Anomalous Magnetic Moment in Discrete Time

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The concept of causal delay in discrete time provides a correction for minimal coupling in electromagnetic interactions. This correction gives energy scale-dependent changes to the charge and mass of the elementary particles. As an application example of these results, this paper attempts to explain the anomaly of the g-factor. In particular, for the muon, $a = 0.001166$ is obtained from the approximation of its energy.

1 Introduction

In my last paper [1], I analyzed the effect of causal delay on the description of a dynamic system from a discrete time perspective. As a result, the dynamics system was divided into two worlds to which fundamentally different dynamical principles were applied, namely, type 1 and type 2. In the case of free particles, type 1 corresponds to ordinary matter that satisfies the existing relativistic quantum mechanics, and type 2 has properties similar to dark matter.

In this paper, I will discuss the interactions of type 1 particles, especially electromagnetic interactions. To do this, first, it is necessary to know the meaning of the result of type 1 in the case of free particles. In the case of free particles, the spinor state of type 1 satisfies the following equation

$$(x^\mu + \Delta x^\mu)\,\Psi(x) - x^\mu \Psi(x+\Delta x) = \Delta x^\mu e^{-i\Delta x^\alpha P_\alpha}\Psi(x)\ . \quad (1)$$

The left side of (1) means the sum of contributions from $x - \Delta x$ and $x + \Delta x$ at x if the second term is translated by $-\Delta x$. The right-hand side means that the spinor change is only in phase, that is, the spinor forms a plane wave. Therefore, the above equation means that the sum of contributions from $x-\Delta x$ and $x+\Delta x$ forms a plane wave, that is, harmonic oscillation.

These facts require a new perspective on the matter field. In the existing field theory, especially quantum field theory, the field is based on the ontological basis of the statistical-mechanical analogy of the gathering of harmonic oscillators. From this point of view, harmonic oscillation is a property inherently immanent in the field. On the other hand, the matter field implied by (1) does not assume the property of harmonic oscillation inherent in nature. That is, harmonic oscillations are simply "formed" by the sum of contributions from the past and future of Δt_d. If we look at harmonic oscillations from this point of view, it can be said that interactions "deform" harmonic oscillations.

In the next section, I will discuss interactions in relativistic quantum mechanics with this new perspective on the matter field.

2 Modified Dirac equation

First, I will try to find the evolution operator equation for interacting particles corresponding to the evolution operator (1) for free particles. If the momentum at x and $x + \Delta x$ is p and $p+\Delta p$, and the spinor state is $\Psi(x, p)$ and $\Psi(x + \Delta x, p + \Delta p)$, respectively, the difference of cause-effect vector is as follows

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\,\Psi(x,p) - x^\mu \Psi(x+\Delta x, p+\Delta p)\\
&= (x^\mu + \Delta x^\mu)\,\Psi(x,p) -\\
&-x^\mu \sum_{m,n=0}^{\infty} \frac{1}{m!}\left(\Delta P^\alpha \frac{\partial}{\partial P^\alpha}\right)^m \frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n \Psi(x,p)\\
&= (x^\mu + \Delta x^\mu)\,\Psi(x,p) -\\
&-e^{\Delta P \frac{\partial}{\partial P}}\, x^\mu \left(1 + \Delta x^\alpha \frac{\partial}{\partial x^\alpha} + \sum_{n=2}^{\infty}\frac{1}{n!}\left(\Delta x^\alpha \frac{\partial}{\partial x^\alpha}\right)^n\right)\Psi(x,p)\\
&= \left(\Delta x^\mu - x^\mu \Delta x \frac{\partial}{\partial x}\right)\Psi - \left(e^{\Delta P \frac{\partial}{\partial P}} - 1\right)\left(x^\mu + x^\mu \Delta x \frac{\partial}{\partial x}\right)\Psi -\\
&-e^{\Delta P \frac{\partial}{\partial P}}\, x^\mu \sum_{n=2}^{\infty}\frac{1}{n!}\left(\Delta x \frac{\partial}{\partial x}\right)^n \Psi\\
&= \left\{\Delta x^\mu - e^{\Delta P \frac{\partial}{\partial P}}\left(x^\mu + x^\mu \Delta x \frac{\partial}{\partial x}\right) + x^\mu\right\}\Psi -\\
&-e^{\Delta P \frac{\partial}{\partial P}}\, x^\mu \sum_{n=2}^{\infty}\frac{1}{n!}\left(\Delta x \frac{\partial}{\partial x}\right)^n \Psi\\
&= \left\{x^\mu + \Delta x^\mu - e^{\Delta P \frac{\partial}{\partial P}}\,(x^\mu + \Delta x^\mu) - e^{\Delta P \frac{\partial}{\partial P}}\left[x^\mu, \Delta x \frac{\partial}{\partial x}\right]\right\}\Psi -\\
&-e^{\Delta P \frac{\partial}{\partial P}}\left[x^\mu, \sum_{n=2}^{\infty}\frac{1}{n!}\left(\Delta x \frac{\partial}{\partial x}\right)^n\right]\Psi\\
&= \left\{(x^\mu + \Delta x^\mu)\left(1 - e^{\Delta P \frac{\partial}{\partial P}}\right) - e^{\Delta P \frac{\partial}{\partial P}}\left[x^\mu, e^{\Delta x \frac{\partial}{\partial x}}\right]\right\}\Psi(x,p)\\
&= \left\{(x^\mu + \Delta x^\mu)\left(1 - e^{\Delta P \frac{\partial}{\partial P}}\right) + e^{\Delta P \frac{\partial}{\partial P}}\,\Delta x^\mu e^{-i\Delta x \cdot P}\right\}\Psi(x,p)\ . \quad (2)
\end{aligned}$$

