The Structure of the Photon in Complex Vector Space

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Considering the complex vector electromagnetic field, the energy of the photon is expressed as an even multivector consisting of a scalar kinetic energy part and a bivector rotational energy part. Since any even multivector can be expressed as a rotor representing internal rotations, the electromagnetic energy even multivector represents internal complex rotations. It has been shown that the spin angular momentum is the generator of rotations in the plane normal to the propagation direction and the orbital angular momentum is the generator of rotations in a plane normal to the spin plane. The internal structure of the photon may be visualized as a superposition of electromagnetic field flow or rotation in two normal orientations in complex vector space. The cause of such complex rotations is attributed to the presence of electromagnetic zeropoint field.

1 Introduction

Even after the photon inception into the field of physics over a century ago, the obscurity in understanding the photon structure persists. The concept of the photon, the energy quanta of electromagnetic radiation, was introduced by Planck in the blackbody radiation formula and Einstein in the explanation of the photoelectric effect. The photon is normally considered as a massless bundle of electromagnetic energy and the photon momentum is defined as the ratio between the energy of the photon and the velocity of light. It is well known from Maxwell's theory that electromagnetic radiation carries both energy and momentum [1]. The linear momentum density is given by the Poynting vector $\mathbf{E} \times \mathbf{B}$ and the angular momentum is the cross product of the Poynting vector with the position vector. Poynting suggested that circularly polarized light must contain angular momentum and showed it as the ratio between the free energy per unit volume and the angular frequency. In 1936, Beth [2] first measured the angular momentum of light from the inference that circularly polarized light should exert torque on a birefringent plate and that the ratio between angular momentum J and linear momentum P was found to be $\lambda/2\pi$, where λ is the wavelength of light. The measured angular momentum agreed in spin magnitude with that predicted by both wave mechanics and quantum mechanics. The Beth angular momentum is in general considered as the photon spin angular momentum.

The energy momentum tensor of the electromagnetic field $T^{\mu\nu}$ is not generally symmetric. By adding a divergence term $\partial_\mu U^{\mu\alpha\nu}$ to $T^{\mu\nu}$, one can construct a symmetric energy momentum tensor $\Theta^{\mu\nu}$ which is normally known as the Belinfante energy momentum tensor [3]. The tensor $U^{\mu\alpha\nu}$ is asymmetric in the last two indices. The symmetric energy momentum tensor satisfies the conservation law $\partial_\mu \Theta^{\mu\nu} = 0$. The advantage of the symmetric energy momentum tensor is that the angular momentum calculated from Θ^{k0} is a conserved quantity. Belinfante established the fact that the spin could be regarded as a circulating flow of energy and this idea was well explained by Ohanian [4]. In an infinite plane wave, the electric and magnetic field vectors are perpendicular to the propagation direction. In a finite transverse extent, the field lines are closed loops and represent circulating energy flow and imply the existence of angular momentum whose orientation is in the plane of circulation and it is the spin angular momentum. Further, as the electromagnetic waves propagate, the energy also flows along the direction of propagation. The translational energy flow implies the existence of additional orbital angular momentum. The magnetic field vector can be expressed as the curl of a vector potential $\mathbf{A}$ and the angular momentum density becomes $\mathbf{E} \times \mathbf{A}$. A close inspection shows that the total angular momentum has two components: one the spin angular momentum associated with the polarization and the other the orbital angular momentum associated with the spatial distribution [1]. The total angular momentum J can be split into a spin angular momentum S and an orbital angular momentum L [5]

$$J = \frac{1}{4\pi} \int \mathbf{E} \times \mathbf{A} \, d^3 r + \frac{1}{4\pi} \int \mathbf{E}^n (\mathbf{r} \times \nabla) \mathbf{A}^n \, d^3 r \,. \quad (1)$$

The first term on the right is dependent on polarization and hence it is called spin angular momentum S and the second term is independent of polarization and depends on spatial distribution and identified with orbital angular momentum L. It has been argued that the photon angular momentum cannot be separated into a spin part and an orbital part in a gauge invariant way and the paradox was a subject for several papers and in standard textbooks for the past few decades [6].

