ_w.

To confirm the existence of massive particles describing this interaction, divide the Eqs. (1) by Δt so that $\mathrm{v}_x \Delta p_x = \hbar/\Delta t = \Delta\varepsilon$ with $\Delta p_x \approx (m' - m)\mathrm{v}_x$ according to the Eq. (3): hence the uncertainty prospects the chance of two kinds of vector bosons of different masses describing the interaction.

Consider first the carrier of mass m and implement the Eq. (24), noting that the volume V defining the density ρ_w can be written as $V = \Delta x^2 \delta x_u$ without loss of generality; introducing indeed V via an arbitrary coefficient ξ is actually irrelevant, because $\xi \Delta x^2 \delta x_u$ would be handled exactly like $V = \Delta x^2 \delta x'_u$ simply rewriting $\delta x'_u = \xi \delta x_u$. So the actual geometric shape of V is waived because the sizes of Δx and δx_u are arbitrary in the conceptual frame based on the uncertainty Eqs. (1) only. Let us write the Eq. (24) as $\varepsilon_u = (n\hbar)^2/m\Delta x^2$ with $\varepsilon_u = F_u \delta x_u$ and then identify ε_u with the energy mc^2 necessary to create just the concerned rest mass m by virtue of the quantum energy fluctuation only; so one finds with $n = 1$ the reduced Compton length associated to m

$$\lambda\!\!\!^{-} = \Delta x, \qquad \lambda\!\!\!^{-} = \frac{\hbar}{mc}. \tag{44}$$

This expression holds for any particle free and neutral: the former condition assumes that m does not directly interact with m', the latter requires that no additional net charge is created during Δt_w because of the total charge conservation with respect to that early concerned by the Maxwell equations before the quantum fluctuation.

Analogous considerations hold for m', in particular as concerns the condition of charge conservation during the fluctuation time of $\mathbf{J}_w$. So m' either describes another neutral particle or it could actually consist of a couple of particles having equal mass and opposite charges; as in the latter case the charges interact to form an electromagnetic interaction driven Coulomb system with gain of energy, let therefore m' consist of two particles of equal reduced mass $m'_r = m'/2$. The energy ε_{em} and Bohr radius r_{em} of a hydrogenlike system are well known: considering the ground energy state with $n = 1$ only, they are $\varepsilon_{em} = -\alpha^2 m'_r c^2/2 = -e^2/2r_{em}$ with $r_{em} = \alpha^{-1}\hbar/m'_r c$; thus ε_{em} is defined by the diametric delocalization distance $2r_{em}$ only of the system of charges orbiting around their centre of mass [18]. Express r_{em} via the condition of steady circular waves $2\pi r_{em} = n_w \lambda_w$ early introduced to account for the stability of the old Bohr atom, whence $\varepsilon_{em} = -\pi e^2/n_w \lambda_w$ with $n_w \geq 1$ an arbitrary integer. Define then the new energy $\varepsilon_w = n_w \varepsilon_{em} = -\pi e^2/\lambda_w$. Clearly $n_w = 1$ still implies the electromagnetic energy $\varepsilon_w = \varepsilon_{em}$, whereas $n_w > 1$ implies $\varepsilon_w > \varepsilon_{em}$ since $\lambda_w < r_{em}$: this shows that actually ε_{em} and ε_w are both allowed and thus coexisting. On the one hand ε_w is hidden into and closely related to ε_{em}: having merely replaced r_{em} with the wavelengths λ_w allowed to the circular waves of charge, ε_w appears as a sort of short range high energy compatible with the electromagnetic interaction from which it differs for $n_w > 1$, rather than the energy of a separate form of interaction. On the other hand, if really the masses of all three particles correspond to the available energy ε_w, it should be true that $\varepsilon_w \approx 3m_w c^2$ for three equal masses m_w. In fact this expectation is compatible with $-\pi e^2/\lambda_w$ putting $m_w c^2 \approx e^2/\lambda_w$ while $\lambda_w \approx \lambda\!\!\!^{-} \approx \lambda\!\!\!^{-}{}'$; the replacement of r_{em} with the smaller λ_w accounts for the increase of energy necessary to create short range massive boson vectors, whereas the factor π replacing the expected factor 3 simply reveals that the masses of the neutral and charged boson vectors should actually be slightly different. Otherwise stated, regarding this result as $(m_0 + m_+ + m_-)c^2 = \pi e^2/\lambda_w$ with obvious meaning of symbols, one infers

