

Novel Gravity Probe B Frame-Dragging Effect

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The Gravity Probe B (GP-B) satellite experiment will measure the precession of on-board gyroscopes to extraordinary accuracy. Such precessions are predicted by General Relativity (GR), and one component of this precession is the “frame-dragging” or Lense-Thirring effect, which is caused by the rotation of the Earth. A new theory of gravity, which passes the same extant tests of GR, predicts, however, a second and much larger “frame-dragging” precession. The magnitude and signature of this larger effect is given for comparison with the GP-B data.

1 Introduction

The Gravity Probe B (GP-B) satellite experiment was launched in April 2004. It has the capacity to measure the precession of four on-board gyroscopes to unprecedented accuracy [1, 2, 3, 4]. Such a precession is predicted by the Einstein theory of gravity, General Relativity (GR), with two components (i) a geodetic precession, and (ii) a “frame-dragging” precession known as the Lense-Thirring effect. The latter is particularly interesting effect induced by the rotation of the Earth, and described in GR in terms of a “gravitomagnetic” field. According to GR this smaller effect will give a precession of 0.042 arcsec per year for the GP-B gyroscopes. However a recently developed theory gives a different account of gravity. While agreeing with GR for all the standard tests of GR this theory gives a dynamical account of the so-called “dark matter” effect in spiral galaxies. It also successfully predicts the masses of the black holes found in the globular clusters M15 and G1. Here we show that GR and the new theory make very different predictions for the “frame-dragging” effect, and so the GP-B experiment will be able to decisively test both theories. While predicting the same earth-rotation induced precession, the new theory has an additional much larger “frame-dragging” effect caused by the observed translational motion of the Earth. As well the new theory explains the “frame-dragging” effect in terms of vorticity in a “substratum flow”. Herein the magnitude and signature of this new component of the gyroscope precession is predicted for comparison with data from GP-B when it becomes available.

2 Theories of gravity

The Newtonian “inverse square law” for gravity,

$$F = \frac{Gm_1m_2}{r^2}, \quad (1)$$

was based on Kepler’s laws for the motion of the planets. Newton formulated gravity in terms of the gravitational ac-

celeration vector field $\mathbf{g}(\mathbf{r}, t)$, and in differential form

$$\nabla \cdot \mathbf{g} = -4\pi G\rho, \quad (2)$$

where $\rho(\mathbf{r}, t)$ is the matter density. However there is an alternative formulation [5] in terms of a vector “flow” field $\mathbf{v}(\mathbf{r}, t)$ determined by

$$\frac{\partial}{\partial t}(\nabla \cdot \mathbf{v}) + \nabla \cdot [(\mathbf{v} \cdot \nabla) \mathbf{v}] = -4\pi G\rho, \quad (3)$$

with \mathbf{g} now given by the Euler “fluid” acceleration

$$\mathbf{g} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{d\mathbf{v}}{dt}. \quad (4)$$

Trivially this \mathbf{g} also satisfies (2). External to a spherical mass M of radius R a velocity field solution of (2) is

$$\mathbf{v}(\mathbf{r}) = -\sqrt{\frac{2GM}{r}} \hat{\mathbf{r}}, \quad r > R, \quad (5)$$

which gives from (4) the usual inverse square law \mathbf{g} field

$$\mathbf{g}(\mathbf{r}) = -\frac{GM}{r^2} \hat{\mathbf{r}}, \quad r > R. \quad (6)$$

However the flow equation (2) is not uniquely determined by Kepler’s laws because

$$\frac{\partial}{\partial t}(\nabla \cdot \mathbf{v}) + \nabla \cdot [(\mathbf{v} \cdot \nabla) \mathbf{v}] + C(\mathbf{v}) = -4\pi G\rho, \quad (7)$$

where

$$C(\mathbf{v}) = \frac{\alpha}{8} [(\text{tr} D)^2 - \text{tr}(D^2)], \quad (8)$$

and

$$D_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \quad (9)$$

also has the same external solution (5), because $C(\mathbf{v})=0$ for the flow in (5). So the presence of the $C(\mathbf{v})$ would not have manifested in the special case of planets in orbit about the massive central sun. Here α is a dimensionless

constant – a new gravitational constant, in addition to usual the Newtonian gravitational constant G . However inside a spherical mass we find [5] that $C(\mathbf{v}) \neq 0$, and using the Greenland borehole g anomaly data [4] we find that $\alpha^{-1} = 139 \pm 5$, which gives the fine structure constant $\alpha = e^2 \hbar / c \approx 1/137$ to within experimental error. From (4) we can write