In recent times the definitions of these angular momenta raised certain controversy. In all these definitions the angular momentum is defined as a vector product containing the position coordinate. The decomposition of total angular momentum of the photon into spin and orbital parts basically involves how we split the vector potential in a gauge invariant way and it has been studied by several authors and a detailed discussion is given in the review article by Leader and Lorce [7]. The absence of any rest frame for the photon suggests that the total angular momentum is observable but not separately as spin angular momentum and orbital angular momentum. Though this separation is normally considered to be unphysical and not observable, Van Enk and Nienhuis [8] argued that both spin and orbital angular momenta are separately measurable quantities and gauge invariant. The gauge invariant spin and angular momentum parts are expressed as

$$J = \frac{1}{4\pi}\int \mathbf{E}\times\mathbf{A}\,d^3r + \frac{1}{4\pi}\int \mathbf{r}\times\mathbf{E}^n\nabla\mathbf{A}^n\,d^3r\,. \qquad (2)$$

In this expression $\mathbf{A} = \mathbf{A}_\perp$ and therefore both terms are gauge invariant. The canonical expression $\mathbf{E}^n\nabla\mathbf{A}^n$ gives pure mechanical momentum which is responsible for the orbital angular momentum of a photon. The azimuthal flow of electromagnetic field is given by $\mathbf{E}^n\nabla\mathbf{A}^n$ which is half of $\mathbf{E}\times\mathbf{B}$ and the other half is spin flow [9]. In an analogous way, in quantum chromodynamics, the gluon angular momentum can be decomposed into a spin part and an angular momentum part which plays an important role in understanding nucleon structure. Recently, Chen *et al.* [10] decomposed the gauge potential into pure and physical parts: $\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys}$, the pure part is related to gauge invariance and the physical part is related to physical degrees of freedom. In the decomposition by Wakamatsu *et al.* [11], the orbital angular momentum is defined similar to a classical expression $\mathbf{r}\times P_{kin}$, where $P_{kin} = -\frac{1}{4\pi}\int \mathbf{A}_{phys}\times\mathbf{E}\,d^3r$ and in this decomposition each term is gauge invariant and observable. Further studies by several authors revealed the fact that there could be infinitely many different ways to perform such decomposition in a gauge invariant way [7, 12]. In Beth's experiment, actually the spin angular momentum was measured. The measurement of orbital angular momentum has been performed in recent times. The amplitude of a Laguerre-Gaussian mode of light wave has an azimuthal angular dependence of $\exp(-il\phi)$, where l is the azimuthal mode index. The ratio between the angular momentum to the energy is $1/\omega$ or $L = l(\mathbf{E}\times\mathbf{B})/\omega$ and for Laguerre-Gaussian laser mode, it has been shown that the angular momentum is equal to $l\hbar$ and the total angular momentum of the whole light beam is $(l+\sigma_z)\hbar$, where σ_z is a unit vector along the direction of propagation [13]. The measurement of orbital angular momentum was reported by several authors [14–16].

Another important aspect of the photon is its internal zitterbewegung motion. It is well known from the first observation of Schrödinger [17] that a Dirac electron possesses zitterbewegung motion which is the oscillatory motion of the electron with very high frequency $\omega = 2mc^2/\hbar$ with internal velocity equal to the velocity of light. Such internal motion arises because of the classical electromagnetic fluctuating zeropoint field present throughout space [18]. The spin angular momentum of the electron is identified as the zeropoint angular momentum [18, 19]. On the basis of electron internal oscillations, classical models of electron were developed [20–22]. It is quite interesting that such zitterbewegung motion for the photon was derived from the relativistic Schrödinger like equation of the photon by Kobe [23]. It has been proved that the photon velocity contains parallel and perpendicular components with respect to the direction of propagation. The time dependent perpendicular component of velocity rotates about the direction of propagation with an angular frequency ω equal to the frequency of the electromagnetic wave. The finite special extension of internal rotation is equal to the reduced wavelength. The photon spin is then identified as the internal angular momentum due to zitterbewegung. Considering internal dynamical variables in the configuration space the zitterbewegung is attributed to the normal component of velocity vector oscillations about the particle centre [24]. In the quantum field theory, it has been shown that the zitterbewegung of a photon is attributed to the virtual transition process corresponding to the continuous creation and annihilation of virtual pairs of elementary excitations [25, 26]. Recently, Zhang [27] proposed that zitterbewegung of the photons may appear near the Dirac point in a two dimensional photonic crystal. In the case of an electron, the spin angular momentum is an intrinsic property. In the same way both spin and orbital momenta of the photon are intrinsic in nature [28, 29]. Thus one can anticipate that the photon is also having an internal spin structure described by the internal oscillations or rotations.

