On the Physical Nature of the de Broglie Wave

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Here is revisited de Broglie's Wave Mechanics Theory of Double Solution wherein a particle endowed with a variable proper mass is required to propagate within a hidden medium in order to describe a physical scalar wave carrying its own associated mass. Since the experiment that detected the wave applied to electrons, we extend the de Broglie's theory to the Dirac spinor, so that we can outline the physical reality of this fermion field.

Introduction

Some hundred years ago, was established the famous relation $E = h\nu$ later verified for the photon. On this basis, in 1924, Louis de Broglie extended the wave dualism to all massive particles. The predicted original wave function associated with a given particle was soon detected in 1927 by Davisson and Germer in their famous experience on electrons diffraction by a nickel crystal lattice [1]. The wave producing physical effects, was an overwhelming evidence of its true existence.

Nevertheless, since the *Brussels Solvay Symposium* was held in 1927, official physics interpretation prevailed which considered quantum mechanics on the pure statistical grounds and then leading to accept the notion of *non-real* wave functions.

Although it is unquestionable that use of a probabilistic wave and its generalization did lead to accurate prediction and fruitful theories, de Broglie could never believe that observable physical phenomena follow from abstract mathematical wave functions. In his opinion, the wave function had to remain an objective physical entity which is intimately related with its mass, rather than the subjective probabilistic representation currently adopted in modern quantum physics. Since the real wave was detected by means of electrons scattering, we will here formally show that there is a strict identity between its phase and the one of its associated wave which therefore physically carries the particle. To make this identity possible, the electron proper mass must be variable according to the Planck-Laue relation [2]. Within this frame, de Broglie's theory inferred a so-called "guidance formulae" which forces the electron to be always in motion. However, because of the stationary property of energy levels inside an atom, a static electron is not compatible with its dynamic guided state. de Broglie then postulated the existence of a *hidden medium* which permanently exchanges energy and momentum with the electron causing it to oscillate and then avoiding a motionless location.

When I first met Louis de Broglie in summer 1966, this issue was debated with a great deal of speculation. Today, another explanation can be pushed forward.

Notations

Space-time Latin indices run from $a = b$: 0, 1, 2, 3, while spatial Greek indices run from $\alpha = \beta$: 1, 2, 3. The space-time signature is -2.

1 Spinor field-electron duality

1.1 The origins of the Double Solution Theory

1.1.1 Basics of the wave mechanics

From standard optics, we first recall the definition of the classical wave with a frequency υ

$$\psi = a(n) \exp\left[i(\nu t - \boldsymbol{k} \cdot \boldsymbol{r})\right] \tag{1.1}$$

which propagates along the direction of the unit vector $\boldsymbol{n}$. (Here $\boldsymbol{k}$ is the 3-wave vector, $\boldsymbol{k} \cdot \boldsymbol{r} = \phi$ is the wave spatial phase, n is the refractive index of the medium.)

Formula (1.1) is a solution of the classical propagation equation

$$\Delta\psi = \frac{1}{w^2} \frac{\partial^2 \psi}{c^2 \partial t^2}, \tag{1.2}$$

where w is the wave phase velocity of the wave moving in a dispersive medium whose refractive index is $n(\nu)$ generally depending on the coordinates, and which is defined by

$$\frac{1}{w} = \frac{n(\nu)}{c}. \tag{1.3}$$

This medium is assumed to be homogeneous and only depends on the frequency ν. The (constant) phase ϕ of the wave is progressing along the given direction with a separation given by a distance $\lambda = w/\nu$, called wavelength.

Consider now the superposition of a group of stationary (monochromatic) waves having each a very close frequency along the x-axis

$$\psi = \int_{\nu_0 - \Delta\nu}^{\nu_0 + \Delta\nu} a(n) \exp\left[i(\nu t - \phi(\nu))\right]. \tag{1.4}$$

Such a group of waves moves with a constant velocity called group velocity υ_g according to the Rayleigh's formula

$$\frac{1}{\upsilon_g} = \frac{d(\nu/\upsilon)}{d\nu} = \frac{1}{\nu_0} \frac{\partial n\nu}{\partial \nu}. \tag{1.5}$$

The wave mechanics eventually shows that the group velocity v_g of waves associated with a particle of rest mass m_0, coincides with the velocity of this particle whose momentum along the x-axis (in vacuum) is given by the famous de Broglie's relation [3]

$$p_x = m_0\, v_x = \frac{h}{\lambda}\,. \tag{1.6}$$

We clearly note that there is an obvious *first* physical link between the particle and its associated wave which will be further substantiated.