$$\nabla \cdot \mathbf{g} = -4\pi G\rho - 4\pi G\rho_{DM}, \quad (10)$$

where

$$\rho_{DM}(\mathbf{r}) = \frac{\alpha}{32\pi G} [(\text{tr}D)^2 - \text{tr}(D^2)], \quad (11)$$

which introduces an effective “matter density” representing the flow dynamics associated with the $C(\mathbf{v})$ term. In [5] this dynamical effect is shown to be the “dark matter” effect. The interpretation of the vector flow field \mathbf{v} is that it is a manifestation, at the classical level, of a quantum substratum to space; the flow is a rearrangement of that substratum, and not a flow *through* space. However (7) needs to be further generalised [5] to include vorticity, and also the effect of the motion of matter through this substratum via

$$\mathbf{v}_R\{\mathbf{r}_0(t), t\} = \mathbf{v}_0(t) - \mathbf{v}\{\mathbf{r}_0(t), t\}, \quad (12)$$

where $\mathbf{v}_0(t)$ is the velocity of an object, at $\mathbf{r}_0(t)$, relative to the same frame of reference that defines the flow field; then \mathbf{v}_R is the velocity of that matter relative to the substratum. The flow equation (7) is then generalised to, with $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ the Euler fluid or total derivative,

$$\begin{aligned} \frac{dD_{ij}}{dt} + \frac{\delta_{ij}}{3} \text{tr}(D^2) + \frac{\text{tr}D}{2} \left(D_{ij} - \frac{\delta_{ij}}{3} \text{tr}D \right) + \\ + \frac{\delta_{ij}}{3} \frac{\alpha}{8} [(\text{tr}D)^2 - \text{tr}(D^2)] + (\Omega D - D\Omega)_{ij} = \end{aligned} \quad (13)$$

$$= -4\pi G\rho \left(\frac{\delta_{ij}}{3} + \frac{v_R^i v_R^j}{2c^2} + \dots \right), \quad i, j = 1, 2, 3,$$

$$\nabla \times (\nabla \times \mathbf{v}) = \frac{8\pi G\rho}{c^2} \mathbf{v}_R, \quad (14)$$

$$\Omega_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} \right) = \quad (15)$$

$$= -\frac{1}{2} \epsilon_{ijk} \omega_k = -\frac{1}{2} \epsilon_{ijk} (\nabla \times \mathbf{v})_k,$$

and the vorticity vector field is $\vec{\omega} = \nabla \times \mathbf{v}$. For zero vorticity and $v_R \ll c$ (13) reduces to (7). We obtain from (14) the Biot-Savart form for the vorticity

$$\vec{\omega}(\mathbf{r}, t) = \frac{2G}{c^2} \int d^3r' \frac{\rho(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|^3} \mathbf{v}_R(\mathbf{r}', t) \times (\mathbf{r} - \mathbf{r}'). \quad (16)$$

The path $\mathbf{r}_0(t)$ of an object through this flow is obtained by extremising the relativistic proper time

$$\tau[\mathbf{r}_0] = \int dt \left(1 - \frac{v_R^2}{c^2} \right)^{1/2} \quad (17)$$

giving, as a generalisation of (4), the acceleration

$$\begin{aligned} \frac{d\mathbf{v}_0}{dt} = \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] + (\nabla \times \mathbf{v}) \times \mathbf{v}_R - \\ - \frac{\mathbf{v}_R}{1 - \frac{v_R^2}{c^2}} \frac{1}{2} \frac{d}{dt} \left(\frac{v_R^2}{c^2} \right). \end{aligned} \quad (18)$$

Formulating gravity in terms of a flow is probably unfamiliar, but General Relativity (GR) permits an analogous result for metrics of the Panlevé-Gullstrand class [7],

$$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - \frac{1}{c^2} [d\mathbf{r} - \mathbf{v}(\mathbf{r}, t) dt]^2. \quad (19)$$