One of the most important applications of the photon angular momentum lies in the exploitation of the photon spin and angular momentum states for quantum computation and quantum information processing [30]. Superposition of polarization states can be used to construct qubits and transmit information. A standard approach to visualise the transformation of qubits is provided by the Poincaré sphere representation. Generally, any completely polarized state can be described as a linear superposition of spin states and corresponds to a point on the surface of a unit sphere. Analogous representation of orbital angular momentum states of the photon was introduced by Padgett and Courtial [31] and Agarwal [32]. Quantum entanglement of states is a consequence of quantum non-locality. The entanglement involving the spatial modes of electromagnetic field carrying orbital angular momentum was studied by Mair *et al.* [33] and Franke-Arnold *et al.* [34]. The phase dependence of angular momentum may provide multi-dimensional entangled states which are of considerable interest in the field of quantum information.

In vector algebra, the angular momentum is defined by a cross product of position and momentum vectors and identified as a vector normal to the plane containing the position and momentum vectors. However, the angular momentum is basically a planar quantity and better defined as a bivector in a plane [35]. Note that the cross product cannot be defined in a plane. In the case of the electron, the classical internal bivector spin was obtained from the multivector valued Lagrangian by Barut and Zhangi [20]. It has been shown that the particle executes internal complex rotations by absorbing zeropoint field and the angular momentum of these internal rotations is identified as the bivector spin of the particle [36]. In an analogous way, the photon spin is a bivector quantity. Therefore, the photon spin may be visualised as a bivector product between an internal finite extension of the photon and an internal momentum. Similarly, one can visualise the orbital angular momentum of the photon as a bivector. Considering the electromagnetic field as a complex vector, it is possible to express the set of Maxwell's equations into a single form. The basics of complex vector algebra have been discussed in detail previously in references [37, 38].

Recently, the nature of the photon was discussed at length by several authors in the book edited by Roychoudhuri *et al.* [39]. The main views of understanding the nature or the structure of the photon are as follows. Einstein viewed the photon as a singular point which is surrounded by electromagnetic fields. In quantum electrodynamics, the photon is introduced as a unit of excitation associated with the quantised mode of the radiation field and it is associated with precise momentum, energy and polarization. In another view, the photon is interpreted as neither a quantum nor a wave but it can be a meson which produces off other hadronic matter and attains physical status. Photons are just fluctuations of random field or wave packets in the form of needles of radiation superimposed in the zeropoint field. However, understanding the photon structure still remains an open question.

The aim of this article is to explore the structure of the photon in complex vector space. To understand the structure of the photon, the electromagnetic field is expressed as a complex vector and the total energy momentum even multivector is developed in section 2. Section 3 deals with the internal angular momentum structure of the photon and conclusions are presented in section 4. Throughout this article, Lorentz–Heaviside units are used, i.e. $\epsilon_0 = \mu_0 = 1$ and energy terms are divided by 4π and conveniently we choose $c = 1$ [1]. However, for clarification sake in some places c is reintroduced.

2 Energy momentum multivector of the electromagnetic field

In the complex vector formalism, we express the electric field as a vector $\mathbf{E}$ and the magnetic field as a bivector $\mathbf{iB}$ and the electromagnetic field F is expressed as a complex vector [37, 38]

$$F = \frac{1}{2}(\mathbf{E} + \mathbf{iB}). \tag{3}$$

Here, $\mathbf{i}$ is a pseudoscalar in geometric algebra of three dimensions [40], it commutes with all elements of the algebra and $\mathbf{i}^2 = -1$. A reversion operation changes the order of vectors and is indicated by an overbar

$$\bar{F} = \frac{1}{2}(\mathbf{E} - \mathbf{iB}). \tag{4}$$

Now, the product $\bar{F}F$ is written as

$$\bar{F}F = \frac{1}{4}(E^2 + B^2) + \frac{1}{2}(\mathbf{E} \wedge \mathbf{iB}). \tag{5}$$