1.1.2 Double nature of the wave function

Like we mentioned above, de Broglie was firmly convinced that the wave associated with a massive particle should be a *real observable quantity*, therefore, he introduced a *true plane wave* of the usual form

$$\psi = a(x^\alpha)\exp\left[\frac{i}{h}\,\phi(x^\alpha)\right], \tag{1.7}$$

which is connected to a probabilistic Ω-wave by the relation

$$\Omega = f\,\psi\,, \tag{1.8}$$

where f is a constant normalizing factor.

The original wave mechanics is thus complemented with the Double Solution Theory [4], for Ω and ψ are two solutions of the same propagation equation. The Ω-wave (normed in the usual quantum machanical formalism), has the nature of a *subjective* probability representation formulated by means of the objective ψ-wave.

Defining ψ^* as the complex conjugate of ψ, it is well known that $\psi^2 dV = \psi\,\psi^*\,dV$ gives the absolute value of finding the particle in the volume element dV so that the normalization condition is adapted with f as

$$\int_V \Omega\,\Omega^*\,dV = 1\,. \tag{1.8 bis}$$

This guarantees that the particle is present in the arbitrary volume V.

The Ω and ψ have the same phase ϕ, but the constant f ought to be much larger than 1. Indeed, the current theory which only uses the Ω-function assumes this quantity to be spread out over the whole wave, i.e. spread out over a related physical quantity b (e.g. energy of the particle) according to

$$\int_V \Omega\,\Omega^*\,dV = b\,. \tag{1.8 ter}$$

In the double solution theory however, b should be concentrated in a very small region occupied by the particle and the integral of $a^2 b\,dV$ taken over the ψ-wave in the volume V is much smaller than b, which eventually leads to $|b| \gg 1$.

2 Extension to the spinor

2.1 The real spinor wave

2.1.1 The Dirac operators and Dirac equation (reminder)

In order to write the Schrödinger equation under a relativistic form, P. A. M. Dirac has defined a specific four-components wave function Ψ^A called *spinor* [5] which must necessarily apply to any spin-1/2 particles thus in our case, the electron. (Capital Latin spinorial indices are: $A = B = 1, 2, 3, 4$.)

To this effect, he introduced a system of (4×4) non local trace free matrices $\gamma_a = (\gamma_a{}^A{}_B)$. (In the classical theory, it is customary to omit the spinorial indices.)

The matrices γ a can display the standard following components [6]:

$$\gamma_0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$
$$\gamma_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{2.1}$$

in order to satisfy the fundamental relation

$$\gamma_a\gamma_b + \gamma_b\gamma_a = -2\eta_{ab}\,\mathbf{I}\,, \tag{2.2}$$

where η_{ab} is the Minkowskian tensor, and $\mathbf{I}$ is the unit matrix.

Formula $W = \gamma^a\,\partial_a$ is known as the Dirac operator where the Planck constant h is absorbed in the ∂_a.

For a free massive spin 1/2-field, the Dirac equation is eventually written as

$$(W - m_0 c)\,\Psi = 0\,, \tag{2.3}$$

where the proper mass m_0 is attributed to the associated spin 1/2-electron.

