Elastodynamics of the Spacetime Continuum

Pierre A. Millette*

Abstract: We develop the Elastodynamics of the Spacetime Continuum (*STCED*) based on the analysis of the deformations of the *STC* within a general relativistic and continuum mechanical framework. We show that *STC* deformations can be decomposed into a massive dilatation and a massless wave distortion reminiscent of wave-particle duality. We show that rest-mass energy density arises from the volume dilatation of the *STC*. We derive Electromagnetism from *STCED* and provide physical explanations for the electromagnetic potential and the current density. We derive the Klein-Gordon equation and show that the quantum mechanical wavefunction describes longitudinal waves propagating in the *STC*. The equations obtained reflect a close integration of gravitational and electromagnetic interactions.

Contents:

*Ottawa, Canada. E-mail: PierreAMillette@alumni.uottawa.ca

§1. Introduction. The theory of General Relativity initially proposed by Einstein [1] is a theory of gravitation based on the geometry of the spacetime continuum (*STC*). The geometry of the spacetime continuum is determined by the energy-momentum present in the *STC*. This can be represented by the relation

$$\textit{Energy-momentum} \longrightarrow \textit{STC Geometry}$$

or, in terms of its mathematical representation,

$$T^{\mu\nu} \longrightarrow G^{\mu\nu}$$

where $G^{\mu\nu}$ is the Einstein tensor and $T^{\mu\nu}$ is the energy-momentum stress tensor. The spacetime continuum is thus warped by the presence of energy-momentum. This is a physical process as shown by the deflection of light by the sun, or the cosmological models resulting in a physical structure of the universe, derived from General Relativity.

Hence the theory of General Relativity leads implicitly to the proposition that the spacetime continuum must be a deformable continuum. This deformation is physical in nature. The "vacuum" that is omnipresent in Quantum Theory, is the spacetime continuum, made more evident by the microscopic scale of quantum phenomena. The physical nature of the spacetime continuum is further supported by the following evidence:

— The physical electromagnetic properties of the vacuum: characteristic impedance of the vacuum $Z_{\mathrm{em}} = 376.73\,\Omega$, electromagnetic permittivity of free space ϵ_{em}, electromagnetic permeability of free space μ_{em}.

— A straightforward explanation of the existence and constancy of the speed of light c: it is the maximum speed at which transverse deformations propagate in the *STC*.

— A physical framework for the vacuum of quantum electrodynamics with its constant creation/annihilation of (virtual) particles, corresponding to a state of constant vibration of the *STC* due to the energy-momentum continuously propagating through it.

— A physical framework to support vacuum quantum effects such as vacuum polarization, zero-point energy, the Casimir force, the Aharonov-Bohm effect.

The assignment of physical dynamic properties to the spacetime of General Relativity has been considered previously. For example, Sakharov [2] considers a "metrical elasticity" of space in which generalized forces oppose the curving of space. Tartaglia *et al* have recently explored "strained spacetime" in the cosmological context, as an extension of the spacetime Lagrangian to obtain a generalized Einstein equation [3, 4].

Considering the spacetime continuum to be a deformable continuum results in an alternative description of its dynamics, represented by the relation

Energy-momentum ⟶ *STC Deformations*

The energy-momentum present in the spacetime continuum, represented by the energy-momentum stress tensor, results in strains in the *STC*, hence the reference to "strained spacetime". The spacetime continuum strains result in displacements of the elements of the spacetime continuum, hence the *STC* deformations. The spacetime continuum itself is the medium that supports those deformations. The spacetime continuum deformations result in the geometry of the *STC*.

This theory is referred to as the Elastodynamics of the Spacetime Continuum (*STCED*) (see Millette [5–8]). In this theory, we analyse the spacetime continuum within the framework of Continuum Mechanics and General Relativity. This allows for the application of continuum mechanical methods and results to the analysis of the *STC* deformations.

Hence, while General Relativity can be described as a top-down theory of the spacetime continuum, the Elastodynamics of the Spacetime Continuum can be described as a bottom-up theory of the spacetime continuum. *STCED* provides a fundamental description of the microscopic processes underlying the spacetime continuum. The relation between *STCED* and General Relativity is represented by the diagram

Energy-momentum ⟶ *STC Geometry*
Energy-momentum ↘ *STC Deformations*
STC Deformations ↑ *STC Geometry*

The combination of all deformations present in the spacetime continuum generates its geometry. *STCED* must thus be a description complementary to that of General Relativity, which is concerned with modeling the resulting geometry of the spacetime continuum rather than the deformations generating that geometry. The value of *STCED* is that it provides a microscopic description of the fundamental *STC* processes expected to reach down to the quantum level.

§1.1. Outline of the paper. We start by demonstrating from first principles that spacetime is strained by the presence of mass. In addition, we find that this provides a natural decomposition of the spacetime metric tensor and of spacetime tensor fields, both of which are still unresolved and are the subject of continuing investigations (see for example [9–13]).

Based on that analysis from first principles of the effect of a test mass on the background metric, we obtain a natural decomposition of the spacetime metric tensor of General Relativity into a background and a dynamical part. We find that the presence of mass results in strains in the spacetime continuum, and that those strains correspond to the dynamical part of the spacetime metric tensor. We note that these results are considered to be local effects in the particular reference frame of the observer. In addition, the applicability of the proposed metric to the Einstein field equations remains open to demonstration.

The presence of strains in the spacetime continuum as a result of the applied stresses from the energy-momentum stress tensor is an expected continuum mechanical result. The strains result in a deformation of the continuum which can be modeled as a change in the underlying geometry of the continuum. The geometry of the spacetime continuum of General Relativity resulting from the energy-momentum stress tensor can thus be seen to be a representation of the deformation of the spacetime continuum resulting from the strains generated by the energy-momentum stress tensor.

We then derive the Elastodynamics of the Spacetime Continuum by applying continuum mechanical results to strained spacetime. Based on this model, a stress-strain relation is derived for the spacetime continuum. We apply that stress-strain relation to show that rest-mass energy density arises from the volume dilatation of the spacetime continuum. Then we propose a natural decomposition of tensor fields in strained spacetime, in terms of dilatations and distortions. We show that dilatations correspond to rest-mass energy density, while distortions correspond to massless shear transverse waves. We note that this decomposition of spacetime continuum deformations into a massive dilatation and a massless transverse wave distortion is somewhat reminiscent of wave-particle duality.

From the kinematic relations and the equilibrium dynamic equation of the spacetime continuum, we derive a series of wave equations: the displacement, dilatational, rotational and strain wave equations. Hence we find that energy propagates in the spacetime continuum as wave-like deformations which can be decomposed into dilatations and distortions.

Dilatations involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation, while distortions correspond to a change of shape of the spacetime continuum without a change in volume and are thus massless. The deformations propagate in the continuum by longitudinal and transverse wave displacements. Again, this is somewhat reminiscent of wave-particle duality, with the transverse mode corresponding to the wave aspects and the longitudinal mode corresponding to the particle aspects. A continuity equation for deformations of the spacetime continuum is derived, where the gradient of the massive volume dilatation acts as a source term. The nature of the spacetime continuum volume force and the inhomogeneous wave equations are areas of further investigation.

We then investigate the strain energy density of the spacetime continuum in the Elastodynamics of the Spacetime Continuum by applying continuum mechanical results to strained spacetime. The strain energy density is a scalar. We find that it is separated into two terms: the first one expresses the dilatation energy density (the "mass" longitudinal term) while the second one expresses the distortion energy density (the "massless" transverse term). The quadratic structure of the energy relation of Special Relativity is found to be present in the theory. In addition, we find that the kinetic energy pc is carried by the distortion part of the deformation, while the dilatation part carries only the rest-mass energy.

Since Einstein first published his Theory of General Relativity in 1915, the problem of the unification of Gravitation and Electromagnetism has been and remains the subject of continuing investigation (see for example [23–31] for recent attempts). Electromagnetism is found to come out naturally from the *STCED* theory in a straightforward manner. This theory thus provides a unified description of the spacetime deformation processes underlying general relativistic Gravitation and Electromagnetism, in terms of spacetime continuum displacements resulting from the strains generated by the energy-momentum stress tensor.

We derive Electromagnetism from the Elastodynamics of the Spacetime Continuum based on the identification of the theory's antisymmetric rotation tensor with the electromagnetic field-strength tensor. The theory provides a physical explanation of the electromagnetic potential, which arises from transverse (shearing) displacements of the spacetime continuum, in contrast to mass which arises from longitudinal (dilatational) displacements. In addition, the theory provides a physical explanation of the current density four-vector, as the 4-gradient of the volume dilatation of the spacetime continuum. The Lorentz condition is obtained directly from the theory. In addition, we obtain a generalization of Electromagnetism for the situation where a volume force is present, in the general non-macroscopic case. Maxwell's equations are found to remain unchanged, but the current density has an additional term proportional to the volume force.

The strain energy density of the electromagnetic energy-momentum stress tensor is then calculated. The dilatation energy density (the rest-mass energy density of the photon) is found to be 0 as expected. The transverse distortion energy density is found to include a longitudinal electromagnetic energy flux term, from the Poynting vector, that is massless as it is due to distortion, not dilatation, of the spacetime continuum. However, because this energy flux is along the direction of propagation (i.e. longitudinal), it gives rise to the particle aspect of the electromagnetic field, the photon.