Similarly, we find

$$F\bar{F} = \frac{1}{4}(E^2 + B^2) - \frac{1}{2}(\mathbf{E} \wedge \mathbf{iB}). \tag{6}$$

The energy density of the electromagnetic field can be obtained from the scalar product

$$\bar{F} \cdot F = \frac{1}{2}(\bar{F}F + F\bar{F}) = \frac{1}{4}(E^2 + B^2). \tag{7}$$

Further, the product $\bar{F} \wedge F$ gives a vector of the form

$$\mathbf{p} = -\frac{1}{c}\bar{F} \wedge F = -\frac{1}{2c}(\bar{F}F - F\bar{F}) = -\frac{1}{2c}(\mathbf{E} \wedge \mathbf{iB}), \tag{8}$$

and the dual of $\mathbf{p}$ is expressed as

$$\mathbf{ip} = \frac{1}{2c}(\mathbf{E} \wedge \mathbf{B}). \tag{9}$$

From the above expression, one can express the energy density of internal electromagnetic flux flow in the bivector plane normal to the propagation direction

$$\mathbf{ip}c = \frac{1}{2}(\mathbf{E} \wedge \mathbf{B}). \tag{10}$$

This energy density of the photon can be identified as the rotational energy density. However, the energy density obtained in (7) represents the energy density of the photon as it propagates and it may be treated as the kinetic energy density of the photon. An even multivector is a sum of a vector and a bivector. The energy terms in (7) and (10) combine to give the total energy of the photon in even multivector form

$$\mathcal{E} = \frac{1}{4\pi}\int \frac{E^2 + B^2}{4} d^3r + \frac{1}{4\pi}\int \frac{\mathbf{E} \wedge \mathbf{B}}{2} d^3r = \mathcal{E}_{kin} + \mathcal{E}_{rot}. \tag{11}$$

The scalar part shows the flow of energy in the direction of propagation which can be identified as the kinetic part of energy $\mathcal{E}_{kin}$ and the bivector part can be identified as the rotational energy $\mathcal{E}_{rot}$ representing circulation of electromagnetic energy in a plane normal to the direction of propagation. In general, twice the kinetic energy is treated as the electromagnetic energy per unit volume and it is the energy of the photon. Since the energy of a photon is expressed as momentum times its velocity, we define kinetic momentum of a photon as $\mathbf{p}_k = \mathcal{E}_{kin}/\mathbf{v}$, where the velocity $\mathbf{v} = \mathbf{n}c$ and $\mathbf{n}$ is a unit vector along the direction of propagation. Introducing an internal velocity $\mathbf{u}$ satisfying the condition $\mathbf{u} \cdot \mathbf{v} = 0$ and $|\mathbf{u}| = c$, the internal momentum representing the rotational flux flow can be defined as $\mathbf{p}_r = \mathcal{E}_{rot}/\mathbf{u}$. From these definitions generalised photon velocity and momentum complex vectors can be constructed as

$$U = \mathbf{v} + \mathbf{iu}, \tag{12}$$

$$P = \mathbf{p}_k + \mathbf{i}\mathbf{p}_r \,. \tag{13}$$

A reversion operation on P gives $\bar{P} = \mathbf{p}_k - \mathbf{i}\mathbf{p}_r$. Since the magnitudes $|\mathbf{p}_k|$ and $|\mathbf{p}_r|$ are equal, we have $P^2 = \bar{P}^2 = 0$. Therefore, the complex vector P is a complex null vector which represents the lightlike nature of the photon. Similarly, the complex velocity vector is also a complex null vector. Now, the total energy of the photon is expressed as

$$\mathcal{E} = \mathbf{p}_k \cdot \mathbf{v} + \mathbf{p}_r \wedge \mathbf{u} \,. \tag{14}$$

