Occurrence and Properties of Low Spin Identical Bands in Normal-Deformed Even-Even Nuclei

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The identical bands (IB's) phenomenon in normally deformed rare-earth nuclei has been studied theoretically at low spins. Six neighboring even-even isotopes (N = 92) and the isotopes ^{166,168,170}Hf are proposed that may represent favorable cases for observation of this phenomenon. A first step has been done by extracting the smoothed excitation energies of the yrast rotational bands in these nuclei using the variable moment of inertia (VMI) model. The optimized parameters of the model have been deduced by using a computer simulated search programm in order to obtain a minimum root mean square deviation between the calculated theoretical excitation energies and the experimental ones. Most of the identical parameters are extracted. It is observed that the nuclei having $N_p N_n/\Delta$ values exhibit identical excitation energies and energy ratio R(4/2), R(6/4) in their ground-state rotational bands, N_p and N_n are the valence proton and neutron number counted as particles or holes from the nearest spherical shell or spherical subshell closure and Δ is the average pairing gap. The nuclear kinematic and dynamic moments of inertia for the ground state rotational bands have been calculated, a smooth gradual increase in both moments of inertia as function of rotational frequency was seen. The study indicates that each pair of conjugate nuclei have moments of inertia nearly identical.

1 Introduction

One of the most remarkable properties so far discovered of rotational bands in superdeformed (SD) nuclei is the extremely close coincidence in the energies of the deexciting γ-ray transitions or rotational frequencies between certain pairs of rotational bands in adjacent even and odd nuclei with different mass number [1–5]. In a considerable number of nuclei in the Dy region as well as in the Hg region one has found different in transition energies E_γ of only 1-3 KeV, i.e there exist sequence of bands in neighboring nuclei, which are virtually identical $\Delta E_\gamma/E_\gamma \sim 10^{-3}$. This means that the rotational frequencies of the two bands are very similar because the rotational frequency (dE/dI) is approximately half the transition energy, and also implies that the dynamical moments of inertia are almost equal. Several groups have tried to understand the phenomenon of SD identical bands (IB's) or twin bands [5–10] assuming the occurrence of such IB's to be a specific property of the SD states of nuclei.

Shortly afterwards, low spin IB's were found in the ground state rotational bands of normally deformed (ND) nuclei [11–14], which showed that the occurrence of IB's is not restricted to the phenomenon of superdeformation and high-spin states. Since then, a vast amount of IB's have been observed both in SD and ND nuclei, and there have been a lot of theoretical works presented based on various nuclear models [15–18]. All explantation to IB's in SD nuclei differing by one or two particle numbers factor to the odd-even difference in the moments of inertia, namely the pair force, is substantially weakened for high-spin SD states. However, these outlines would fail to explain IB's at low spin, where the blocking of the pairing contributions of the odd nucleon is predicted to reduce the nuclear superfluidity, there by increasing the moment of inertia of the odd-A nucleus. Because of the known spins, configurations and excitations energies of the ND bands, the systematic analysis of IB's in ND nuclei would be useful in investigation of the origin of IB's.

It is the purpose of this paper to point out that existence of low-spin IB's in the well deformed rare-earth region is a manifestation of a more general property of nuclear excitation mechanism in this region, i.e, almost linear dependence of the moment of inertia on a simple function of the valence proton and neutron number. The properties of rotational bands in our selected normal deformed nuclei have been systematically analyzed by using the variables moment of inertia (VIM) model [19, 20].

2 Description of VMI model

The excitation energy of the rotational level with angular moment I for an axially symmetric deformed nucleus is given by

$$E(I) = \frac{\hbar^2}{2J} I(I+1), \quad (1)$$

with J being the rigid moment of inertia. This rigid rotor formula violated at high angular momenta. Bohr and Mottelson [21] introduced a correction term

$$\Delta E(I) = -B\,[I\,(I+1)]^2 \quad (2)$$

which is attributed to rotation-vibration interaction where J and B are the model parameters.

In the variable moment of inertia (VMI) model [19] the level energy is given by

$$E(I, J, J_0, c) = \frac{\hbar^2}{2J} I(I+1) + \frac{c}{2}(J - J_{00})^2 \tag{3}$$

where J_0 is the ground-state moment of inertia. The second term represents the harmonic term with c in the stiffness parameter. The moment of inertia J is a function of the spin $I(J(I))$.