$$m_0 c^2 + 2m_\pm c^2 = \pi \frac{e^2}{\lambda_w}, \qquad m_\pm c^2 = \frac{e^2}{\lambda_w},$$

$$m_0 c^2 = (\pi - 2)\frac{e^2}{\lambda_w}, \qquad m_+ = m_- = m_\pm. \tag{45}$$

Hence, it should be true that

$$m_0/(m_0 + m_+ + m_-) = (\pi - 2)/\pi,$$

$$m_\pm/(m_0 + m_+ + m_-) = 1/\pi.$$

Compare this last conclusion with the experimental data

$$m_{Z^0} = 91.19 \text{ GeV}, \qquad m_{W^\pm} = 80.39 \text{ GeV},$$

$$m_{tot} = m_{Z^0} + 2m_{W^\pm} = 251.97 \text{ GeV}.$$

Indeed $m_0/m_{tot} = 0.36$ and $m_\pm/m_{tot} = 0.32$ agree well with $(\pi - 2)/\pi = 0.363$ and $1/\pi = 0.318$; despite the non-relativistic approach, this agreement supports the idea that the energy gain ε_w due to the charge system accounts for the creation of its own mass plus a further neutral particle as well. The experimental energies support the idea that contracting λ_w from $2\pi r_{em}$ down to $2\pi r_{em}/n_w$ implies the chance of a new form of interaction correlated to and coexisting with the familiar electromagnetic interaction at increasing values of the quantum number n_w.

Let us put now

$$m'c^2 \approx \frac{\hbar}{\Delta t_w} \tag{46}$$

being Δt_w the characteristic lifetime of the vector bosons.

This result is reasonable, as m' is proportional to the characteristic energy $\hbar/\Delta t_w$. To calculate this expression, let us also assume $m' \propto \Delta t_w$: as any process in nature requires a definite time to be completed, it is natural to expect that the amount of mass creatable during the fluctuation of $\mathbf{J}_w$ is proportional to the time length of this fluctuation. In other words: the longer the fluctuation, the greater the transient amount of energy and thus of mass that can be created. Putting then $m' = k_w \Delta t_w$, where k_w is an appropriate proportionality constant, there are two chances: either $k_w \approx 1$ or $k_w \neq 1$. In general the latter chance means that some physical effect is still hidden in k_w, whereas the former chance means that in fact k_w accounts for the concerned physical correlation without need of further considerations. Let us guess that $k_w \approx 1$ effectively represents the fluctuation lifetime; then, replacing into the Eq. (46), one finds

$$k_w (c\Delta t_w)^2 = \hbar, \qquad k_w \approx 1\mathrm{g/s}, \tag{47}$$

which yields $\Delta t_w \approx 10^{-24}$s. Note that the second Eqs. (45) reads $m_\pm c^2 = \hbar\alpha c/\lambda_w$, which suggests that αc is the actual displacement rate of the charged vector bosons having energy $\hbar \mathrm{v}/\lambda_w$ and that the same holds for the neutral boson. Assuming therefore that $\mathrm{v} = \alpha c$ is the actual displacement rate of the massive bosons, the characteristic range of this interaction should be of the order of $\Delta x_w \approx \alpha c \Delta t_w = 2 \times 10^{-16}$cm, whereas $\hbar c/\Delta x_w \approx 0.15$ erg = 98 GeV in agreement with the Eq. (43) previously found.

In conclusion we have introduced three particles of comparable mass, of the order of 90 GeV, two of which with opposite charges and the third neutral, that propagate the interaction within the sub-nuclear space range Δx_w during a characteristic time range Δt_w. These results are the fingerprint of the weak interaction, which has been inferred as a generalization of the Maxwell equations inherent the homogeneous diffusion equation (30) via the transient fluctuation term $\mathbf{J}_w(t)$ appearing in the more general Eq. (29). So this kind of interaction differs in principle from, but it is strictly related to, the electromagnetic interactions of the Maxwell equations; it is simply an extension of these latter to the transient formation of three further short range carriers consistent with the time flux function $\mathbf{J}_w$ additional to the electric and magnetic fields described by $\mathbf{J}_2^*$ and $\mathbf{J}_1^*$, consequences themselves of the early Fick diffusion equations. It is worth emphasizing once again that the existence of magnetic monopoles does not conflict with, rather comes directly from, all of these outcomes and their quantum origin.

