# Theorem of Non-Returning and Time Irreversibility of Tachyon Kinematics

Yaroslav I. Grushka
Institute of Mathematics NAS of Ukraine, 3, Tereschenkivska st., 01601, Kyiv, Ukraine.
E-mail: grushka@imath.kiev.ua

Using the recently developed mathematical apparatus of the theory of universal kinematic sets, we prove that the hypothesis of the existence of material objects and inertial reference frames moving with superluminal velocities in the general case does not lead to the violation of the principle of causality, that is, to a possibility of the returning to the own past. This result is obtained as the corollary of the abstract theorem on irreversibility, which gives the sufficient condition of time irreversibility for universal kinematic sets.

## 1 Introduction

Subject of constructing the theory of super-light movement, had been posed in the papers [1, 2] more than 50 years ago. Despite the fact that on today tachyons (ie objects moving at a velocity greater than the velocity of light) are not experimentally detected, this subject remains being actual.

It is well known that among physicists it is popular the belief that the hypothesis of the existence of tachyons leads to temporal paradoxes, connected with the possibility of changing the own past. Conditions of appearing these time paradoxes were carefully analyzed in [3]. It should be noted, that in [3] superluminal motion is allowed only for particles or signals whereas superluminal motion for reference frames is forbidden. This fact does not give the possibility to bind the own time with tachyon particle, and, therefore to determine real direction of motion of the particle. In the paper [4] for tachyon particles the own reference frames are axiomatically introduced only for the case of one space dimension. Such approach allows to determine real direction of motion of the tachyon particle by more correct way, and so to obtain more precise results.

In particular, in the paper [4] it was shown, that the hypothesis of existence of material objects, moving with the velocity, greater than the velocity of light, does not lead to formal possibility of returning to the own past in general. Meanwhile in the papers of E. Recami, V. Olkhovsky and R. Goldoni [5–7], and and later in the papers of S. Medvedev [8] as well as J. Hill and B. Cox [9] the generalized Lorentz transforms for superluminal reference frames are deduced in the case of three-dimension space of geometric variables. In the paper [10] it was proven, that the above generalized Lorentz transforms may be easy introduced for the more general case of arbitrary (in particular infinity) dimension of the space of geometric variables.

Further, in [11], using theory of kinematic changeable sets, on the basis of the transformations [10], the mathematically strict models of kinematics, allowing the superluminal motion for particles as well as for inertial reference frames, had been constructed. Thus, the tachyon kinematics in the sense of E.Recami, V. Olkhovsky and R. Goldoni are surely mathematically strict objects. But, these kinematics are impossible to analyze on the subject of time irreversibility (that is on existence the formal possibility of returning to the own past), using the results of the paper [4], because in [4] complete, multidimensional superluminal reference frames are missing.

Moreover, it can be proved, that the axiom "AxSameFuture" from [4, subsection 2.1] for these tachyon kinematics is not satisfied. The paper [12] [1] is based on more general mathematical apparatus in comparison with the paper [4], namely on mathematical apparatus of the theory of kinematic changeable sets. In [12] the strict definitions of time reversibility and time irreversibility for universal kinematics were given, moreover in this paper it was proven, that all tachyon kinematics, constructed in the paper [11], are time reversible in principle. In connection with the last fact the following question arises:

Is it possible to build the certainly time-irreversible universal kinematics, which allows for reference frames moving with any speed other than the speed of light, using the generalized Lorentz-Poincare transformations in terms of E. Recami, V. Olkhovsky and R. Goldoni?

In the present paper we prove the abstract theorem on non-returning for universal kinematics and, using this theorem, we give the positive answer on the last question.

For further understanding of this paper the main concepts and denotation system of the theories of changeable sets, kinematic sets and universal kinematics, are needed. These theories were developed in [11, 13–17]. Some of these papers were published in Ukrainian. That is why, for the convenience of readers, main results of these papers were "converted" into English and collected in the preprint [18], where one can find the most complete and detailed explanation of these theories. Hence, we refer to [18] all readers who are not familiar with the essential concepts. So, during citation of needed main results we sometimes will give the dual reference of these results (in one of the papers [11, 13–17] as well as in [18]).

[1] Note, that main results of the paper [12] were announced in [19].

## 2 Elementary-time states and changeable systems of universal kinematics

**Definition 1.** *Let $\mathcal{F}$ be any universal kinematics*[1]*, $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$ and $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ be any elementary-time state in the reference frame $\mathfrak{l}$. The set*

$$\omega^{\{\mathfrak{l},\mathcal{F}\}} = \{(\mathfrak{m}, \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega) \mid \mathfrak{m} \in \mathcal{L}k(\mathcal{F})\}$$

*(where $(x,y)$ is the ordered pair, composed of $x$ and $y$) is called by* ***elementary-time state of the universal kinematics*** *$\mathcal{F}$, generated by $\omega$ in the reference frame $\mathfrak{l}$.*

*Remark* 1. In the case, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\omega^{\{\mathfrak{l}\}}$ instead of the denotation $\omega^{\{\mathfrak{l},\mathcal{F}\}}$.

**Assertion 1.** *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$. Then for arbitrary elementary-time states $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ and $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ the following assertions are equivalent:*
*1) $\omega^{\{\mathfrak{l}\}} = \omega_1^{\{\mathfrak{m}\}}$; 2) $\omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega$.*

*Proof.* **1.** First, we prove, that statement 2) leads to the statement 1). Consider any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ and $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ such that $\omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega$. Applying Definition 1 and [18, Property 1.12.1(3)][2], we deduce

$$\begin{aligned}\omega_1^{\{\mathfrak{m}\}} &= \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m} \rangle\, \omega_1) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \\ &= \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m} \rangle \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \\ &= \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{l} \rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \omega^{\{\mathfrak{l}\}}.\end{aligned}$$

**2.** Inversely, suppose, that $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$, $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ and $\omega^{\{\mathfrak{l}\}} = \omega_1^{\{\mathfrak{m}\}}$. Then, by Definition 1, we have

$$\begin{aligned}&\{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{l} \rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\} = \\ &\qquad = \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m} \rangle\, \omega_1) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\}. \qquad (1)\end{aligned}$$

According to [18, Property 1.12.1(1)], we have, $\langle !\, \mathfrak{l} \leftarrow \mathfrak{l} \rangle\, \omega = \omega$. Hence, in accordance with (1), for element $(\mathfrak{l}, \omega) = (\mathfrak{l}, \langle !\, \mathfrak{l} \leftarrow \mathfrak{l} \rangle\, \omega) \in \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{l} \rangle\, \omega) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\}$ we obtain the correlation, $(\mathfrak{l}, \omega) \in \{(\mathfrak{p}, \langle !\, \mathfrak{p} \leftarrow \mathfrak{m} \rangle\, \omega_1) \mid \mathfrak{p} \in \mathcal{L}k(\mathcal{F})\}$. Therefore, there exists the reference frame $\mathfrak{p}_0 \in \mathcal{L}k(\mathcal{F})$ such that $(\mathfrak{l}, \omega) = (\mathfrak{p}_0, \langle !\, \mathfrak{p}_0 \leftarrow \mathfrak{m} \rangle\, \omega_1)$. Hence we deduce $\mathfrak{l} = \mathfrak{p}_0$, as well $\omega = \langle !\, \mathfrak{p}_0 \leftarrow \mathfrak{m} \rangle\, \omega_1 = \langle !\, \mathfrak{l} \leftarrow \mathfrak{m} \rangle\, \omega_1$. So, based on [18, Properties 1.12.1(1,3)], we conclude, $\omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{m} \rangle\, \omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \langle !\, \mathfrak{l} \leftarrow \mathfrak{m} \rangle\, \omega_1 = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega$. □

The next corollary follows from Assertion 1.

**Corollary 1.** *Let $\mathcal{F}$ be any universal kinematics. Then for every $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ and $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the following equality holds:*

$$\omega^{\{\mathfrak{l}\}} = (\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega)^{\{\mathfrak{m}\}}.$$

[1] Definition of universal kinematics can be found in [11, page 89] or [18, page 156].

[2] Reference to Property 1.12.1(3) means reference to the item 3 from the group of properties "Properties 1.12.1".

**Assertion 2.** *Let $\mathcal{F}$ be any universal kinematics. Then the set*

$$\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \left\{\omega^{\{\mathfrak{l},\mathcal{F}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} \qquad (2)$$

*does not depend of the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (ie $\forall \mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ $\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \mathbb{B}\mathfrak{s}[\mathfrak{m}, \mathcal{F}]$).*

*Proof.* Consider arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$. Using Corollary 1, we have

$$\begin{aligned}\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] &= \left\{\omega^{\{\mathfrak{l}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} = \\ &= \left\{(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega)^{\{\mathfrak{m}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\}.\end{aligned}$$

Hence, according to [18, Corollary 1.12.6], we obtain

$$\begin{aligned}\mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] &= \left\{(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega)^{\{\mathfrak{m}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\} = \\ &= \left\{\omega_1^{\{\mathfrak{m}\}} \mid \omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{m})\right\} = \mathbb{B}\mathfrak{s}[\mathfrak{m}, \mathcal{F}].\end{aligned}$$ □

**Definition 2.** *Let $\mathcal{F}$ be any universal kinematics.*

*1. The set $\mathbb{B}\mathfrak{s}(\mathcal{F}) = \mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}]$ $(\forall\, \mathfrak{l} \in \mathcal{L}k(\mathcal{F}))$ is called by the set of all elementary-time states of $\mathcal{F}$.*

*2. Any subset $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ is called by the (common)* ***changeable system of the universal kinematics*** *$\mathcal{F}$.*

**Assertion 3.** *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$. Then for every element $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ only one element $\omega_0 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ exists such, that $\hat{\omega} = \omega_0^{\{\mathfrak{l}\}}$.*

