The Origin of Inertial Mass in the Spacetime Continuum

Pierre A. Millette
E-mail: PierreAMillette@alumni.uottawa.ca, Ottawa, Canada

In this paper, we revisit the nature of inertial mass as provided by the Elastodynamics of the Spacetime Continuum (*STCED*). We note that, in addition to providing a physical explanation for inertial mass and for wave-particle duality, it answers unresolved questions pertaining to mass: It provides a direct physical definition of mass independent of the operational definition of mass currently used. It shows that, in general, a singular "point" particle is not physically valid and that particles need to be given a finite volume to avoid invalid results and give physically realistic ones. It confirms theoretically the equivalence of inertial and gravitational mass. It demonstrates that Mach's principle (or conjecture) is incorrect in that inertia originates from the massive dilatation associated with a spacetime deformation, not from interaction with the average mass of the universe. It shows that the electromagnetic field is transverse and massless, and that it contributes to the particle's total energy, but not to its inertial mass.

It must also be said that the origin of inertia is and remains ***the*** *most obscure subject in the theory of particles and fields. A. Pais, 1982 [1, p. 288]*

... the notion of mass, although fundamental to physics, is still shrouded in mystery. M. Jammer, 2000 [2, p. ix]

1 Introduction

In this paper, we revisit the nature of inertial mass as provided by the Elastodynamics of the Spacetime Continuum (*STCED*) [3, 4]. *STCED* is a natural extension of Einstein's General Theory of Relativity which blends continuum mechanical and general relativistic descriptions of the spacetime continuum. The introduction of strains in the spacetime continuum as a result of the energy-momentum stress tensor allows us to use, by analogy, results from continuum mechanics, in particular the stress-strain relation, to provide a better understanding of the general relativistic spacetime.

2 Elastodynamics of the Spacetime Continuum

The stress-strain relation for an isotropic and homogeneous spacetime continuum is given by [3, 4]

$$2\bar{\mu}_0\, \varepsilon^{\mu\nu} + \bar{\lambda}_0\, g^{\mu\nu}\varepsilon = T^{\mu\nu} \tag{1}$$

where $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the spacetime continuum: $\bar{\mu}_0$ is the shear modulus (the resistance of the spacetime continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus (the resistance of the spacetime continuum to *dilatations*):

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \bar{\mu}_0/2 \tag{2}$$

in a four-dimensional continuum. $T^{\mu\nu}$ is the general relativistic energy-momentum stress tensor, $\varepsilon^{\mu\nu}$ the spacetime continuum strain tensor resulting from the stresses, and

$$\varepsilon = \varepsilon^{\alpha}{}_{\alpha}\,, \tag{3}$$

the trace of the strain tensor obtained by contraction, is the volume dilatation ε defined as the change in volume per original volume [9, see pp. 149–152] and is an invariant of the strain tensor. It should be noted that the structure of (1) is similar to that of the field equations of general relativity,

$$R^{\mu\nu} - \tfrac{1}{2}\, g^{\mu\nu}R = -\varkappa\, T^{\mu\nu} \tag{4}$$

where $R^{\mu\nu}$ is the Ricci curvature tensor, R is its trace, $\varkappa = 8\pi G/c^4$ and G is the gravitational constant (see [3, Ch. 2] for more details).

3 Inertial mass in *STCED*

In *STCED*, as shown in [3, 4], energy propagates in the spacetime continuum (*STC*) as wave-like deformations which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape (shearing) of the spacetime continuum without a change in volume and are thus massless.

Thus deformations propagate in the spacetime continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements. This provides a natural explanation for wave-particle duality, with the massless transverse mode corresponding to the wave aspects of the deformations and the massive longitudinal mode corresponding to the particle aspects of the deformations.