4.3 The gravity force

Exploit the dimensional relationship

$$\pm\mathbf{J}\cdot\mathbf{v} = \frac{|\mathbf{F}|}{surface}; \tag{48}$$

of course $\mathbf{v}$ is the rate with which propagate the carriers of the force $\mathbf{F}$ at right hand side and $\mathbf{J}$ their flux. The double sign takes into account either chance of sign in principle possible at left hand side, being the modulus of force positive by definition. The gravitons are acknowledged to be the carriers of the gravity force at the light speed; anyway, whatever the actual physical nature of these boson vectors and their displacement rate might specifically be, is enough for the present purposes to introduce a one-dimensional reference system R to which will be referred the scalars of the Eq. (48). This assumption on R is consistent with the chance of describing the gravitational interaction between two masses placed arbitrarily apart along one coordinate. Imposing this condition and thus introducing an arbitrary x-axis, write $|\mathbf{F}| = \xi F_x$: the x-component of $\mathbf{F}$ has been related to its modulus $|\mathbf{F}|$ via the dimensionless proportionality factor ξ, which obviously is an unknown variable quantity. Moreover, being $J_x = \hbar/\Delta x^4$, it is possible to write in an analogous way $\mathbf{J}\cdot\mathbf{v} = \pm\zeta J_x \mathrm{v}_x = \pm\zeta\hbar c/\Delta x^4$: once more the dimensionless proportionality factor ζ relating the scalar $\mathbf{J}\cdot\mathbf{v}$ to its arbitrary component $J_x\mathrm{v}_x$ is an unknown variable quantity. In this way, whatever v_x and the interaction carriers might be, $J_x\mathrm{v}_x$ can be expressed via ζ as a function of the constant quantity $\hbar c$. Of course, even $surface$ reduces to Δx^2 in R. These positions are useful to rewrite the initial Eq. (48) as $\zeta\hbar c/\Delta x^4 = \pm\xi F_x/\Delta x^2$ and thus $\zeta m_o^2 G/\Delta x^4 = \pm\xi F_x/\Delta x^2$ in R, having put $\hbar c = G m_o^2$ by dimensional reasons; this is surely possible by defining appropriately the value of the constant mass m_o. Yet the specific value of m_o is not essential: the term $m_o^2\zeta/\xi$ yields indeed $m_1 m_2$, with $m_1 = m_o\zeta$ and $m_2 = m_o/\xi$ because of the arbitrary values of the proportionality factors ζ and ξ. In this way m_1 and m_2 are two arbitrary inputs defining F_x, which indeed owing to the Eq. (48) reads

$$F_x = \pm G\frac{m_1 m_2}{\Delta x^2}.$$

Note that the Δx^{-2} law could be directly inferred from the Eqs. (1), since in the present model the derivatives are defined as mere ratios of uncertainty ranges. Differentiating the Eqs. (1) at constant n yields $\delta\Delta p_x = -(n\hbar/\Delta x^2)\delta\Delta x$, then dividing both sides by $\delta\Delta t$ corresponding to $\delta\Delta x$ one finds $\delta\Delta p_x/\delta\Delta t = -n\hbar\mathrm{v}_x/\Delta x^2$ with $\mathrm{v}_x = \delta\Delta x/\delta\Delta t$: at left hand side appears the x-component of a force, at right hand side the concept of mass is hidden in the physical dimensions of the factor $\hbar\mathrm{v}_x$, which reveals its physical meaning of space-time deformation rate of $\delta\Delta x$ during $\delta\Delta t$. Of course v_x is positive or negative depending on whether $\delta\Delta x$ represents expansion or contraction of Δx.