2.1.2 The normed spinor density

Since we are here considering a spin 1/2-fermion particle we must look for a wave which is a *real* spinor Ψ that physically carries the electron. From the classical Dirac theory, it is well known that the probability density of the electron's presence. is the time component of the (real) Dirac current vector density [7]

$$(J^a)_\mathrm{D} = i\,({}^\sharp\Psi\gamma^a\Psi)\,, \tag{2.4}$$

where ${}^\sharp\Psi$ is the Dirac adjoint spinor $\Psi^+\gamma^0$, and Ψ^+ is the (complex) conjugate transpose of Ψ. So, this density of the electron reads

$$(J^0)_\mathrm{D} = i\,({}^\sharp\Psi\gamma^0\Psi) \tag{2.4 bis}$$

which is easily shown to be always definite and positive. Without the loss of generality, we could express Ψ under the form of a plane wave spinor [8] as

$$\Psi = \varpi(x^a)\exp\left[\frac{i}{h}\,\phi(x^a)\right],$$

where the wave spinor amplitude ϖ and the phase ϕ are *real local* functions. The Dirac spinor amplitudes ϖ could then be tuned so as to possess the orthogonality and completeness properties that guarantee that the plane waves Ψ have the adequate *normalization* to delta functions [9]. However, only a single spinor Ψ can be considered as a physical wave function, whereas we are left with 4-components Ψ^A. Then, at first glance, one might be tempted to consider the simple combination

$$\boldsymbol{\Psi} = \Psi^1 + \Psi^2 + \Psi^3 + \Psi^4.$$

Unfortunately, the Ψ-components are defined with respect to a spinorial frame $S(V_4)$ distinct from the structural Minkowski space, which renders those physically irrelevant. Instead, we will follow another extremely simple way: since ρ is here a real value, we have always the freedom to define a scalar wave function Φ such that

$$\Phi\,\Phi^* = \rho\,. \tag{2.5}$$

Moreover, we assume that this wave function has the same form as ψ (1.7)

$$\Phi = \omega\,(x^a)\exp\left[\frac{i}{h}\,\phi\,(x^a)\right]. \tag{2.6}$$

We state that Φ is the true wave function of the electron which was actually detected in the Davisson and Germer experiment upon a given set of gamma matrices γ^a, simply because it is derived from a real quantity which is itself inferred from the 1/2-spinor definition (2.4 bis) as it should.

Thus, we apply the same hypothesis conjectured by de Broglie (1.8 bis), and we are now able to write the normed expression as

$$\int_V \Xi\,\Xi^*\,dV = 1\,, \tag{2.7}$$

where

$$\Xi = g\,\Phi \tag{2.7 bis}$$

is the subjective wave function and g is a normalizing factor which satisfies (2.7).

In all the following text, Φ will be denoted as the "spinor wave".

2.1.3 Internal frequency of the electron

From (2.6), the energy and momentum of the electron located at x^a are

$$E = \partial_t\phi\,, \tag{2.8}$$

$$\boldsymbol{P} = P_a = -\mathrm{grad}\,\phi\,. \tag{2.9}$$

In order to outline the physical nature of the Φ-spinor wave, we start from the following consideration: in the framework of the Special Theory of Relativity, the frequency of a plane monochromatic wave is transformed as

$$\nu = \frac{\nu_0}{\sqrt{1-\mathbf{v}^2/c^2}}\,, \qquad \mathbf{v} = \mathrm{v}_a\,, \tag{2.10}$$

whereas the clock's frequency ν_c is transformed according to

$$\nu_c = \nu_0\,\sqrt{1-\mathbf{v}^2/c^2}\,. \tag{2.11}$$

If an electron is assumed to contain a rest energy $m_0c^2 = h\nu_0$, it is likened to a small clock of frequency ν_0, so that when moving with velocity $\mathbf{v}$, its frequency ν_c differs from that of the wave which is here noted ν.

In this concept, our main task will consist of showing that the electron is permanently in phase with its associated spinor wave, thus justifying the true nature of Φ that *physically* carries the electron

2.2 The physical nature of the spinor-electron duality

2.2.1 The Planck-Laue relation

We now postulate that the electron possesses a variable proper mass m_0' from which an important useful equation will be inferred.

Let us first write the Lagrange function for an observer who sees the electron of variable proper mass m_0' moving at the 3-velocity $\mathbf{v}$

$$L = -\,m_0'c^2\,\sqrt{1-\mathbf{v}^2/c^2} \tag{2.12}$$

so that the least action principle applied to this Lagrangian be still expressed by

$$\delta\int_{t_0}^{t_1} L dt = \delta\int_{t_0}^{t_1}\left(-\,m_0'\,c^2\,\sqrt{1-\mathbf{v}^2/c^2}\right)dt = 0\,. \tag{2.13}$$

From this principle are inferred the equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_a} = \frac{\partial L}{\partial x_a} \tag{2.14}$$

with $\dot{x}_a = dx_a/dt$. It leads to

$$\frac{d\boldsymbol{P}'}{dt} = -\,c^2\,\sqrt{1-\mathbf{v}^2/c^2}\;\;\mathrm{grad}\,m_0' \tag{2.15}$$

(since m_0' is now variable).