The equiblirium condition

$$\frac{\partial E}{\partial J} = 0 \tag{4}$$

determines the values of the variable moment of inertia J_I, one obtains

$$J_I^3 - J_0 J_I^2 = \frac{1}{2c} I(I+1). \tag{5}$$

This equation has one real root for any finite positive value of J_0 and c can be solved algebraically to yield

$$\begin{aligned} J(J_0, c, I) &= \frac{J_0}{3} \\ &+ \left\{ \frac{1}{2}\frac{I(I{+}1)}{2C} + \frac{J_0^3}{27} + \left[\frac{1}{4}\frac{I^2(I{+}1)^2}{4c^2} + \frac{J_0^3}{27}\frac{I(I{+}1)}{2c} \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}} \\ &+ \left\{ \frac{1}{2}\frac{I(I{+}1)}{2c} + \frac{J_0^3}{27} - \left[\frac{1}{4}\frac{(I{+}1)^2}{4c^2} + \frac{J_0^3}{27}\frac{I(I{+}1)}{2c} \right]^{\frac{1}{2}} \right\}^{\frac{1}{3}}. \end{aligned} \tag{6}$$

A softness parameter σ was introduced, which measures the relative initial variation of J with respect to I. This quantity is obtained from the equation (3)

$$\sigma = \frac{1}{J}\frac{dJ}{dI}\,|_{I=0} = \frac{1}{2cJ_0^3}. \tag{7}$$

To find the rotational frequency $\hbar\omega$, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia for VMI model, let $\hat{I} = [I(I+1)]^{\frac{1}{2}}$. Equations (3,5) can be written in the form

$$E = \frac{\hbar^2}{2J}\hat{I}^2 + \frac{c}{2}(J - J_0)^2, \tag{8}$$

$$J^3 - J_0 J^2 - \frac{\hat{I}^2}{2c} = 0. \tag{9}$$

Differentiating these two equations with respect to $\hat{I}$ and using the chain rule, we get

$$\frac{dE}{d\hat{I}} = \frac{\hat{I}}{J} + \left[c(J - J_0) - \frac{\hat{I}^2}{2J^2} \right] \frac{dJ}{d\hat{I}}, \tag{10}$$

$$\begin{aligned} \frac{d^2E}{d\hat{I}^2} = &\ \frac{1}{J} - \frac{2\hat{I}}{J^2}\frac{dJ}{d\hat{I}} + \left(c + \frac{\hat{I}^2}{J^3}\right)\left(\frac{dJ}{d\hat{I}}^2\right) \\ &+ \left[c(J - J_0) - \frac{\hat{J}^2}{2J^2} \right] \frac{d^2J}{d\hat{I}^2}, \end{aligned} \tag{11}$$

$$\frac{dJ}{d\hat{I}} = \frac{\hat{I}}{cJ(3J - 2J_0)}, \tag{12}$$

$$\frac{d^2J}{d\hat{I}^2} = \frac{1 - 2c(3J - J_0)}{cJ(3J - 2J_0)} \left(\frac{dJ}{d\hat{I}}\right)^2. \tag{13}$$

Using the above differentiations, we can extract $\hbar\omega$, $J^{(1)}$ and $J^{(2)}$ from their definitions:

$$\hbar\omega = \frac{dE}{d\hat{I}}, \tag{14}$$

$$J^{(1)} = \hbar^2 \hat{I} \left(\frac{dE}{d\hat{I}^2}\right)^{-1} \simeq \frac{2I - 1}{E_\gamma(I \to I - 2)}, \tag{15}$$

$$J^{(2)} = \hbar^2 \left(\frac{d^2E}{d\hat{I}^2}\right)^{-1} \simeq \frac{4}{E_\gamma(I + 2 \to I) - E_\gamma(I \to I - 2)}. \tag{16}$$

The $J^{(1)}$ moment of inertia is a direct measure of the transition energies while $J^{(2)}$ is obtained from differences in transitions energies (relative change in transition energies).

3 Identical bands parameters

In the concept of F-spin [22], the N_π proton bosons and N_ν neutron bosons are assigned intrinsic quantum number called F-spin $F = \frac{1}{2}$, with projection $F_0 = +\frac{1}{2}$ for proton bosons and $F_0 = -\frac{1}{2}$ for neutrons bosons.