*Proof.* Consider any $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$. By Definition 2 and Assertion 2 (formula (2)), we have

$$\mathbb{B}\mathfrak{s}(\mathcal{F}) = \mathbb{B}\mathfrak{s}[\mathfrak{l}, \mathcal{F}] = \left\{\omega^{\{\mathfrak{l}\}} \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\}.$$

So, since $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$, the element $\omega_0 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ must exist such that the following equality is performed:

$$\hat{\omega} = \omega_0^{\{\mathfrak{l}\}}. \qquad (3)$$

Let us prove that such element $\omega_0$ is unique. Assume that $\hat{\omega} = \omega_1^{\{\mathfrak{l}\}}$, where $\omega_1 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$. Then, from the equality (3) we deduce, $\omega_0^{\{\mathfrak{l}\}} = \omega_1^{\{\mathfrak{l}\}}$. Hence, according to Assertion 1 and [18, Property 1.12.1(1)], we obtain, $\omega_1 = \langle !\, \mathfrak{l} \leftarrow \mathfrak{l} \rangle\, \omega_0 = \omega_0$. □

**Definition 3.** *Let $\mathcal{F}$ be any universal kinematics, $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ be any elementary-time state of $\mathcal{F}$ and $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$. Elementary-time state $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ is named by* ***image*** *of elementary-time state $\hat{\omega}$* ***in the reference frame*** *$\mathfrak{l}$ if and only if $\hat{\omega} = \omega^{\{\mathfrak{l}\}}$.*

In accordance with Assertion 3, every elementary-time state $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ always has only one image in any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$. Image of elementary-time state $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ will be denoted via $\hat{\omega}_{\{\mathfrak{l},\mathcal{F}\}}$ (in the cases, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\hat{\omega}_{\{\mathfrak{l}\}}$).

Thus, according to Definition 3, for arbitrary $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ the following equality holds:

$$\left(\hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \hat{\omega}. \tag{4}$$

From the other hand, if for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and any fixed elementary-time state $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$, we denote $\hat{\omega} := \omega^{\{\mathfrak{l}\}}$, then by Definition 3, we will receive, $\omega = \hat{\omega}_{\{\mathfrak{l}\}}$. Therefore we have:

$$\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{l}\}} = \omega \quad (\forall \mathfrak{l} \in \mathcal{L}k(\mathcal{F})\ \forall \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})). \tag{5}$$

From equalities (4) and (5) we deduce the following corollary:

**Corollary 2.** *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be any reference frame of $\mathcal{F}$. Then:*

*1. The mapping $(\cdot)^{\{\mathfrak{l}\}}$ is bijection from $\mathbb{B}\mathfrak{s}(\mathfrak{l})$ onto $\mathbb{B}\mathfrak{s}(\mathcal{F})$.*

*2. The mapping $(\cdot)_{\{\mathfrak{l}\}}$ is bijection from $\mathbb{B}\mathfrak{s}(\mathcal{F})$ onto $\mathbb{B}\mathfrak{s}(\mathfrak{l})$.*

*3. The mapping $(\cdot)_{\{\mathfrak{l}\}}$ is inverse to the mapping $(\cdot)^{\{\mathfrak{l}\}}$.*

**Assertion 4.** *Let $\mathcal{F}$ be any universal kinematics and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ be any reference frames $\mathcal{F}$. Then the following statements are performed:*

*1. For every $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ the equality $\hat{\omega}_{\{\mathfrak{m}\}} = \langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \hat{\omega}_{\{\mathfrak{l}\}}$ holds.*

*2. For each $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ the equality $\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{m}\}} = \langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \omega$ is true.*

*Proof.* 1) Chose any $\hat{\omega} \in \mathbb{B}\mathfrak{s}(\mathcal{F})$. Applying Corollary 1 to the elementary-time state $\hat{\omega}_{\{\mathfrak{l}\}} \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ and using equality (4), we obtain

$$\left(\langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{m}\}} = \left(\hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \hat{\omega}.$$

Thence, using equality (5), we have

$$\hat{\omega}_{\{\mathfrak{m}\}} = \left(\left(\langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \hat{\omega}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{m}\}}\right)_{\{\mathfrak{m}\}} = \langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \hat{\omega}_{\{\mathfrak{l}\}}.$$

2) Consider any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$. Applying Corollary 1 as well as equality (5), we deliver

$$\left(\omega^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{m}\}} = \left(\left(\langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \omega\right)^{\{\mathfrak{m}\}}\right)_{\{\mathfrak{m}\}} = \langle ! \, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \, \omega. \qquad \square$$

Let $\mathcal{F}$ be any universal kinematics. The set $\widehat{\mathbf{A}}_{\{\mathfrak{l},\mathcal{F}\}} = \left\{\hat{\omega}_{\{\mathfrak{l},\mathcal{F}\}} \mid \hat{\omega} \in \widehat{\mathbf{A}}\right\}$ is called ***image of changeable system*** $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$.

Any changeable system $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ always generates the (common) changeable system $A^{\{\mathfrak{l},\mathcal{F}\}} := \left\{\omega^{\{\mathfrak{l},\mathcal{F}\}} \mid \omega \in A\right\} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$.

*Remark* 2. In the cases, where universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotations $\widehat{\mathbf{A}}_{\{\mathfrak{l}\}}$ and $A^{\{\mathfrak{l}\}}$ instead of $\widehat{\mathbf{A}}_{\{\mathfrak{l},\mathcal{F}\}}$ and $A^{\{\mathfrak{l},\mathcal{F}\}}$ (correspondingly).

Applying equalities (4) and (5), we obtain the equalities:

$$\left(\widehat{\mathbf{A}}_{\{\mathfrak{l}\}}\right)^{\{\mathfrak{l}\}} = \widehat{\mathbf{A}} \quad \text{and} \quad \left(A^{\{\mathfrak{l}\}}\right)_{\{\mathfrak{l}\}} = A$$

(for arbitrary universal kinematics $\mathcal{F}$, reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and changeable systems $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ as well $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$).

## 3 Chain paths of universal kinematics and definition of time irreversibility

**Definition 4.** *Let $\mathcal{F}$ be any universal kinematics. Changeable system $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ is called **piecewise chain** changeable system if and only if there exist the sequences of changeable systems $\widehat{\mathbf{A}}_1, \cdots, \widehat{\mathbf{A}}_n \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ and reference frames $\mathfrak{l}_1, \cdots, \mathfrak{l}_n \in \mathcal{L}k(\mathcal{F})$ $(n \in \mathbb{N})$ satisfying the following conditions:*

**(a)** $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}l(\mathfrak{l}_k)$ $\left(\forall k \in \overline{1,n}\right)$,[1] *where definition of set $\mathbb{L}l(\mathfrak{l}_k) = \mathbb{L}l((\mathfrak{l}_k)^\wedge)$ can be found in* [18, pages 63, 88, 156]*;*

**(b)** $\bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k = \widehat{\mathbf{A}}$,

*and, moreover, in the case $n \geq 2$ the following additional conditions are satisfied:*

**(c)** $\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1} \neq \emptyset$ $\left(\forall k \in \overline{1,n-1}\right)$*;*

**(d)** *For each $k \in \overline{1,n-1}$ and arbitrary $\omega_1 \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$, $\omega_2 \in \left(\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$ the inequality $\mathsf{tm}(\omega_1) <_{\mathfrak{l}_k} \mathsf{tm}(\omega_2)$ holds.*

**(e)** *For every $k \in \overline{2,n}$ and arbitrary $\omega_1 \in \left(\widehat{\mathbf{A}}_{k-1} \cap \widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}}$, $\omega_2 \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$ the inequality $\mathsf{tm}(\omega_1) <_{\mathfrak{l}_k} \mathsf{tm}(\omega_2)$ is performed.*

*In this case the ordered composition $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ will be named by the **chain path** of universal kinematics $\mathcal{F}$.*

**Definition 5.** *Let $\mathcal{F}$ be any universal kinematics.*

**(a)** *Changeable system $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l})$ is refereed to as **geometrically-stationary** in the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ if and only if $A \in \mathbb{L}l(\mathfrak{l})$ and for arbitrary $\omega_1, \omega_2 \in A$ the equality $\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega_1)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega_2)\right)$ holds.*

**(b)** *The set of all geometrically-stationary changeable systems in the reference frame $\mathfrak{l}$ is denoted via $\mathbb{L}g(\mathfrak{l}, \mathcal{F})$. In the cases, where the universal kinematics $\mathcal{F}$ is known in advance, we use the abbreviated denotation $\mathbb{L}g(\mathfrak{l})$.*

**(c)** *The chain path $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ in $\mathcal{F}$ $(n \in \mathbb{N})$ is called by **piecewise geometrically-stationary** if and only if $\forall k \in \overline{1,n}$ $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g(\mathfrak{l}_k)$.*

From the physical point of view piecewise geometrically-stationary chain path may be interpreted as process of "vagrancy" of observer (or some material particle or signal), which moves by means of "jumping" from previous reference frame to the next frame with a finite number of times.

**Definition 6.** *Let $\mathcal{F}$ be any universal kinematics and let $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ be arbitrary chain path in $\mathcal{F}$.*

[1] Further we denote via $\overline{m,n}$ $(m, n \in \mathbb{N},\ m \leq n)$ the set $\overline{m,n} = \{m, \cdots, n\}$.

