LETTERS TO PROGRESS IN PHYSICS

Mansouri-Sexl Test Theory: The Question of Equivalence between Special Relativity and Ether Theories

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The goal of this paper is drawing attention to a mistake confusing discussion upon the alternatives to special theory of relativity (STR). In the Mansouri-Sexl test theory utilized as a mathematical framework for testing the preferred frame theories, the Lorentz transformation of time has an erroneous form. This generates a false conclusion, namely that a theory based on Tangherlini transformation is empirically equivalent to STR.

Before the advent of STR, FitzGerald [1] and Lorentz [2] proposed a solution to the Michelson-Morley experiment, different from that resulting from the Einstein's theory. Their idea, extensively developed in the Lorentz's theory of electrons [3, 4] (later known as Lorentz ether theory — LET) consisted in assumption that objects moving with respect to a postulated preferred frame of reference, determined by motionless "aether", are contracted in the direction of their motion. This idea, together with the introduced by Larmor assumption that clocks moving through ether slow down by a velocity dependent factor, sufficed also to explain the modified M-M experiment, i.e. the Kennedy-Thorndike experiment. Defined in these terms, length contraction and time dilation constitute real processes of dynamic origin, connected with the impact of absolute motion on molecular forces. However, after appearing of Einstein's 1905 paper on STR [5], this idea has been ignored and abandoned by the overwhelming majority of physicists. The reason was that, in spite of its different ontology LET did not formally differ from STR, neither led to specific empirical predictions. The underlying cause binds to the space-time transformations, in fact determining the shape of theory. Namely, the Lorentz transformation (to which Voigt, Larmor, Poincare and Lorentz contributed in various degree) evolved to a symmetrical form reflecting the STR founding postulates instead of the Lorentz's assumptions. Thus, paradoxically, Lorentz transformation became the main obstacle in evolving the original Lorentz's idea to a form of consistent autonomic theory. Eventually, LET gained the status of a superfluous ontology put upon the STR formalism (so-called "Lorentzian approach to relativity"), which made the choice between LET and STR the question of simplicity ruled by the Occam's razor. Neither the (much later) space-time transformation consistent with original assumptions (Tangherlini [6]), nor the Bell's exact calculations (Bell [7]) deriving "relativistic" effects from Maxwell's equations by means of classical physics and quantum mechanics, did alter this general opinion.

The today's version of LET takes the form of test theories verifying STR by introducing free parameters instead of these resulting from definite assumptions. They are in particular the Robertson's test theory [8] and Mansouri-Sexl theory [9–11] for their basic equivalence known by the common name of Robertson-Mansouri-Sexl test theory (RMS). We shall focus on the Mansouri-Sexl (M-S) transformation presented in [9], considered to be a proper mathematical framework for experiments verifying special relativity. While the Lorentz transformation (boost) is

$$\left.\begin{aligned} t' = \gamma\left(t - \frac{vx}{c^2}\right), \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right) \\ x' = \gamma(x - vt), \qquad x = \gamma(x' + vt') \end{aligned}\right\}, \tag{1}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

(while $y' = y$, $z' = z$, in all transformations here considered), Mansouri & Sexl introduced a generalization:

$$t = a\,T + \epsilon X, \qquad x = b\,(X - vT) \tag{2}$$

The coordinates, X, T are the ones measured in the postulated preferred frame Σ in which the speed of light is axiomatically isotropic. Instead, x, t are the coordinates measured in frame S being in standard configuration with Σ. The idea consists in measuring independently the factors a and b (functions of v) in experiments, and to choose one of two alternative values of ϵ: $-v/c^2$ or 0, corresponding to the alternative synchronization conventions. The first, Poincare-Einstein (P-E) "internal" synchronization, based on the axiom of isotropic one-way speed of light in any inertial frame (i.e. based on the postulate of invariant speed of light), relates to $\epsilon = -v/c^2$, a factor responsible for the relativity of simultaneity. The second, "external" synchronization, related to $\epsilon = 0$, consists in adjusting all inertial clocks to the clocks

synchronized in the preferred frame Σ according to the P-E synchronization, which entails absolute simultaneity. Beside these two, there exists a third possible convention, the (slow) clock transport, which can be classified as internal procedure. The clock-transport convention confirms P-E synchronization provided STR is correct; instead its relation with the theories involving absolute simultaneity is not unambiguous inasmuch as they basically may, or may not predict time dilation and length contraction.

