

# Qualitative Prediction of Isotope Abundances with the Bipolar Model of Oscillations in a Chain System

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We analyzed the individual masses of non-radioactive isotopes of the chemical elements with an extended version of the bipolar model of oscillations in a chain system. When defining a small set of appropriate rules, the model is able to predict the isotope which possesses the highest abundance. This information can be read out from the continued fraction representations of the isotope masses. Isotopes with enhanced nuclear stability due to a magic number of neutrons in the nucleus were frequently found as exceptions from the model. The model is applicable to the di-, tri- and tetranuclidic chemical elements; it fails completely as soon as a chemical element is composed of 5 or more stable isotopes. From this we conclude that the bipolar model of oscillations in a chain system – in its present form – is not yet the final version; the model must still be extended.

## 1 Introduction

In a previous paper [1], the bipolar model of oscillations in a chain system was applied to the standard atomic weights of the chemical elements. The atomic weights of the 19 mononuclidic elements and Helium, which have the lowest standard deviations, were expressed in continuous fraction form without any outliers. This was the calibration (and determination of the phase shift) of the model. It was then found that the vast majority of atomic weights of the polynuclidic elements could be reproduced through continued fractions as well.

The underlying mathematical formalism worked as follows: the mean atomic weights were transformed into a continued fraction according to the equations

$$\ln \frac{m}{m_{electron}} = p_e + S, \quad \ln \frac{m}{m_{proton}} = p_p + S, \quad (1)$$

where  $p$  is the phase shift (it must hold  $p_p = -p_e$ ) and  $S$  is the continued fraction ( $e$  is Euler's number)

$$S = n_0 + \frac{e}{n_1 + \frac{e}{n_2 + \frac{e}{n_3 + \dots}}}. \quad (2)$$

Numerically (if  $\neq 0$ ),  $p_p$  was found to be  $-1.7918229$  for the calibrating (low standard deviation) data set.

In this article we extend this previously established version of the model and demonstrate how to predict with an adequate set of rules, which isotope of a given chemical element has the highest abundance.

## 2 Data sources and computational details

All masses and percentage abundances of isotopes were taken from the web-site of the National Institute of Standards (NIST). An isotope mass is understood as the mass of the neutral atom in its nuclear and electronic ground state.

As in previous articles, the continued fraction representation  $p + S$  is abbreviated as  $[p; n_0 | n_1, n_2, n_3, \dots]$ , where the free link  $n_0$  is allowed to be  $0, \pm 3, \pm 6, \pm 9, \pm 12, \pm 15 \dots$  and all partial denominators  $n_i$  can take the values  $e+1, -e-1, \pm 6, \pm 9, \pm 12, \pm 15 \dots$

## 3 Results and discussion

### 3.1 Model extension

Within the originally presented form of the bipolar model (eq. 1) it is not possible to express all the nuclide masses through continued fractions within the accuracy of their standard deviations. Two adjustments are mandatory, one is related to the model itself, the other one to the data set.

First we introduce an additional phase shift  $\delta$ , as it was already done in a previous article dealing with the electron density distribution in the Hydrogen atom [2]. We write

$$\ln \frac{m}{m_{electron}} = \delta_e + p_e + S, \quad \ln \frac{m}{m_{proton}} = \delta_p + p_p + S. \quad (3)$$

In the same manner as holds  $p_p = -p_e$ , must consequently hold  $\delta_p = -\delta_e$ , which means the bipolarity is strictly conserved. The only difference between  $\delta$  and  $p$  is the fact that  $\delta$  is a small phase shift ( $\neq 0$ , with either positive or negative sign) applying to *all* isotope masses, while the phase shift  $p$  varies among the data points. Some of the masses are associated to the phase shift zero, others to its non-zero value.

Second, in order to be able to express (almost) all the nuclide masses through continued fractions, we have to split the data set of non-radioactive nuclide masses into groups:

Group zero is the set of 19 mononuclidic elements, which was already analyzed in a previous article. Here the phase shift  $p$  was determined ( $p_p = -1.7918229$ ) and a  $\delta$  parameter was not considered, which means  $\delta_p = 0$ .

Group 1 is the set of dinuclidic elements. We require that the phase shift  $p$  remains the same for all nuclides, so only  $\delta$

must be adjusted in such a way that ideally all isotopes can be expressed through a continued fraction.

Group 2 is composed of all stable isotopes of the set of the trinuclear chemical elements.

Analogously the remaining chemical elements can be grouped. Every group of masses leads to the determination of a different numerical value of the parameter  $\delta$ .