We then investigate the volume force and its impact on the equations of the Elastodynamics of the Spacetime Continuum. First we consider a linear elastic volume force which leads to equations which are of the Klein-Gordon type. Based on the results obtained, we then consider a variation of that linear elastic volume force based on the Klein-Gordon quantum mechanical current density. We find that the quantum mechanical wavefunction describes longitudinal wave propagations in the *STC* corresponding to the volume dilatation associated with the particle property of an object. The longitudinal wave equation is then found to correspond to the Klein-Gordon equation with an interaction term of the form $\boldsymbol{A} \cdot \boldsymbol{j}$, further confirming that the quantum mechanical wavefunction describes longitudinal wave propagations in the *STC*. The transverse wave equation is found to be a new equation of the electromagnetic field strength $F^{\mu\nu}$, which includes an interaction term of the form $\boldsymbol{A} \times \boldsymbol{j}$ corresponding to the volume density of the magnetic torque (magnetic torque density). The equations obtained reflect a close integration of gravitational and electromagnetic interactions at the microscopic level.

This paper presents a linear elastic theory of the Elastodynamics of the Spacetime Continuum based on the analysis of the deformations of the spacetime continuum. It is found to provide a microscopic description of gravitational and electromagnetic phenomena and some quantum results, based on the framework of General Relativity and Continuum Mechanics. Based on the model, the theory should in principle be able to explain the basic physical theories from which the rest of physical theory can be built, without the introduction of inputs external to the theory. A summary of the physical phenomena derived from *STCED* in this paper are summarized in the conclusion. The direction of the next steps to further extend this theory are discussed in the concluding section of this paper.

§1.2. A note on units and constants. In General Relativity and in Quantum Electrodynamics, it is customary to use "geometrized units" and "natural units" respectively, where the principal constants are set equal to 1. The use of these units facilitates calculations since cumbersome constants do not need to be carried throughout derivations. In this paper, all constants are retained in the derivations, to provide insight into the nature of the equations being developed.

In addition, we use rationalized MKSA units for Electromagnetism, as the traditionally used Gaussian units are gradually being replaced by rationalized MKSA units in more recent textbooks (see for example [32]). Note that the electromagnetic permittivity of free space $\epsilon_{\rm em}$, and the electromagnetic permeability of free space $\mu_{\rm em}$ are written with "*em*" subscripts as the "0" subscripts are used in *STCED* constants. This allows us to differentiate between for example $\mu_{\rm em}$, the electromagnetic permeability of free space, and μ_0, the Lamé elastic constant for the shear modulus of the spacetime continuum.

§1.3. Glossary of physical symbols. This analysis uses symbols across the fields of continuum mechanics, elasticity, general relativity, electromagnetism and quantum mechanics. The symbols used need to be applicable across these disciplines and be self-consistent. A glossary of the physical symbols is included below to facilitate the reading of this paper.

α	Fine-structure constant.
$\epsilon^{\alpha\beta\mu\nu}$	Permutation symbol in four-dimensional spacetime.
$\epsilon_{\rm em}$	Electromagnetic permittivity of free space (*STC*).
ε	Volume dilatation.
$\varepsilon^{\mu\nu}$	Strain tensor.
$\eta_{\mu\nu}$	Flat spacetime metric tensor.
$\Theta^{\mu\nu}$	Symmetric electromagnetic stress tensor.
κ_0	Bulk modulus of the *STC*.
λ_0	Lamé elastic constant of the *STC*.
$\lambda_{\rm c}$	Compton wavelength of the electron.

μ_0	Shear modulus Lamé elastic constant of the *STC*.
μ_{em}	Electromagnetic permeability of free space (*STC*).
μ_{B}	Bohr magneton.
ρ	Rest-mass density.
ρ_0	*STC* density.
ϱ	Charge density.
$\sigma^{\mu\nu}$	Stress tensor.
ϕ	Phase of the quantum mechanical wavefunction.
φ_0	*STC* electromagnetic shearing potential constant.
$\omega^{\mu\nu}$	Rotation tensor.
ψ	Quantum mechanical wavefunction.
$\boldsymbol{A}$	Vector potential.
A^μ	Four-vector potential.
$\bar{A}^\mu$	Reduced four-vector potential.
$\boldsymbol{B}$	Magnetic field.
c	Speed of light.
e	Electrical charge of the electron.
e_s	Strain scalar.
$e^{\mu\nu}$	Strain deviation tensor.
$\boldsymbol{E}$	Electric field.
$\hat{E}$	Total energy density.
$\mathcal{E}$	Strain energy density of the spacetime continuum.
$E^{\mu\nu\alpha\beta}$	Elastic moduli tensor of the *STC*.
$F^{\mu\nu}$	Electromagnetic field strength tensor.
$g_{\mu\nu}$	Metric tensor.
G	Gravitational constant.
h	Planck's constant.
$\hbar$	Planck's reduced constant.
$\boldsymbol{j}$	Current density vector.
j^μ	Current density four-vector.
$\bar{j}^\mu$	Reduced current density four-vector.
k_0	Elastic force constant of the *STC* volume force.
k_{L}	*STC* longitudinal dimensionless ratio.

k_T	STC transverse dimensionless ratio.
m	Mass of the electron.
$\boldsymbol{p}$	Momentum 3-vector.
$\hat{p}$	Momentum density.
R	Contracted Ricci curvature tensor.
$R^{\mu\nu}$	Ricci curvature tensor.
$R^{\mu}{}_{\nu\alpha\beta}$	Curvature tensor.
$\boldsymbol{S}$	Poynting vector (electromagnetic field energy flux).
S^μ	Poynting four-vector.
t_s	Stress scalar.
$t^{\mu\nu}$	Stress deviation tensor.
$T^{\mu\nu}$	Energy-momentum stress tensor.
u^μ	Displacement four-vector.
U_em	Electromagnetic field energy density.
x^μ	Position four-vector.
X^ν	Volume (or body) force.
Z_em	Characteristic impedance of the vacuum (STC).

§2. Elastodynamics of the Spacetime Continuum

§2.1. Strained spacetime and the natural decomposition of the spacetime metric tensor. There is no straightforward definition of local energy density of the gravitational field in General Relativity, [14, see p. 84, p. 286] and [12, 15, 16]. This arises because the spacetime metric tensor includes both the background spacetime metric and the local dynamical effects of the gravitational field. No natural way of decomposing the spacetime metric tensor into its background and dynamical parts is known.

In this section, we propose a natural decomposition of the spacetime metric tensor into a background and a dynamical part. This is derived from first principles by introducing a test mass in the spacetime continuum described by the background metric, and calculating the effect of this test mass on the metric.

Consider the diagram of Figure 1. Points A and B of the spacetime continuum, with coordinates x^μ and $x^\mu + dx^\mu$ respectively, are separated by the infinitesimal line element

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor describing the background state of the spacetime continuum.

We now introduce a test mass in the spacetime continuum. This results in the displacement of point A to $\widetilde{A}$, where the displacement is written as u^μ. Similarly, the displacement of point B to $\widetilde{B}$ is written as $u^\mu + du^\mu$. The infinitesimal line element between points $\widetilde{A}$ and $\widetilde{B}$ is given by $d\widetilde{s}^2$.

By reference to Figure 1, the infinitesimal line element $d\widetilde{s}^2$ can be expressed in terms of the background metric tensor as

$$d\widetilde{s}^2 = g_{\mu\nu}(dx^\mu + du^\mu)(dx^\nu + du^\nu)\,. \tag{2}$$

Multiplying out the terms in parentheses, we get

$$d\widetilde{s}^2 = g_{\mu\nu}(dx^\mu dx^\nu + dx^\mu du^\nu + du^\mu dx^\nu + du^\mu du^\nu)\,. \tag{3}$$

Expressing the differentials du as a function of x, this equation becomes

$$\begin{aligned} d\widetilde{s}^2 = g_{\mu\nu}(dx^\mu dx^\nu + dx^\mu\, u^\nu{}_{;\alpha}\, dx^\alpha + u^\mu{}_{;\alpha}\, dx^\alpha dx^\nu + \\ + u^\mu{}_{;\alpha}\, dx^\alpha\, u^\nu{}_{;\beta}\, dx^\beta)\,, \end{aligned} \tag{4}$$

where the semicolon (;) denotes covariant differentiation. Rearranging the dummy indices, this expression can be written as

$$d\widetilde{s}^2 = (g_{\mu\nu} + g_{\mu\alpha}\, u^\alpha{}_{;\nu} + g_{\alpha\nu}\, u^\alpha{}_{;\mu} + g_{\alpha\beta}\, u^\alpha{}_{;\mu} u^\beta{}_{;\nu})\, dx^\mu dx^\nu \tag{5}$$

and lowering indices, the equation becomes

$$d\widetilde{s}^2 = (g_{\mu\nu} + u_{\mu;\nu} + u_{\nu;\mu} + u^\alpha{}_{;\mu} u_{\alpha;\nu})\, dx^\mu dx^\nu. \tag{6}$$