This short note aims to emphasize that in the present model the concept of gravity force is still linked to that of space-time deformation; yet the force also explicitly follows from the diffusion equations. In conclusion, taking the minus sign, we have found the Newton gravity law. Note however three remarks:

(i) this result is not new, it has been inferred in different ways directly from the Eqs. (1) in [20, 23];

(ii) here even the anti-gravity with the plus sign is allowed, as it has been repeatedly found elsewhere [22, 23];

(iii) the Newton law is actually an approximation of a more general gravity law, as found previously when concerning $\mathbf{F} \approx m\mathbf{a}$.

In fact one could guess an expression of *surface* like $\Delta x'^2 = \Delta x^2(1 + a_1\Delta x_o/\Delta x + a_2(\Delta x_o/\Delta x)^2 + \cdot\cdot)$; the series expansion is dimensionally compatible with the Eq. (48) and reduces to Δx^2 previously considered for $\Delta x \to \infty$ only, i.e. for weak gravity fields at large distances between the masses. This expansion defines a more general scalar component $\zeta J'_x \mathrm{v}_x = \pm\xi F'_x/\Delta x'^2$ defining a more complex force component $\pm F'_x$ that coincides, as a particular case, with that F_x previously found simply putting equal to zero the higher order coefficients $a_{j\geq 1}$ of the series expansion. Note that $F_x \to 0$ for $\Delta x \to \infty$. The present choice to express the series expansions of *surface* has been purposely assumed in order that even the non-Newtonian $F'_x \to 0$ satisfies the same condition of the Newtonian F_x.

4.4 The strong interaction

The starting point and the subsequent reasoning are still that of the subsection 4.3. Note however that the dimensional equation (48) does not compel defining *force* as purposely done before; as a subtle and possible alternative, nothing hinders defining in the one dimensional R the right hand side as

$$\pm\mathbf{J}\cdot\mathbf{v} = \frac{|\mathbf{F}|}{\Delta x^2} + \frac{energy}{\Delta x^3}. \qquad (49)$$

Proceeding as before, we merge again $\mathbf{J}\cdot\mathbf{v}$ with the concerned force per unit surface at the right hand side of the Eq. (48); one finds $\pm\xi\hbar c/\Delta x^4 = F_x/\Delta x^2 + \varepsilon_o/\Delta x^3$ i.e. $F_x = \pm\xi\hbar c/\Delta x^2 - \varepsilon_o/\Delta x$, where ε_o is a constant. This force component is derivable from a potential energy U having the form

$$U = \pm\frac{\xi\hbar c}{\Delta x} + \varepsilon_o \log(\Delta x/\Delta x_o), \qquad (50)$$

which in turn, putting $\Delta x = \Delta x_o \pm \delta x$, reads

$$U \approx \pm\left(\frac{a}{\Delta x} \pm b\delta x\right), \qquad \Delta x = \Delta x_o \pm \delta x,$$

$$a = \xi\hbar c, \qquad b = \frac{\varepsilon_o}{\Delta x_o}, \qquad \frac{\delta x}{\Delta x_o} << 1. \qquad (51)$$

This is certainly possible because, being both Δx and Δx_o arbitrary, the necessary inequality can be actually verified at short distances $\Delta x \gtrsim \Delta x_o$ or $\Delta x \lesssim \Delta x_o$. This result with the minus sign at right hand side reads

$$U \approx -\frac{a}{\Delta x} + b\delta x,$$

i.e. it leads to the sought interaction energy of interest here.

It is however also interesting to note that attractive and repulsive strong forces are in principle allowed in this model.

The physical dimensions of the constants a and b are $energy \times length$ and $energy/length$, so that $ab = energy^2$ and $a/b = length^2$: write then $\hbar/\sqrt{ab} = \Delta t_s$ whence $\hbar c/\sqrt{ab} = \lambda_s = c\Delta t_s$. The chance of introducing the characteristic range λ_s directly via c agrees with the idea of massless vector bosons mediating this kind of interaction, which follows in turn from the lack of a compelling motivation to introduce a slower velocity of heavy particles. Thus, putting reasonably $\lambda_s = \sqrt{a/b}$ too, one finds