The even multivector form given in the above equation can be compared to the symmetric energy momentum tensor $\Theta^{\mu\nu}$ with the identification of the scalar part with Θ^{00} and bivector part with Θ^{ij} . In three dimensions, the property of an even multivector is that it represents rotations in the bivector plane [35]. Then, the energy multivector can be expressed as a rotor with angular frequency ω

$$\mathcal{E} = \mathcal{E}_0 \, e^{\hat{J}\omega t} \,, \tag{15}$$

where $\hat{J}$ is a unit bivector in the plane normal to the propagation direction. This relation shows that the photon contains internal complex rotations and these rotations are analogous to the internal complex rotations or zitterbewegung of the electron. The cause of these internal rotations is attributed to the fluctuations of the zeropoint field [38]. In (15) the internal rotation represents the clockwise or right-handed rotation. A reversion operation on $\mathcal{E}$ gives

$$\bar{\mathcal{E}} = \mathcal{E}_0 \, e^{-\hat{J}\omega t} \,. \tag{16}$$

In this case, the internal rotation represents counterclockwise or left-handed rotation. The frequency of internal rotation is the rate per unit energy flux flow within the photon

$$\Omega = -\hat{J}\omega t = -\frac{1}{\mathcal{E}}\frac{d\mathcal{E}}{dt} \,. \tag{17}$$

Here, the frequency of internal rotation represents the counterclockwise direction. The internal complex rotations suggest that there exists an internal complex structure of the photon.

3 Internal structure of the photon

In general, the internal complex rotations represent the angular momentum of the photon. The angular momentum of a photon is defined as the ratio between the rotational energy of the photon and the frequency of internal rotation. Since the energy of the photon is a sum of kinetic and rotational energy components, we expect that the angular momentum of the photon contains two parts: one corresponding to the rotational flow of energy and the other to the translational flow of energy. According to the definition given in (2), the spin angular momentum bivector is in the orientation of the plane $\mathbf{A} \wedge \mathbf{E}$ which is a plane normal to the propagation direction. Let us consider a set of orthogonal unit vectors $\{\sigma_k; k = 1, 2, 3\}$ along x, y and z axes. If we choose the propagation direction along the z-axis, then the unit bivector along the spin orientation is $\mathbf{i}\sigma_3$. To understand the orientation of spin and orbital angular momenta, let us consider circularly polarized light waves propagating along the z-direction and the waves have finite extent in the $x-$ and y-directions. The propagating wave has cylindrical symmetry about the z-axis. The energy of the wave can be visualised as a sum of circulating energy flow in the x-y plane and a translational energy flow in the z-direction. In the case of circularly polarized light the vector potential $\mathbf{A}$ contains only two components

$$\mathbf{A} = \frac{\mathcal{E}_0}{\omega} \left[\sigma_1 cos(\mathbf{k}\cdot\mathbf{r} - \omega t) + \sigma_2 sin(\mathbf{k}\cdot\mathbf{r} - \omega t)\right] \,. \tag{18}$$

Here, $\mathbf{k}$ is the wave vector. The three vectors $\mathbf{E}$, $\mathbf{B}$ and $\mathbf{A}$ rotate in a plane normal to the propagation direction. Differentiation of (18) with respect to time gives the electric field vector

$$\mathbf{E} = \mathcal{E}_0 \left[-\sigma_1 sin(\mathbf{k}\cdot\mathbf{r} - \omega t) + \sigma_2 cos(\mathbf{k}\cdot\mathbf{r} - \omega t)\right] \,. \tag{19}$$

Then, the bivector product $\mathbf{A} \wedge \mathbf{E}$ becomes

$$\mathbf{A} \wedge \mathbf{E} = \mathbf{i}\sigma_3 \, \frac{\mathcal{E}_0^2}{\omega} \,, \tag{20}$$

where $\sigma_1\sigma_2 = \mathbf{i}\sigma_3$. The spin angular momentum of electromagnetic field or the photon is expressed as

$$S = \mathbf{i}\sigma_3 \, \frac{1}{4\pi} \int \frac{\mathcal{E}_0^2}{\omega} \, d^3 r = \mathbf{i}\sigma_3 \hbar \,, \tag{21}$$

where the energy density of the electromagnetic wave is normalized so that the energy is one quantum. Normally, because of the fact $\sigma_3 \wedge \mathbf{k} = 0$, the z-component of angular momentum goes to zero but not the other components of the orbital angular momentum. From the second term on the right of (2), the angular momentum density is expressed as a sum of two terms

$$\mathbf{r} \wedge \mathbf{E}^n \nabla \mathbf{A}^n = \mathbf{r} \wedge \mathbf{E}^x \nabla \mathbf{A}^x + \mathbf{r} \wedge \mathbf{E}^y \nabla \mathbf{A}^y \,. \tag{22}$$