The external-Schwarzschild metric belongs to this class [8], and when expressed in the form of (19) the \mathbf{v} field is identical to (5). Substituting (19) into the Einstein equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}, \quad (20)$$

gives

$$\begin{aligned} G_{00} = \sum_{i,j=1,2,3} v_i \mathcal{G}_{ij} v_j - c^2 \sum_{j=1,2,3} \mathcal{G}_{0j} v_j - \\ - c^2 \sum_{i=1,2,3} v_i \mathcal{G}_{i0} + c^2 \mathcal{G}_{00}, \end{aligned} \quad (21)$$

$$G_{i0} = - \sum_{j=1,2,3} \mathcal{G}_{ij} v_j + c^2 \mathcal{G}_{i0},$$

$$G_{ij} = \mathcal{G}_{ij}, \quad i, j = 1, 2, 3,$$

where the $\mathcal{G}_{\mu\nu}$ are given by

$$\mathcal{G}_{00} = \frac{1}{2} [(\text{tr}D)^2 - \text{tr}(D^2)],$$

$$\mathcal{G}_{i0} = \mathcal{G}_{0i} = -\frac{1}{2} [\nabla \times (\nabla \times \mathbf{v})]_i, \quad (22)$$

$$\begin{aligned} \mathcal{G}_{ij} = \frac{d}{dt} \left(D_{ij} - \delta_{ij} \text{tr}D \right) + \left(D_{ij} - \frac{1}{2} \delta_{ij} \text{tr}D \right) \text{tr}D - \\ - \frac{1}{2} \delta_{ij} \text{tr}(D^2) + (\Omega D - D\Omega)_{ij}, \quad i, j = 1, 2, 3 \end{aligned}$$

and so GR also uses the Euler “fluid” derivative, and we obtain a set of equations analogous but not identical to (13)–(14). In vacuum, with $T_{\mu\nu} = 0$, we find that (22) demands that

$$[(\text{tr}D)^2 - \text{tr}(D^2)] = 0. \quad (23)$$

This simply corresponds to the fact that GR does not permit the “dark matter” dynamical effect, namely that $\rho_{DM} = 0$, according to (10). This happens because GR was forced to agree with Newtonian gravity, in the appropriate limits, and that theory also has no such effect. The predictions from (13)–(14) and from (22) for the Gravity Probe B experiment are different, and provide an opportunity to test both gravity theories.

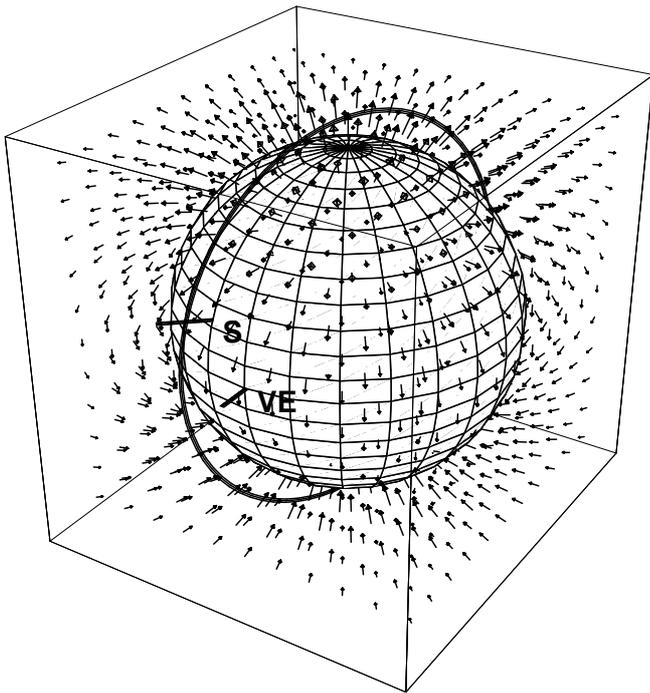


Fig. 1: Shows the Earth (N is up) and vorticity vector field component $\vec{\omega}$ induced by the rotation of the Earth, as in (24). The polar orbit of the GP-B satellite is shown, \mathbf{S} is the gyroscope starting spin orientation, directed towards the guide star IM Pegasi, $RA = 22^h 53' 2.26''$, $Dec = 16^\circ 50' 28.2''$, and \mathbf{VE} is the vernal equinox.