The first task (before making any abundance prediction) is the determination of  $\delta$ , so that from the continued fraction representations (ideally) every isotope mass can be reproduced with a numerical error smaller than its standard deviation.

This means for every isotope mass we obtain 4 different continued fraction representations (eq. 3): two of them interpret the mass as a proton resonance and two others as electron resonances. In the case of no outliers, at least one of these continued fractions reproduces the mass value with an error smaller than its standard deviation.

### 3.2 Prediction rules

The following simple rules lead to a prediction of nature's preference for the one or the other isotope.

#### Rule 1:

The electrons contribute very little to the isotope mass, therefore the electron resonances are not decisive and we express the nuclide masses only as proton resonances, according to the equations

$$\ln \frac{m(\text{nuclide})}{m_{\text{proton}}} = \delta_p + 0 + S_0$$

and

$$\ln \frac{m(\text{nuclide})}{m_{\text{proton}}} = \delta_p + (-1.7918229) + S_p.$$

This means we calculate two continued fractions  $S_0$  and  $S_p$ . In all the fractions below, the number -1.7918229 is abbreviated as  $p$ .

#### Rule 2:

It is obvious that now, due to the elimination of the electron resonances, many nuclide masses cannot be expressed anymore through a continued fraction with a numerical error smaller than the standard deviation. Consequently we ignore the standard deviation criterion and consider continued fractions leading to a numerical error up to 0.3 u as valid; whenever this error is greater, the result is interpreted as "no continued fraction found".

The choice of 0.3 u as the allowed numerical error is not fully arbitrary. It was adjusted in such a way to make it possible to express at least 95% of the masses through *valid continued fractions*. If the allowed error is too small, many masses fall out of the model, so the model automatically does not work for them. However, with increasing error also rises the probability that the continued fraction has no physical relation to the mass.

#### Rule 3:

The priority rule for continued fractions with different phase shifts: the fractions with phase shift zero have priority.

#### Rule 4:

Comparison rule: we can compare only continued fractions (of different masses) which were calculated considering the same phase shift.

#### Rule 5:

Abundant isotopes accumulate in nodes and sub-nodes with *high positive denominator*.

#### Rule 6:

A nuclide mass which cannot be expressed through a continued fraction is not abundant.

### 3.3 Model verification

These rules are now applied to the different groups of isotope masses. For simplicity, only the first four denominators of the fractions are given, which is sufficient for comparison purposes.

**Group 1:** dinuclear chemical elements,  $\delta_p = 0.002919$ .

#### 1. Hydrogen:

$^1\text{H}$ : [0; 0 | -1146, e+1, -e-1, e+1], 99.9885%

$^2\text{D}$ : [0; 0 | e+1, 12, 9, 6], 0.0115%

Here we compare the first denominators:  $e+1 > -1146$ , so the model predicts that the isotope  $^2\text{D}$  is more abundant than the isotope  $^1\text{H}$ , which is not observed. The reason for the failure of the model is simply the fact that the isotope  $^1\text{H}$  is directly linked to the proton, the reference mass of the model, always more abundant than any other nuclide mass.

#### 2. Helium:

$^3\text{He}$ : [p; 3 | -24, 12, -e-1, -9], 0.000134%

$^4\text{He}$ : [p; 3 | 15, e+1, -e-1, e+1], 99.999866%

It is not possible to express the Helium isotope masses through continued fractions with phase shift zero. According to the priority rule for phase shifts we now consider the phase shifted fractions. As the first denominator (15) is higher than (-24), the isotope  $^4\text{He}$  should be preferred by nature.

#### 3. Lithium:

$^6\text{Li}$ : [p; 3 | e+1, e+1, -e-1, e+1], 7.59%

$^7\text{Li}$ : [p; 3 | e+1, 441, -6, -e-1], 92.41%

$441 > e+1$ , therefore the isotope  $^7\text{Li}$  should have the higher abundance, as observed. None of the Li isotope masses can be expressed via a continued fraction with phase shift zero.

#### 4. Boron:

$^{10}\text{B}$ : [0; 3 | -e-1, -21, 18, -15], 19.9%

$^{11}\text{B}$ : [0; 3 | -e-1, -e-1, -150, 15], 80.1%

$-e-1 > -21$ , therefore preference to  $^{11}\text{B}$ .