Substituting individual components of $\mathbf{E}$ and $\nabla\mathbf{A}$ in the above equation, we find the orbital angular momentum density

$$\mathbf{r} \wedge \mathbf{E}^n \nabla \mathbf{A}^n = \frac{\mathcal{E}_0^2}{\omega} \, \mathbf{r} \wedge \mathbf{k} \,. \tag{23}$$

Then the orbital angular momentum of the photon is expressed as

$$L = \mathbf{r} \wedge \mathbf{k} \, \frac{1}{4\pi} \int \frac{\mathcal{E}_0^2}{\omega} \, d^3 r \,. \tag{24}$$

In the above equation, the vector $\mathbf{r}$ is restricted to the plane $\mathbf{i}\sigma_3$ and contains only x and y components. If the magnitude of $\mathbf{r}$ is equal to the reduced wavelength, then the product

$|\mathbf{r}||\mathbf{k}| = 1$ for circularly polarized light. The orbital angular momentum is now expressed as

$$L = \hat{\mathbf{r}} \wedge \sigma_3 \frac{1}{4\pi} \int \frac{\mathcal{E}_0^2}{\omega} d^3 r = \mathbf{i m} \hbar \,, \tag{25}$$

where the unit vector $\hat{\mathbf{r}}$, in an arbitrary direction, lies in the plane $\mathbf{i}\sigma_3$, the unit vector $\mathbf{m}$ is chosen normal to the orientation of $\hat{\mathbf{r}} \wedge \sigma_3$ and the integral term in (25) represents the ratio between the energy of the photon and the frequency. Thus the orientation of the orbital angular momentum is always normal to the orientation of spin angular momentum in a photon. In the case the photon is propagating in an arbitrary direction say $\mathbf{n}$ then from the above analysis, the spin angular momentum and orbital angular momentum are expressed as

$$S = \mathbf{i n} \hbar \,, \tag{26}$$

$$L = \mathbf{i m} \hbar \,. \tag{27}$$

The vectors $\mathbf{n}$ and $\mathbf{m}$ satisfy the condition $\mathbf{n} \cdot \mathbf{m} = 0$ and the vector $\mathbf{m}$ lies in the plane of the unit bivector $\mathbf{in}$. Since, the direction of unit vector $\mathbf{m}$ or the orientation of the plane $\mathbf{im}$ is arbitrary, the rotation of $\mathbf{m}$ is expressed by the relation $\mathbf{m}' = \bar{R}\mathbf{m}R$. Here, $R = e^{\mathbf{in}\phi/2}$ is a rotor and in this way the orbital angular momentum depends on the angle ϕ. The spin angular momentum describes the intrinsic angular momentum of a photon and commutes with the generator of translation $\mathbf{n}|\mathbf{k}|$. The spin angular momentum causes the complex vector field F to rotate in the $\mathbf{E} \wedge \mathbf{B}$ plane without changing the direction of propagation vector $\mathbf{k}$. The photon spin is the generator of rotations in the plane normal to the propagation direction. Whereas, the orbital angular momentum causes the plane having orientation defined by the bivector $\mathbf{r} \wedge \mathbf{k}$ to rotate without changing the direction of the vector $\mathbf{k}$ and the orientation of the plane $\mathbf{E} \wedge \mathbf{B}$. The orbital angular momentum does not commute with the generator of translation. The photon orbital angular momentum is the generator of rotations in a plane normal to the spin plane. Thus one can conclude that both the spin and orbital angular momenta of a photon are intrinsic. The intrinsic nature of orbital angular momentum was discussed by Berry [28]. Further, Allen and Padgett [29] argued that the spin and the orbital angular momenta are intrinsic in nature in the case when the transverse momentum is zero for the helical wave fronts. The spin and orbital angular momenta of the photon are fundamental quantities and produce complex rotations in space and such rotations are actually produced by the fluctuating zeropoint fields present throughout space [38, 41]. The internal complex rotations are not only limited to the rotations pertaining to the plane of spin angular momentum but also exists in the plane of orbital angular momentum. In the Laguerre-Gaussian modes of laser beams it has been shown explicitly in the quantum mechanical approach that the orbital angular momentum of light beams resembles the angular momentum of the harmonic oscillator [42].