The rest-mass energy density of the longitudinal mode is given by [4, see Eq. (32)]

$$\rho c^2 = 4\bar{\kappa}_0\varepsilon \tag{5}$$

where ρ is the rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC*, and ε is the volume dilatation given by (3). Integrating over the 3-D space volume,

$$\int_{V_3} \rho c^2\, \mathrm{d}V_3 = 4\bar{\kappa}_0 \int_{V_3} \varepsilon\, \mathrm{d}V_3\,, \tag{6}$$

and using

$$m = \int_{V_3} \rho \, dV_3 \tag{7}$$

in (6), where m is the rest mass (often denoted as m_0) of the deformation, we obtain

$$mc^2 = 4\bar{\kappa}_0 \, V_{\varepsilon s} \tag{8}$$

where

$$V_{\varepsilon s} = \int_{V_3} \varepsilon \, dV_3 \tag{9}$$

is the space volume dilatation corresponding to rest-mass m, and spacetime continuum volume dilatation ε is the solution of the 4-D dilatational (longitudinal) wave equation [3, see Eq. (3.35)]

$$(2\bar{\mu}_0 + \bar{\lambda}_0) \, \nabla^2 \varepsilon = -\partial_\nu X^\nu \tag{10}$$

where ∇ and ∂ are the 4-D operators and X^ν is the spacetime continuum volume force.

This demonstrates that mass is not independent of the spacetime continuum, but rather mass is part of the spacetime continuum fabric itself. Hence mass results from the dilatation of the spacetime continuum in the longitudinal propagation of energy-momentum in the spacetime continuum. Matter does not warp spacetime, but rather, matter *is* warped spacetime (*i.e.* dilated spacetime). The universe consists of the spacetime continuum and energy-momentum that propagates in it by deformation of its structure.

It is interesting to note that Pais, in his scientific biography of Einstein *'Subtle is the Lord...'*, mentions [1, p. 253]

> The trace of the energy momentum tensor does vanish for electromagnetic fields but not for matter.

which is correct, as shown in [5, 6], where the zero trace of the electromagnetic field energy-momentum stress tensor is reflected in the zero mass of the photon. The missing link in general relativity is the understanding that the trace of the energy-momentum stress tensor is related to the trace of the spacetime continuum strain tensor and is proportional to the mass of matter as given by (5) and (8).

There are basic questions of physics that can be resolved given this understanding of the origin of inertial mass. The following sections deal with many of these unresolved questions.

3.1 Definition of mass

An important consequence of relations (5) and (8) is that they provide a definition of mass. The definition of mass is still one of the open questions in physics, with most authors adopting an indirect definition of mass based on the ratio of force to acceleration [15, see Ch. 8]. However, mass is one of the fundamental dimensions of modern systems of units, and as such, should be defined directly, not indirectly. This is a reflection of the current incomplete understanding of the nature of mass in physics. *STCED* provides a direct physical definition of mass: *mass is the invariant change in volume of spacetime in the longitudinal propagation of energy-momentum in the spacetime continuum.*

Note that the operational definition of mass ($m = F/a$) is still needed to measure the mass of objects and compare them. Jammer covers the various operational and philosophical definitions of mass that have been proposed [2, Ch. 1].

3.2 Point particles

The fact that the mass of a particle corresponds to a finite spacetime volume dilatation $V_{\varepsilon s}$ shows that a singular "point" particle is not physically valid. All particles occupy a finite volume, even if that volume can be very small. Problems arising from point particles are thus seen to result from the abstraction of representing some particles as point objects. Instead, particles need to be given a finite volume to give physically realistic results and avoid invalid results.

3.3 Equivalence of inertial and gravitational mass

Einstein's general relativistic principle of equivalence of inertial and gravitational mass can be given added confirmation in *STCED*. As shown in [5,7], the Ricci tensor can also be decomposed into dilatation and distortion components. The dilatation component can be shown to result in Poisson's equation for a newtonian gravitational potential [3, see Eq. (2.44)] where the gravitational mass density is identical to the rest-mass density identified in *STCED*. This confirms theoretically the equivalence of inertial mass and gravitational mass, as demonstrated experimentally within the accuracy currently achievable [10].