Therefore, a given nucleus is then characterized by two quantum numbers $F = \sum_i F_i = \frac{1}{2}(N_\pi + N_\nu) = \frac{1}{4}(N_p + N_n)$ and it's projection $F_0 = \frac{1}{2}(N_\pi - N_\nu) = \frac{1}{4}(N_p - N_n)$. Squaring and subtracting, yield $4(F^2 - F_0^2) = 4N_\pi N_\nu = N_p N_n$.

That is any pairs of conjugate nuclei with the same F-spin and $\pm F_0$ values in any F-spin multiplet have identical $N_p N_n$ values [23]. The product $N_p N_n$ was used in classification the changes occur in nuclear structure of transitional region [13, 24].

It was assumed that [14], the moment of inertia J has a simple dependence on the product of valence proton and neutron numbers ($N_p N_n$) written in the form

$$J \propto SF \cdot SP \tag{17}$$

where SF and SP are called the structure factor and saturation parameter given by

$$SF = N_p N_n (N_p + N_n), \tag{18}$$

$$SP = \left[1 + \frac{SF}{(SF)_{\max}} \right]^{-1}. \tag{19}$$

Computing by taking

$$N_p = \min\left[(Z-50),(82-Z)\right], \tag{20}$$

$$N_n = \min\left[(N-82),(126-N)\right], \tag{21}$$

it was found that the low spin dynamical moment of inertia defined as

$$J^{(2)}(I=2) = \frac{4}{E_\gamma(4^+ \to 2^+) - E_\gamma(2^+ \to 0^+)} \tag{22}$$

shows an approximate dependence on SF

$$J^{(2)}(I=2) \propto (SF)^{\frac{1}{2}}. \tag{23}$$

Since the nuclei having identical N_pN_n and $|N_p - N_n|$ values are found to have identical moment of inertia, the structure factor SF is related not only to the absolute value of ground state moment of inertia but also to its angular momentum dependence.

Also it was shown [11, 25, 26] that the development of collectivity and deformation in medium and heavy nuclei is very smoothly parameterized by the p-factor defined as

$$P = \frac{N_pN_n}{N_p + N_n}. \tag{24}$$

The p-factor can be viewed as the ratio of the number of valence p-n residual interaction to the number of valence like-nucleon-pairing interaction, or, if the p-n and pairing interactions are orbit independent, then p is proportional to the ratio of the integrated p-n interaction strength.

Observables such as $E(4_1^+)/E(2_1^+)$ or $B(E_2, 0_1^+ \to 2_1^+)$ that are associated with the mean field vary smoothly with p-factor.

The square of deformation parameter β^2 is invariant under rotations of the coordinate system fixed in the space. In the SU(3) limit of the interacting boson model (IBM) [27], the matrix elements of β^2 in a state with angular momenta I are given by

$$\langle\beta^2\rangle_I = \frac{1}{6(2N-1)}\left[I(I+1) + 8N^2 + 22N - 15\right] \tag{25}$$

where N is the total number of the valence bosons For the expectations value of β^2 in the ground state $I = 0$, yielding

$$\langle\beta^2\rangle_{I=0} = \frac{1}{6(2N-1)}\left[8N^2 + 22N - 15\right] \tag{26}$$

which is increasing function of N.

In order to determine β from equation (26) to a given rotational region or grouped of isotopes, one should normalize it, then

$$\beta_0 = \alpha\left[\frac{8N^2 + 22N - 15}{6(2N-1)}\right]^{\frac{1}{2}} \tag{27}$$

where α is the normalization constant ($\alpha = 0.101$ for rare earth nuclei.)

Table 1: The simulated adopted best VMI parameters used in the calculations for the identical bands in normal deformed even-even ^{158}Dy, ^{160}Er, ^{162}Yb and $^{166-170}$Hf nuclei. σ denoting the softness parameter of the VMI model. We also list the total percent root mean square deviation.