4 Conclusions

The electromagnetic field per unit volume is represented by an energy momentum even multivector and expressed as a sum of scalar and bivector components, and we identify the scalar part as the kinetic part which shows the flow of energy in the direction of propagation and the bivector part as the rotational energy flow in the plane normal to the direction of propagation over a finite extent. The even multivector form of energy shows that there exist internal complex rotations of the electromagnetic field. The cause of these internal rotations is attributed to the fluctuations of the zeropoint field. In general, the internal complex rotations represent the angular momentum of the photon. The angular momentum of the photon is defined as the ratio between the rotational energy of the photon and the angular frequency of rotation. The spin angular momentum bivector represents a plane normal to the propagation direction. We find that the orientation of orbital angular momentum is always normal to the orientation of spin angular momentum in a photon. The photon spin is the generator of rotations in the plane normal to the propagation direction. The photon orbital angular momentum is the generator of rotations in a plane normal to the spin plane. Thus, one can conclude that both spin and orbital angular momenta of a photon are intrinsic in nature. The internal structure of the photon may be visualized as the superposition of electromagnetic field flow or rotation in two normal orientations in complex vector space. Because of the formal similarity between gluons and photons, the conclusions obtained here may be extended to the gluon structure.

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References

1. Jackson J. D. Classical Electrodynamics. Wiley Eastern Limited, New Delhi, 1978.
2. Beth R. A. Mechanical Detection and Measurement of the Angular Momentum of Light. *Phys. Rev.*, 1936, v. 50, 115.
3. Belinfante F. J. On the spin angular momentum of mesons. *Physica*, 1939, v. 6, 887–898.
4. Ohanian H. C. What is spin? *Am. J. Phys.*, 1986, v. 54, 500.
5. Cohen-Tannoudji C., Dupont-Roc J., Grynberg G. Photons and Atoms. John Wiley & Sons, New York, 1989.
6. Jauch J. M., Rohlich F. The Theory of Photons and Electrons. Springer-Verlag, Berlin, 1976.
7. Leader E., Lorce C. The angular momentum controversy: What's it all about and does it matter? *Physics Reports*, 2014, v. 541, 163–248.
8. Van Enk, Nienhuis G. Spin and Orbital Angular Momentum of Photons. *Euro. Phys. Lett.*, 1994, v. 25, 497.
9. Chen X. S., Chen G. B., Sun W. M., Wang F. Testing the correctness of the Poynting vector $\mathbf{E} \times \mathbf{B}$ as the momentum density of gauge fields. arXiv: hep-ph/0710.1427v1.
10. Chen X. S., Lu X. F., Sun W. M., Wang F., Goldman T. Spin and Orbital Angular Momentum in Gauge Theories: Nucleon Spin Structure and Multipole Radiation Revisited. *Phys. Rev. Lett.*, 2008, v. 100, 232002.
11. Wakamatsu M. Gauge-invariant decomposition of nucleon spin. *Phys. Rev. D.*, 2010, v. 81, 114010.