3.4 Mach's principle

Mach's principle, a terminology first used by Einstein [1, p. 287], was not explicitly stated by Mach, and hence various takes on its statement exist. One of the better formulation holds that one can determine rotation and hence define inertial frames with respect to the fixed stars [11, see pp. 86–88]. By extension, inertia would then be due to an interaction with the average mass of the universe [11, see p. 17].

This principle played an important role in the initial development of general relativity by Einstein which is well documented by Pais [1, pp. 283–287]. It also had an impact on the initial work performed in cosmology by Einstein who was searching for a cosmological model that would be in accord with Mach's principle. Einstein's evolving perspective on Mach's work is best summarized by Pais [1, p. 287]:

> So strongly did Einstein believe at that time in the relativity of inertia that in 1918 he stated as being on equal footing three principles on which a satisfactory theory of gravitation should rest [Mach's principle was the third] ... In later years, Einstein's enthusiasm for Mach's principle waned and finally vanished.

Modifications of Einstein's Theory of General Relativity have been proposed in an attempt to incorporate Mach's principle into general relativity (see for example [12, 13]).

The book *Gravitation and Inertia* by Ciufolini and Wheeler [14], with its emphasis on geometrodynamics and its well-known sayings "spacetime tells mass how to move and mass tells spacetime how to curve" and "inertia here arises from mass there", explores these ideas in detail. However, it is important to realize that this perspective is an interpretation of Einstein's field equations of general relativity (4). These equations are simply a relation between the geometry of the spacetime continuum and the energy-momentum present in its structure. *STCED* shows that mass is not outside of the spacetime continuum telling it how to curve (so to speak), but rather mass is part of the spacetime continuum fabric itself participating in the curvature of the spacetime continuum. The geometry of the spacetime continuum is generated by the combination of all spacetime continuum deformations which are composed of longitudinal massive dilatations and transverse massless distortions.

As shown in [3, §2.5], the geometry of spacetime used in (4) can thus be considered to be a linear composition (represented by a sum) of *STC* deformations, starting with the total energy-momentum generating the geometry of general relativity, $T^{\mu\nu}_{GR}$, being a composition of the energy-momentum of the individual deformations of *STCED*, $T^{\mu\nu}_{STCED}$:

$$T^{\mu\nu}_{GR} = \sum T^{\mu\nu}_{STCED} \,. \tag{11}$$

Substituting into (11) from (1) and (4), we obtain

$$-\frac{1}{\varkappa}\left[R^{\mu\nu} - \tfrac{1}{2}\, g^{\mu\nu} R\right] = \sum\left[2\bar{\mu}_0\, \varepsilon^{\mu\nu} + \bar{\lambda}_0\, g^{\mu\nu}\varepsilon\right] . \tag{12}$$

Contraction of (12) yields the relation

$$\frac{1}{\varkappa}\, R = \sum 2(\bar{\mu}_0 + 2\bar{\lambda}_0)\, \varepsilon \tag{13}$$

which, using (2) and (5), simplifies to

$$\frac{1}{\varkappa}\, R = \sum 4\, \bar{\kappa}_0\, \varepsilon = \sum \rho\, c^2 \tag{14}$$

i.e. the curvature of the spacetime continuum arises from the composition of the effect of individual deformations and is proportional to the rest-mass energy density present in the spacetime continuum. Substituting for $R/\varkappa$ from (14) into (12), and rearranging terms, we obtain

$$\frac{1}{\varkappa}\, R^{\mu\nu} = \sum\left[(\bar{\lambda}_0 + \bar{\mu}_0) g^{\mu\nu}\varepsilon - 2\bar{\mu}_0\, \varepsilon^{\mu\nu}\right] . \tag{15}$$

Eqs. (14) and (15) give the relation between the microscopic description of the strains (*i.e.* deformations of the spacetime continuum) and the macroscopic description of the gravitational field in terms of the curvature of the spacetime continuum resulting from the combination of the many microscopic displacements of the spacetime continuum from equilibrium. The source of the inertia is thus in the massive dilatation associated with each deformation, and Mach's principle (or conjecture as it is also known) is seen to be incorrect.