$$a = \hbar c, \qquad \xi = 1, \qquad (52)$$

i.e. a sensible value of the proportionality constant ξ. Moreover holds also now the reasoning previously introduced about the proportionality between mass and characteristic lifetime of particles mediating the interaction. Let us repeat therefore an identical approach, concerning however the energy of the messengers instead of their mass to rewrite the proportionality condition $m \propto \Delta t$ as $\sqrt{ab}/c^2 \propto \Delta t_s$; introducing once more a proportionality constant k one finds $\sqrt{ab} = kc^2\Delta t_s$, which reads in turn $\sqrt{ab} = kc^2\hbar/\sqrt{ab}$ so that $ab = k\hbar c^2$. Hence, owing to the Eq. (52),

$$b = kc, \qquad k \approx 1\text{g/s}. \qquad (53)$$

The last position, coherent with that of the Eq. (47), is justified by the same hint of the previous section about the physical meaning of any proportionality constant correlating two physical amounts. The values of these constants are therefore

$$a = 3\times 10^{-17}\text{erg cm} = 0.2\text{ GeV fm},$$

$$b \approx 10^{10}\text{dyn} = 10^5\text{N}. \qquad (54)$$

These figures yield therefore the characteristic length Δx_o defined by $a/\Delta x_o = b\Delta x_o$ and the characteristic interaction time as a function of the characteristic energy $\sqrt{ab}$; one obtains

$$\Delta x_o = \sqrt{a/b} \approx 10^{-13}\text{cm}, \qquad \Delta t_s = \hbar/\sqrt{ab} \approx 10^{-24}\text{s},$$

$$\sqrt{ab} = \sqrt{k\hbar c^2} \approx 10^{-3}\text{erg} = 0.6\text{GeV}.$$

Note that $a/\Delta x$ reads $\hbar c/\Delta x_o = \alpha^{-1}e^2/\Delta x_o$, i.e. the strength of this kind of interaction is α^{-1} times greater than that of the electromagnetic interaction. The form of U in the Eq. (51) and these figures are fingerprints of the strong interaction.

5 Connection between gravity and electromagnetism

Note that in the cgs system $(charge/mass)^2$ has physical dimensions l^3/mt^2, i.e. the same as the gravity constant. Yet, what has to do the electromagnetism with the gravity force? The possible answer relies just on the hint suggested by the question itself, i.e. the link between $(e/m_G)^2$ and G. It is interesting the possibility of specifying m_G directly as follows

$$G = \frac{\hbar c}{m_G^2} = \frac{1}{\alpha}\left(\frac{e}{m_G}\right)^2,$$

which defines $m_G = 2.2 \times 10^{-5}$g as a function of the value of G assumed known; moreover, introducing m_G via its reduced Compton length $\bar{\lambda}_G$, one finds

$$G = \frac{1}{\alpha}\left(\frac{e\bar{\lambda}_G c}{\hbar}\right)^2 = \frac{e}{\alpha}\frac{e}{m_G^2}, \qquad \bar{\lambda}_G = \frac{\hbar}{m_G c}. \tag{55}$$

It is interesting the fact that the gravity constant is linked: (i) to the electromagnetism via the electric charge, (ii) to the relativity via c and (iii) to the quantum theory via $\hbar$; also, $\bar{\lambda}_G$ results to be of the order of the Planck length. However we acknowledge gravity and electromagnetism as two separate forces despite their common origin from the diffusion equations, whence the question: how and why does actually the nature split the electromagnetic and gravity forces? The starting point to answer this question is the Newton law itself previously found. Rewrite first the Newton law with the help of the Eq. (55) as

$$F = G\frac{m_1 m_2}{\Delta x^2} = \frac{e}{\alpha}\frac{e}{\Delta x^2}\frac{m_1}{m_G}\frac{m_2}{m_G}. \tag{56}$$

The only term of the second equality that does not depend neither upon Δx nor upon m_1 and m_2 is e/α. Let us split therefore this equation via a proportionality constant k as follows

$$G = k\frac{e}{\alpha}, \qquad \frac{m_1 m_2}{\Delta x^2} = \frac{F}{G} = \frac{1}{k}\frac{e}{\Delta x^2}\frac{m_1}{m_G}\frac{m_2}{m_G}. \tag{57}$$