Nucleus	J_0 ($\hbar^2$ MeV^{-1})	c (10^{-1}MeV^3)	$\sigma = 1/2cJ_0^3$ (10^{-1})	% rmsd
^{158}Dy	28.8866	2.37364	8.7372	0.57
^{170}Hf	29.9116	1.93836	9.6386	0.87
^{160}Er	22.7538	2.65536	15.9839	0.86
^{168}Hf	22.8761	2.48160	16.8303	0.70
^{162}Yb	16.8587	2.83884	36.7584	0.60
^{166}Hf	17.6941	2.76559	32.6359	0.82

4 Results and discussion

A fitting procedure has been applied to all measured values of excitation energies $E(I)$ in a given band. The parameters J_0, c and σ of the VMI model results from the fitting procedure for our selected three pairs IB's are listed in Table 1. The percentage root mean square (rms) deviation of the calculated from the experimental level energies is also given in the Table and is within a fraction of 1%. To illustrate the quantitative agreement obtained from the excitation energies, we present in Table 2 the theoretical values of energies, transition energies, rotational frequencies kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia and the variable moment of inertia J_{VMI} as a function of spin for our three pairs of IB's which each pair has identical $N_p\,N_n$ product. The calculated kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are plotted against rotational frequency $\hbar\omega$ in Figure 1.

The similarities are striking, although the frequency range covered in each two IB's is smaller than that observed in the

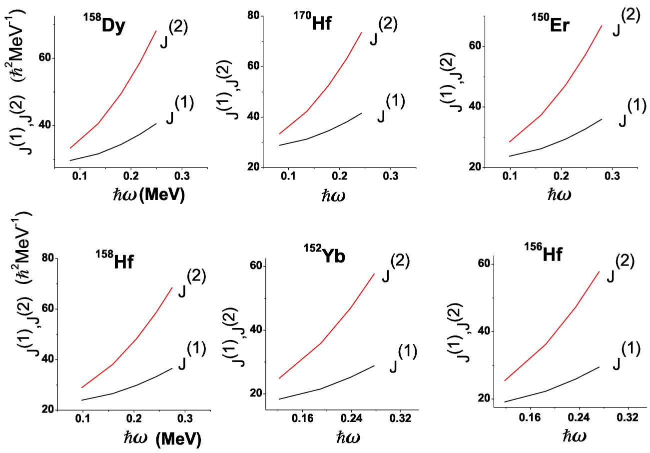

Fig. 1: Plot of the calculated kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia versus the rotational frequency $\hbar\omega$ for the low lying states in the conjugate pairs (^{158}Dy, ^{170}Hf), (^{160}Er, ^{168}Hf) and (^{162}Yb, ^{166}Hf).

Table 2: Theoretical calculations to outline the properties of our selected even rare-earth nuclei in framework of VMI model for each nucleus we list the energy E(I), the gamma ray transition energy $E_\gamma(I \rightarrow I-2)$, the rotational frequency $\hbar\omega$, the dynamic moment of inertia $J^{(2)}$, the kinematic moment of inertia $J^{(1)}$ and the variable moment of inertia J_{VMI}