1. *Element $\hat{\omega}_s \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ is called by **start** element of the path $\mathscr{A}$, if and only if $\hat{\omega}_s \in \widehat{\mathbf{A}}_1$ and for every $\hat{\omega} \in \widehat{\mathbf{A}}_1$ the inequality $\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\,(\hat{\omega}_{\{\mathfrak{l}_1\}})$ is performed.*
2. *Element $\hat{\omega}_f \in \mathbb{B}\mathfrak{s}(\mathcal{F})$ is called by **final** element of the path $\mathscr{A}$, if and only if $\hat{\omega}_f \in \widehat{\mathbf{A}}_n$ and for every $\hat{\omega} \in \widehat{\mathbf{A}}_n$ the inequality $\mathsf{tm}\,(\hat{\omega}_{\{\mathfrak{l}_n\}}) \leq_{\mathfrak{l}_n} \mathsf{tm}\left(\left(\hat{\omega}_f\right)_{\{\mathfrak{l}_n\}}\right)$ holds.*
3. *The chain path $\mathscr{A}$, which owns (at least one) start element and (at least one) final element, is called by **closed**.*

**Assertion 5.** *Any chain path $\mathscr{A}$ of arbitrary universal kinematics $\mathcal{F}$ can not have more, than one start element and more, than one final element.*

*Proof.* **(a)** Let $\hat{\omega}_s$, $\hat{\omega}_x$ be two start elements of the chain path $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$. Then, by Definition 6, we have $\hat{\omega}_s, \hat{\omega}_x \in \widehat{\mathbf{A}}_1$, $\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)$ and $\mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)$. Therefore we get

$$\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) = \mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right). \tag{6}$$

Since $\hat{\omega}_s, \hat{\omega}_x \in \widehat{\mathbf{A}}_1$, then $(\hat{\omega}_s)_{\{\mathfrak{l}_1\}}, (\hat{\omega}_x)_{\{\mathfrak{l}_1\}} \in \left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}$, where, in accordance with Definition 4 (subitem (a)), we have, $\left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}} \in \mathbb{L}l\,(\mathfrak{l}_1)$. That is, according to [18, Assertion 1.7.5 (item 1)], $\left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}$ is a function from $\mathbf{Tm}\,(\mathfrak{l}_1)$ into $\mathfrak{Bs}\,(\mathfrak{l}_1)$. So, using equality $\omega = (\mathsf{tm}\,(\omega), \mathsf{bs}\,(\omega))$ $(\omega \in \mathbb{B}\mathfrak{s}\,(\mathfrak{l}_1))$ as well as formula (6), we obtain

$$\mathsf{bs}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right) = \left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}\left(\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)\right) = \\ = \left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}}\left(\mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)\right) = \mathsf{bs}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right).$$

Using the last equality and equality (6), we deduce, $(\hat{\omega}_s)_{\{\mathfrak{l}_1\}} = \left(\mathsf{tm}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right), \mathsf{bs}\left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)\right) = \left(\mathsf{tm}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right), \mathsf{bs}\left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)\right) = (\hat{\omega}_x)_{\{\mathfrak{l}_1\}}$. Hence, according to formula (4), we deliver $\hat{\omega}_s = \left((\hat{\omega}_s)_{\{\mathfrak{l}_1\}}\right)^{\{\mathfrak{l}_1\}} = \left((\hat{\omega}_x)_{\{\mathfrak{l}_1\}}\right)^{\{\mathfrak{l}_1\}} = \hat{\omega}_x$.

**(c)** Similarly it can be proven that the chain path $\mathscr{A}$ can not have more, than one final element. □

Further the start element of the chain path $\mathscr{A}$ of the universal kinematics $\mathcal{F}$ will be denoted via $\mathsf{po}\,(\mathscr{A}, \mathcal{F})$, or via $\mathsf{po}\,(\mathscr{A})$. The final element of the chain path $\mathscr{A}$ will be denoted via $\mathsf{ki}\,(\mathscr{A}, \mathcal{F})$, or via $\mathsf{ki}\,(\mathscr{A})$. Where the denotations $\mathsf{po}\,(\mathscr{A})$ and $\mathsf{ki}\,(\mathscr{A})$ are used in the cases when they do not cause misunderstanding. Thus, for every closed chain path $\mathscr{A}$ both start and final elements ($\mathsf{po}\,(\mathscr{A})$ and $\mathsf{ki}\,(\mathscr{A})$) always exist.

**Definition 7.** *Closed chain path $\mathscr{A}$ of universal kinematics $\mathcal{F}$ is refereed to as **geometrically-cyclic** in the reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$ if and only if* $\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right)$.

**Definition 8.** *Universal kinematics $\mathcal{F}$ is called **time irreversible** if and only if for every reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$ and for each chain path $\mathscr{A}$, geometrically-cyclic in the frame $\mathfrak{l}$ and piecewise geometrically-stationary in $\mathcal{F}$, it is performed the inequality* $\mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)$.

*Universal kinematics $\mathcal{F}$ is called **time reversible** if and only if it is not time irreversible.*

The physical sense of time irreversibility notion is that in time irreversible kinematics there is not any process or object which returns to the begin of the own path at the past, moving by means of "jumping" from previous reference frame to the next frame. So, there are not temporal paradoxes in these kinematics.

## 4 Direction of time between reference frames of universal kinematics

For formulation main theorem we need some notions, connected with direction of time between reference frames.

**Definition 9.** *Let $\mathcal{F}$ be any universal kinematics.*

1. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k\,(\mathcal{F})$ is **time-nonnegative** relatively the reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$ (in the universal kinematics $\mathcal{F}$) (denotation is $\mathfrak{m} \Uparrow_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k\,(\mathfrak{l})$ such that $\mathsf{bs}\,(\mathrm{w}_1) = \mathsf{bs}\,(\mathrm{w}_2)$ and $\mathsf{tm}\,(\mathrm{w}_1) \leq_{\mathfrak{l}} \mathsf{tm}\,(\mathrm{w}_2)$ it is performed the inequality,* $\mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) \leq_{\mathfrak{m}} \mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.
2. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k\,(\mathcal{F})$ is **time-positive** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$ (denotation is $\mathfrak{m} \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k\,(\mathfrak{l})$ such that $\mathsf{bs}\,(\mathrm{w}_1) = \mathsf{bs}\,(\mathrm{w}_2)$ and $\mathsf{tm}\,(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}\,(\mathrm{w}_2)$ it is performed the inequality,* $\mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{m}} \mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.
3. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k\,(\mathcal{F})$ is **time-nonpositive** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$ (denotation is $\mathfrak{m} \Downarrow_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k\,(\mathfrak{l})$ such that $\mathsf{bs}\,(\mathrm{w}_1) = \mathsf{bs}\,(\mathrm{w}_2)$ and $\mathsf{tm}\,(\mathrm{w}_1) \leq_{\mathfrak{l}} \mathsf{tm}\,(\mathrm{w}_2)$ it is performed the inequality,* $\mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) \geq_{\mathfrak{m}} \mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.
4. *We say that reference frame $\mathfrak{m} \in \mathcal{L}k\,(\mathcal{F})$ is **time-negative** in $\mathcal{F}$ relatively the reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$ (denotation is $\mathfrak{m} \Downarrow^{-}_{\mathcal{F}} \mathfrak{l}$) if and only if for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k\,(\mathfrak{l})$ such that $\mathsf{bs}\,(\mathrm{w}_1) = \mathsf{bs}\,(\mathrm{w}_2)$ and $\mathsf{tm}\,(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}\,(\mathrm{w}_2)$ it is performed the inequality,* $\mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) >_{\mathfrak{m}} \mathsf{tm}\,([\mathfrak{m} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$.
5. *The universal kinematics $\mathcal{F}$ is named by **weakly time-positive** if and only if there exist at least one reference frame $\mathfrak{l}_0 \in \mathcal{L}k\,(\mathcal{F})$ such that the correlation $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$ holds for every reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{F})$.*

*Remark* 3. Apart from weak time-positivity we can introduce other, more strong, form of time-positivity. We say that universal kinematics $\mathcal{F}$ is **time-positive** if and only if for arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\,(\mathcal{F})$ the correlation $\mathfrak{l} \Uparrow^{+}_{\mathcal{F}} \mathfrak{m}$

holds. It is not hard to prove that every kinematics of kind $\mathcal{F} = \mathfrak{UP}(\mathfrak{H}, \mathcal{B}, c)$ (connected with classical special relativity and introduced in [11] and [18, Section 24]) is time-positive.

**Assertion 6.** *For arbitrary reference frames* $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ *of any universal kinematics* $\mathcal{F}$ *the following statements are performed.*

*1) If* $\mathfrak{m} \upuparrows^{+}_{\mathcal{F}} \mathfrak{l}$, *then* $\mathfrak{m} \upuparrows_{\mathcal{F}} \mathfrak{l}$.

*2) If* $\mathfrak{m} \downdownarrows^{-}_{\mathcal{F}} \mathfrak{l}$, *then* $\mathfrak{m} \downdownarrows_{\mathcal{F}} \mathfrak{l}$.

*Proof.* **1)** Indeed, let $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ and $\mathfrak{m} \upuparrows^{+}_{\mathcal{F}} \mathfrak{l}$. Then for every $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l})$ such, that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) \leq_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$, we deduce the following:

**(a)** In the case $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$, by Definition 9, item 2, we get, $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\, \mathrm{w}_1) <_{\mathfrak{m}} \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\, \mathrm{w}_2)$.

**(b)** In the case $\mathsf{tm}(\mathrm{w}_1) = \mathsf{tm}(\mathrm{w}_2)$, we have $\mathrm{w}_1 = (\mathsf{tm}(\mathrm{w}_1), \mathsf{bs}(\mathrm{w}_1)) = (\mathsf{tm}(\mathrm{w}_2), \mathsf{bs}(\mathrm{w}_2)) = \mathrm{w}_2$, and so $\mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\, \mathrm{w}_1) = \mathsf{tm}([\mathfrak{m} \leftarrow \mathfrak{l}]\, \mathrm{w}_2)$.

**2)** Second item of this Assertion can be proven similarly. □

## 5 Theorem of Non-Returning

**Theorem 1.** *Any weakly time-positive universal kinematics* $\mathcal{F}$ *is time irreversible.*

To prove Theorem 1 we need a few auxiliary assertions.