3.5 Electromagnetic mass

The advent of Maxwell's theory of electromagnetism in the second half of the nineteenth century led to the possibility of inertia resulting from electromagnetism, first proposed in 1881 by J. J. Thomson [15, see Chapter 11]. The application of the concept of electromagnetic mass to the electron discovered by J. J. Thomson in 1897, by modelling it as a small charged sphere, led to promising results [16, see Chapter 28]. One can then calculate the energy in the electron's electric field and divide the result by c^2. Alternatively, the electromagnetic momentum of a moving electron can be calculated from Poynting's vector and the electromagnetic mass set equal to the factor multiplying the electron's velocity vector. Different methods give different results.

Using the classical electron radius

$$r_0 = \frac{e^2}{m_e c^2} \tag{16}$$

where e is the electronic charge and m_e the mass of the electron, then the electromagnetic mass of the electron can be written as

$$m_{em} = k_e\, \frac{e^2}{r_0 c^2} \tag{17}$$

where the factor k_e depends on the assumed charge distribution in the sphere and the method of calculation used. For a surface charge distribution, $k_e = 2/3$, while for a uniform volume distribution, $k_e = 4/5$. Numerous modifications were attempted to get $m_{em} = m_e$ [15, 16] with Poincaré introducing non-electrical forces known as "Poincaré stresses" to get the desired result. This is a classical treatment that does not take relativistic or quantum effects into consideration.

It should be noted that the simpler classical treatment of the electromagnetic mass of the electron based purely on the electric charge density of the electron is a calculation of the static mass of the electron. In *STCED*, the charge density ϱ can be calculated from the current density four-vector j^ν (see [3, §4.3])

$$j^\nu = \frac{\varphi_0}{\mu_0}\, \frac{2\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0}\, \varepsilon^{;\nu} \tag{18}$$

where φ_0 is the *STC electromagnetic shearing potential constant*, which has units of $[\mathrm{V \cdot s \cdot m^{-2}}]$ or equivalently [T], μ_0 is the electromagnetic permeability of free space, and $\varepsilon^{;\nu}$ can be written as the dilatation current $\xi^\nu = \varepsilon^{;\nu}$. Substituting for j^ν from (18) in the relation [23, see p. 94]

$$j^\nu j_\nu = \varrho^2 c^2, \tag{19}$$

we obtain the expression for the charge density

$$\varrho = \frac{1}{2}\,\frac{\varphi_0}{\mu_0 c}\,\frac{2\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0}\,\sqrt{\varepsilon^{;\nu}\varepsilon_{;\nu}}\,. \tag{20}$$

Note the difference between the electromagnetic permeability of free space μ_0 and the Lamé elastic constant $\bar{\mu}_0$ used to denote the spacetime continuum shear modulus.

We see that the charge density derives from the norm of the gradient of the volume dilatation ε, *i.e.*

$$\begin{aligned} \|\varepsilon^{;\nu}\| &= \sqrt{\varepsilon^{;\nu}\varepsilon_{;\nu}} \\ &= \sqrt{\left(\frac{\partial\varepsilon}{\partial x}\right)^2 + \left(\frac{\partial\varepsilon}{\partial y}\right)^2 + \left(\frac{\partial\varepsilon}{\partial z}\right)^2 + \frac{1}{c^2}\left(\frac{\partial\varepsilon}{\partial t}\right)^2} \end{aligned} \tag{21}$$

in cartesian coordinates, and from the above, (20) becomes

$$\varrho = \frac{1}{2}\,\frac{\varphi_0}{\mu_0 c}\,\frac{2\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0}\,\|\varepsilon^{;\nu}\|\,. \tag{22}$$

The charge density is a manifestation of the spacetime fabric itself, however it does not depend on the volume dilatation ε, only on its gradient, and it does not contribute to inertial mass as given by (5). The electromagnetic mass calculation is based on the energy in the electron's electric field and we now consider electromagnetic field energy in *STCED* to clarify its contribution, if any, to inertial mass. This also covers the calculation of electromagnetic mass from the Poynting vector.