3 “Frame-dragging” as a vorticity effect

Here we consider one difference between the two theories, namely that associated with the vorticity part of (18), leading to the “frame-dragging” or Lense-Thirring effect. In GR the vorticity field is known as the “gravitomagnetic” field $\mathbf{B} = -c \vec{\omega}$. In both GR and the new theory the vorticity is given by (16) but with a key difference: in GR \mathbf{v}_R is *only* the rotational velocity of the matter in the Earth, whereas in (13)–(14) \mathbf{v}_R is the vector sum of the rotational velocity and the translational velocity of the Earth through the substratum. At least seven experiments have detected this translational velocity; some were gas-mode Michelson interferometers and others coaxial cable experiments [8, 9, 10], and the translational velocity is now known to be approximately 430 km/s in the direction $RA = 5.2^h$, $Dec = -67^\circ$. This direction has been known since the Miller [11] gas-mode interferometer experiment, but the RA was more recently confirmed by the 1991 DeWitte coaxial cable experiment performed in the Brussels laboratories of Belgacom [9]. This flow is related to galactic gravity flow effects [8, 9, 10], and so is different to that of the velocity of the Earth with respect to the Cosmic Microwave Background (CMB), which is 369 km/s in the direction $RA = 11.20^h$, $Dec = -7.22^\circ$.

First consider the common but much smaller rotation

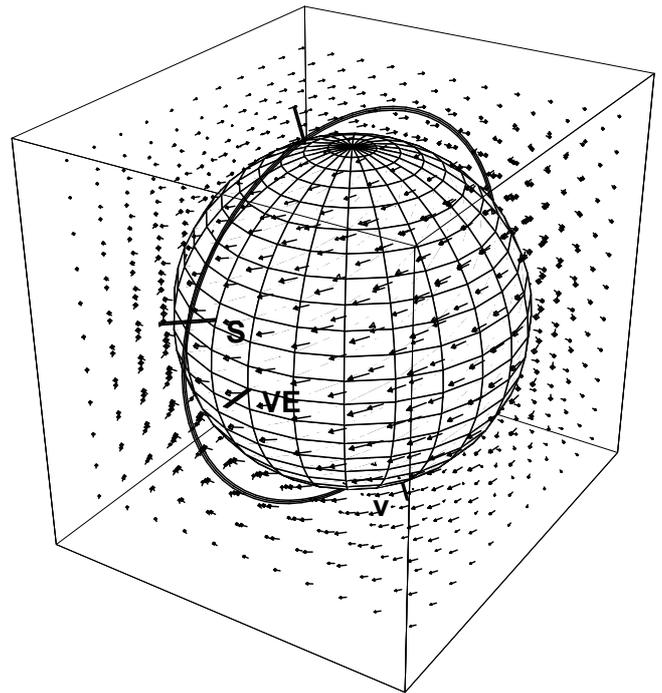


Fig. 2: Shows the Earth (N is up) and the much larger vorticity vector field component $\vec{\omega}$ induced by the translation of the Earth, as in (27). The polar orbit of the GP-B satellite is shown, and \mathbf{S} is the gyroscope starting spin orientation, directed towards the guide star IM Pegasi, $RA = 22^h 53' 2.26''$, $Dec = 16^\circ 50' 28.2''$, \mathbf{VE} is the vernal equinox, and \mathbf{V} is the direction $RA = 5.2^h$, $Dec = -67^\circ$ of the translational velocity \mathbf{v}_c .

induced “frame-dragging” or vorticity effect. Then $\mathbf{v}_R(\mathbf{r}) = \mathbf{w} \times \mathbf{r}$ in (16), where \mathbf{w} is the angular velocity of the Earth, giving

$$\vec{\omega}(\mathbf{r}) = 4 \frac{G}{c^2} \frac{3(\mathbf{r} \cdot \mathbf{L})\mathbf{r} - r^2 \mathbf{L}}{2r^5}, \quad (24)$$

where \mathbf{L} is the angular momentum of the Earth, and \mathbf{r} is the distance from the centre. This component of the vorticity field is shown in Fig. 1. Vorticity may be detected by observing the precession of the GP-B gyroscopes. The vorticity term in (18) leads to a torque on the angular momentum \mathbf{S} of the gyroscope,

$$\vec{\tau} = \int d^3r \rho(\mathbf{r}) \mathbf{r} \times [\vec{\omega}(\mathbf{r}) \times \mathbf{v}_R(\mathbf{r})], \quad (25)$$

where ρ is its density, and where \mathbf{v}_R is used here to describe the rotation of the gyroscope. Then $d\mathbf{S} = \vec{\tau} dt$ is the change in \mathbf{S} over the time interval dt . In the above case $\mathbf{v}_R(\mathbf{r}) = \mathbf{s} \times \mathbf{r}$, where \mathbf{s} is the angular velocity of the gyroscope. This gives

$$\vec{\tau} = \frac{1}{2} \vec{\omega} \times \mathbf{S} \quad (26)$$

and so $\vec{\omega}/2$ is the instantaneous angular velocity of precession of the gyroscope. This corresponds to the well known fluid