**Assertion 7.** *Let* $\widehat{\mathbf{A}} \subseteq \mathbb{B}\mathfrak{s}(\mathcal{F})$ *be changeable system of universal kinematics* $\mathcal{F}$ *such, that* $\widehat{\mathbf{A}}_{\{\mathfrak{l}_0\}} \in \mathbb{L}g(\mathfrak{l}_0)$ *for some reference frame* $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$. *Let* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be reference frame, satisfying condition* $\mathfrak{l} \upuparrows_{\mathcal{F}} \mathfrak{l}_0$.

*Then for arbitrary* $\hat{\omega}_1, \hat{\omega}_2 \in \widehat{\mathbf{A}}$ *the inequality* $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)$ *assures the the inequality* $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}\}}\right)$.

*Proof.* Suppose that, under conditions of the assertion, we have $\hat{\omega}_1, \hat{\omega}_2 \in \widehat{\mathbf{A}}$ and $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)$. According to Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)]), we have $\mathsf{tm}(\omega) = \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega)\right)$ $(\forall\, \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}_0))$. So, we get

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right). \tag{7}$$

Since $(\hat{\omega}_1)_{\{\mathfrak{l}_0\}}, (\hat{\omega}_2)_{\{\mathfrak{l}_0\}} \in \widehat{\mathbf{A}}_{\{\mathfrak{l}_0\}}$ (where $\widehat{\mathbf{A}}_{\{\mathfrak{l}_0\}} \in \mathbb{L}g(\mathfrak{l}_0)$) then, by Definition 5 (items (a),(b)), we have

$$\mathsf{bs}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right). \tag{8}$$

Taking into account that $\mathfrak{l} \upuparrows_{\mathcal{F}} \mathfrak{l}_0$ and using Definition 9 (item 1) as well as formulas (7), (8), we get the inequality:

$$\mathsf{tm}\left([\mathfrak{l} \leftarrow \mathfrak{l}_0]\, \mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) \leq_{\mathfrak{l}} \mathsf{tm}\left([\mathfrak{l} \leftarrow \mathfrak{l}_0]\, \mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right).$$

Thence, using [18, formula (3.2)], we obtain

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left(\langle !\, \mathfrak{l} \leftarrow \mathfrak{l}_0 \rangle\, (\hat{\omega}_1)_{\{\mathfrak{l}_0\}}\right)\right) \leq_{\mathfrak{l}} \\ \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left(\langle !\, \mathfrak{l} \leftarrow \mathfrak{l}_0 \rangle\, (\hat{\omega}_2)_{\{\mathfrak{l}_0\}}\right)\right).$$

Applying the last inequality as well as Assertion 4, we deduce the inequality:

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left((\hat{\omega}_1)_{\{\mathfrak{l}\}}\right)\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}\left((\hat{\omega}_2)_{\{\mathfrak{l}\}}\right)\right). \tag{9}$$

According to Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)]), for every $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ we have the equality $\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right) = \mathsf{tm}(\omega)$. That is why from the inequality (9) it follows the desired inequality $\mathsf{tm}\left((\hat{\omega}_1)_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left((\hat{\omega}_2)_{\{\mathfrak{l}\}}\right)$. □

**Assertion 8.** *Let,* $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ $(n \in \mathbb{N})$ *be closed, piecewise geometrically-stationary chain path of universal kinematics* $\mathcal{F}$ *and* $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ *be reference frame such that* $\mathfrak{l} \upuparrows_{\mathcal{F}} \mathfrak{l}_i$ *for every* $i \in \overline{1,n}$. *Then for arbitrary* $\hat{\omega} \in \widehat{\mathbf{A}}$ *the following inequality holds:*

$$\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}(\mathscr{A})_{\{\mathfrak{l}\}}\right). \tag{10}$$

*Proof.* Let $\mathcal{F}$ be universal kinematics and $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ $(n \in \mathbb{N})$ be closed, piecewise geometrically-stationary chain path of $\mathcal{F}$. Let, $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ be reference frame such that $\mathfrak{l} \upuparrows_{\mathcal{F}} \mathfrak{l}_i$ $(\forall\, i \in \overline{1,n})$.

**1)** First we prove that for any $\hat{\omega} \in \widehat{\mathbf{A}}$ it holds the inequality:

$$\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right). \tag{11}$$

By Definition 4 (item (b)), $\widehat{\mathbf{A}} = \bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k$. So, it is sufficient to prove the inequality (11) for the cases $\hat{\omega} \in \widehat{\mathbf{A}}_k$ $(k \in \overline{1,n})$.

**1.a)** First we prove the inequality (11) for $\hat{\omega} \in \widehat{\mathbf{A}}_1$. According to Definition 6 (item 1), for $\hat{\omega} \in \widehat{\mathbf{A}}_1$ we obtain that $\mathsf{po}(\mathscr{A}) \in \widehat{\mathbf{A}}_1$ and

$$\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}_1\}}\right) \leq_{\mathfrak{l}_1} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_1\}}\right). \tag{12}$$

According to the above, we have $\hat{\omega} \in \widehat{\mathbf{A}}_1$ and $\mathsf{po}(\mathscr{A}) \in \widehat{\mathbf{A}}_1$. Moreover, by Definition 5 (item (c)), we get, $\left(\widehat{\mathbf{A}}_1\right)_{\{\mathfrak{l}_1\}} \in \mathbb{L}g(\mathfrak{l}_1)$. By conditions of Assertion, we have, $\mathfrak{l} \upuparrows_{\mathcal{F}} \mathfrak{l}_1$. So, in accordance with Assertion 7, the correlation (12) stipulates the inequality $\mathsf{tm}\left(\mathsf{po}(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right)$. Hence, in the case $\hat{\omega} \in \widehat{\mathbf{A}}_1$, the inequality (11) has been proven. Moreover, the last inequality has been proven for all $\hat{\omega} \in \widehat{\mathbf{A}}$ in the case $n = 1$. So, further we consider, that $n > 1$.

**1.b)** Assume, that inequality (11) is performed for all $\hat{\omega} \in \widehat{\mathbf{A}}_{k-1}$, where $k \in \overline{2,n}$. And, let us prove, that then this inequality is true for each $\hat{\omega} \in \widehat{\mathbf{A}}_k$.

In the case $\hat{\omega} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1}$ the inequality (11) is true in accordance with inductive hypothesis. Hence, it remains to prove the last inequality for every $\hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}$. According to item (c) of Definition 4, we have $\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1} \neq \emptyset$. Hence, at least one element $\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1}$ exists. Since,

$$\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1} \quad \text{and} \quad \hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}, \tag{13}$$

then we get $\hat{\eta}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$, $\hat{\omega}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k-1}\right)_{\{\mathfrak{l}_k\}}$. Therefore, according to item (e) of Definition 4, we deliver

$$\mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}_k\}}\right) \leq_{\mathfrak{l}_k} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_k\}}\right). \tag{14}$$

According to (13), we have $\hat{\eta}, \hat{\omega} \in \widehat{\mathbf{A}}_k$, where, by item (c) of Definition 5, $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g\left(\mathfrak{l}_k\right)$. Since $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_k$, then taking into account inequality (14) and Assertion 7 we deduce

$$\mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right). \tag{15}$$

According to (13), we have $\hat{\eta} \in \widehat{\mathbf{A}}_{k-1}$. So, by inductive hypothesis, we deliver

$$\mathsf{tm}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right). \tag{16}$$

Inequalities (15) and (16) assure inequality (11).

Thus, by Principle of mathematical induction, inequality (11) is true for arbitrary $\hat{\omega} \in \bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k = \widehat{\mathbf{A}}$.

**2)** Now we are aiming to prove, that for any $\hat{\omega} \in \widehat{\mathbf{A}}$ it holds the inequality:

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right). \tag{17}$$

**2.a)** First we prove the inequality (17) for $\omega \in \widehat{\mathbf{A}}_n$. According to Definition 6 (item 2), for $\hat{\omega} \in \widehat{\mathbf{A}}_n$ we obtain that $\mathsf{ki}\left(\mathscr{A}\right) \in \widehat{\mathbf{A}}_n$ and

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_n\}}\right) \leq_{\mathfrak{l}_n} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}_n\}}\right). \tag{18}$$

According to the above, we have $\hat{\omega} \in \widehat{\mathbf{A}}_n$ and $\mathsf{ki}\left(\mathscr{A}\right) \in \widehat{\mathbf{A}}_n$. Moreover, by Definition 5 (item (c)), we get $\left(\widehat{\mathbf{A}}_n\right)_{\{\mathfrak{l}_n\}} \in \mathbb{L}g\left(\mathfrak{l}_n\right)$. By conditions of Assertion, we have $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_n$. So, in accordance with Assertion 7, the correlation (18) stipulates the inequality (17). Hence, in the case $\hat{\omega} \in \widehat{\mathbf{A}}_n$, the inequality (17) is proven. Moreover, the last inequality is proven for all $\hat{\omega} \in \widehat{\mathbf{A}}$ in the case $n = 1$. So, further we consider, that $n > 1$.

**2.b)** Assume, that inequality (17) is performed for all $\hat{\omega} \in \widehat{\mathbf{A}}_{k+1}$, where $k \in \overline{1, n-1}$. And, let us prove, that then this inequality is true for each $\hat{\omega} \in \widehat{\mathbf{A}}_k$.