The right-hand side of (2) is the evolution operator of interacting particles. If we apply operator $e^{-\Delta P \frac{\partial}{\partial P}}$ to both sides of (2) to get a simpler expression, it is as follows

$$\begin{aligned}
&(x^\mu + \Delta x^\mu)\,\Psi(x, p-\Delta p) - x^\mu \Psi(x+\Delta x, p)\\
&= \left\{\Delta x^\mu e^{-i\Delta x \cdot P} + (x^\mu + \Delta x^\mu)\left(e^{-\Delta P \frac{\partial}{\partial P}} - 1\right)\right\}\Psi(x,p)\ .
\end{aligned} \quad (3)$$

In (3), the first term on the right side is the evolution operator of a free particle and the second term is the interaction term. Since interaction is a local phenomenon, x in the interaction term can be set to Δx. And if

$$e^{-\Delta P \frac{\partial}{\partial P}} e^{i\Delta x \cdot P} = e^{-i\Delta p \cdot \Delta x} e^{i\Delta x \cdot P} \tag{4}$$

is used, the final evolution operator O becomes

$$O\Psi(x,p) = \left\{ e^{-i\Delta x \cdot P} + 2\left(e^{-i\Delta x \cdot \Delta p} - 1\right)\right\} \Psi(x,p)\,. \tag{5}$$

A little trick is needed here. Although not in practice, it is assumed that Ψ is analytic in order to maintain the conventional dynamics view. Then evolution operator O also needs to be defined as a locally continuous variable. What we need here is $\Delta x = x$. Therefore, the evolution operator O is equal to

$$O = e^{-ix\cdot P} + 2\left(e^{-ix\cdot\Delta p} - 1\right). \tag{6}$$

The evolution operator of free particles satisfies the Dirac equation. Therefore, the modified Dirac equation that the interaction evolution operator (6) must satisfy can be put as

$$\left(i\gamma^\mu \partial_\mu - m + A\right)\left\{ e^{-ix\cdot P} + 2\left(e^{-ix\cdot\Delta p} - 1\right)\right\}\Psi(x,p) = 0\,. \tag{7}$$

By finding A in (7), the following modified Dirac equation can be obtained

$$D_m \Psi = \left(i\gamma^\mu \partial_\mu - f_{1r}\gamma^\mu p_\mu - f_{2r}\gamma^\mu \Delta p_\mu\right)\Psi = 0 \tag{8}$$

where

$$\begin{aligned} f_{1r} &= \mathrm{Re} f_1 = \frac{1}{3}\mathrm{Re}\frac{e^{-ix\cdot p}}{e^{-ix\cdot p} + 2\left(e^{-ix\cdot\Delta p} - 1\right)} \\ f_{2r} &= \mathrm{Re} f_2 = \frac{1}{3}\mathrm{Re}\frac{2e^{-ix\cdot\Delta p}}{e^{-ix\cdot p} + 2\left(e^{-ix\cdot\Delta p} - 1\right)}\,. \end{aligned} \tag{9}$$