Note now that the masses m_1 and m_2 appear in this equation as dimensionless ratios m_1/m_G and m_2/m_G; these pure numbers yield therefore

$$\frac{F}{G} = \frac{r_2}{k}\frac{Q_{e1}}{\Delta x^2} = \frac{1}{\alpha G}\frac{Q_{e2}Q_{e1}}{\Delta x^2}, \qquad Q_{e1} = r_1 e, \qquad Q_{e2} = r_2 e,$$

$$\frac{m_1}{m_G} = r_1, \qquad \frac{m_2}{m_G} = r_2. \tag{58}$$

In practice we have eliminated the concept of mass from the right hand side of F: the arbitrary variable r_1, which depends on the arbitrary value of m_1, converts the fixed charge e of the second equation (57) into the arbitrary total charge Q_{e1}. The ratio r_2/k involves an arbitrary number r_2 and a factor k that is reasonably related to the measure units of the modulus $Q_{e1}/\Delta x^2$ of a new quantity we call electric field strength due to the charge Q_{e1} at a distance Δx: hold indeed for Q_{e2} the same considerations highlighted for Q_{e1}, i.e. Q_{e2} is an arbitrary charge in the field of Q_{e1}. In fact the first Eq. (58) turns into

$$F = \frac{Q_{e2}}{\alpha}\frac{Q_{e1}}{\Delta x^2}. \tag{59}$$

From numerical and dimensional points of view, the factor α^{-1} is immaterial: since both Q_{e1} and Q_{e2} are arbitrary, one could identically write F as $Q'_{e2}Q_{e1}/\Delta x^2$ with $Q'_{e2} = Q_{e2}/\alpha$ without loss of generality. Conceptually, however, α^{-1} replaces in fact G: the latter describes the interaction between m_1 and m_2, the former that between Q_{e1} and Q_{e2}. This also shows that the analogous analytical form of the Coulomb and Newton laws is not at all accidental, as already shown in [23]. It is clear that the key step of this conclusion is the position $G = k(e/\alpha)$ of the Eq. (57). It is instructive to calculate e/α and compare it with the experimental values of G in the cgs and SI systems

$$G = 6.68 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2} = 6.68 \times 10^{-11}\text{m}^3\text{Kg}^{-1}\text{s}^{-2};$$

while being

$$e_{cgs} = 4.8 \times 10^{-10}\text{esu}, \qquad e_{SI} = -1.6 \times 10^{-19}\text{C}.$$

One finds

$$k_{cgs}\frac{e_{cgs}}{\alpha} = k_{cgs}6.6 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2},$$

$$k_{SI}\frac{e_{SI}}{\alpha} = k_{SI}2.1 \times 10^{-12}\text{m}^3\text{Kg}^{-1}\text{s}^{-2}.$$

Of course $k_{SI} \neq k_{cgs}$ for two reasons: (i) because of the different measure units and (ii) because in the cgs system the charge is directly defined via the electric force, in the SI the charge is defined in an independent way via the Ampere; thus k_{SI} requires an additional multiplicative factor k_0 to match G calculated simply changing the mass and length units of the proportionality constants k_{cgs} and k_{SI}. As the physical dimensions of k_{cgs} are $(length/mass)^{3/2}/time$, one expects $k_{SI} = (10^{3/2}k_{cgs})k_0$; the factor in parenthesis accounts for the different metric units only. Hence

$$G = k_{cgs}6.6 \times 10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2},$$

$$G = k_{cgs}k_0 6.6 \times 10^{-11}\text{m}^3\text{Kg}^{-1}\text{s}^{-2}. \tag{60}$$

This result clearly shows that the actual value of the gravity constant is well described by the dimensionless proportionality constant $k_{cgs} \approx 1$ and that $k_{cgs}k_0 \approx 1$ is also true; actually $k_0 \approx 1$ is not surprising, it is consequence of having implemented e_{SI} by including the Coulomb factor in the second Eq. (60). As repeatedly stated, a proportionality factor of the order of the unity shows that the correlation between two quantities is physically correct; no hidden effect is to be expected. What is significant is that the dimensionless values $k_{cgs} \approx 1$ and $k_0 \approx 1$ fit the experimental values of G in both systems.