In the case $\omega \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}$ the inequality (17) is true in accordance with inductive hypothesis. Hence, it remains to prove the last inequality for every $\hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}$. According to item (c) of Definition 4, we have $\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1} \neq \emptyset$. Hence, at least one element $\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}$ exists. Taking into account that

$$\hat{\eta} \in \widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1} \quad \text{and} \quad \hat{\omega} \in \widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}, \tag{19}$$

we get $\hat{\eta}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \cap \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$, $\hat{\omega}_{\{\mathfrak{l}_k\}} \in \left(\widehat{\mathbf{A}}_k \setminus \widehat{\mathbf{A}}_{k+1}\right)_{\{\mathfrak{l}_k\}}$. Therefore, according to item (d) of Definition 4, we deliver

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}_k\}}\right) \leq_{\mathfrak{l}_k} \mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}_k\}}\right). \tag{20}$$

According to (19), we have $\hat{\eta}, \hat{\omega} \in \widehat{\mathbf{A}}_k$, where $\left(\widehat{\mathbf{A}}_k\right)_{\{\mathfrak{l}_k\}} \in \mathbb{L}g\left(\mathfrak{l}_k\right)$ by item (c) of Definition 5. Since $\mathfrak{l} \Uparrow_{\mathcal{F}} \mathfrak{l}_k$ then, taking into account inequality (20) and Assertion 7, we deduce

$$\mathsf{tm}\left(\hat{\omega}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right). \tag{21}$$

According to (19), we have $\hat{\eta} \in \widehat{\mathbf{A}}_{k+1}$. So, by inductive hypothesis, we deliver

$$\mathsf{tm}\left(\hat{\eta}_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right). \tag{22}$$

Inequalities (21) and (22) assure inequality (17). Thus, by Principle of mathematical induction, inequality (17) is true for arbitrary $\hat{\omega} \in \bigcup_{k=1}^{n} \widehat{\mathbf{A}}_k = \widehat{\mathbf{A}}$.

Inequality (10) follows from (11) and (17). □

*Proof of Theorem 1.* Let $\mathcal{F}$ be weakly time-positive universal kinematics. Then, by Definition 9, there exists the reference frame $\mathfrak{l}_0 \in \mathcal{L}k\left(\mathcal{F}\right)$ such that

$$\forall\, \mathfrak{m} \in \mathcal{L}k\left(\mathcal{F}\right) \quad \mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{m}. \tag{23}$$

Let $\mathscr{A} = \left(\widehat{\mathbf{A}}, \left(\widehat{\mathbf{A}}_1, \mathfrak{l}_1\right), \cdots, \left(\widehat{\mathbf{A}}_n, \mathfrak{l}_n\right)\right)$ ($n \in \mathbb{N}$) be piecewise geometrically-stationary chain path in $\mathcal{F}$ and, moreover, $\mathscr{A}$ is geometrically-cyclic relatively some reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathcal{F}\right)$. By Definition 7, $\mathscr{A}$ is closed chain path. According to Assertion 6, correlation (23) leads to the correlation $\mathfrak{l}_0 \Uparrow_{\mathcal{F}} \mathfrak{l}_k$ $(\forall k \in \overline{1, n})$. Hence, applying Assertion 8, we ensure

$$\mathsf{tm}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}_0\}}\right) \leq_{\mathfrak{l}_0} \mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}_0\}}\right). \tag{24}$$

Assume, that $\mathsf{tm}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right) <_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)$. Then, by Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)]), we obtain

$$\mathsf{tm}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right) <_{\mathfrak{l}} \mathsf{tm}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right). \tag{25}$$

Since the path $\mathscr{A}$ is geometrically-cyclic relatively the reference frame $\mathfrak{l}$, then, by Definition 7, we have

$$\mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right) = \mathsf{bs}\left(\mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right). \tag{26}$$

Since (in accordance with (23)) $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$, then, by Definition 9 (item 2), from the correlations (25), and (26), we get the inequality:

$$\mathsf{tm}\left(\left[\mathfrak{l}_0 \leftarrow \mathfrak{l}\right] \mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{ki}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right) <_{\mathfrak{l}_0} \\ <_{\mathfrak{l}_0} \mathsf{tm}\left(\left[\mathfrak{l}_0 \leftarrow \mathfrak{l}\right] \mathbf{Q}^{\langle\mathfrak{l}\rangle}\left(\mathsf{po}\left(\mathscr{A}\right)_{\{\mathfrak{l}\}}\right)\right).$$

Thence, using [18, formula (3.2)] , we deduce the inequality:

$$\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0\rangle}\left(\langle !\ \mathfrak{l}_0 \leftarrow \mathfrak{l}\rangle\, \mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right) <_{\mathfrak{l}_0} \\ <_{\mathfrak{l}_0} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0\rangle}\left(\langle !\ \mathfrak{l}_0 \leftarrow \mathfrak{l}\rangle\, \mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)\right).$$

Taking into account Assertion 4, the last inequality can be reduced to the form, $\mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0\rangle}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right)\right) <_{\mathfrak{l}_0} \mathsf{tm}\left(\mathbf{Q}^{\langle \mathfrak{l}_0\rangle}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right)\right)$, and, by Definition of Minkowski coordinates (see [11, formula (2)] or [18, formula (2.3)])), we assure

$$\mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right) <_{\mathfrak{l}_0} \mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}_0\}}\right).$$

But, the last inequality contradicts to the correlation (24). Therefore, hypothesis affirming, that $\mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right) <_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right)$ is false. Consequently we have

$$\mathsf{tm}\left(\mathsf{po}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right) \leq_{\mathfrak{l}} \mathsf{tm}\left(\mathsf{ki}\,(\mathscr{A})_{\{\mathfrak{l}\}}\right). \tag{27}$$

Thus, for each reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ and for each chain path $\mathscr{A}$, geometrically-cyclic in the frame $\mathfrak{l}$ and piecewise geometrically-stationary in $\mathcal{F}$, it holds the inequality (27). So, by Definition 8, kinematics $\mathcal{F}$ is time irreversible, which must be proved. □

## 6 Certainly time irreversibility. Strengthened version of theorem of non-returning

Recall, that in the papers [17, Definition 6], [18, Definition 3.25.2] the notion of equivalence of universal kinematics relatively coordinate transform had been introduced. According to these papers, we denote equivalent relatively coordinate transform kinematics $\mathcal{F}_1$ and $\mathcal{F}_2$ via $\mathcal{F}_1\,[\equiv]\,\mathcal{F}_2$.

**Definition 10.** *We say that universal kinematics $\mathcal{F}$ is **certainly time irreversible** if and only if arbitrary universal kinematics $\mathcal{F}_1$ such, that $\mathcal{F}\,[\equiv]\,\mathcal{F}_1$ is time irreversible. In the opposite case we will say that universal kinematics $\mathcal{F}$ is **conditionally time reversible**.*

Since, according to [17, Assertion 3] (see also [18, Assertion 3.25.1]), for each universal kinematics $\mathcal{F}$ it is fulfilled the correlation $\mathcal{F}\,[\equiv]\,\mathcal{F}$, then we receive the following Corollary from Definition 10:

**Corollary 3.** *Any certainly time irreversible universal kinematics $\mathcal{F}$ is time irreversible.*

The physical sense of certain time irreversibility notion is that in certainly time irreversible kinematics temporal paradoxes are impossible basically, that is there is not potential possibility to affect the own past by means of "traveling" and "jumping" between reference frames. Whereas, in time irreversible, but conditionally time reversible kinematics such potential possibility exists, but it is not realized in the scenario of evolution, acting in this kinematics.

**Assertion 9.** *Let universal kinematics $\mathcal{F}$ be weakly time-positive. Then every universal kinematics $\mathcal{F}_1$ such that $\mathcal{F}_1\,[\equiv]\,\mathcal{F}$ is weakly time-positive also.*

*Proof.* Let $\mathcal{F}$ be weakly time-positive universal kinematics and $\mathcal{F}_1\,[\equiv]\,\mathcal{F}$. Recall, that in [18, Definition 3.27.3] for every reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ it was introduced the reference frame $\mathfrak{m}\,\downharpoonright_{\mathcal{F}_1}$, related with $\mathfrak{m}$ in the universal kinematics $\mathcal{F}_1$:

$$\mathfrak{m}\,\downharpoonright_{\mathcal{F}_1} := \mathbf{lk}_{\mathtt{ind}(\mathfrak{m})}\,(\mathcal{F}_1)\,. \tag{28}$$

Since kinematics $\mathcal{F}$ is weakly time-positive then, by Definition 9, the reference frame $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$ exists such that for each reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ the correlation $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$ holds. Denote:

$$\mathfrak{l}_0^{(1)} := \mathfrak{l}_0\,\downharpoonright_{\mathcal{F}_1}\,.$$

Let us consider any reference frame $\mathfrak{l}^{(1)} \in \mathcal{L}k(\mathcal{F}_1)$. Denote: $\mathfrak{l} := \mathfrak{l}^{(1)}\,\downharpoonright_{\mathcal{F}} \in \mathcal{L}k(\mathcal{F})$. Then, according to [18, Properties 3.27.1] and formula (28), we have

$$\mathfrak{l}^{(1)} = \mathfrak{l}\,\downharpoonright_{\mathcal{F}_1} = \mathbf{lk}_{\mathtt{ind}(\mathfrak{l})}\,(\mathcal{F}_1)\,.$$

Hence, taking into account [18, Definition 3.25.2 (item 2)], formula (28) and [18, Property 3.25.1(1)], we get

$$\begin{aligned} \mathbb{M}k\left(\mathfrak{l}_0^{(1)};\mathcal{F}_1\right) &= \mathbb{M}k\left(\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}\,(\mathcal{F}_1)\,;\mathcal{F}_1\right) = \\ &= \mathbb{M}k\left(\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}\,(\mathcal{F})\,;\mathcal{F}\right) = \mathbb{M}k\,(\mathfrak{l}_0;\mathcal{F})\,; \\ \mathbb{M}k\left(\mathfrak{l}^{(1)};\mathcal{F}_1\right) &= \mathbb{M}k\,(\mathfrak{l};\mathcal{F})\,. \end{aligned} \tag{29}$$

Similarly applying [18, Definition 3.25.2 (item 2)] we ensure the equalities:

$$\mathbb{T}\mathbf{m}\left(\mathfrak{l}_0^{(1)}\right) = \mathbb{T}\mathbf{m}\,(\mathfrak{l}_0)\,; \quad \mathbb{T}\mathbf{m}\left(\mathfrak{l}^{(1)}\right) = \mathbb{T}\mathbf{m}\,(\mathfrak{l}) \tag{30}$$