The expression $u_{\mu;\nu} + u_{\nu;\mu} + u^\alpha{}_{;\mu} u_{\alpha;\nu}$ is equivalent to the definition of the strain tensor $\varepsilon^{\mu\nu}$ of Continuum Mechanics. The strain $\varepsilon^{\mu\nu}$ is expressed in terms of the displacements u^μ of a continuum through the kinematic relation, [17, see p. 149] and [18, see pp. 23–28]:

$$\varepsilon^{\mu\nu} = \frac{1}{2}\,(u^{\mu;\nu} + u^{\nu;\mu} + u^{\alpha;\mu} u_\alpha{}^{;\nu})\,. \tag{7}$$

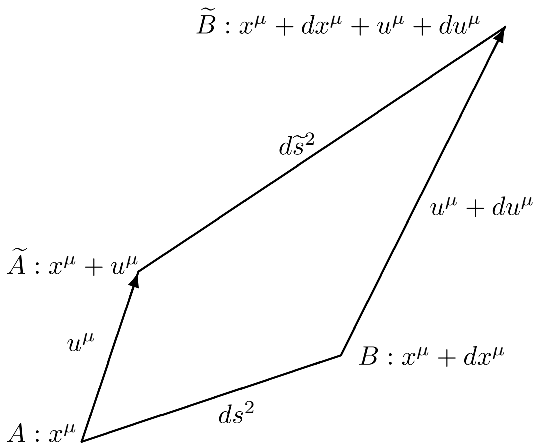
Substituting for $\varepsilon^{\mu\nu}$ from (7) into (6), we get

$$d\widetilde{s}^2 = (g_{\mu\nu} + 2\,\varepsilon_{\mu\nu})\, dx^\mu dx^\nu. \tag{8}$$

Setting [18, see p. 24]

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} + 2\,\varepsilon_{\mu\nu} \tag{9}$$

Fig. 1: Effect of a test mass on the background metric tensor.



then (8) becomes

$$d\widetilde{s}^2 = \widetilde{g}_{\mu\nu}\, dx^\mu dx^\nu, \tag{10}$$

where $\widetilde{g}_{\mu\nu}$ is the metric tensor describing the spacetime continuum with the test mass.

Given that $g_{\mu\nu}$ is the background metric tensor describing the background state of the continuum, and $\widetilde{g}_{\mu\nu}$ is the spacetime metric tensor describing the final state of the continuum with the test mass, then $2\,\varepsilon_{\mu\nu}$ must represent the dynamical part of the spacetime metric tensor due to the test mass:

$$g^{\rm dyn}_{\mu\nu} = 2\,\varepsilon_{\mu\nu}\,. \tag{11}$$

We are thus led to the conclusion that the presence of mass results in strains in the spacetime continuum. Those strains correspond to the dynamical part of the spacetime metric tensor. Hence the applied stresses from mass (i.e. the energy-momentum stress tensor) result in strains in the spacetime continuum, that is strained spacetime.

§2.2. Model of the Elastodynamics of the Spacetime Continuum. The spacetime continuum (STC) is modelled as a four-dimensional differentiable manifold endowed with a metric $g_{\mu\nu}$. It is a continuum that can undergo deformations and support the propagation of such deformations. A continuum that is deformed is strained.

An infinitesimal element of the unstrained continuum is characterized by a four-vector x^μ, where $\mu = 0, 1, 2, 3$. The time coordinate is $x^0 \equiv ct$.

A *deformation* of the spacetime continuum corresponds to a state of the *STC* in which its infinitesimal elements are displaced from their unstrained position. Under deformation, the infinitesimal element x^μ is displaced to a new position $x^\mu + u^\mu$, where u^μ is the displacement of the infinitesimal element from its unstrained position x^μ.

The spacetime continuum is approximated by a deformable linear elastic medium that obeys Hooke's law. For a general anisotropic continuum in four dimensions [18, see pp. 50–53],

$$E^{\mu\nu\alpha\beta}\varepsilon_{\alpha\beta} = T^{\mu\nu}, \tag{12}$$

where $\varepsilon_{\alpha\beta}$ is the strain tensor, $T^{\mu\nu}$ is the energy-momentum stress tensor, and $E^{\mu\nu\alpha\beta}$ is the elastic moduli tensor.

The spacetime continuum is further assumed to be isotropic and homogeneous. This assumption is in agreement with the conservation laws of energy-momentum and angular momentum as expressed by Noether's theorem [21, see pp. 23–30]. For an isotropic medium, the elastic moduli tensor simplifies to [18]:

$$E^{\mu\nu\alpha\beta} = \lambda_0(g^{\mu\nu}g^{\alpha\beta}) + \mu_0(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}), \tag{13}$$

where λ_0 and μ_0 are the Lamé elastic constants of the spacetime continuum. μ_0 is the shear modulus (the resistance of the continuum to *distortions*) and λ_0 is expressed in terms of κ_0, the bulk modulus (the resistance of the continuum to *dilatations*) according to

$$\lambda_0 = \kappa_0 - \frac{1}{2}\,\mu_0 \tag{14}$$

in a four-dimensional continuum. A *dilatation* corresponds to a change of volume of the spacetime continuum without a change of shape while a *distortion* corresponds to a change of shape of the spacetime continuum without a change in volume.

§2.3. Stress-strain relation of the spacetime continuum. By substituting (13) into (12), we obtain the stress-strain relation for an isotropic and homogeneous spacetime continuum

$$2\,\mu_0\varepsilon^{\mu\nu} + \lambda_0\,g^{\mu\nu}\varepsilon = T^{\mu\nu}, \tag{15}$$

where
$$\varepsilon = \varepsilon^{\alpha}{}_{\alpha} \tag{16}$$
is the trace of the strain tensor obtained by contraction. The volume dilatation ε is defined as the change in volume per original volume [17, see pp. 149–152] and is an invariant of the strain tensor.

It is interesting to note that the structure of (15) is similar to that of the field equations of General Relativity, viz.
$$R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R = -\varkappa T^{\mu\nu}, \tag{17}$$
where $\varkappa = 8\pi G/c^4$ and G is the gravitational constant. This strengthens our conjecture that the geometry of the spacetime continuum can be seen to be a representation of the deformation of the spacetime continuum resulting from the strains generated by the energy-momentum stress tensor.

§3. Rest-mass energy relation. The introduction of strains in the spacetime continuum as a result of the energy-momentum stress tensor allows us to use by analogy results from Continuum Mechanics, in particular the stress-strain relation, to provide a better understanding of strained spacetime. As derived in (15), the stress-strain relation for an isotropic and homogeneous spacetime continuum can be written as:
$$2\,\mu_0\, \varepsilon^{\mu\nu} + \lambda_0\, g^{\mu\nu} \varepsilon = T^{\mu\nu}.$$
The contraction of (15) yields the relation
$$2\,(\mu_0 + 2\,\lambda_0)\,\varepsilon = T^{\alpha}{}_{\alpha} \equiv T. \tag{18}$$

The time-time component T^{00} of the energy-momentum stress tensor represents the total energy density given by [19, see pp. 37–41]
$$T^{00}(x^k) = \int d^3\boldsymbol{p}\, E_{\rm p}\, f(x^k, \boldsymbol{p})\,, \tag{19}$$
where $E_{\rm p} = \sqrt{\rho^2 c^4 + p^2 c^2}$, ρ is the rest-mass energy density, c is the speed of light, $\boldsymbol{p}$ is the momentum 3-vector and $f(x^k, \boldsymbol{p})$ is the distribution function representing the number of particles in a small phase space volume $d^3\boldsymbol{x}\, d^3\boldsymbol{p}$. The space-space components T^{ij} of the energy-momentum stress tensor represent the stresses within the medium given by
$$T^{ij}(x^k) = c^2 \int d^3\boldsymbol{p}\, \frac{p^i p^j}{E_{\rm p}}\, f(x^k, \boldsymbol{p}). \tag{20}$$

They are the components of the net force acting across a unit area of a surface, across the x^i planes in the case where $i = j$. In the simple case of a particle, they are given by [20, see p. 117]

$$T^{ii} = \rho\, v^i v^i, \tag{21}$$

where v^i are the spatial components of velocity. If the particles are subject to forces, these stresses must be included in the energy-momentum stress tensor.

The energy-momentum stress tensor thus includes the energy density, momentum density and stresses from all matter and all fields, such as for example the electromagnetic field.

Explicitly separating the time-time and the space-space components, the trace of the energy-momentum stress tensor is written as

$$T^\alpha{}_\alpha = T^0{}_0 + T^i{}_i\,. \tag{22}$$

Substituting from (19) and (20), using the metric $\eta^{\mu\nu}$ of signature $(+\,-\,-\,-)$, we obtain:

$$T^\alpha{}_\alpha(x^k) = \int d^3\boldsymbol{p}\left(E_\mathrm{p} - \frac{p^2c^2}{E_\mathrm{p}}\right) f(x^k, \boldsymbol{p}) \tag{23}$$

which simplifies to

$$T^\alpha{}_\alpha(x^k) = \rho^2 c^4 \int d^3\boldsymbol{p}\; \frac{f(x^k,\boldsymbol{p})}{E_\mathrm{p}}\,. \tag{24}$$

Using the relation [19, see p. 37]

$$\frac{1}{\overline{E}_\mathrm{har}(x^k)} = \int d^3\boldsymbol{p}\; \frac{f(x^k,\boldsymbol{p})}{E_\mathrm{p}} \tag{25}$$

in equation (24), we obtain the relation

$$T^\alpha{}_\alpha(x^k) = \frac{\rho^2 c^4}{\overline{E}_\mathrm{har}(x^k)}\,, \tag{26}$$

where $\overline{E}_\mathrm{har}(x^k)$ is the Lorentz invariant harmonic mean of the energy of the particles at x^k.