Any observed deviations from the exact relativistic values of a and b in the first or second order experiments (according to Mansouri and Sexl, resulting in deviations from the isotropic two-way speed of light) would speak for the preferred frame alternatives to STR. Mansouri and Sexl state that for $a = b = 1$, $\epsilon = 0$, the Galilean transformation is obtained, which is correct. If, after employing the external synchronization, a and b equal to unity, it would mean that mechanical phenomena are ruled by Newtonian physics and subject to the Galilean principle of relativity, while the Maxwell equations (and the relevant constant speed of light) refer to the preferred frame (ether) only. This is exactly what Michelson and Morley (ineffectively) expected to detect in their experiments.

However, Mansouri and Sexl also claim that for $1/a = b = \gamma$ and $\epsilon = -v/c^2$, their transformation turns into the Lorentz transformation, which is obviously wrong. This mistake is coupled with the incorrect notation of the Lorentz transformation of time, written in their paper as:

$$t' = \frac{t}{\gamma} - \frac{vx}{c^2}\,, \tag{3}$$

whereas the correct form is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right). \tag{4}$$

In fact, this mistake is not simply accidental; being trivial, it has however a dipper cause. Namely, Mansouri and Sexl intended to treat separately the questions of time dilation and simultaneity. This, however, is infeasible with respect to the Lorentz transformation in which relativity of simultaneity and relativistic effects are inseparably connected. This mistake entails false conclusion as to the question of equivalence between STR and the postulated ether theory. It also maintains a persistent myth, according to which the Michelson-Morley experiment, together with the Kennedy-Thorndike experiment provides an evidence for the invariant speed of light. What these (and other) experiments proved in fact with a high degree of probability is the isotropy of the two-way speed of light, which however is not tantamount to isotropy of the one-way speed of light. Mansouri and Sexl came to a false conclusion that the difference in one-way speed of light is a sole matter of choice of the synchronization convention. Consequently, they concluded that only violation of the two-way isotropy resulting in deviations from the relativistic values of a and b constitutes a challenge to STR.

From among various alternatives to special relativity, the preferred frame theory (PFT) here considered seems to be the only one consistent with the Lorentz's original idea (we treat PFT as a specific formulation of "ether theory"). It is based on the general assumption according to which there exists a physically substantial preferred frame of reference, of which the properties are:

1. In the preferred frame, the one-way speed of light is isotropic;
2. The bodies moving in the preferred frame shrink by the Lorentz factor in the direction of their motion; the clocks moving in the preferred frame slow down by the Lorentz factor.

The effects mentioned in the second postulate are interpreted as "real", which means that their relation to the preferred frame does not depend on the choice of reference frame in which they are described. Provided that, from these postulates one derives the following asymmetrical transformation between the preferred frame Σ (coordinates T, X) and frame S moving with respect to the preferred frame (coordinates t, x):

$$\left.\begin{aligned} t &= \frac{T}{\gamma}\,, & T &= t\gamma \\ x &= \gamma(X - vt)\,, & X &= \frac{x}{\gamma} + vt\gamma \end{aligned}\right\}. \tag{5}$$

While using the notation used in M-S transformation, this would mean: $1/a = b = \gamma$, $\epsilon = 0$. Transformation (5) determines all dynamic and kinematical properties of PFT. Formally, the above transformation and Lorentz transformation do not convert to each other. Mansouri and Sexl quote this transformation in their paper [9], rightly attributing it to Tangherlini. However, they erroneously claim Tangherlini transformation differs from Lorentz transformation only with respect to the synchronization convention employed, which is a direct consequence of a basic mistake above-mentioned. They conclude that theories determined by these transformations (i.e. STR and PFT) are empirically equivalent to each other. According to this viewpoint, the ether system can be singled out in an arbitrary manner and thus respective predictions concerning experimental results in any inertial system are identical. This false conclusion confuses discussion on the Lorentzian approach for nearly forty years.

As a matter of fact, PFT shares some empirical predictions with STR. The main similarity is that PFT predicts length contraction and time dilation by the usual Lorentz factor, provided measurements are executed in the preferred frame (in more detail Rybicki [12]). It predicts e.g. the elongation of lifetime of muons crossing the atmosphere since the Earth frame is nearly identical (compared with the muon's speed) with the postulated preferred frame. It also gives identical to STR prediction (although different interpretation) to

the twin paradox, irrespectively of the choice of the observer's "rest" reference frame. This also refers to the "realistic" version of twin paradox, namely the Hafele-Keating experiment.