(where (in accordance with [18, Subsection 6.3]) $\mathbb{T}\mathbf{m}(\mathfrak{m}) = (\mathbf{Tm}\,(\mathfrak{m})\,, \leq_{\mathfrak{m}})$ $(\forall\, \mathfrak{m} \in \mathcal{L}k(\mathcal{F}) \cup \mathcal{L}k(\mathcal{F}_1))$). Moreover, according to [18, Property 3.25.1(1) and Definition 3.25.2 (item 3)], we obtain

$$\begin{aligned} [\mathfrak{l}_0 \leftarrow \mathfrak{l},\,\mathcal{F}] &= \left[\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}\,(\mathcal{F}) \leftarrow \mathbf{lk}_{\mathtt{ind}(\mathfrak{l})}\,(\mathcal{F})\,,\,\mathcal{F}\right] = \\ &= \left[\mathbf{lk}_{\mathtt{ind}(\mathfrak{l}_0)}\,(\mathcal{F}_1) \leftarrow \mathbf{lk}_{\mathtt{ind}(\mathfrak{l})}\,(\mathcal{F}_1)\,,\,\mathcal{F}_1\right] = \\ &= \left[\mathfrak{l}_0\,\downharpoonright_{\mathcal{F}_1} \leftarrow \mathfrak{l}\,\downharpoonright_{\mathcal{F}_1},\,\mathcal{F}_1\right] = \left[\mathfrak{l}_0^{(1)} \leftarrow \mathfrak{l}^{(1)},\,\mathcal{F}_1\right]. \end{aligned} \tag{31}$$

Taking into account (29), let us consider any elements $w_1, w_2 \in \mathbb{M}k\left(\mathfrak{l}^{(1)};\mathcal{F}_1\right) = \mathbb{M}k\,(\mathfrak{l};\mathcal{F})$ such that $\mathsf{bs}\,(w_1) = \mathsf{bs}\,(w_2)$ and $\mathsf{tm}\,(w_1) <_{\mathfrak{l}^{(1)}} \mathsf{tm}\,(w_2)$. Then, in accordance with (30), we obtain the inequality $\mathsf{tm}\,(w_1) <_{\mathfrak{l}} \mathsf{tm}\,(w_2)$. Since (as it was mentioned before) $\mathfrak{l}_0 \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$, then, by Definition 9 (item 2), we obtain the inequality $\mathsf{tm}\,([\mathfrak{l}_0 \leftarrow \mathfrak{l},\,\mathcal{F}]\,w_1) <_{\mathfrak{l}_0} \mathsf{tm}\,([\mathfrak{l}_0 \leftarrow \mathfrak{l},\,\mathcal{F}]\,w_2)$. Thence, using (31) and (30), we ensure the inequality, $\mathsf{tm}\left(\left[\mathfrak{l}_0^{(1)} \leftarrow \mathfrak{l}^{(1)},\,\mathcal{F}_1\right] w_1\right) <_{\mathfrak{l}_0^{(1)}} \mathsf{tm}\left(\left[\mathfrak{l}_0^{(1)} \leftarrow \mathfrak{l}^{(1)},\,\mathcal{F}_1\right] w_2\right)$. By Definition 9 (item 2), taking into account the arbitrariness of choice elements $w_1, w_2 \in \mathbb{M}k\left(\mathfrak{l}^{(1)}; \mathcal{F}_1\right)$ such, that $\mathsf{bs}(w_1) = \mathsf{bs}(w_2)$ and $\mathsf{tm}(w_1) <_{\mathfrak{l}^{(1)}} \mathsf{tm}(w_2)$, we obtain the correlation $\mathfrak{l}_0^{(1)} \Uparrow_{\mathcal{F}_1}^{+} \mathfrak{l}^{(1)}$ (for every reference frame $\mathfrak{l}^{(1)} \in \mathcal{L}k(\mathcal{F}_1)$). Hence, by Definition 9, kinematics $\mathcal{F}_1$ is weakly time-positive. □

Applying Assertion 9 as well as Theorem 1, we obtain the following (strengthened) variant of Theorem of Non-Returning:

**Theorem 2.** *Any weakly time-positive universal kinematics $\mathcal{F}$ is* ***certainly*** *time irreversible.*

## 7 Example of certainly time irreversible tachyon kinematics

In this section we build the certainly time-irreversible universal kinematics, which allows for reference frames moving with any speed other than the speed of light, using the generalized Lorentz-Poincare transformations in terms of E. Recami, V. Olkhovsky and R. Goldoni.

Let $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ be a Hilbert space over the real field such, that $\dim(\mathfrak{H}) \geq 1$, where $\dim(\mathfrak{H})$ is dimension of the space $\mathfrak{H}$. Emphasize, that the condition $\dim(\mathfrak{H}) \geq 1$ should be interpreted in a way that the space $\mathfrak{H}$ may be infinite-dimensional. Let $\mathcal{L}(\mathfrak{H})$ be the space of (homogeneous) linear continuous operators over the space $\mathfrak{H}$. Denote by $\mathcal{L}^{\times}(\mathfrak{H})$ the space of all operators of affine transformations over the space $\mathfrak{H}$, that is $\mathcal{L}^{\times}(\mathfrak{H}) = \{\mathbf{A}_{[\mathbf{a}]} \mid \mathbf{A} \in \mathcal{L}(\mathfrak{H}),\, \mathbf{a} \in \mathfrak{H}\}$, where $\mathbf{A}_{[\mathbf{a}]}x = \mathbf{A}x + \mathbf{a}$, $x \in \mathfrak{H}$. The ***Minkowski space*** over the Hilbert space $\mathfrak{H}$ is defined as the Hilbert space $\mathcal{M}(\mathfrak{H}) = \mathbb{R} \times \mathfrak{H} = \{(t, x) \mid t \in \mathbb{R},\ x \in \mathfrak{H}\}$, equipped by the inner product and norm: $\langle w_1, w_2\rangle = \langle w_1, w_2\rangle_{\mathcal{M}(\mathfrak{H})} = t_1 t_2 + \langle x_1, x_2\rangle$, $\|w_1\| = \|w_1\|_{\mathcal{M}(\mathfrak{H})} = \left(t_1^2 + \|x_1\|^2\right)^{1/2}$ (where $w_i = (t_i, x_i) \in \mathcal{M}(\mathfrak{H})$, $i \in \{1, 2\}$) ( [10, 18]). In the space $\mathcal{M}(\mathfrak{H})$ we select the next subspaces: $\mathfrak{H}_0 := \{(t, \mathbf{0}) \mid t \in \mathbb{R}\}$, $\mathfrak{H}_1 := \{(0, x) \mid x \in \mathfrak{H}\}$ with $\mathbf{0}$ being zero vector. Then, $\mathcal{M}(\mathfrak{H}) = \mathfrak{H}_0 \oplus \mathfrak{H}_1$, where $\oplus$ means the orthogonal sum of subspaces. Denote: $\mathbf{e}_0 := (1, \mathbf{0}) \in \mathcal{M}(\mathfrak{H})$. Introduce the orthogonal projectors on the subspaces $\mathfrak{H}_1$ and $\mathfrak{H}_0$:

$$\mathbf{X}w = (0, x) \in \mathfrak{H}_1;\ \widehat{\mathbf{T}}w = (t, \mathbf{0}) = \mathcal{T}(w)\,\mathbf{e}_0 \in \mathfrak{H}_0,$$
$$\text{where } \mathcal{T}(w) = t \quad (w = (t, x) \in \mathcal{M}(\mathfrak{H})).$$

Let $\mathbf{B}_1(\mathfrak{H}_1)$ be the unit sphere in the space $\mathfrak{H}_1$ ($\mathbf{B}_1(\mathfrak{H}_1) = \{x \in \mathfrak{H}_1 \mid \|x\| = 1\}$). Any vector $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ generates the following orthogonal projectors, acting in $\mathcal{M}(\mathfrak{H})$:

$$\mathbf{X}_1[\mathbf{n}]\,w = \langle \mathbf{n}, w\rangle\,\mathbf{n} \quad (w \in \mathcal{M}(\mathfrak{H}));$$
$$\mathbf{X}_1^{\perp}[\mathbf{n}] = \mathbf{X} - \mathbf{X}_1[\mathbf{n}].$$

Recall, that an operator $U \in \mathcal{L}(\mathfrak{H})$ is referred to as ***unitary*** on $\mathfrak{H}$, if and only if $\exists\, U^{-1} \in \mathcal{L}(\mathfrak{H})$ and $\forall\, x \in \mathfrak{H}\ \|Ux\| = \|x\|$. Let $\mathfrak{U}(\mathfrak{H}_1)$ be the set of all ***unitary*** operators over the space $\mathfrak{H}_1$. Fix some real number $c$ such, that $0 < c < \infty$. Denote:

$$\mathfrak{P}\mathfrak{T}_{\mathsf{fin}}^{\mp}(\mathfrak{H}, c) := \left\{ \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}] \;\middle|\; \begin{array}{l} \lambda \in [0,\infty) \setminus \{c\}, \\ s = \operatorname{sign}(c - \lambda), \\ J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1), \\ \mathbf{a} \in \mathcal{M}(\mathfrak{H}) \end{array} \right\}, \quad (32)$$

where $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}] \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$ ($\lambda \in [0,\infty) \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$) are operators of generalized Lorentz-Poincare Transformations in the sense of E. Recami, V. Olkhovsky and R. Goldoni, introduced in [10, 11, 18]:

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]w = \mathbf{W}_{\lambda,c}[s, \mathbf{n}, J](w + \mathbf{a}), \quad \text{where}$$
$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]w = \frac{\left(s\mathcal{T}(w) - \frac{\lambda}{c^2}\langle \mathbf{n}, w\rangle\right)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}}\mathbf{e}_0 + {}$$
$$+ J\left(\frac{\lambda\mathcal{T}(w) - s\langle \mathbf{n}, w\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}}\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,w\right). \quad (33)$$

According to [18, 20], every operator of kind $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]$ belongs to $\mathbf{Pk}(\mathfrak{H})$, where $\mathbf{Pk}(\mathfrak{H})$ is the set of all operators $\mathbf{S} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$, which have the continuous inverse operator $\mathbf{S}^{-1} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$. Using results of the papers [18, 20], we can calculate the operators, inverse to the operators of kind $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$ and $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J; \mathbf{a}]$.