In the harmonic mean of the energy of the particles $\overline{E}_\mathrm{har}$, the momentum contribution $\boldsymbol{p}$ will tend to average out and be dominated by the mass term ρc^2, so that we can write

$$\overline{E}_\mathrm{har}(x^k) \simeq \rho c^2. \tag{27}$$

Substituting for $\overline{E}_{\text{har}}$ in (26), we obtain the relation

$$T^{\alpha}{}_{\alpha}(x^k) \simeq \rho c^2. \tag{28}$$

The total rest-mass energy density of the system is obtained by integrating over all space:

$$T^{\alpha}{}_{\alpha} = \int d^3\boldsymbol{x}\; T^{\alpha}{}_{\alpha}(x^k). \tag{29}$$

The expression for the trace derived from (22) depends on the composition of the sources of the gravitational field. Considering the energy-momentum stress tensor of the electromagnetic field, we can show that $T^{\alpha}{}_{\alpha} = 0$ as expected for massless photons, while

$$T^{00} = \frac{\epsilon_{\text{em}}}{2}\left(E^2 + c^2 B^2\right)$$

is the total energy density, where ϵ_{em} is the electromagnetic permittivity of free space, and E and B have their usual significance (see Page 250 for details).

Hence $T^{\alpha}{}_{\alpha}$ corresponds to the invariant rest-mass energy density and we write

$$T^{\alpha}{}_{\alpha} = T = \rho c^2, \tag{30}$$

where ρ is the rest-mass energy density. Using (30) into (18), the relation between the invariant volume dilatation ε and the invariant rest-mass energy density becomes

$$2\left(\mu_0 + 2\lambda_0\right)\varepsilon = \rho c^2 \tag{31}$$

or, in terms of the bulk modulus κ_0,

$$4\kappa_0\,\varepsilon = \rho c^2. \tag{32}$$

This equation demonstrates that rest-mass energy density arises from the volume dilatation of the spacetime continuum. The rest-mass energy is equivalent to the energy required to dilate the volume of the spacetime continuum, and is a measure of the energy stored in the spacetime continuum as volume dilatation. κ_0 represents the resistance of the spacetime continuum to dilatation. The volume dilatation is an invariant, as is the rest-mass energy density.

§4. Decomposition of tensor fields in strained spacetime. As opposed to vector fields which can be decomposed into longitudinal (irrotational) and transverse (solenoidal) components using the Helmholtz representation theorem [17, see pp. 260–261], the decomposition of spacetime tensor fields can be done in many ways (see for example [9–11, 13]).

The application of Continuum Mechanics to a strained spacetime continuum offers a natural decomposition of tensor fields, in terms of dilatations and distortions [18, see pp. 58–60]. A *dilatation* corresponds to a change of volume of the spacetime continuum without a change of shape while a *distortion* corresponds to a change of shape of the spacetime continuum without a change in volume. Dilatations correspond to longitudinal displacements and distortions correspond to transverse displacements [17, see p. 260].

The strain tensor $\varepsilon^{\mu\nu}$ can thus be decomposed into a strain deviation tensor $e^{\mu\nu}$ (the *distortion*) and a scalar e_s (the *dilatation*) according to [18, see pp. 58–60]:

$$\varepsilon^{\mu\nu} = e^{\mu\nu} + e_s\, g^{\mu\nu}, \tag{33}$$

where

$$e^{\mu}{}_{\nu} = \varepsilon^{\mu}{}_{\nu} - e_s\, \delta^{\mu}{}_{\nu}\,, \tag{34}$$

$$e_s = \frac{1}{4}\, \varepsilon^{\alpha}{}_{\alpha} = \frac{1}{4}\, \varepsilon\,. \tag{35}$$

Similarly, the energy-momentum stress tensor $T^{\mu\nu}$ is decomposed into a stress deviation tensor $t^{\mu\nu}$ and a scalar t_s according to

$$T^{\mu\nu} = t^{\mu\nu} + t_s\, g^{\mu\nu}, \tag{36}$$

where similarly

$$t^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - t_s\, \delta^{\mu}{}_{\nu}\,, \tag{37}$$

$$t_s = \frac{1}{4}\, T^{\alpha}{}_{\alpha}\,. \tag{38}$$

Using (33) to (38) into the strain-stress relation of (15) and making use of (18) and (14), we obtain separated dilatation and distortion relations respectively:

$$\begin{aligned} &\text{dilatation} : t_s = 2\,(\mu_0 + 2\,\lambda_0)\, e_s = 4\,\kappa_0\, e_s = \kappa_0\, \varepsilon \\ &\text{distortion} : t^{\mu\nu} = 2\,\mu_0\, e^{\mu\nu}. \end{aligned} \tag{39}$$

The distortion-dilatation decomposition is evident in the dependence of the dilatation relation on the bulk modulus κ_0 and of the distortion relation on the shear modulus μ_0. The dilatation relation of (39) corresponds to rest-mass energy, while the distortion relation is traceless and thus massless, and corresponds to shear transverse waves.

This decomposition of spacetime continuum deformations into a massive dilatation and a massless transverse wave distortion is somewhat reminiscent of wave-particle duality. This could explain why dilatation-measuring apparatus measure the massive "particle" properties of the deformation, while distortion-measuring apparatus measure the massless transverse "wave" properties of the deformation.

§5. Kinematic relations. The strain $\varepsilon^{\mu\nu}$ can be expressed in terms of the displacement u^{μ} through the kinematic relation [17, see pp. 149–152]:

$$\varepsilon^{\mu\nu} = \frac{1}{2}\left(u^{\mu;\nu} + u^{\nu;\mu} + u^{\alpha;\mu} u_{\alpha}{}^{;\nu}\right) \tag{40}$$

where the semicolon (;) denotes covariant differentiation. For small displacements, this expression can be linearized to give the symmetric tensor

$$\varepsilon^{\mu\nu} = \frac{1}{2}\left(u^{\mu;\nu} + u^{\nu;\mu}\right) = u^{(\mu;\nu)}. \tag{41}$$

We use the small displacement approximation in this analysis.

An antisymmetric tensor $\omega^{\mu\nu}$ can also be defined from the displacement u^{μ}. This tensor is called the rotation tensor and is defined as [17]:

$$\omega^{\mu\nu} = \frac{1}{2}\left(u^{\mu;\nu} - u^{\nu;\mu}\right) = u^{[\mu;\nu]}. \tag{42}$$

Where needed, displacements in expressions derived from (41) will be written as $u_{\parallel}$ while displacements in expressions derived from (42) will be written as $u_{\perp}$. Using different symbolic subscripts for these displacements provides a reminder that symmetric displacements are along the direction of motion (longitudinal), while antisymmetric displacements are perpendicular to the direction of motion (transverse).

In general, we have [17]

$$u^{\mu;\nu} = \varepsilon^{\mu\nu} + \omega^{\mu\nu} \tag{43}$$

where the tensor $u^{\mu;\nu}$ is a combination of symmetric and antisymmetric tensors. Lowering index ν and contracting, we get the volume dilatation of the spacetime continuum

$$u^{\mu}{}_{;\mu} = \varepsilon^{\mu}{}_{\mu} = u_{\parallel}{}^{\mu}{}_{;\mu} = \varepsilon \tag{44}$$

where the relation

$$\omega^{\mu}{}_{\mu} = u_{\perp}{}^{\mu}{}_{;\mu} = 0 \tag{45}$$

has been used.

§6. Dynamic equation

§6.1. Equilibrium condition. Under equilibrium conditions, the dynamics of the spacetime continuum is described by the equation [18, see pp. 88–89],

$$T^{\mu\nu}{}_{;\mu} = -X^{\nu}, \tag{46}$$

where X^{ν} is the volume (or body) force. As Wald [14, see p. 286] points out, in General Relativity the local energy density of matter as measured by a given observer is well-defined, and the relation

$$T^{\mu\nu}{}_{;\mu} = 0 \tag{47}$$

can be taken as expressing local conservation of the energy-momentum of matter. However, it does not in general lead to a global conservation law. The value $X^{\nu} = 0$ is thus taken to represent the macroscopic local case, while (46) provides a more general expression.

At the microscopic level, energy is conserved within the limits of the Heisenberg Uncertainty Principle. The volume force may thus be very small, but not exactly zero. It again makes sense to retain the volume force in the equation, and use (46) in the general case, while (47) can be used at the macroscopic local level, obtained by setting the volume force X^{ν} equal to zero.