Hence, by differentiating the well know relativistic relation

$$\frac{E'^{\,2}}{c^2} = \boldsymbol{P}'^{\,2} + m_0'^{\,2}\,c^2 \tag{2.16}$$

we obtain

$$\frac{dE'}{dt} = c^2\,\sqrt{1-\mathbf{v}^2/c^2}\,\frac{\partial m_0'}{\partial t}\,. \tag{2.17}$$

Combining (2.15) and (2.17) readily gives

$$\frac{dE'}{dt} - \frac{\mathbf{v}\,d\boldsymbol{P}'}{dt} = c^2\,\sqrt{1-\mathbf{v}^2/c^2}\,\frac{dm_0'}{dt} \tag{2.18}$$

where

$$\frac{dm_0'}{dt} = \frac{\partial m_0'}{\partial t} + \mathrm{grad}\,m_0'$$

is the variation of the mass in the course of its motion.

On the other hand, we have

$$\frac{d\,(\boldsymbol{P}'\cdot\mathbf{v})}{dt} = \frac{\mathbf{v}\,d\boldsymbol{P}'}{dt} + \frac{m_0'\,c^2}{\sqrt{1-\mathbf{v}^2/c^2}}\,\frac{\mathbf{v}}{c}\,\frac{d\frac{\mathbf{v}}{c}}{dt} = $$
$$= \frac{\mathbf{v}\,d\boldsymbol{P}'}{dt} - m_0'\,c^2\,\frac{d}{dt}\,\sqrt{1-\mathbf{v}^2/c^2} \tag{2.19}$$

or

$$\frac{d}{dt}\left(m_0'\,\sqrt{1-\mathbf{v}^2/c^2}\right) =$$
$$= c^2\,\sqrt{1-\mathbf{v}^2/c^2}\,\frac{dm_0'}{dt} + m_0'\,c^2\,\frac{d}{dt}\,\sqrt{1-\mathbf{v}^2/c^2}\,.$$

Hence, (2.18) can be written as

$$\frac{d}{dt}\left(E' - \mathbf{v}\cdot\boldsymbol{P}' - m_0'c^2\,\sqrt{1-\mathbf{v}^2/c^2}\right) = 0 \tag{2.20}$$

which is satisfied when the electron is at rest (that is $\mathbf{v}=0$, $E_0' = m_0'c^2$).

Therefore, we must have always

$$E' = \frac{m_0'c^2}{\sqrt{1-\mathbf{v}^2/c^2}} = m_0'c^2\,\sqrt{1-\mathbf{v}^2/c^2} + \frac{m_0'\,\mathbf{v}^2}{\sqrt{1-\mathbf{v}^2/c^2}}\,. \tag{2.21}$$

This is known as the Planck-Laue formula which plays a central rôle in our present theory.

2.2.2 Phase identity of the electron and its spinor wave

Let us first recall the relativistic form of the Doppler formula

$$\nu_0 = \nu\,\frac{1-\mathbf{v}/w}{\sqrt{1-\mathbf{v}^2/c^2}}\,, \tag{2.22}$$

where ν_0 is the wave's frequency in the frame attached to the electron, ν and w are respectively the frequency and phase velocity of the spinor wave in a reference frame where this electron has a velocity $\mathbf{v}$.