In (9), the 1/3 factor is introduced under the condition that the sum of the coefficients in (8) is 0, that is $f_{1r} + f_{2r} = 1$. Now let's find the Hamiltonian using $\gamma^\mu p_\mu = m$. From (8),

$$\begin{aligned} H\Psi &= i\partial_0 \Psi \\ &= \left(-i\vec{\alpha}\cdot\vec{\nabla} + f_{1r}\beta m + f_{2r}\beta\gamma^\mu \Delta p_\mu\right)\Psi \\ &= \left\{\bar{\alpha}\cdot\vec{p} + f_{1r}\beta m + f_{2r}\left(\Delta p_0 - \vec{\alpha}\cdot\Delta\vec{p}\right)\right\}\Psi \\ &= \left\{\vec{\alpha}\cdot\left(\vec{p} - f_{2r}\Delta\vec{p}\right) + f_{1r}\beta m + f_{2r}\Delta p_0\right\}\Psi\,. \end{aligned} \tag{10}$$

Comparing the meaning of (10) with Hamiltonian $H_0 = \vec{\alpha}\cdot\vec{p} + \beta m$ of free particles, it means that when there is a change in momentum and energy due to interactions, correction by $-f_{2r}\Delta\vec{p}$ and $-f_{2r}\Delta E$ is required, respectively. According to the existing minimal coupling theory, when a charge q interacts with an electromagnetic field, the momentum and energy become $\vec{p} - q\vec{A}$ and $E - q\phi$. Here, it can be inferred that the momentum change $\Delta\vec{p}$ and the energy change ΔE are $-q\vec{A}$ and $-q\phi$. Therefore, the combined momentum and energy of minimal coupling and causal delay are $\vec{p} - q\vec{A} + f_{2r}q\vec{A}$ and $E - q\phi + f_{2r}q\phi$. Rewriting, the resulting Hamiltonian is

$$H - (1 - f_{2r})\,q\phi = \vec{\alpha}\cdot\left\{\vec{p} - (1 - f_{2r})\,q\vec{A}\right\} + \beta f_{1r} m\,. \tag{11}$$

Comparing (11) with the existing minimal coupling Hamiltonian, mass and charge can be newly defined as shown in (12) below. That is, the causal delay gives a modified mass and charge concept dependent on the energy scale:

$$\begin{aligned} m' &= f_{1r} m \\ q' &= (1 - f_{2r})\,q\,. \end{aligned} \tag{12}$$

3 Anomalous magnetic moment

The discovery of the Dirac equation made it possible to understand the property of spin of elementary particles, and predicted that the g-factor was 2. But, as a result of the measurement, anomaly exists, which was explained by a completely different paradigm of quantum field theory. However, according to the discussion in the previous section, considering the change in charge and mass due to the effect of causal delay, there is a possibility that the anomaly can be explained from the perspective of modified relativistic quantum mechanics.

When a particle with mass m' and charge q' is placed in an external field $A^\mu = \left(\phi, \vec{A}\right)$, the equation for calculating the magnetic moment is as follows

$$(H - q'\phi)^2 = \left(\vec{p} - q'\vec{A}\right)^2 + m'^2 - q'\vec{\Sigma}\cdot\vec{B}. \tag{13}$$

where

$$\Sigma_j = \begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}.$$

If $q' = -e' = -(1 - f_{2r})\,e$, the nonrelativistic limit of (13) is obtained as follows

$$H \cong m' + \frac{\left(\vec{p} + e'\vec{A}\right)^2}{2m'} - e'\phi + \frac{e'}{2m'}\vec{\Sigma}\cdot\vec{B} \tag{14}$$

where

$$\frac{e'}{2m'}\vec{\Sigma}\cdot\vec{B} = \frac{e}{2m}\frac{(1 - f_{2r})}{f_{1r}}2\vec{S}\cdot\vec{B} \equiv g\frac{e}{2m}\vec{S}\cdot\vec{B}\,. \tag{15}$$