§6.2. Displacement wave equation. Substituting for $T^{\mu\nu}$ from (15), (46) becomes

$$2\,\mu_0\,\varepsilon^{\mu\nu}{}_{;\mu} + \lambda_0\,g^{\mu\nu}\varepsilon_{;\mu} = -X^{\nu} \tag{48}$$

and, using (41),

$$\mu_0\,(u^{\mu;\nu}{}_{\mu} + u^{\nu;\mu}{}_{\mu}) + \lambda_0\,\varepsilon^{;\nu} = -X^{\nu}. \tag{49}$$

Interchanging the order of differentiation in the first term and using (44) to express ε in terms of u, this equation simplifies to

$$\mu_0\,u^{\nu;\mu}{}_{\mu} + (\mu_0 + \lambda_0)\,u^{\mu}{}_{;\mu}{}^{\nu} = -X^{\nu}, \tag{50}$$

which can also be written as

$$\mu_0\,\nabla^2 u^{\nu} + (\mu_0 + \lambda_0)\,\varepsilon^{;\nu} = -X^{\nu}. \tag{51}$$

This is the *displacement wave equation.*

Setting X^{ν} equal to zero, we obtain the macroscopic displacement wave equation

$$\nabla^2 u^{\nu} = -\frac{\mu_0 + \lambda_0}{\mu_0}\,\varepsilon^{;\nu}. \tag{52}$$

§6.3. Continuity equation. Taking the divergence of (43), we obtain

$$u^{\mu;\nu}{}_{\mu} = \varepsilon^{\mu\nu}{}_{;\mu} + \omega^{\mu\nu}{}_{;\mu}\,. \tag{53}$$

Interchanging the order of partial differentiation in the first term, and using (44) to express u in terms of ε, this equation simplifies to

$$\varepsilon^{\mu\nu}{}_{;\mu} + \omega^{\mu\nu}{}_{;\mu} = \varepsilon^{;\nu}. \tag{54}$$

Hence the divergence of the strain and rotation tensors equals the gradient of the massive volume dilatation, which acts as a source term. This is the continuity equation for deformations of the spacetime continuum.

§7. Wave equations

§7.1. Dilatational (longitudinal) wave equation. Taking the divergence of (50) and interchanging the order of partial differentiation in the first term, we obtain

$$(2\,\mu_0 + \lambda_0)\,u^{\mu}{}_{;\mu}{}^{\nu}{}_{\nu} = -X^{\nu}{}_{;\nu}\,. \tag{55}$$

Using (44) to express u in terms of ε, this equation simplifies to

$$(2\,\mu_0 + \lambda_0)\,\varepsilon^{;\nu}{}_{\nu} = -X^{\nu}{}_{;\nu} \tag{56}$$

or

$$(2\,\mu_0 + \lambda_0)\,\nabla^2\varepsilon = -X^{\nu}{}_{;\nu}\,. \tag{57}$$

Setting X^{ν} equal to zero, we obtain the macroscopic longitudinal wave equation

$$(2\mu_0 + \lambda_0)\nabla^2\varepsilon = 0\,. \tag{58}$$

The volume dilatation ε satisfies a wave equation known as the dilatational wave equation [17, see p. 260]. The solutions of the homogeneous equation are dilatational waves which are longitudinal waves, propagating along the direction of motion. Dilatations thus propagate in the spacetime continuum as longitudinal waves.

§7.2. Rotational (transverse) wave equation. Differentiating (50) with respect to x^{α}, we obtain

$$\mu_0\,u^{\nu;\mu}{}_{\mu}{}^{\alpha} + (\mu_0 + \lambda_0)\,u^{\mu}{}_{;\mu}{}^{\nu\alpha} = -X^{\nu;\alpha}. \tag{59}$$

Interchanging the dummy indices ν and α, and subtracting the resulting equation from (59), we obtain the relation

$$\mu_0\,(u^{\nu;\mu}{}_{\mu}{}^{\alpha} - u^{\alpha;\mu}{}_{\mu}{}^{\nu}) = -(X^{\nu;\alpha} - X^{\alpha;\nu})\,. \tag{60}$$

Interchanging the order of partial differentiations and using the definition of the rotation tensor $\omega^{\nu\alpha}$ of (42), the following wave equation is obtained:

$$\mu_0 \nabla^2 \omega^{\mu\nu} = -X^{[\mu;\nu]} \tag{61}$$

where $X^{[\mu;\nu]}$ is the antisymmetrical component of the gradient of the volume force defined as

$$X^{[\mu;\nu]} = \frac{1}{2}\left(X^{\mu;\nu} - X^{\nu;\mu}\right). \tag{62}$$

Setting X^ν equal to zero, we obtain the macroscopic transverse wave equation

$$\mu_0 \nabla^2 \omega^{\mu\nu} = 0\,. \tag{63}$$

The rotation tensor $\omega^{\mu\nu}$ satisfies a wave equation known as the rotational wave equation [17, see p. 260]. The solutions of the homogeneous equation are rotational waves which are transverse waves, propagating perpendicular to the direction of motion. Massless waves thus propagate in the spacetime continuum as transverse waves.

§7.3. Strain (symmetric) wave equation. A corresponding symmetric wave equation can also be derived for the strain $\varepsilon^{\mu\nu}$. Starting from (59), interchanging the dummy indices ν and α, adding the resulting equation to (59), and interchanging the order of partial differentiation, the following wave equation is obtained:

$$\mu_0 \nabla^2 \varepsilon^{\mu\nu} + (\mu_0 + \lambda_0)\,\varepsilon^{;\mu\nu} = -X^{(\mu;\nu)} \tag{64}$$

where $X^{(\mu;\nu)}$ is the symmetrical component of the gradient of the volume force defined as

$$X^{(\mu;\nu)} = \frac{1}{2}\left(X^{\mu;\nu} + X^{\nu;\mu}\right). \tag{65}$$

Setting X^ν equal to zero, we obtain the macroscopic symmetric wave equation

$$\nabla^2 \varepsilon^{\mu\nu} = -\frac{\mu_0 + \lambda_0}{\mu_0}\,\varepsilon^{;\mu\nu}. \tag{66}$$

This strain wave equation is similar to the displacement wave equation (52).

§8. Strain energy density of the spacetime continuum. The strain energy density of the spacetime continuum is a scalar given by [18, see p. 51]

$$\mathcal{E} = \frac{1}{2}\,T^{\alpha\beta}\varepsilon_{\alpha\beta}\,, \tag{67}$$

where $\varepsilon_{\alpha\beta}$ is the strain tensor and $T^{\alpha\beta}$ is the energy-momentum stress tensor. Introducing the strain and stress deviators from (33) and (36), this equation becomes

$$\mathcal{E} = \frac{1}{2}\left(t^{\alpha\beta} + t_s\, g^{\alpha\beta}\right)\left(e_{\alpha\beta} + e_s\, g_{\alpha\beta}\right). \tag{68}$$

Multiplying and using relations $e^{\alpha}{}_{\alpha} = 0$ and $t^{\alpha}{}_{\alpha} = 0$ from the definition of the strain and stress deviators, we obtain

$$\mathcal{E} = \frac{1}{2}\left(4\, t_s\, e_s + t^{\alpha\beta} e_{\alpha\beta}\right). \tag{69}$$

Using (39) to express the stresses in terms of the strains, this expression becomes

$$\mathcal{E} = \frac{1}{2}\,\kappa_0\,\varepsilon^2 + \mu_0\, e^{\alpha\beta} e_{\alpha\beta} \tag{70}$$

where the Lamé elastic constant of the spacetime continuum μ_0 is the shear modulus (the resistance of the continuum to *distortions*) and κ_0 is the bulk modulus (the resistance of the continuum to *dilatations*). Alternatively, again using (39) to express the strains in terms of the stresses, this expression can be written as

$$\mathcal{E} = \frac{1}{2\,\kappa_0}\, t_s^2 + \frac{1}{4\mu_0}\, t^{\alpha\beta} t_{\alpha\beta}. \tag{71}$$

§8.1 Physical interpretation of the strain energy density. The strain energy density is separated into two terms: the first one expresses the dilatation energy density (the "mass" longitudinal term) while the second one expresses the distortion energy density (the "massless" transverse term):

$$\mathcal{E} = \mathcal{E}_{\parallel} + \mathcal{E}_{\perp}, \tag{72}$$

where

$$\mathcal{E}_{\parallel} = \frac{1}{2}\,\kappa_0\,\varepsilon^2 \equiv \frac{1}{2\,\kappa_0}\, t^2 \tag{73}$$

and

$$\mathcal{E}_{\perp} = \mu_0\, e^{\alpha\beta} e_{\alpha\beta} \equiv \frac{1}{4\mu_0}\, t^{\alpha\beta} t_{\alpha\beta}. \tag{74}$$

Using (32) into (73), we obtain

$$\mathcal{E}_{\parallel} = \frac{1}{32\,\kappa_0}\left(\rho c^2\right)^2. \tag{75}$$

The rest-mass energy density divided by the bulk modulus κ_0, and the transverse energy density divided by the shear modulus μ_0, have dimensions of energy density as expected.

Multiplying (71) by $32\kappa_0$ and using (75), we obtain

$$32\,\kappa_0\,\mathcal{E} = \rho^2 c^4 + 8\,\frac{\kappa_0}{\mu_0}\,t^{\alpha\beta}t_{\alpha\beta}\,. \tag{76}$$

Noting that $t^{\alpha\beta}t_{\alpha\beta}$ is quadratic in structure, we see that this equation is similar to the energy relation of Special Relativity [22, see p. 51] for energy density

$$\hat{E}^2 = \rho^2 c^4 + \hat{p}^{\,2} c^2, \tag{77}$$

where $\hat{E}$ is the total energy density and $\hat{p}$ the momentum density.