3.6 Electromagnetic field energy in the spacetime continuum

As shown in [8], the correct special relativistic relation for momentum p is given by

$$p = m_0\, u\,, \tag{23}$$

where m_0 is the proper or rest mass, u is the velocity with respect to the proper time τ, given by $u = \gamma v$, where

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}}\,, \tag{24}$$

$\beta = v/c$, and v is the velocity with respect to the local time t. When dealing with dynamic equations in the local time t instead of the invariant proper time τ, momentum p is given by

$$p = m^* v\,, \tag{25}$$

where the relativistic mass m^* is given by

$$m^* = \gamma m_0\,. \tag{26}$$

Eq. (25), compared to (23), shows that relativistic mass m^* is an effective mass which results from dealing with dynamic equations in the local time t instead of the invariant proper time τ. The relativistic mass energy $m^* c^2$ corresponds to the total energy of an object (invariant proper mass plus kinetic energy) measured with respect to a given frame of reference [8]. As noted by Jammer [2, p. 41],

> Since [velocity v] depends on the choice of [reference frame] S relative to which it is being measured, [relativistic mass m^*] also depends on S and is consequently a relativistic quantity and not an intrinsic property of the particle.

Using the effective mass, we can write the energy E as the sum of the proper mass and the kinetic energy K of the body, which is typically written as

$$E = m^* c^2 = m_0\, c^2 + K\,. \tag{27}$$

If the particles are subjected to forces, these stresses must be included in the energy-momentum stress tensor, and hence added to K. Thus we see that the inertial mass corresponds to the proper or rest mass of a body, while relativistic mass does not represent an actual increase in the inertial mass of a body, just its total energy (see Taylor and Wheeler [17], Okun [18–20], Oas [21, 22]).

Considering the energy-momentum stress tensor of the electromagnetic field, we can show that $T^{\alpha}{}_{\alpha} = 0$ as expected for massless photons, while

$$T^{00} = \frac{\epsilon_0}{2}\,(E^2 + c^2 B^2) = U_{em} \tag{28}$$

is the total energy density, where U_{em} is the electromagnetic field energy density, ϵ_0 is the electromagnetic permittivity of free space, and E and B have their usual significance for the electric and magnetic fields (see [3, §5.3]). As $m_0 = 0$ for the electromagnetic field, the electromagnetic field energy then needs to be included in the K term in (27).

In general, the energy relation in special relativity is quadratic, given by

$$E^2 = m_0^2\, c^4 + p^2 c^2\,, \tag{29}$$

where p is the momentum. Making use of the effective mass (26) allows us to obtain (25) from (29) [6], starting from

$$m^{*2} c^4 = \gamma^2 m_0^2\, c^4 = m_0^2\, c^4 + p^2 c^2\,. \tag{30}$$

This section provides a description of the electromagnetic field energy using a quadratic energy relation which corresponds to the more complete classical treatment of the electromagnetic mass of the electron based on the Poynting vector of the electron in motion.

In *STCED*, energy is stored in the spacetime continuum as strain energy [6]. As seen in [4, see Section 8.1], the strain energy density of the spacetime continuum is separated into two terms: the first one expresses the dilatation energy density (the mass longitudinal term) while the second one expresses the distortion energy density (the massless transverse term):

$$\mathcal{E} = \mathcal{E}_{\parallel} + \mathcal{E}_{\perp} \tag{31}$$

where

$$\mathcal{E}_{\parallel} = \frac{1}{2}\,\bar{\kappa}_0 \varepsilon^2 \equiv \frac{1}{32\bar{\kappa}_0}\,\rho^2 c^4\,, \tag{32}$$

ρ is the rest-mass density of the deformation, and

$$\mathcal{E}_\perp = \bar{\mu}_0\, e^{\alpha\beta} e_{\alpha\beta} = \frac{1}{4\bar{\mu}_0}\, t^{\alpha\beta} t_{\alpha\beta}\,, \tag{33}$$

with the strain distortion

$$e^{\alpha\beta} = \varepsilon^{\alpha\beta} - e_s\, g^{\alpha\beta} \tag{34}$$

and the strain dilatation $e_s = \frac{1}{4}\,\varepsilon^\alpha{}_\alpha$. Similarly for the stress distortion $t^{\alpha\beta}$ and the stress dilatation t_s. Then the dilatation (massive) strain energy density of the deformation is given by the longitudinal strain energy density (32) and the distortion (massless) strain energy density of the deformation is given by the transverse strain energy density (33).