With this formula, and taking the classical Planck relation $E = h\nu$ into account, we find

$$E = E_0\,\frac{1-\mathbf{v}^2/c^2}{1-\mathbf{v}/w}\,. \tag{2.23}$$

However, inspection shows that the usual equation

$$E = \frac{E_0}{\sqrt{1-\mathbf{v}^2/c^2}} \tag{2.24}$$

holds only if

$$1 - \mathbf{v}/w = 1 - \mathbf{v}^2/c^2 \tag{2.25}$$

that implies

$$w\mathbf{v} = c^2. \tag{2.26}$$

This latter relation is satisfied provided we set up

$$E' = \frac{m_0'c^2}{\sqrt{1-\mathbf{v}^2/c^2}}\,, \tag{2.27}$$

$$\boldsymbol{P}' = \frac{m_0'\,\mathbf{v}}{\sqrt{1-\mathbf{v}^2/c^2}}\,. \tag{2.28}$$

A variable proper mass is then required to insure that the electron as it moves, remains constantly in phase with that of the associated spinor wave. To see this, let us first multiply the Planck-Laue equation by dt

$$\left(\frac{m_0'c^2}{\sqrt{1-\mathbf{v}^2/c^2}} - \frac{m_0'\,\mathbf{v}^2}{\sqrt{1-\mathbf{v}^2/c^2}}\right)dt = m_0'c^2\,\sqrt{1-\mathbf{v}^2/c^2}\,dt. \tag{2.29}$$

If $\boldsymbol{n}$ is the unit vector normal to the phase surface, we then consider that the electron whose internal frequency is $\nu_0 = m_0'c^2/h$ has travelled a distance $d\boldsymbol{n}$ during a time interval dt, so that its internal phase ϕ_i has been changed by

$$d\phi_i = h\nu_0\,\sqrt{1-\mathbf{v}^2/c^2}\,dt = m_0'c^2\,\sqrt{1-\mathbf{v}^2/c^2}\,dt\,. \tag{2.30}$$

At the same time, the corresponding spinor wave phase variation is

$$d\phi = \partial_t\phi\,dt + \partial_{\boldsymbol{n}}\phi\,d\boldsymbol{n} = (\partial_t\phi + \mathbf{v}\,\mathrm{grad}\,\phi)\,dt$$

and, by analogy with the classical formulae (2.8) and (2.9), one can write

$$\boldsymbol{P}' = -\,\mathrm{grad}\,\phi = \frac{m_0'\mathbf{v}}{\sqrt{1-\mathbf{v}^2/c^2}}\,,$$

$$E' = \partial_t\phi = \frac{m_0'c^2}{\sqrt{1-\mathbf{v}^2/c^2}}\,,$$

so we find

$$d\phi = \left(\frac{m_0'c^2}{\sqrt{1-\mathbf{v}^2/c^2}} - \frac{m_0'\,\mathbf{v}^2}{\sqrt{1-\mathbf{v}^2/c^2}}\right)dt. \tag{2.31}$$

Hence, from (2.29) we obtain the fundamental result which states that the internal phase of the electron is identical to that of its associated spinor wave

$$d\phi = d\phi_i\,. \tag{2.32}$$

With (1.6), there is an obvious *second physical link* between the electron and the spinor wave Φ which clearly *carries* the lepton.

This is what we wanted to show.

Conclusions and outlook

Within the above theory, the electron is guided by its spinor wave which means that it is always in motion. In this case, the electron doesnot apparently comply with atomic quantum stationary states for which the electron is required to have zero velocity. De Broglie et al. [10] thus postulated a vacuum *hidden thermostat* whereby the electron is permanently exchanging energy and momenta. According to the authors this sub-quantum medium would cause the electron to fluctuate in a *Brownian-like* manner so as to exhibit a static situation only at the atomic level. In this way, the wavy-electron would be allowed to undergo perpetual infinitesimal propagation. Our opinion however differs from this hypothesis which we believe, would mark the limitation of the Double Solution theory. Preferably, we suggest that each energy level of an atom be characterized by a stationary limited spinor *wave packet* carrying a dynamical electron: the mean energy of the pair *wavepacket-moving electron* would then represent the quantized energy level of the atom.

"Squeezing" stepwise the wave packet (i.e. increasing the frequency) would mean jumping to a higher energy level and vice versa, which actually could reflect the excited/desexcited states of the atom. This process tends to validate the spectroscopic sharpness of the atomic rays as it is observed. All in all, the exposed theory seems to cope with an electron whose *physical* wave interacts with a *physical* diffraction device, and yet satisfies the established relativistic features of Dirac's theory.

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