The quadratic structure of the energy relation of Special Relativity is thus found to be present in the Elastodynamics of the Spacetime Continuum. Equations (76) and (77) also imply that the kinetic energy pc is carried by the distortion part of the deformation, while the dilatation part carries only the rest mass energy.

This observation is in agreement with photons which are massless ($\mathcal{E}_\parallel = 0$), as will be shown on Page 250, but still carry kinetic energy in the transverse electromagnetic wave distortions ($\mathcal{E}_\perp = \frac{1}{4\mu_0}\,t^{\alpha\beta}t_{\alpha\beta}$).

§9. Theory of Electromagnetism from *STCED*

§9.1. Electromagnetic field strength. In the Elastodynamics of the Spacetime Continuum, the antisymmetric rotation tensor $\omega^{\mu\nu}$ is given by (42), viz.

$$\omega^{\mu\nu} = \frac{1}{2}\,(u^{\mu;\nu} - u^{\nu;\mu}) \tag{78}$$

where u^μ is the displacement of an infinitesimal element of the spacetime continuum from its unstrained position x^μ. This tensor has the same structure as the electromagnetic field tensor $F^{\mu\nu}$ [33, see p. 550]:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{79}$$

where A^μ is the electromagnetic potential four-vector $(\phi, \boldsymbol{A})$, ϕ is the scalar potential and $\boldsymbol{A}$ the vector potential.

Identifying the rotation tensor $\omega^{\mu\nu}$ with the electromagnetic field-strength tensor according to

$$F^{\mu\nu} = \varphi_0\,\omega^{\mu\nu} \tag{80}$$

leads to the relation

$$A^\mu = -\frac{1}{2}\,\varphi_0\,u_\perp^\mu\,, \tag{81}$$

where the symbolic subscript $\perp$ of the displacement u^μ indicates that the relation holds for a transverse displacement (orthogonal to the direction of motion). The constant of proportionality φ_0 will be referred to as the "*STC* electromagnetic shearing potential constant".

Due to the difference in the definition of $\omega^{\mu\nu}$ and $F^{\mu\nu}$ with respect to their indices, a negative sign is introduced, and is attributed to (81). This relation provides a physical explanation of the electromagnetic potential: it arises from transverse (shearing) displacements of the spacetime continuum, in contrast to mass which arises from longitudinal (dilatational) displacements of the spacetime continuum. Sheared spacetime is manifested as electromagnetic potentials and fields.

§9.2. Maxwell's equations and the current density four-vector. Taking the divergence of the rotation tensor of (78), gives

$$\omega^{\mu\nu}{}_{;\mu} = \frac{1}{2}\left(u^{\mu;\nu}{}_{\mu} - u^{\nu;\mu}{}_{\mu}\right). \tag{82}$$

Recalling (50), viz.

$$\mu_0\, u^{\nu;\mu}{}_{\mu} + (\mu_0 + \lambda_0)\, u^{\mu}{}_{;\mu}{}^{\nu} = -X^{\nu},$$

where X^ν is the volume force and λ_0 and μ_0 are the Lamé elastic constants of the spacetime continuum, substituting for $u^{\nu;\mu}{}_{\mu}$ from (50) into (82), interchanging the order of partial differentiation in $u^{\mu;\nu}{}_{\mu}$ in (82), and using the relation $u^{\mu}{}_{;\mu} = \varepsilon^{\mu}{}_{\mu} = \varepsilon$ from (44), we obtain

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \frac{1}{2\mu_0}\,X^{\nu}. \tag{83}$$

As seen previously on Page 239, in the macroscopic local case, the volume force X^ν is set equal to zero to obtain the macroscopic relation

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu}. \tag{84}$$

Using (80) and comparing with the covariant form of Maxwell's equations [34, see pp. 42–43]

$$F^{\mu\nu}{}_{;\mu} = \mu_{\rm em}\, j^{\nu}, \tag{85}$$

where j^ν is the current density four-vector $(c\varrho, \boldsymbol{j})$, ϱ is the charge density scalar, and $\boldsymbol{j}$ is the current density vector, we obtain the relation

$$j^{\nu} = \frac{\varphi_0}{\mu_{\rm em}}\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu}. \tag{86}$$

This relation provides a physical explanation of the current density four-vector: it arises from the 4-gradient of the volume dilatation of the spacetime continuum. A corollary of this relation is that massless (transverse) waves cannot carry an electric charge or produce a current.

Substituting for j^ν from (86) in the relation [35, see p. 94]

$$j^\nu j_\nu = \varrho^2 c^2, \tag{87}$$

we obtain the expression for the charge density

$$\varrho = \frac{1}{2}\,\frac{\varphi_0}{\mu_{\rm em} c}\,\frac{2\mu_0+\lambda_0}{2\mu_0}\,\sqrt{\varepsilon^{;\nu}\varepsilon_{;\nu}} \tag{88}$$

or, using the relation $c = 1/\sqrt{\epsilon_{\rm em}\mu_{\rm em}}$,

$$\varrho = \frac{1}{2}\,\varphi_0\,\epsilon_{\rm em}\,c\,\frac{2\mu_0+\lambda_0}{2\mu_0}\,\sqrt{\varepsilon^{;\nu}\varepsilon_{;\nu}}\ . \tag{89}$$

Up to now, our identification of the rotation tensor $\omega^{\mu\nu}$ of the Elastodynamics of the Spacetime Continuum with the electromagnetic field-strength tensor $F^{\mu\nu}$ has generated consistent results, with no contradictions.

§9.3. The Lorentz condition. The Lorentz condition can be derived directly from the theory. Taking the divergence of (81), we obtain

$$A^\mu{}_{;\mu} = -\frac{1}{2}\,\varphi_0\, u_\perp{}^\mu{}_{;\mu}\,. \tag{90}$$

From (45), (90) simplifies to

$$A^\mu{}_{;\mu} = 0\,. \tag{91}$$

The Lorentz condition is thus obtained directly from the theory. The reason for the value of zero is that transverse displacements are massless because such displacements arise from a change of shape (distortion) of the spacetime continuum, not a change of volume (dilatation).

§9.4. Four-vector potential. Substituting (81) into (82) and rearranging terms, we obtain the equation

$$\nabla^2 A^\nu - A^{\mu;\nu}{}_\mu = \varphi_0 \omega^{\mu\nu}{}_{;\mu} \tag{92}$$

and, using (80) and (85), this equation becomes

$$\nabla^2 A^\nu - A^{\mu;\nu}{}_\mu = \mu_{\rm em}\, j^\nu. \tag{93}$$

Interchanging the order of partial differentiation in the term $A^{\mu;\nu}{}_\mu$ and using the Lorentz condition of (91), we obtain the well-known wave equation for the four-vector potential [34, see pp. 42–43]

$$\nabla^2 A^\nu = \mu_{\rm em}\, j^\nu. \tag{94}$$

The results we obtain are thus consistent with the macroscopic theory of Electromagnetism, with no contradictions.

§10. Electromagnetism and the volume force X^ν. We now investigate the impact of the volume force X^ν on the equations of Electromagnetism. Recalling (83), Maxwell's equation in terms of the rotation tensor is given by

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\, \varepsilon^{;\nu} + \frac{1}{2\mu_0}\, X^\nu. \tag{95}$$

Substituting for $\omega^{\mu\nu}$ from (80), this equation becomes

$$F^{\mu\nu}{}_{;\mu} = \varphi_0\, \frac{2\mu_0 + \lambda_0}{2\mu_0}\, \varepsilon^{;\nu} + \frac{\varphi_0}{2\mu_0}\, X^\nu. \tag{96}$$

The additional X^ν term can be allocated in one of two ways:

1) either j^ν remains unchanged as given by (86) and the expression for $F^{\mu\nu}{}_{;\mu}$ has an additional term as developed in the first section below,
2) or $F^{\mu\nu}{}_{;\mu}$ remains unchanged as given by (85) and the expression for j^ν has an additional term as developed in the second section below.

Option 2 is shown in the following derivation to be the logically consistent approach.

§10.1. j^ν unchanged (contradiction). Using (86) (j^ν unchanged) into (96), Maxwell's equation becomes

$$F^{\mu\nu}{}_{;\mu} = \mu_{\rm em}\, j^\nu + \frac{\varphi_0}{2\mu_0}\, X^\nu. \tag{97}$$

Using (95) into (92) and making use of the Lorentz condition, the wave equation for the four-vector potential becomes

$$\nabla^2 A^\nu - \frac{\varphi_0}{2\mu_0}\, X^\nu = \mu_{\rm em}\, j^\nu. \tag{98}$$

In this case, the equations for $F^{\mu\nu}{}_{;\mu}$ and A^ν both contain an additional term proportional to X^ν.

We show that this option is not logically consistent as follows. Using (86) into the continuity condition for the current density [34]

$$\partial_\nu j^\nu = 0 \tag{99}$$

yields the expression

$$\nabla^2 \varepsilon = 0\,. \tag{100}$$

This equation is valid in the macroscopic case where $X^\nu = 0$, but disagrees with the general case (non-zero X^ν) given by (57), viz.

$$(2\mu_0 + \lambda_0)\nabla^2 \varepsilon = -X^\nu{}_{;\nu}\,.$$

This analysis leads to a contradiction and consequently is not valid.