5. Carbon:  
 $^{12}\text{C}$ : [0; 3 | -6, **e+1**, -6, -6], 98.93%  
 $^{13}\text{C}$ : [0; 3 | -6, **-24**, -e-1, e+1], 1.07%  
 e+1 > -24, therefore preference to  $^{12}\text{C}$ .
6. Nitrogen:  
 $^{14}\text{N}$ : [0; 3 | -6, -e-1, e+1, -e-1], 99.636%  
 $^{15}\text{N}$ : [0; 3 | -9, 1137, -e-1, e+1], 0.364%  
 -6 > -9, therefore preference to  $^{14}\text{N}$ .
7. Chlorine:  
 $^{35}\text{Cl}$ : [0; 3 | **6**, -e-1, e+1, -e-1], 75.76%  
 $^{37}\text{Cl}$ : [0; 3 | **e+1**, e+1, -6, -e-1], 24.24%  
 6 > e+1, therefore preference to  $^{35}\text{Cl}$ .
8. Vanadium:  
 $^{50}\text{V}$ : [0; 3 | e+1, -e-1, **-18**, e+1], 0.25%  
 $^{51}\text{V}$ : [0; 3 | e+1, -e-1, **15**, e+1], 99.75%  
 15 > -18, therefore preference to  $^{51}\text{V}$ .
9. Copper:  
 $^{63}\text{Cu}$ : [p; 6 | -**36**, 6, e+1, -e-1], 69.15%  
 $^{65}\text{Cu}$ : [p; 6 | -**60**, -9, 9, e+1], 30.85%  
 -36 > -60, therefore preference to  $^{63}\text{Cu}$ .
10. Gallium:  
 $^{69}\text{Ga}$ : [p; 6 | **186**, -e-1, 6, -6], 60.108%  
 $^{71}\text{Ga}$ : [p; 6 | **63**, -15, 30, 6], 39.892%  
 186 > 63, therefore preference to  $^{69}\text{Ga}$ .
11. Bromine:  
 $^{79}\text{Br}$ : [p; 6 | **18**, 24, -27, 21], 50.69%  
 $^{81}\text{Br}$ : [p; 6 | **15**, 6, -e-1, 6], 49.31%  
 18 > 15, therefore preference to  $^{79}\text{Br}$ .
12. Rubidium:  
 $^{85}\text{Rb}$ : [p; 6 | 12, **15**, 6, -e-1], 72.17%  
 $^{87}\text{Rb}$ : [p; 6 | 12, **-e-1**, e+1, -e-1], 27.83%  
 15 > -e-1, therefore preference to  $^{85}\text{Rb}$ .
13. Silver:  
 $^{107}\text{Ag}$ : [p; 6 | 6, **-375**, 12, e+1], 51.839%  
 $^{109}\text{Ag}$ : [p; 6 | 6, **-12**, e+1, -9], 48.161%  
 As -12 > -375, the model predicts the higher abundance for the isotope  $^{109}\text{Ag}$ , which is not observed. So the element Silver is the first and only unexplained outlier where our model fails.  
 It is completely impossible to express these masses through continued fractions with  $p = 0$ .
14. Indium:  
 $^{113}\text{In}$ : [p; 6 | 6, -e-1, **-6**, 54], 4.29%  
 $^{115}\text{In}$ : [p; 6 | 6, -e-1, **6**, 18], 95.71%  
 6 > -6, preference to  $^{115}\text{In}$ , as observed.
15. Antimony:  
 $^{121}\text{Sb}$ : [p; 6 | e+1, e+1, -e-1, **e+1**], 57.21%  
 $^{123}\text{Sb}$ : [p; 6 | e+1, e+1, -e-1, **-e-1**], 42.79%  
 e+1 > -e-1, preference to  $^{121}\text{Sb}$ , as observed.
16. Lanthanum:  
 $^{138}\text{La}$ : [p; 6 | e+1, **24**, -e-1, e+1], 0.09%
- $^{139}\text{La}$ : [p; 6 | e+1, **33**, 6, -e-1], 99.91%  
 33 > 24, preference to  $^{139}\text{La}$ , as observed.
17. Europium:  
 $^{151}\text{Eu}$ : [0; 6 | -e-1, e+1, -e-1, **e+1**], 47.81%  
 $^{153}\text{Eu}$ : [0; 6 | -e-1, e+1, -e-1, **6**], 52.19%  
 6 > e+1, preference to  $^{153}\text{Eu}$ , as observed.
18. Lutetium:  
 $^{175}\text{Lu}$ : [0; 6 | -e-1, 6, **-e-1**, -e-1], 97.41%  
 $^{176}\text{Lu}$ : [0; 6 | -e-1, 6, **-6**, e+1], 2.59%  
 -e-1 > -6, preference to  $^{175}\text{Lu}$ , as observed.
19. Tantalum:  
 $^{180}\text{Ta}$ : [p; 6 | e+1, -e-1, e+1, **-9**], 0.012%  
 $^{181}\text{Ta}$ : [p; 6 | e+1, -e-1, e+1, **-6**], 99.988%  
 -6 > -9, preference to  $^{181}\text{Ta}$ , as observed.
20. Rhenium:  
 $^{185}\text{Re}$ : [0; 6 | -e-1, **9**, e+1, -9], 37.40%  
 $^{187}\text{Re}$ : [0; 6 | -e-1, **12**, -15, e+1], 62.60%  
 12 > 9, preference to  $^{187}\text{Re}$ , as observed.
21. Iridium:  
 $^{191}\text{Ir}$ : [0; 6 | -e-1, **21**, -6, e+1], 37.3%  
 $^{193}\text{Ir}$ : [0; 6 | -e-1, **33**, -27, -e-1], 62.7%  
 33 > 21, preference to  $^{193}\text{Ir}$ , as observed.
22. Thallium:  
 $^{203}\text{Tl}$ : [0; 6 | -e-1, **-15**, -396, -e-1], 29.52%  
 $^{205}\text{Tl}$ : [0; 6 | -e-1, **-12**, 6, e+1], 70.48%  
 -12 > -15, preference to  $^{205}\text{Tl}$ , as observed.