12. Sun W. M. Physical Decomposition of Photon Angular Momentum. arXiv: quant-ph/1407.2305v1.
13. Allen L., Beijersbergen M. W., Spreeuw R. J. C., Woerdman J. P. Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes. *Phys. Rev. A*, 1992, v. 45, 8185.
14. Leach J., Padgett M. J., Barnett S. M., Franke-Arnold S., Courtial J. Measuring the orbital angular momentum of a single photon. *Phys. Rev. Lett.*, 2002, v. 88, 257901.
15. Allen L., Barnett S. M., Padgett M. J. Optical Angular Momentum. IOP Publishing, Bristol and Philadelphia, 2003.
16. Padgett M., Courtial J., Allen L. Light's Orbital Angular Momentum. *Physics Today*, May 2004, 35.
17. Barut A. O., Bracken A. J. Zitterbewegung and the internal geometry of electron. *Phys. Rev. D*, 1981, v. 23, 2454.
18. Hestenes D. Spin and uncertainty in the interpretation of quantum mechanics. *Am. J. Phys.*, 1979, v. 47, 399–415.
19. Hestenes D. Zitterbewegung in Quantum Mechanics. *Found. Phys.*, 2010, v. 40, 1–54.
20. Barut A. O., Zanghi A. J. Classical Model of the Dirac Electron. *Phys. Rev. Lett.*, 1984, v. 52, 2009–2012.
21. Hovarthy P. A. Mathisson's spinning electron: Noncommutative mechanics and exotic Galilean symmetry, 66 years ago. *Acta. Phys. Pol. B*, 2003, v. 34, 2611–2622.
22. Vaz J., Rodriguez W. A., Zitterbewegung and the electromagnetic field of the electron. *Phys. Lett. B*, 1993, v. 319, 243.
23. Kobe D. H. Zitterbewegung of a photon. *Phys. Lett. A*, 1999, v. 253, 7–11.
24. Unal N. A Simple Model of Classical Zitterbewegung: Photon Wave Function. *Found. Phys.*, 1997, v. 27, 731.
25. Wang Z. Y., Xiong C. D. Zitterbewegung by quantum field-theory considerations. *Phys. Rev. A*, 2008, v. 77, 045402.
26. Wang Z. Y., Xiong C. D., Qiu Q. Photon wave function and Zitterbewegung. *Phys. Rev, A*, 2009, v. 80, 032118.
27. Zhang X. Observing Zitterbewegung for Photon near the Dirac point of a Two-Dimensional Photonic Crystal. *Phys. Rev. Lett.*, 2008, v. 100, 113903.
28. Berry M. V. Paraxial beams of spinning light. In: Allen S., Barnett S. M., Padgett, M. J., eds. Optical Angular Momentum. IOP Publishing, Bristol, Philadelphia, 2003.
29. Allen L., Padgett M. J. The Poynting vector in Laguerre-Gaussian beams and the interpretation of their angular momentum density. *Opt. Commun.*, 2000, v. 184, 6771.
30. Calvo G. F., Picón A., Bagan E. Quantum field theory of photons with orbital angular momentum. *Phys. Rev. A*, 2006, v. 73, 01385.
31. Padgett M. J., Courtial J. Poincare-sphere equivalent for light beams containing orbital angular momentum. *Opt. Lett.*, 1999, v. 24, 430.
32. Agarwal G. S. SU(2) structure of the Poincaré sphere for light beams with orbital angular momentum. *J. Opt. Soc. Am. A*, 1999, v. 16, 2914–2916.
33. Mair A., Vaziri A., Weihs G., Zeilinger A. Entanglement of the orbital angular momentum states of photons. *Nature*, 2001, v. 412, 313–316.
34. Franke-Arnold S., Barnett S. M., Padgett M. J., Allen L. Two-photon entanglement of orbital angular momentum states. *Phys. Rev. A*, 2002, v. 65, 033823-1.
35. Hestenes D. Oersted Medal Lecture 2002: Reforming the Mathematical Language of Physics. *Am. J. Phys.*, v. 71, 104.
36. Muralidhar K. The Spin Bivector and Zeropoint Energy in Geometric Algebra. *Adv. Studies Theor. Phys.*, 2012, v. 6, 675–686.
37. Muralidhar K. Complex Vector Formalism of Harmonic Oscillator in Geometric Algebra: Particle Mass, Spin and Dynamics in Complex Vector Space. *Found. Phys.*, 2014, v. 44, 265–295.
38. Muralidhar K. Algebra of Complex vectors and Applications in electromagnetic Theory and Quantum Mechanics. *Mathematics*, 2015, v. 3, 781–842.
39. Roychoudhuri C., Kracklauer A. F., Creath K. The Nature of Light. What is a Photon? C. R. C. Press, Taylor and Francis Group, Boca Raton, FL, 2008.
40. Doran C., Lasenby A. Geometric Algebra for Physicists. Cambridge University Press, Cambridge, 2003.
41. Kobe D. H. A relativistic Schrödinger-like Equation for a Photon and Its second Quantization. *Found. Phys.*, 1999, v. 29, 1203–1231.
42. Nienhuis G., Allen L. Paraxial wave optics and harmonic oscillators. *Phys. Rev. A*, 1993, v. 48, 656–665.