**Lemma 1.** *For arbitrary $c \in (0,\infty)$, $\lambda \in [0,\infty) \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$ and $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ the following equality holds:*

$$(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J])^{-1} =$$
$$= \mathbf{W}_{\lambda,c}\left[s \operatorname{sign}(c - \lambda), \operatorname{sign}(c - \lambda)J\mathbf{n}, J^{-1}\right]. \quad (34)$$

*Proof.* Consider arbitrary $0 < c < \infty$, $\lambda \in [0,\infty) \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$ and $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$. According to [10, page 143] or [18, formula (2.86)], operator $\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]$ may be represented in the form:

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J] = \mathbf{U}_{\theta,c}[s, \mathbf{n}, J], \quad (35)$$

where

$$\theta = \frac{1 - \frac{\lambda}{c}}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} \quad \left(\lambda = c\frac{1 - \theta|\theta|}{1 + \theta|\theta|}\right),\ -1 \leq \theta \leq 1.$$

Hence, according to [20, Corollary 5.1] or [18, Corollary 2.18.3], we obtain, that $(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J])^{-1} \in \mathcal{L}(\mathcal{M}(\mathfrak{H}))$, and moreover:

$$(\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J])^{-1} = (\mathbf{U}_{\theta,c}[s, \mathbf{n}, J])^{-1} =$$
$$= \mathbf{U}_{\theta^s,c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right], \quad (36)$$

$$\text{where } s_\theta = \mathfrak{S}(s, \theta) = \begin{cases} 1, & s, \theta > 0 \\ -1, & s < 0 \text{ or } \theta < 0. \end{cases}$$

In the case $s = 1$ we have, $s_\theta = \operatorname{sign}\theta = \operatorname{sign}\left(\frac{1-\frac{\lambda}{c}}{\sqrt{\left|1-\frac{\lambda^2}{c^2}\right|}}\right) = \operatorname{sign}(c-\lambda)$. Hence, in this case, using (36) and (35), we obtain

$$\begin{aligned}(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J])^{-1} &= \mathbf{U}_{\theta,c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{W}_{\lambda,c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{W}_{\lambda,c}\left[\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right] \quad (s=1). \end{aligned} \tag{37}$$

Now we consider the case $s = -1$ $(\theta^s = \theta^{-1})$. Applying (36) and [18, formula (2.90)], in this case we deduce

$$\begin{aligned}(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J])^{-1} &= \mathbf{U}_{\theta^{-1},c}\left[s_\theta, s_\theta J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{U}_{\theta,c}\left[s_\theta \operatorname{sign}\theta, -s_\theta(\operatorname{sign}\theta)J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{U}_{\theta,c}\left[-\operatorname{sign}\theta, (\operatorname{sign}\theta)J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{U}_{\theta,c}\left[-\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right] = \\ &= \mathbf{W}_{\lambda,c}\left[-\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right] \\ &\qquad (s=-1). \end{aligned} \tag{38}$$

Taking into account (37) and (38) in the both cases we obtain (34). □

Using Lemma 1, we obtain the following corollary.

**Corollary 4.** *For arbitrary $c \in (0,\infty)$, $\lambda \in [0,\infty)\setminus\{c\}$, $s \in \{-1,1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ the following equality is fulfilled:*

$$\begin{aligned}(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}])^{-1}\,\mathrm{w} &= \\ = \mathbf{W}_{\lambda,c}&\left[s\operatorname{sign}(c-\lambda), \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w} - \mathbf{a} \\ &\qquad (\mathrm{w} \in \mathcal{M}(\mathfrak{H})).\end{aligned}$$

Let $\mathcal{B}$ be any base changeable set such, that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ and $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R}, \leq)$, where $\leq$ is the standard order in the field of real numbers $\mathbb{R}$. Denote:

$$\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c) := \mathfrak{Ku}\left(\mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c), \mathcal{B};\, \mathfrak{H}\right), \tag{39}$$

where the denotation $\mathfrak{Ku}(\cdot,\cdot;\,\cdot)$ is introduced in [11], [18, page 166]. From [18, Assertion 2.17.5] it follows, that in the case $\dim(\mathfrak{H}) = 3$ universal kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)$ may be considered as tachyon extension of kinematics of classical special relativity, which allows for reference frames moving with arbitrary speed other than the speed of light.

According to [18, Property 3.23.1(1)], the set $\mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right)$ of all reference frames of universal kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)$, defined by (39), can be represented in the form:

$$\begin{aligned}\mathcal{L}k&\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right) = \\ &= \left\{(\mathbf{U}, \mathbf{U}[\mathcal{B}, \mathbb{T}\mathbf{m}(\mathcal{B})]) \mid \mathbf{U} \in \mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)\right\} = \\ &= \left\{(\mathbf{U}, \mathbf{U}[\mathcal{B}]) \mid \mathbf{U} \in \mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)\right\}. \end{aligned} \tag{40}$$

In accordance with [18, Corollary 2.19.5], subclass of operators

$$\begin{aligned}&\mathfrak{P}_+(\mathfrak{H},c) = \\ &= \left\{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \;\middle|\; \begin{array}{l}\lambda \in [0,c),\ s=1, \\ J \in \mathfrak{U}(\mathfrak{H}_1), \\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H})\end{array}\right\} \subseteq \\ &\qquad\qquad \subseteq \mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)\end{aligned}$$

is group of operators over the space $\mathcal{M}(\mathfrak{H})$. So, the identity operator $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}\mathrm{w} = \mathrm{w}$ $(\forall \mathrm{w} \in \mathcal{M}(\mathfrak{H}))$ belongs to the class $\mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},c)$. Hence, in accordance with (40), we may define the following reference frame:

$$\begin{aligned}\mathfrak{l}_{0,\mathcal{B}} :&= (\mathbb{I}_{\mathcal{M}(\mathfrak{H})}, \mathbb{I}_{\mathcal{M}(\mathfrak{H})}[\mathcal{B}]) = \\ &= (\mathbb{I}_{\mathcal{M}(\mathfrak{H})}, \mathcal{B}) \in \mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right)\end{aligned} \tag{41}$$

(recall, that, according to [18, Remark 1.11.3], $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}[\mathcal{B}] = \mathcal{B}$).

**Lemma 2.** *For each reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)\right)$ the following correlation holds:*

$$\mathfrak{l}_{0,\mathcal{B}} \Uparrow^{+}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}.$$

*Proof.* Consider any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{UPT}^{\mp}(\mathfrak{H},\mathcal{B},c))$. According to (40) and (32), frame $\mathfrak{l}$ can be represented in the form:

$$\mathfrak{l} = (\mathbf{U}, \mathbf{U}[\mathcal{B}]), \quad \text{where} \tag{42}$$

$$\begin{aligned}\mathbf{U} = \mathbf{W}_{\lambda,c}&[\operatorname{sign}(c-\lambda), \mathbf{n}, J; \mathbf{a}], \\ &0 \leq \lambda < +\infty,\ \lambda \neq c, \\ \mathbf{n} \in \mathbf{B}_1&(\mathfrak{H}_1),\ J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H}).\end{aligned} \tag{43}$$

Applying [18, Properties 3.23.1(3,4,7)] as well (42), (43), (41) and Corollary 4 we obtain

$$\mathbb{T}\mathbf{m}(\mathfrak{l}) = \mathbb{T}\mathbf{m}(\mathfrak{l}_{0,\mathcal{B}}) = \mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R}, \leq); \tag{44}$$

$$\begin{aligned}\mathbb{M}k(\mathfrak{l}) &= \mathbb{M}k(\mathfrak{l}_{0,\mathcal{B}}) = \mathbf{Tm}(\mathcal{B}) \times \mathfrak{H} = \\ &= \mathbb{R} \times \mathfrak{H} = \mathcal{M}(\mathfrak{H}); \\ [\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w} &= \mathbb{I}_{\mathcal{M}(\mathfrak{H})}\mathbf{U}^{-1}\mathrm{w} = \\ &= (\mathbf{W}_{\lambda,c}[\operatorname{sign}(c-\lambda), \mathbf{n}, J; \mathbf{a}])^{-1}\,\mathrm{w} = \\ &= \mathbf{W}_{\lambda,c}\left[(\operatorname{sign}(c-\lambda))^2, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w} - \mathbf{a} = \\ &= \mathbf{W}_{\lambda,c}\left[1, \operatorname{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w} - \mathbf{a} \\ &\qquad (\mathrm{w} \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})).\end{aligned} \tag{45}$$

Now we consider any $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})$ such that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$. According to (44), inequality $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ is equivalent to the inequality $\mathsf{tm}(\mathrm{w}_1) < \mathsf{tm}(\mathrm{w}_2)$. From the equality $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ it follows that

$$\mathbf{X}(\mathrm{w}_2 - \mathrm{w}_1) = \\ = \mathbf{X}(\mathsf{tm}(\mathrm{w}_2) - \mathsf{tm}(\mathrm{w}_1), \mathsf{bs}(\mathrm{w}_2) - \mathsf{bs}(\mathrm{w}_1)) = \\ = (0, \mathsf{bs}(\mathrm{w}_2) - \mathsf{bs}(\mathrm{w}_1)) = \mathbf{0}.$$