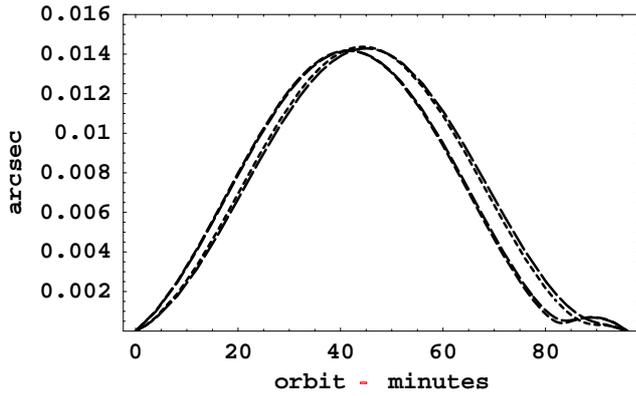


Fig. 3: Predicted variation of the precession angle $\Delta\Theta = |\Delta\mathbf{S}(t)|/|\mathbf{S}(0)|$, in arcsec, over one 97 minute GP-B orbit, from the vorticity induced by the translation of the Earth, as given by (28). The orbit time begins at location \mathbf{S} . Predictions are for the months of April, August, September and February, labeled by increasing dash length. The “glitches” near 80 minutes are caused by the angle effects in (28). These changes arise from the effects of the changing orbital velocity of the Earth about the Sun. The GP-B expected angle measurement accuracy is 0.0005 arcsec. Novel gravitational waves will affect these plots.

result that the vorticity vector is twice the angular velocity vector. For GP-B the direction of \mathbf{S} has been chosen so that this precession is cumulative and, on averaging over an orbit, corresponds to some 7.7×10^{-6} arcsec per orbit, or 0.042 arcsec per year. GP-B has been superbly engineered so that measurements to a precision of 0.0005 arcsec are possible.

However for the unique translation-induced precession if we use $v_R \approx v_C = 430$ km/s in the direction $RA = 5.2^{\text{hr}}$, $Dec = -67^\circ$, namely ignoring the effects of the orbital motion of the Earth, the observed flow past the Earth towards the Sun, and the flow into the Earth, and effects of the gravitational waves, then (16) gives

$$\vec{\omega}(\mathbf{r}) = \frac{2GM}{c^2} \frac{\mathbf{v}_C \times \mathbf{r}}{r^3}. \quad (27)$$

This much larger component of the vorticity field is shown in Fig. 2. The maximum magnitude of the speed of this precession component is $\omega/2 = gv_C/c^2 = 8 \times 10^{-6}$ arcsec/s, where here g is the gravitational acceleration at the altitude of the satellite. This precession has a different signature: it is not cumulative, and is detectable by its variation over each single orbit, as its orbital average is zero, to first approximation. Fig. 3 shows $\Delta\Theta = |\Delta\mathbf{S}(t)|/|\mathbf{S}(0)|$ over one orbit, where, as in general,

$$\begin{aligned} \Delta\mathbf{S}(t) &= \left[\int_0^t dt' \frac{1}{2} \vec{\omega}(\mathbf{r}(t')) \right] \times \mathbf{S}(t') \approx \\ &\approx \left[\int_0^t dt' \frac{1}{2} \vec{\omega}(\mathbf{r}(t')) \right] \times \mathbf{S}(0). \end{aligned} \quad (28)$$

Here $\Delta\mathbf{S}(t)$ is the integrated change in spin, and where the approximation arises because the change in $\mathbf{S}(t')$ on the RHS of (28) is negligible. The plot in Fig. 3 shows this effect to be some $30 \times$ larger than the expected GP-B errors, and so easily detectable, if it exists as predicted herein. This precession is about the instantaneous direction of the vorticity $\vec{\omega}(\mathbf{r}(t))$ at the location of the satellite, and so is neither in the plane, as for the geodetic precession, nor perpendicular to the plane of the orbit, as for the earth-rotation induced vorticity effect.

Because the yearly orbital rotation of the Earth about the Sun slightly effects \mathbf{v}_C [9] predictions for four months throughout the year are shown in Fig. 3. Such yearly effects were first seen in the Miller [11] experiment.

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