§10.2. $F^{\mu\nu}{}_{;\mu}$ unchanged (logically consistent). Proper treatment of the general case requires that the current density four-vector be proportional to the RHS of (96) as follows ($F^{\mu\nu}{}_{;\mu}$ unchanged):

$$\mu_{\mathrm{em}}\, j^\nu = \varphi_0\, \frac{2\mu_0 + \lambda_0}{2\mu_0}\, \varepsilon^{;\nu} + \frac{\varphi_0}{2\mu_0}\, X^\nu. \tag{101}$$

This yields the following general form of the current density four-vector:

$$j^\nu = \frac{1}{2}\, \frac{\varphi_0}{\mu_{\mathrm{em}}\, \mu_0} \left[(2\mu_0 + \lambda_0)\, \varepsilon^{;\nu} + X^\nu\right]. \tag{102}$$

Using this expression in the continuity condition for the current density given by (99) yields (57) as required.

Using (102) into (96) yields the same covariant form of the Maxwell equations as in the macroscopic case:

$$F^{\mu\nu}{}_{;\mu} = \mu_{\mathrm{em}}\, j^\nu \tag{103}$$

and the same four-vector potential equation

$$\nabla^2 A^\nu = \mu_{\mathrm{em}}\, j^\nu \tag{104}$$

in the Lorentz gauge.

§10.3. Homogeneous Maxwell equation. The validity of this analysis can be further demonstrated from the homogeneous Maxwell equation [34]

$$\partial^\alpha F^{\beta\gamma} + \partial^\beta F^{\gamma\alpha} + \partial^\gamma F^{\alpha\beta} = 0\,. \tag{105}$$

Taking the divergence of this equation over α,

$$\partial_\alpha \partial^\alpha F^{\beta\gamma} + \partial_\alpha \partial^\beta F^{\gamma\alpha} + \partial_\alpha \partial^\gamma F^{\alpha\beta} = 0\,. \tag{106}$$

Interchanging the order of differentiation in the last two terms and making use of (103) and the antisymmetry of $F^{\mu\nu}$, we obtain

$$\nabla^2 F^{\beta\gamma} + \mu_{\rm em}\,(j^{\beta;\gamma} - j^{\gamma;\beta}) = 0\,. \tag{107}$$

Substituting for j^{ν} from (102),

$$\nabla^2 F^{\beta\gamma} = -\frac{\varphi_0}{2\mu_0}\left[(2\mu_0+\lambda_0)(\varepsilon^{;\beta\gamma} - \varepsilon^{;\gamma\beta}) + (X^{\beta;\gamma} - X^{\gamma;\beta})\right]. \tag{108}$$

Equation (64), viz.

$$\mu_0\,\nabla^2\varepsilon^{\mu\nu} + (\mu_0+\lambda_0)\,\varepsilon^{;\mu\nu} = -X^{(\mu;\nu)}$$

shows that $\varepsilon^{;\mu\nu}$ is a symmetrical tensor. Consequently the difference term $(\varepsilon^{;\beta\gamma} - \varepsilon^{;\gamma\beta})$ disappears and (108) becomes

$$\nabla^2 F^{\beta\gamma} = -\frac{\varphi_0}{2\mu_0}\,(X^{\beta;\gamma} - X^{\gamma;\beta})\,. \tag{109}$$

Expressing $F^{\mu\nu}$ in terms of $\omega^{\mu\nu}$ using (80), the resulting equation is identical to (61), viz.

$$\mu_0\,\nabla^2\omega^{\mu\nu} = -X^{[\mu;\nu]}$$

confirming the validity of this analysis of Electromagnetism including the volume force.

Equations (102) to (104) are the self-consistent electromagnetic equations derived from the Elastodynamics of the Spacetime Continuum with the volume force. In conclusion, Maxwell's equations remain unchanged. The current density four-vector is the only quantity affected by the volume force, with the addition of a second term proportional to the volume force.

It is interesting to note that the current density obtained from the quantum mechanical Klein-Gordon equation with an electromagnetic field also consists of the sum of two terms [36, see p. 35].

§11. Electromagnetic strain energy density. The strain energy density of the electromagnetic energy-momentum stress tensor is calculated. Starting from the symmetric electromagnetic stress tensor [34, see pp. 64–66], which has the form

$$\Theta^{\mu\nu} = \frac{1}{\mu_{\rm em}}\left(F^{\mu}{}_{\alpha}F^{\alpha\nu} + \frac{1}{4}\,g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \equiv \sigma^{\mu\nu}, \tag{110}$$

with $g^{\mu\nu} = \eta^{\mu\nu}$ of signature $(+\,-\,-\,-)$, and the field-strength tensor components [34, see p. 43]

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & B_z & -B_y \\ E_y/c & -B_z & 0 & B_x \\ E_z/c & B_y & -B_x & 0 \end{pmatrix} \tag{111}$$

and

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{pmatrix}, \tag{112}$$

we obtain $\sigma^{\mu\nu} \equiv \Theta^{\mu\nu}$ which is a generalization of the σ^{ij} Maxwell stress tensor (here S^j is the Poynting vector, see [34, p. 66], [37, p. 141])

$$\begin{aligned} \sigma^{00} &= \frac{1}{2}\left(\epsilon_{\rm em} E^2 + \frac{1}{\mu_{\rm em}} B^2\right) = \frac{1}{2}\,\epsilon_{\rm em}\left(E^2 + c^2 B^2\right), \\ \sigma^{0j} &= \sigma^{j0} = \frac{1}{c\,\mu_{\rm em}}\,(E\times B)^j = \epsilon_{\rm em}\, c\,(E\times B)^j = \frac{1}{c}\, S^j, \\ \sigma^{jk} &= -\left(\epsilon_{\rm em} E^j E^k + \frac{1}{\mu_{\rm em}} B^j B^k\right) + \frac{1}{2}\,\delta^{jk}\left(\epsilon_{\rm em} E^2 + \frac{1}{\mu_{\rm em}} B^2\right) = \\ &= -\,\epsilon_{\rm em}\left[\left(E^j E^k + c^2 B^j B^k\right) - \frac{1}{2}\,\delta^{jk}\left(E^2 + c^2 B^2\right)\right]. \end{aligned} \tag{113}$$

Hence the electromagnetic stress tensor is given by [34, see p. 66]:

$$\sigma^{\mu\nu} = \begin{pmatrix} \frac{1}{2}\,\epsilon_{\rm em}\left(E^2 + c^2 B^2\right) & S_x/c & S_y/c & S_z/c \\ S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{pmatrix}, \tag{114}$$

where σ^{ij} is the Maxwell stress tensor. Using $\sigma_{\alpha\beta} = \eta_{\alpha\mu}\eta_{\beta\nu}\,\sigma^{\mu\nu}$ to lower the indices of $\sigma^{\mu\nu}$, we obtain

$$\sigma_{\mu\nu} = \begin{pmatrix} \frac{1}{2}\,\epsilon_{\rm em}\left(E^2 + c^2 B^2\right) & -S_x/c & -S_y/c & -S_z/c \\ -S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ -S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ -S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{pmatrix}. \tag{115}$$

§11.1. Calculation of the longitudinal (mass) term. The mass term is calculated from (73) and (38):

$$\mathcal{E}_{\parallel} = \frac{1}{2\,\kappa_0}\, t_s^2 = \frac{1}{32\,\kappa_0}\,(\sigma^{\alpha}{}_{\alpha})^2. \tag{116}$$

The term $\sigma^{\alpha}{}_{\alpha}$ is calculated from:

$$\left.\begin{aligned} \sigma^{\alpha}{}_{\alpha} &= \eta_{\alpha\beta}\,\sigma^{\alpha\beta} \\ &= \eta_{\alpha 0}\,\sigma^{\alpha 0} + \eta_{\alpha 1}\,\sigma^{\alpha 1} + \eta_{\alpha 2}\,\sigma^{\alpha 2} + \eta_{\alpha 3}\,\sigma^{\alpha 3} \\ &= \eta_{00}\,\sigma^{00} + \eta_{11}\,\sigma^{11} + \eta_{22}\,\sigma^{22} + \eta_{33}\,\sigma^{33} \end{aligned}\right\}. \tag{117}$$

Substituting from (114) and the metric $\eta^{\mu\nu}$ of signature $(+\,-\,-\,-)$, we obtain:

$$\sigma^{\alpha}{}_{\alpha} = \frac{1}{2}\,\epsilon_{\rm em}\left(E^2 + c^2B^2\right) + \sigma_{xx} + \sigma_{yy} + \sigma_{zz}\,. \tag{118}$$

Substituting from (113), this expands to:

$$\begin{aligned} \sigma^{\alpha}{}_{\alpha} = \frac{1}{2}\,\epsilon_{\rm em}\left(E^2 + c^2B^2\right) + \epsilon_{\rm em}\left(E_x^2 + c^2B_x^2\right) + \\ + \,\epsilon_{\rm em}\left(E_y^2 + c^2B_y^2\right) + \epsilon_{\rm em}\left(E_z^2 + c^2B_z^2\right) - \\ - \frac{3}{2}\,\epsilon_{\rm em}\left(E^2 + c^2B^2\right) \end{aligned} \tag{119}$$

and further,

$$\begin{aligned} \sigma^{\alpha}{}_{\alpha} = \frac{1}{2}\,\epsilon_{\rm em}\left(E^2 + c^2B^2\right) + \epsilon_{\rm em}\left(E^2 + c^2B^2\right) - \\ - \frac{3}{2}\,\epsilon_{\rm em}\left(E^2 + c^2B^2\right). \end{aligned} \tag{120}$$

Hence

$$\sigma^{\alpha}{}_{\alpha} = 0 \tag{121}$$

and, substituting into (116),

$$\mathcal{E}_{\parallel} = 0 \tag{122}$$

as expected [34, see pp. 64–66]. This derivation thus shows that the rest-mass energy density of the photon is 0.