Thence, using (45) and (33) we deduce

$$\begin{aligned}
&\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2) - \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) = \\
&= \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2 - [\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) = \\
&= \mathsf{tm}\Big(\mathbf{W}_{\lambda,c}\left[1, \mathrm{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w}_2 - \\
&\quad - \mathbf{W}_{\lambda,c}\left[1, \mathrm{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right]\mathrm{w}_1\Big) = \\
&= \mathsf{tm}\Big(\mathbf{W}_{\lambda,c}\left[1, \mathrm{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right](\mathrm{w}_2 - \mathrm{w}_1)\Big) = \\
&= \mathcal{T}\Big(\mathbf{W}_{\lambda,c}\left[1, \mathrm{sign}(c-\lambda)J\mathbf{n}, J^{-1}\right](\mathrm{w}_2 - \mathrm{w}_1)\Big) = \\
&= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1) - \frac{\lambda}{c^2}\langle \mathrm{sign}(c-\lambda)J\mathbf{n}, \mathrm{w}_2 - \mathrm{w}_1\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \\
&= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1) - \frac{\lambda}{c^2}\langle \mathrm{sign}(c-\lambda)\mathbf{X}J\mathbf{n}, \mathrm{w}_2 - \mathrm{w}_1\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \\
&= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1) - \frac{\lambda}{c^2}\langle \mathrm{sign}(c-\lambda)J\mathbf{n}, \mathbf{X}(\mathrm{w}_2 - \mathrm{w}_1)\rangle}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \\
&= \frac{\mathcal{T}(\mathrm{w}_2 - \mathrm{w}_1)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} = \frac{\mathcal{T}(\mathrm{w}_2) - \mathcal{T}(\mathrm{w}_1)}{\sqrt{\left|1 - \frac{\lambda^2}{c^2}\right|}} > 0
\end{aligned}$$

Therefore, $\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) < \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$, ie, according to (44), we have, $\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{l}_{0,\mathcal{B}}} \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$. Thus, for arbitrary $\mathrm{w}_1, \mathrm{w}_2 \in \mathbb{M}k(\mathfrak{l}) = \mathcal{M}(\mathfrak{H})$ such, that $\mathsf{bs}(\mathrm{w}_1) = \mathsf{bs}(\mathrm{w}_2)$ and $\mathsf{tm}(\mathrm{w}_1) <_{\mathfrak{l}} \mathsf{tm}(\mathrm{w}_2)$ it is true the inequality $\mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_1) <_{\mathfrak{l}_{0,\mathcal{B}}} \mathsf{tm}([\mathfrak{l}_{0,\mathcal{B}} \leftarrow \mathfrak{l}]\,\mathrm{w}_2)$. And, taking into account Definition 9 (item 2), we have seen, that $\mathfrak{l}_{0,\mathcal{B}} \Uparrow^{+}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}$. □

**Corollary 5.** *Every universal kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ $(0 < c < \infty)$ is certainly time irreversible.*

*Proof.* According to Lemma 2 and Definition 9 (item 5), kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ $(0 < c < \infty)$ is weakly time-positive. Hence, by Theorem 2, kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ is certainly time irreversible. □

*Remark* 4. Kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ $(0 < c < \infty)$ is weakly time-positive, but it is not time-positive. Similarly to Lemma 2 it can be proved, that for any (superluminal) reference frame of kind:

$$\mathfrak{l} = (\mathbf{U}, \mathbf{U}[\mathcal{B}]) \in \mathcal{L}k\left(\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)\right), \quad \text{where}$$
$$\mathbf{U} = \mathbf{W}_{\lambda,c}[\mathrm{sign}(c-\lambda), \mathbf{n}, J; \mathbf{a}] = \mathbf{W}_{\lambda,c}[-1, \mathbf{n}, J; \mathbf{a}],$$
$$c < \lambda < +\infty,\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1),\ J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H})$$

the correlation $\mathfrak{l} \Downarrow^{-}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}_{0,\mathcal{B}}$ is true despite the fact that $\mathfrak{l}_{0,\mathcal{B}} \Uparrow^{+}_{\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H},\mathcal{B},c)} \mathfrak{l}$ (according to Lemma 2).

*Remark* 5. It is easy to see that the binary relation $\Uparrow^{+}_{\mathcal{F}}$ is reflexive on the set $\mathcal{L}k(\mathcal{F})$ of all reference frames of arbitrary universal kinematics $\mathcal{F}$. From Remark 4 it follows that in the general case this relation is not symmetric. Using the results of [10, Section 7, paragraph 4] it can be proven that this relation is not transitive in the general case.

## 8 On the physical interpretation of main result

The aim of this section is to explain main Theorem 2 in the physical language. We can imagine, that any universal kinematics $\mathcal{F}$ is some abstract "world", which not necessarily coincides with the our. In every such "world" $\mathcal{F}$ there exists the fixed for this "world" set of reference frames $\mathcal{L}k(\mathcal{F})$. We reach the agreement that for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ the arrows of the clock, fixed in the frame $\mathfrak{l}$ are rotating clockwise relatively the frame $\mathfrak{l}$. We say, that the reference frame $\mathfrak{m} \in \mathcal{L}k(\mathcal{F})$ is time-positive relatively the reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$ (ie $\mathfrak{m} \Uparrow^{+}_{\mathcal{F}} \mathfrak{l}$) if and only if the observer in the reference frame $\mathfrak{m}$ (fixed relatively $\mathfrak{m}$) observes that the arrows of the clock, fixed in the frame $\mathfrak{l}$ are rotating clockwise in the frame $\mathfrak{m}$ as well (cf. Definition 9, item 2). We abandon the physical question, how can the observer in $\mathfrak{m}$ "see" the clock, fixed in the other frame $\mathfrak{l}$. From the mathematical point of view, the possibility of observation the clock, attached to another reference frame, is guaranteed by existence of universal coordinate transform between every two reference frames (see definition of universal kinematics in [11, 18]). According to Remark 5, the binary relation $\Uparrow^{+}_{\mathcal{F}}$ always is reflexive, but, in the general case, it is not symmetric and is not transitive on the set $\mathcal{L}k(\mathcal{F})$ of all reference frames of the "world" $\mathcal{F}$.

We also suppose, that in the "world" $\mathcal{F}$ the interframe voyagers can exist. Such voyagers may move from one reference frame to the another frame, passing near them (similarly as, standing near the tram track, we can jump into the tram, passing near us).

From the physical point of view Theorem 2 asserts, that *if in the "world" $\mathcal{F}$ there exists at least one reference frame $\mathfrak{l}_0 \in \mathcal{L}k(\mathcal{F})$, which is time-positive relatively the every frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{F})$, then in this "world" the temporal paradoxes, connected with the possibility of the returning to the own past are impossible*. This means, that any interframe voyager, starting in some reference frame $\mathfrak{l}$ in some fixed point x can not finish its travel in the frame $\mathfrak{l}$ and in the point x at the past time.

## 9 Conclusions

1. According to Corollary 5, kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ (in the case $\dim(\mathfrak{H}) = 3$) gives the example of certainly time-irreversible tachyon extension of kinematics of classical special relativity, which

allows for reference frames moving with arbitrary velocity other than the velocity of light. Thus, the main conclusion of Theorem 2 is the following:

*In the general case the hypothesis of existence of material objects and inertial reference frames, moving with the velocity, greater than the velocity of light, does not lead to temporal paradoxes, connected with existence of formal possibility of returning to the own past.*

2. In [9] authors have deduced two variants of generalized superluminal Lorentz transforms for the case, when two inertial frames are moving along the common $x$-axis:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad x' = \frac{x - vt}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad \begin{array}{l} y' = y, \\ z' = z, \end{array} \tag{46}$$

where $v \in \mathbb{R}$, $|v| > c$ (see [9, formula (3.16)]) and:

$$t' = \frac{-t + \frac{vx}{c^2}}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad x' = \frac{-x + vt}{\sqrt{\left(\frac{v}{c}\right)^2 - 1}}, \quad \begin{array}{l} y' = y, \\ z' = z \end{array} \tag{47}$$

(see [9, formula (3.18)]). Transforms (46) are particular cases of the transforms of kind (33) for the case, where $\dim(\mathfrak{H}) = 3$, $\lambda > c$ and $s = 1$, whereas transforms (47) belong to the transforms of kind (33) for the case, where $\dim(\mathfrak{H}) = 3$, $\lambda > c$ and $s = -1$. If we chose in (33) the value $s = 1$ for subluminal as well as superluminal diapason, we obtain the class of operators $\mathfrak{PT}_+(\mathfrak{H}, c)$, defined in [13, 18] and based on this class of operators universal kinematics of kind $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$. According to results, announced in [19] and published in [12], this kinematics is conditionally time reversible [1]. But, if we chose in (33) the value $s = 1$ for subluminal diapason and value $s = -1$ for superluminal diapason, we reach the class of operators $\mathfrak{PT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, c)$, defined in (32) and based on this class of operators universal kinematics of kind $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$. According to Corollary 5, kinematics $\mathfrak{UPT}^{\mp}_{\mathsf{fin}}(\mathfrak{H}, \mathcal{B}, c)$ is certainly time irreversible. Thus we can formulate the following conclusion, concerning two variants of superluminal Lorentz transforms, deduced in [9]:

*From the standpoint of time-irreversibility, transforms (47) or [9, formula (3.18)] are more suitable for representation of the tachyon continuation of Einstein's special theory of relativity than (46) or [9, formula (3.16)].*

Main results of this paper had been announced in [19].

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[1] In fact, class of operators $\mathfrak{PT}_+(\mathfrak{H}, c)$ contains apart from operators of kind (33) (with $s = 1$) also operators, corresponding tachyon inertial reference frames with infinite velocities. However, using results of the paper [12], it is not hard to deduce that the "subkinematics" of kinematics $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$, which includes only all reference frames from $\mathfrak{UPT}(\mathfrak{H}, \mathcal{B}, c)$ with finite velocities, also is conditionally time reversible.

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