To conclude this section, it is worth noticing that the value of G had been correctly calculated in several ways as a function of the fundamental constants of nature in the previous paper [20]; moreover more details about the connection between gravity and electric forces have been emphasized in a recent paper [23].

6 Discussion

The idea of linking the diffusion laws to the fundamental interactions was suggested by their generality and by the various implications inherent their basic concepts. Regarding the formulae of the section 2 as strictly related to the mere displacement of chemical elements, thus with outcomes pertinent to the solid state physics only, is certainly reductive. Actually some concepts can be extrapolated beyond the plain domain of the materials science, e.g. as they concern even the fields. This aspect, evidenced by the first and last Eqs. (4), has been emphasized considering for instance that the heat transfer Fourier law has formal physical analogy with the displacement of matter [14]. The connection with the fundamental interactions appears thus natural once acknowledging that these latter consist of the exchange of messenger particles, the vector bosons, that propagate throughout the space-time.

Follow the idea that any body of matter is surrounded by a cloud of bosons randomly flowing towards another body with which it interacts, and that in general both bodies are moving by effect of the interaction itself; consequently transients of local concentration gradients of these carriers throughout the space-time are also allowed to form. If so, the ability of the carriers to mediate the pertinent interaction reduces basically to the diffusion laws governing the displacement of clusters of these carriers. It has been evidenced that the concept of particle flux is crucial in finding the correlation between density gradient of the carriers and strength and kind of interaction; as the flux related to the concept of diffusion concerns intrinsically a non-equilibrium situation, even the interactions fit the idea of dynamical universe evolving towards a thermodynamic steady state.

Obviously the results introduced here are not exhaustive in describing themselves all features of the fundamental forces of the nature; this detailed investigation about each form of interactions is not the actual purpose of the model, which instead aims merely to identify their common root only by merging diffusion laws and quantum uncertainty only. On the one hand, the present conclusions must be regarded having already in mind also previous results, obtained starting directly from the Eqs. (1) to explain the significant features of the various interactions [15]. On the other hand, the fact that the same results are also obtainable via the diffusion laws is informative of the physical mechanism upon which these latter rely: otherwise stated, all interactions are consequences of the second law, i.e. the vector bosons transfer the interaction moving likewise chemical elements of a non-equilibrium thermodynamic system to increase the global internal entropy of the system. Are significant in this respect the considerations of the section 3. A further implication of the present model relies on the possibility of demonstrating that the magnetic monopoles can in fact exist, being compatible with the basic ideas from which the interactions are inferred: at the present stage of development, the model does not prospect any reason to reject their existence. The isotropy of the space-time is essential to introduce the pertinent diffusion coefficient as a numerical value D without requiring instead a tensor matrix; even without excluding that actually this position could be an oversimplification only, the results indicate that the assumption is acceptable at least at the present level of development of the model. Moreover no necessity of extra-dimensions appears in this context, which however does not exclude that these latter might actually exist.

A short remark is useful to explain why the diffusion equations are the key to infer contextually and in a surprisingly simple way the basic aspects of the fundamental interactions. A partial answer is that the concept of uncertainty does not require hypotheses or information about the kind of diffusion medium, kind of vector bosons and strength and range of the interactions; as the Eqs. (1) have a primary significance regardless of any ancillary information, their consequences are expected to match different kinds of interaction just because of their generality. Yet a more comprehensive answer is that the quantum Eqs. (1) are inherently consistent with the general relativity [17], so any reasoning based on these equations leads consequently to relativistic conclusions as well; this explains why some valuable relativistic implications have been contextually found as side outcomes throughout the paper. Previous and present results demonstrate the validity of the theoretical model where uncertainty ranges replace the local values of the dynamical variables; ignoring these latter means accepting that the former only have true physical meaning. On the one hand, it is worth recalling the key role of the arbitrary boundaries of the uncertainty ranges to demonstrate that the quantum origin of the Maxwell equations and related consequences, e.g. the Gauss theorem and the Faraday law, rely on the concept of space-time ranges: **E** and **H** were contextually introduced implementing just both boundaries of ranges to express via the Eqs. (1) the flux of vector bosons that mediate the electromagnetic interaction between charged particles. On the other hand, the most interesting aspect of the formalism based on ranges concerns its conceptual meaning that merges quantum theory and relativity: so the usefulness of the results presently achievable is not the only support to their validity.