**Group 2:** trinucleidic chemical elements,  $\delta_p = -0.016544$ .

Now we apply the same system to the set of 6 trinucleidic chemical elements. We see that (with one magic number exception) the model identifies the most abundant isotope.

- Oxygen:  
 $^{16}\text{O}$ : [0; 3 | -**12**, -6, -24, e+1], 99.757%  
 $^{17}\text{O}$ : [0; 3 | -**18**, e+1, -36, -e-1], 0.038%  
 $^{18}\text{O}$ : [0; 3 | -**27**, -33, -e-1, e+1], 0.205%  
 -12 > (-18 or -27), preference to  $^{16}\text{O}$ , as observed; however the model does not explain why the isotope  $^{18}\text{O}$  is more abundant than  $^{17}\text{O}$ .
- Neon:  
 $^{20}\text{Ne}$ : [0; 3 | **585**, -15, 18, 6], 90.48%  
 $^{21}\text{Ne}$ : [0; 3 | **51**, -12, -e-1, 21], 0.27%  
 $^{22}\text{Ne}$ : [0; 3 | **27**, 15, -e-1, e+1], 9.25%  
 585 > (51 or 27), preference to  $^{20}\text{Ne}$ , as observed.
- Magnesium:  
 $^{24}\text{Mg}$ : [0; 3 | **15**, -6, -18, -e-1], 78.99%  
 $^{25}\text{Mg}$ : [0; 3 | **12**, -48, 12, -e-1], 10.00%  
 $^{26}\text{Mg}$ : [0; 3 | **9**, e+1, -e-1, e+1], 11.01%  
 15 > (12 or 9), preference to  $^{24}\text{Mg}$ , as observed.
- Silicon:  
 $^{28}\text{Si}$ : [0; 3 | **9**, -e-1, e+1, -e-1], 92.223%

<sup>29</sup>Si: no continued fraction found, 4.685%

<sup>30</sup>Si: [0; 3 | 6, e+1, 6, -e-1], 3.092%

9 > 6, preference to <sup>28</sup>Si, as observed.

5. Argon:

<sup>36</sup>Ar: [0; 3 | e+1, **e+1**, -e-1, e+1], 0.3365%

<sup>38</sup>Ar: [0; 3 | e+1, **6**, -6, 93], 0.0632%

<sup>40</sup>Ar: [0; 3 | e+1, **15**, 39, 6], 99.6003%

15 > (6 or e+1), preference to <sup>40</sup>Ar, as observed.

6. Potassium:

<sup>39</sup>K: [0; 3 | e+1, **9**, -e-1, -12], 93.2581%

<sup>40</sup>K: [0; 3 | e+1, **15**, 30, e+1], 0.0117%

<sup>41</sup>K: [0; 3 | e+1, **57**, e+1, -6], 6.7302%

57 > (9 or 15), preference expected to <sup>41</sup>K, which is against the experimental observations. Reason: Potassium is the element with atomic number 19. The isotope <sup>39</sup>K has 39 – 19 = 20 neutrons, which means a magic number of neutrons. This explains the increased abundance.