Then, under the condition of $p \gg \Delta p$, the g-factor and the anomalous magnetic moment are as follows

$$\begin{aligned} \frac{g}{2} &= \frac{1 - f_{2r}}{f_{1r}} = 3 - 2\cos(x\cdot p) \\ a &= \frac{g}{2} - 1 = 2 - 2\cos(x\cdot p)\,. \end{aligned} \tag{16}$$

Previously local variable $x = \Delta x$. Then the phase value is

$$\Delta x\cdot p = E\,\Delta t - \vec{p}\cdot\Delta\vec{x} = \Delta t\left(E - \frac{\vec{p}^2}{m}\right). \tag{17}$$

In (17), Δt is the causal delay time. If $\Delta t = 0$, i.e. continuous time, there is no anomaly. Now let's define the physical meaning of Δt as follows.

Assumption: For a particle, the causal delay is the time it takes for light to pass through the particle's reduced Compton wavelength

$$\Delta t \equiv \frac{\lambda\!\!\!^{-}_c}{c} = \frac{\hbar}{mc^2}\,. \tag{18}$$

The Compton wavelength of a particle is a certain "region" of that particle. Therefore, the time it takes for light to pass through the region is the difference in time between cause and effect in the interaction between the particle and light.

Equation (18) is related to the "Penrose clock" [5]. According to him, *any individual stable massive particle plays a role as a virtually perfect clock.* And since $E = mc^2 = h\nu$, the frequency becomes $\nu = m\left(c^2/h\right)$. This can be said to be the same as (18). Since $\Delta t = 1/\nu$, the frequency becomes $\nu/2\pi = m\left(c^2/h\right)$. Therefore, it can be said that the causal delay time of massive particles plays the role of a fundamental clock that exists in nature.*

By the definition of Δt, the phase value of (17) can be calculated, and consequently the anomalous magnetic moment can be determined. An easy way to do this is the anomalous magnetic moment of the muon. In the case of the muon, at the so-called "magic momentum" $p_0 = 3.094$ GeV/c, the effect of the applied electric field for confinement of the muon is negligible. This means that the potential term in the energy value of (17) can be neglected. Therefore, the phase value of (17) is as follows

$$\frac{\vec{p}^2}{m} = \frac{E^2 - m_\mu^2}{m} = \frac{m_\mu^2\left(\gamma^2 - 1\right)}{m_\mu \gamma} = \beta^2 E$$

$$\text{thus}\quad E - \frac{\vec{p}^2}{m} = \frac{E}{\gamma^2} = \frac{m_\mu}{\gamma} \tag{19}$$

$$\text{and}\quad \therefore \Delta t\left(E - \frac{\vec{p}^2}{m}\right) = \frac{1}{\gamma}\,.$$

Therefore, the anomalous magnetic moment of the muon is

$$a_\mu = 2 - 2\cos\frac{1}{\gamma}\,. \tag{20}$$

Using (20) to find a_μ, γ corresponding to $E = 3.094$ GeV is 29.28, so

$$a_\mu = 0.001166\,. \tag{21}$$

Meanwhile, the value of a_μ recently announced by Fermilab is as follows.

$$a_\mu\,(\text{FNAL}) = 0.00116592040\,(54)\,. \tag{22}$$

The value of (21) is only calculated as an approximation of the muon energy. Thus, how accurately (20) predicts a_μ depends on the determination of γ, which is possible as an independent measurement of the cyclotron frequency $\omega_c = eB/m\gamma$.

*Except for (18) and this paragraph, the rest use natural units.

4 Conclusions

Type 1 has a different view from the existing ones on the concept of field. It is that the harmonic oscillation of the field is formed by the sum of contributions from the past and future by Δt, not inherent in nature. And the interactions deform these harmonic oscillations. The result of analyzing the interaction from this point of view shows that it is more than the description of the interaction of the existing relativistic quantum mechanics.

The causal delay effect in discrete time corrects the existing minimal coupling theory, which leads to the result that the charge and mass of elementary particles change depending on the energy scale. As an example of such a result, it is partially shown that the anomalous magnetic moment of the muon can also be explained from this new perspective.

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