As shown in [3, §5.3.1] for the electromagnetic field, the longitudinal term is given by

$$\mathcal{E}_\parallel = 0 \tag{35}$$

as expected [24, see pp. 64–66]. This result thus shows that the rest-mass energy density of the electromagnetic field, and hence of the photon is zero, *i.e.* the photon is massless. The transverse term is given by [3, §5.3.2]

$$\mathcal{E}_\perp = \frac{1}{4\bar{\mu}_0}\left[\epsilon_0^2\left(E^2 + c^2B^2\right)^2 - \frac{4}{c^2}\, S^2\right] \tag{36}$$

or

$$\mathcal{E}_\perp = \frac{1}{\bar{\mu}_0}\left[{U_{em}}^2 - \frac{1}{c^2}\, S^2\right] \tag{37}$$

where $U_{em} = \frac{1}{2}\,\epsilon_0(E^2 + c^2B^2)$ is the electromagnetic field energy density as before and S is the magnitude of the Poynting vector. The Poynting four-vector is defined as [3, §5.4]

$$S^\nu = (cU_{em}, \boldsymbol{S})\,, \tag{38}$$

where U_{em} is the electromagnetic field energy density, and $\boldsymbol{S}$ is the Poynting vector. Furthermore, S^ν satisfies

$$\partial_\nu S^\nu = 0\,. \tag{39}$$

Using definition (38) in (37), we obtain the transverse massless energy density of the electromagnetic field

$$\mathcal{E}_\perp = \frac{1}{\bar{\mu}_0 c^2}\, S_\nu S^\nu\,. \tag{40}$$

The indefiniteness of the location of the field energy referred to by Feynman [16, see p. 27-6] is thus resolved: the electromagnetic field energy resides in the distortions (transverse displacements) of the spacetime continuum.

Hence the electromagnetic field is transverse and massless, and has no massive longitudinal component. The electromagnetic field has energy, but no rest mass, and hence no inertia. From *STCED*, we see that electromagnetism as the source of inertia is not valid.

Electromagnetic mass is thus seen to be an unsuccessful attempt to account for the inertial mass of a particle from its electromagnetic field energy. The electromagnetic field contributes to the particle's total energy, but not to its inertial mass which *STCED* shows originates in the particle's dilatation energy density (the mass longitudinal term) which is zero for the electromagnetic field.

4 Discussion and conclusion

In this paper, we have revisited the nature of inertial mass as provided by the Elastodynamics of the Spacetime Continuum (*STCED*) which provides a better understanding of general relativistic spacetime. Mass is shown to be the invariant change in volume of spacetime in the longitudinal propagation of energy-momentum in the spacetime continuum. Hence mass is not independent of the spacetime continuum, but rather mass is part of the spacetime continuum fabric itself.

STCED provides a direct physical definition of mass. In addition, it answers many of the unresolved questions that pertain to the nature of mass:

- The mass of a particle corresponds to a finite spacetime volume dilatation $V_{\varepsilon s}$ and particles need to be given a finite volume (as opposed to "point particles") to give physically realistic results and avoid invalid results.
- It confirms theoretically the equivalence of inertial and gravitational mass.
- The source of inertia is in the massive dilatation associated with each deformation, and Mach's principle (or conjecture), which holds that inertia results from interaction with the average mass of the universe, is seen to be incorrect.
- The electromagnetic field is transverse and massless, and has no massive longitudinal component. It has energy, but no rest mass, and hence no inertia. The electromagnetic field contributes to the particle's total energy, but not to its inertial mass.