**Group 3:** tetranuclidic chemical elements,  $\delta_p = 0.025770$ .

1. Sulfur:

<sup>32</sup>S: [0; 3 | 6, **9**, 12, -429], 94.99%

<sup>33</sup>S: [0; 3 | 6, **-21**, -e-1, e+1], 0.75%

<sup>34</sup>S: [0; 3 | 6, **-6**, 9, -e-1], 4.25%

<sup>36</sup>S: [0; 3 | 6, **-e-1**, e+1, -e-1], 0.01%

9 is the highest denominator, preference to the isotope <sup>32</sup>S, which is indeed observed.

2. Chromium:

<sup>50</sup>Cr: [0; 3 | e+1, -e-1, **-e-1**, -6], 4.345%

<sup>52</sup>Cr: [0; 3 | e+1, -e-1, **24**, -15], 83.789%

<sup>53</sup>Cr: [0; 3 | e+1, -e-1, **6**, e+1], 9.501%

<sup>54</sup>Cr: [0; 3 | e+1, -e-1, **e+1**, e+1], 2.365%

24 is the highest denominator, therefore preference to the isotope <sup>52</sup>Cr, as observed.

3. Iron:

When considering the phase shift zero, for both isotopes, <sup>57</sup>Fe and <sup>58</sup>Fe, no continued fraction is found. This is the only case where two isotopes of a chemical element could not be expressed as proton resonance simultaneously. A better description is found for the phase shifted fractions, here only <sup>54</sup>Fe turns out to be an outlier. The model is correct when going down the priority hierarchy and analyze these phase shifted fractions:

<sup>54</sup>Fe: no continued fraction found, 5.845%

<sup>56</sup>Fe: [p; 6 | **-12**, -6, e+1, -6], 91.754%

<sup>57</sup>Fe: [p; 6 | **-15**, e+1, -e-1, e+1], 2.119%

<sup>58</sup>Fe: [p; 6 | **-15**, 48, 150, 12], 0.282%

-12 > -15, therefore <sup>56</sup>Fe has the highest abundance.

4. Strontium:

<sup>84</sup>Sr: [p; 6 | **15**, -e-1, -e-1, e+1], 0.56%

<sup>86</sup>Sr: [p; 6 | **12**, e+1, -6, -e-1], 9.86%

<sup>87</sup>Sr: [p; 6 | **12**, 18, -9, -6], 7.00%

<sup>88</sup>Sr: [p; 6 | **12**, -6, -12, 9], 82.58%

15 > 12, so the model predicts the highest abundance for the isotope <sup>84</sup>Sr, which is not observed. Reason: Strontium is the element with atomic number 38. The most abundant nuclide <sup>88</sup>Sr has 88 – 38 = 50, a magic number of neutrons, which explains the failure of our model.

5. Cerium:

<sup>136</sup>Ce: [p; 6 | e+1, **9**, -e-1, e+1], 0.185%

<sup>138</sup>Ce: [p; 6 | e+1, **12**, -e-1, e+1], 0.251%

<sup>140</sup>Ce: [p; 6 | e+1, **15**, e+1, -e-1], 88.450%

<sup>142</sup>Ce: [p; 6 | e+1, **30**, e+1, e+1], 11.114%

Our model predicts the highest abundance for the isotope <sup>142</sup>Ce. However, the most abundant isotope <sup>140</sup>Ce has a magic number of 140 – 58 = 82 neutrons, so its abundance is increased.

6. Lead:

<sup>204</sup>Pb: [0; 6 | -e-1, **-33**, 6, e+1], 1.4%

<sup>206</sup>Pb: [0; 6 | -e-1, **-21**, e+1, -e-1], 24.1%

<sup>207</sup>Pb: [0; 6 | -e-1, **-18**, e+1, -e-1], 22.1%

<sup>208</sup>Pb: [0; 6 | -e-1, **-15**, e+1, 6], 52.4%

-15 is the highest denominator, the model predicts the highest abundance for <sup>208</sup>Pb, as observed.

**Higher groups:** unfortunately, the model fails completely when predicting the most abundant nuclide for all chemical elements consisting of more than four isotopes. Despite the fact that the grouping scheme still allows the expression of the nuclide masses through continued fractions (with few outliers), no correlation between the maximum abundance and the denominators is visible.

## 4 Conclusions

We have shown that a minor extension of the bipolar model of oscillations in a chain system allows a satisfactory prediction of the most abundant isotope for a given chemical element. Most outliers occur when one of the isotopes has a magic number of neutrons in the nucleus. From its total failure for elements with 5 or more stable isotopes, we conclude that our model is still incomplete and must be extended.

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