$E_{exp}(I)$ (keV)	I^π ($\hbar$)	$E_{cal}(I)$ (keV)	$E_{\gamma(I\rightarrow I-2)}$ keV	$\hbar\omega$ (MeV)	$J^{(2)}$ ($\hbar^2$ MeV^{-1})	$J^{(1)}$ ($\hbar^2$ MeV^{-1})	J_{VMI} ($\hbar^2$ MeV^{-1})
			$^{158}Dy_{92}$				
99	2^+	101.379	101.379	0.0807	33.2515	29.5919	30
317	4^+	323.053	221.674	0.1354	40.5724	31.5779	33
638	6^+	643.316	320.263	0.1803	49.4620	34.3467	36
1044	8^+	1044.449	401.133	0.2175	58.8001	37.3940	39
1520	10^+	1513.609	469.160	0.2492	68.1419	40.4979	42
2050	12^+	2041.470	527.861			43.5720	45
			$^{170}Hf_{92}$				
100.8	2^+	104.135	104.135	0.0820	33.3828	28.8087	30
321.99	4^+	328.092	223.957	0.1356	42.2275	31.2560	33
642.9	6^+	646.774	318.682	0.1783	52.5513	34.5171	36
1043.3	8^+	1041.572	394.798	0.2132	63.1123	37.9941	40
1505.5	10^+	1499.749	458.177	0.2426	73.5077	41.4686	43
2016.4	12^+	2012.342	512.593			44.8699	47
			$^{160}Er_{68}$				
126	2^+	126.476	126.476	0.0983	28.4620	23.7199	25
390	4^+	393.490	267.014	0.1603	37.3148	26.2158	28
765	6^+	767.700	374.210	0.2082	47.2533	29.3452	31
1229	8^+	1226.560	458.860	0.2469	57.1845	32.6897	34
1761	10^+	1755.369	528.809	0.2793	66.8337	35.9297	37
2340	12^+	2344.028	588.659			39.0718	41
			$^{168}Hf_{96}$				
124	2^+	125.554	125.544	0.0974	28.8591	23.8941	25
386	4^+	389.712	264.158	0.1583	38.0709	26.4992	28
757	6^+	758.937	369.225	0.2052	48.3412	29.7921	31
1214	8^+	1210.907	451.970	0.2430	58.5677	33.1880	35
1736	10^+	1731.174	520.267	0.2747	68.4802	36.5194	38
2306	12^+	2309.582	578.678			39.7457	41
			$^{162}Yb_{92}$				
166	2^+	163.728	163.728	0.1220	24.9036	18.3230	20
487	4^+	488.075	324.347	0.1900	35.9266	21.5818	23
923	6^+	923.760	435.685	0.2390	47.0494	25.2475	27
1445	8^+	1444.462	520.702	0.2777	57.6036	28.8072	30
2023	10^+	2034.604	590.142			32.1936	34
			$^{166}Hf_{94}$				
159	2^+	157.173	157.173	0.1179	25.4281	19.0872	20
470	4^+	471.652	314.479	0.1848	36.2236	22.2590	24
897	6^+	896.556	424.904	0.2336	47.2768	25.8882	28
1406	8^+	1406.068	509.512	0.2720	57.8285	29.4399	31
1970	10^+	1984.750	578.682			32.8332	34

SD nuclei. The $J^{(2)}$ is significantly larger than $J^{(1)}$ over a large rotational frequency range.For our three IB pairs, the IB parameters are listed in Table 3.

5 Conclusion

The problem of identical bands(IB's) in normal deformed nuclei is treated. We investigated three pairs of conjugate normal deformed nuclei in rare-earth region (^{158}Dy,^{170}Hf), (^{160}Er, ^{168}Hf) and (^{162}Yb, ^{166}Hf) with the same F spin and projections $\pm F_0$ values have identical product of valence proton and neutron numbers N_pN_n values. Also the values of dynamical moment of inertia $J^{(2)}$ for each IB pair are approximately the same. We extracted all the IB symmetry parameters like p-factor, saturation factor SF, structure factor SP etc. which all depending on the valence proton and neutron numbers. By using the VMI model, we find agreement between experimental excitation energies and theoretical ones.

The optimized model free parameters for each nucleus have been deduced by using a computer simulation search programm to fit the calculated theoretical excitation energies with the experimental energies.

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Table 3: The calculated correlation factors for selected three pairs of even-even rare-earth nuclei having nearly identical bands.

	^{158}Dy	^{170}Hf	^{160}Er	^{168}Hf	^{162}Yb	^{166}Hf
$(N_\pi,\ N_\nu)$	(8,5)	(5,8)	(7,5)	(5,7)	(6,5)	(5,6)
N_pN_n	160	160	140	140	120	120
F	6.5	6.5	6	6	5.5	5.5
F_0	1.5	-1.5	1	-1	0.5	-0.5
P	6.1538	6.1538	5.8333	5.8333	5.4545	5.4545
SF	4160	4160	3360	3360	2640	2640
SP	0.6176	0.6176	0.666	0.666	0.7179	0.7179
$J^{(2)}_{SF}$	32.2643	32.2645	28.9966	28.9966	25.7027	25.7027
$E^{(2)}_{SF}$	103.0160	103.0160	127.5437	127.5437	162.3283	162.3283
R(4/2)	3.2060	3.1943	3.0993	3.1096	2.9230	2.9671
R(6/2)	6.4468	6.3771	6.0866	6.1040	5.5430	5.6586
β_0	0.3322	0.3322	0.3218	0.3218	0.3110	0.3110
Δ	0.9546	0.9203	0.9486	0.9258	0.9428	0.9313
N_pN_n/Δ	167.6094	173.8563	147.5859	151.2205	127.2804	128.8521

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