In the wave mechanics the dynamical variables of the classical formulae are replaced by operators that constitute the wave equations, whose solutions provides the eigenvalues of the observables; in the present model the dynamical variables are replaced by the respective uncertainty ranges, the eigenvalues are inferred by elementary manipulations of the classical formulae while the quantization is introduced via n. The present model reverts thus fundamental inputs and outcomes of the standard wave mechanics: the uncertainty is no longer consequence of the commutation rules of postulated quantum operators, it becomes instead the fundamental statement as a function of which the operator formalism is inferred by consequence of the range formalism. Several papers, e.g.

[18, 19] show that this way of thinking is a valid alternative to the standard wave mechanics: the expressions of the eigenvalues are identical in all cases where the wave equations can be solved analytically without the need of numerical procedures. The intriguing advantage of the present approach is thus that it not only agrees with the wave formalism, in fact inferable as a corollary so that the present model is in principle compliant with any quantum results today known, but contextually implies even the conceptual foundations of the special and general relativity [17]; so are not surprising the chance of having obtained the Eq. (19) and recognized the approximate character of the Netwton law $\mathbf{F} \approx m\mathbf{a}$, preliminarily obtainable as in the Eq. (20), without the relativistic correction involving the space-time deformation in the presence of mass.

The quantum space-time uncertainty has profound implications in relativity, whose formulae result indeed expressed themselves via uncertainty ranges; although the formulae are seemingly identical, however their physical meaning is definitely different. E.g., it has been emphasized that the Eq. (2) entails the functional dependence $p_x = \mathrm{v}_x\varepsilon/c^2$ of the local dynamical variables: the latter equation is well known, the former seems a redundant and pretextuous attempt to rewrite the standard relativistic result. Yet just in this way, introducing ranges that replace local variables, the relativity is made compliant with the quantum theory. The local dynamical variables are incompatible with the Heisenberg principle, the uncertainty ranges do by definition; so the usual formulae of the standard relativity are mere classical limit cases of range sizes tending to zero, in agreement with the classical character of the relativity itself.

In short, the present paper is a further contribution confirming that the Eqs. (1) represent the common root underlying quantum theory and relativity.

7 Conclusion

The necessity of skipping a detailed analysis about the specific features of all forms of interaction, outside of the scope of this paper, ranks the significance of the essential outcomes provided by the model; the value of results already known relies on the fact of being obtained contextually in the frame of a unique idea, which emphasizes the validity of the theoretical basis so far implemented. The approach proposed here suggests that an appropriate basic assumption about the displacement mechanism of the vector bosons has prioritary importance with respect to the detailed speculation about the single interactions themselves; moreover the scalar $\mathbf{J} \cdot \mathbf{v}$ was proven effective as a common basis to infer distinguishing information even without introducing explicit hypotheses on the pertinent vector bosons. The analytical form of the gravity force was inferred waiving the specific nature of the gravitons; the well known form (51) of the strong force has been inferred waiving the features of the gluons and their property of exchanging the colour force between quarks, whereas the electromagnetic interaction was found related to the photons as a particular case of a more general electro-weak interaction involving massive vector bosons. The weak interaction only required considering explicitly the displacement velocity of the carriers, which cannot travel at the light speed as their masses affect the characteristic space range and lifetime. Yet the basic features of all interactions depend primarily on the diffusion like behaviour of vector bosons described case by case through the form of the respective scalars $\mathbf{J} \cdot \mathbf{v}$. Although such theoretical approach is seemingly classical, indeed the section 2 exploits standard vector calculus, relativistic implications are anyway evident and occasionally even unexpected; this is because the Eqs. (1) contain an obvious quantum character that however encloses also relativistic implications, which therefore appear by consequence while implementing them. Considering the quantum origin of the diffusion laws, it is not surprising that the implications of the model are general enough to span not only the solid state physics but also the fundamental interaction physics.

Submitted on February 11, 2015 / Accepted on February 14, 2015

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