STCED thus provides a physical model of the nature of inertial mass, which also includes an explanation for wave-particle duality. This model leads to the clarification and resolution of unresolved and contentious questions pertaining to inertial mass and its nature.

Received on May 27, 2019

References

1. Pais A. 'Subtle is the Lord...': The Science and the Life of Albert Einstein. Oxford University Press, Oxford, 1982.
2. Jammer M. Concepts of Mass in Contemporary Physics and Philosophy. Princeton University Press, Princeton, 2000.
3. Millette P. A. Elastodynamics of the Spacetime Continuum: A Spacetime Physics Theory of Gravitation, Electromagnetism and Quantum Physics. American Research Press, Rehoboth, NM, 2017.

4. Millette P. A. Elastodynamics of the Spacetime Continuum. *The Abraham Zelmanov Journal*, 2012, vol. 5, 221–277.
5. Millette P. A. On the Decomposition of the Spacetime Metric Tensor and of Tensor Fields in Strained Spacetime. *Progress in Physics*, 2012, vol. 8 (4), 5–8.
6. Millette P. A. Strain Energy Density in the Elastodynamics of the Spacetime Continuum and the Electromagnetic Field. *Progress in Physics*, 2013, vol. 9 (2), 82–86.
7. Millette P. A. Dilatation–Distortion Decomposition of the Ricci Tensor. *Progress in Physics*, 2013, vol. 9 (4), 32–33.
8. Millette P. A. On Time Dilation, Space Contraction, and the Question of Relativistic Mass. *Progress in Physics*, 2017, vol. 13 (4), 202–255.
9. Segel L. A. Mathematics Applied to Continuum Mechanics. Dover Publications, New York, 1987.
10. Roll P. G., Krotkov R., and Dicke R. H. The Equivalence of Inertial and Passive Gravitational Mass. *Annals of Physics*, 1964, vol. 26, 442–517.
11. Weinberg S. Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. John Wiley & Sons, New York, 1972.
12. Brans C. and Dicke R. H. Mach's Principle and a Relativistic Theory of Gravitation. *Physical Review*, 1961, vol. 124 (3), 925–935.
13. Goenner H. F. M. Mach's Principle and Theories of Gravitation. In Barbour J. B., Pfister, H., Eds. Mach's Principle: From Newton's Bucket to Quantum Gravity. Birkhäuser, Boston, 1995, pp. 442–457.
14. Ciufolini I. and Wheeler J. A. Gravitation and Inertia. Princeton University Press, Princeton, NJ, 1995.
15. Jammer M. Concepts of Mass in Classical and Modern Physics. Dover Publications, New York, (1961) 1997.
16. Feynman R. P., Leighton R. B., Sands M. Lectures on Physics, Volume II: Mainly Electromagnetism and Matter. Addison-Wesley Publishing Company, Reading, Massachusetts, 1964.
17. Taylor E. F., Wheeler J. A. Spacetime Physics: Introduction to Special Relativity, 2nd ed. Freeman, New York, 1992, pp. 250–251.
18. Okun L. B. The Concept of Mass. *Physics Today*, 1989, v. 42 (6), 31–36.
19. Okun L. B. The Einstein Formula: $E_0 = mc^2$. "Isn't the Lord Laughing?". *Physics–Uspekhi*, 2008, v. 51 (5), 513–527. arXiv: physics.hist-ph/0808.0437.
20. Okun L. B. Energy and Mass in Relativity Theory. World Scientific, New Jersey, 2009.
21. Oas G. On the Abuse and Use of Relativistic Mass. arXiv: physics.ed-ph/0504110v2.
22. Oas G. On the Use of Relativistic Mass in Various Published Works. arXiv: physics.ed-ph/0504111.
23. Barut A. O. Electrodynamics and Classical Theory of Fields and Particles. Dover Publications, New York, 1980.
24. Charap J. M. Covariant Electrodynamics: A Concise Guide. The John Hopkins University Press, Baltimore, 2011.