PFT predicts the isotropic two-way speed of light, which makes the M-S theory ineffective in testing this alternative to STR. To show this question in details, let us return to the usually used notation with primed and non-primed coefficients, here the latter attributed to the preferred frame (thus, below, S denotes the preferred frame and S' the frame in motion). From the fact that clocks and measuring rods moving with respect to S are distorted in the definite way by the Lorentz factor it follows that, in S', the speed of light traveling along x-axis is, dependently on the (positive or negative) direction:

$$c'_1 = (c - v)\,\gamma^2, \qquad c'_2 = (c + v)\,\gamma^2, \tag{6}$$

where v denotes the velocity of the observer with respect to the preferred frame along x-axis. (In the 2D and 3D depictions, the light wave front form ellipse and ellipsoid, respectively). The averaged two-way speed of light on path l' parallel to x-axis is constant (isotropic) since the respective time is

$$t' = \frac{l'}{(c - v)\,\gamma^2} + \frac{l'}{(c + v)\,\gamma^2}\,. \tag{7}$$

After simple algebra, one gets $t' = 2l'/c$, a result identical to that predicted by STR. While the speed of light defined according to STR determines the relativity of simultaneity, the speed of light defined according to Eq. (6) forms an alternative solution, in the sense that it determines absolute simultaneity.

In general, the concept of "relative velocity" between two frames, defined in STR as identical speed (the same for the observers in S' and S), is replaced in PFT by the concept of "mutual velocities". While S' moves against S with the velocity v, the speed of S measured in S' becomes

$$v' = v\gamma^2. \tag{8}$$

This involves significant consequences, e.g. such as the following one. Assume S' and S'' are the frames in motion to each other, and that their velocities with respect to the preferred frame S are identical. Since also the Lorentz factors described in S for the frames S' and S'' are identical, the mutual velocities measured in both frames must be identical either, thus constituting the "relative velocity" in the STR sense. However, contrary to the STR predictions, neither of these frames will manifest "relativistic effects" (length contraction and time dilation) when observed (measured) from the other one, since

$$\frac{\gamma'}{\gamma} = \frac{\gamma''}{\gamma} \Longrightarrow \frac{\gamma'}{\gamma''} = 1. \tag{9}$$

This specific prediction of PFT, together with the characteristic "position" of the Earth with respect to the assumed preferred frame enables experiment settling between STR and PFT. Namely, one assumes that, if the preferred frame exists, it is likely identical with the (local) frame in which the cosmic microwave background radiation (CMBR) is isotropic. Meanwhile, from the observed Doppler effect obtained from WMAP known as "dipole anisotropy" one deduces that Solar System moves with respect to isotropic CMBR with the velocity 368 ± 2 km/sec in the direction of galactic longitude $l = 263.85^\circ$ and latitude $b = 48.25^\circ$. This translates to the Lorentz factor:

$$\gamma = (1 - 1.52 \times 10^{-6})^{-1} \approx 1 + 7.6 \times 10^{-7}. \tag{10}$$

PFT predicts that an object moving with equal velocity with respect to the isotropic CMBR, in the direction (e.g.) opposite to that of Solar System (i.e. $l = 83.85^\circ$ and $b = 228.25^\circ$) will not exhibit any relativistic effects since $\gamma'/\gamma'' = \gamma''/\gamma' = 1$. This prediction is absolute, i.e. does not depend on the choice of synchronization conventions or any other assumptions. It is quite obvious that in the lab experiments with γ reaching the value of 1,000 (thousand) and higher, the difference between 7.6×10^{-7} and zero is not identifiable. To be detected, it thus demands employing subtle methods in the specially aimed experiments. Nevertheless, it does not seem to lie beyond the scope of the today's experimental capabilities.

Conclusion

We have shown that an incorrect notation of the Lorentz transformation of time in the Mansouri-Sexl test theory entails false claims, namely:

1. Only the theories predicting anisotropic two-way speed of light differ from STR;
2. A theory maintaining absolute simultaneity is equivalent to special relativity (Mansouri and Sexl call this a "remarkable result");
3. As far as prediction of experimental results is concerned, Tangherlini transformation is completely equivalent to Lorentz transformation.

These claims confuse the discussion upon the preferred frame alternatives to special relativity. Contrary to a common belief, a theory based on the preferred frame postulate and formalized by Tangherlini transformation (i.e. PFT) is not in whole experimentally equivalent to STR. Thus settling between them two in experiments is a feasible task. The present author aims to develop this subject in the subsequent papers.

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