§11.2. Calculation of the transverse (massless) term. The transverse term is calculated from (74), viz.

$$\mathcal{E}_{\perp} = \frac{1}{4\mu_0}\, t^{\alpha\beta}t_{\alpha\beta}\,. \tag{123}$$

Given that $t_s = \frac{1}{4}\,\sigma^\alpha{}_\alpha = 0$, then $t^{\alpha\beta} = \sigma^{\alpha\beta}$ and the terms $\sigma^{\alpha\beta}\sigma_{\alpha\beta}$ are calculated from the components of the electromagnetic stress tensors of (114) and (115). Substituting for the diagonal elements and making use of the symmetry of the Poynting component terms and of the Maxwell stress tensor terms from (114) and (115), this expands to:

$$\begin{aligned}
\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \frac{1}{4}\,\epsilon_{\text{em}}^2 \left(E^2 + c^2B^2\right)^2 + \\
&+ \epsilon_{\text{em}}^2 \left[\left(E_xE_x + c^2B_xB_x\right) - \frac{1}{2}\,\left(E^2 + c^2B^2\right)\right]^2 + \\
&+ \epsilon_{\text{em}}^2 \left[\left(E_yE_y + c^2B_yB_y\right) - \frac{1}{2}\,\left(E^2 + c^2B^2\right)\right]^2 + \\
&+ \epsilon_{\text{em}}^2 \left[\left(E_zE_z + c^2B_zB_z\right) - \frac{1}{2}\,\left(E^2 + c^2B^2\right)\right]^2 - \\
&- 2\left(\frac{S_x}{c}\right)^2 - 2\left(\frac{S_y}{c}\right)^2 - 2\left(\frac{S_z}{c}\right)^2 + \\
&+ 2\,(\sigma_{xy})^2 + 2\,(\sigma_{yz})^2 + 2\,(\sigma_{zx})^2. \qquad (124)
\end{aligned}$$

The E-B terms expand to:

$$\begin{aligned}
\text{EBterms} = \epsilon_{\text{em}}^2 \Bigg[\frac{1}{4}\,\left(E^2 + c^2B^2\right)^2 + \left(E_x^2 + c^2B_x^2\right)^2 - \\
&- \left(E_x^2 + c^2B_x^2\right)\left(E^2 + c^2B^2\right) + \left(E_y^2 + c^2B_y^2\right)^2 - \\
&- \left(E_y^2 + c^2B_y^2\right)\left(E^2 + c^2B^2\right) + \left(E_z^2 + c^2B_z^2\right)^2 - \\
&- \left(E_z^2 + c^2B_z^2\right)\left(E^2 + c^2B^2\right) + \frac{3}{4}\,\left(E^2 + c^2B^2\right)^2 \Bigg]. \qquad (125)
\end{aligned}$$

Simplifying,

$$\begin{aligned}
\text{EBterms} = \epsilon_{\text{em}}^2 \Big[\left(E^2 + c^2B^2\right)^2 - \big(E_x^2 + c^2B_x^2 + \\
&+ E_y^2 + c^2B_y^2 + E_z^2 + c^2B_z^2\big)\left(E^2 + c^2B^2\right) + \\
&+ \left(E_x^2 + c^2B_x^2\right)^2 + \left(E_y^2 + c^2B_y^2\right)^2 + \left(E_z^2 + c^2B_z^2\right)^2 \Big], \qquad (126)
\end{aligned}$$

which gives

$$\begin{aligned}
\text{EBterms} = \epsilon_{\text{em}}^2 \Big[\left(E^2 + c^2B^2\right)^2 - \left(E^2 + c^2B^2\right)^2 + \\
&+ \left(E_x^2 + c^2B_x^2\right)^2 + \left(E_y^2 + c^2B_y^2\right)^2 + \left(E_z^2 + c^2B_z^2\right)^2 \Big], \qquad (127)
\end{aligned}$$

and finally

$$\text{EBterms} = \epsilon_{\text{em}}^2 \Big[\left({E_x}^4 + {E_y}^4 + {E_z}^4\right) + c^4 \left({B_x}^4 + {B_y}^4 + {B_z}^4\right) + + 2\,c^2 \left(E_x^2 B_x^2 + E_y^2 B_y^2 + E_z^2 B_z^2\right) \Big]. \tag{128}$$

Including the E-B terms in (124), substituting from (113), expanding the Poynting vector and rearranging, we obtain

$$\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \epsilon_{\text{em}}^2 \Big[\left({E_x}^4 + {E_y}^4 + {E_z}^4\right) + c^4\left({B_x}^4 + {B_y}^4 + {B_z}^4\right) + + 2\,c^2 \left(E_x^2 B_x^2 + E_y^2 B_y^2 + E_z^2 B_z^2\right) \Big] - - 2\,\epsilon_{\text{em}}^2 c^2 \Big[\left(E_y B_z - E_z B_y\right)^2 + \left(-E_x B_z + E_z B_x\right)^2 + + \left(E_x B_y - E_y B_x\right)^2 \Big] + 2\,\epsilon_{\text{em}}^2 \Big[\left(E_x E_y + c^2 B_x B_y\right)^2 + + \left(E_y E_z + c^2 B_y B_z\right)^2 + \left(E_z E_x + c^2 B_z B_x\right)^2 \Big]. \tag{129}$$

Expanding the quadratic expressions,

$$\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \epsilon_{\text{em}}^2 \Big[\left({E_x}^4 + {E_y}^4 + {E_z}^4\right) + c^4\left({B_x}^4 + {B_y}^4 + {B_z}^4\right) + + 2\,c^2 \left(E_x^2 B_x^2 + E_y^2 B_y^2 + E_z^2 B_z^2\right) \Big] - - 2\,\epsilon_{\text{em}}^2 c^2 \Big[E_x^2 B_y^2 + E_y^2 B_z^2 + E_z^2 B_x^2 + B_x^2 E_y^2 + + B_y^2 E_z^2 + B_z^2 E_x^2 - 2\left(E_x E_y B_x B_y + E_y E_z B_y B_z + + E_z E_x B_z B_x\right) \Big] + 2\,\epsilon_{\text{em}}^2 \Big[\left(E_x^2 E_y^2 + E_y^2 E_z^2 + E_z^2 E_x^2\right) + + 2\,c^2 \left(E_x E_y B_x B_y + E_y E_z B_y B_z + E_z E_x B_z B_x\right) + + c^4 \left(B_x^2 B_y^2 + B_y^2 B_z^2 + B_z^2 B_x^2\right) \Big]. \tag{130}$$

Grouping the terms in powers of c together,

$$\frac{1}{\epsilon_{\text{em}}^2}\,\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \Big[\left({E_x}^4 + {E_y}^4 + {E_z}^4\right) + 2\big(E_x^2 E_y^2 + + E_y^2 E_z^2 + E_z^2 E_x^2\big)\Big] + 2\,c^2 \Big[\left(E_x^2 B_x^2 + E_y^2 B_y^2 + + E_z^2 B_z^2\right) - \big(E_x^2 B_y^2 + E_y^2 B_z^2 + E_z^2 B_x^2 +$$

$$+ B_x^2 E_y^2 + B_y^2 E_z^2 + B_z^2 E_x^2\big) + 4\big(E_x E_y B_x B_y +$$
$$+ E_y E_z B_y B_z + E_z E_x B_z B_x\big)\Big] + c^4 \Big[\big({B_x}^4 + {B_y}^4 + {B_z}^4\big) +$$
$$+ 2\left(B_x^2 B_y^2 + B_y^2 B_z^2 + B_z^2 B_x^2\right)\Big]. \tag{131}$$

Simplifying,

$$\frac{1}{\epsilon_{\rm em}^2}\,\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \left(E_x^2 + E_y^2 + E_z^2\right)^2 +$$
$$+ 2c^2\left(E_x^2 + E_y^2 + E_z^2\right)\left(B_x^2 + B_y^2 + B_z^2\right) -$$
$$- 2c^2\Big[2\big(E_x^2 B_y^2 + E_y^2 B_z^2 + E_z^2 B_x^2 +$$
$$+ B_x^2 E_y^2 + B_y^2 E_z^2 + B_z^2 E_x^2\big) - 4\big(E_x E_y B_x B_y +$$
$$+ E_y E_z B_y B_z + E_z E_x B_z B_x\big)\Big] + c^4\left(B_x^2 + B_y^2 + B_z^2\right)^2, \tag{132}$$

which is further simplified to

$$\frac{1}{\epsilon_{\rm em}^2}\,\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \left(E^4 + 2c^2E^2B^2 + c^4B^4\right) - 4c^2\Big[\left(E_yB_z - B_yE_z\right)^2 +$$
$$+ \left(E_zB_x - B_zE_x\right)^2 + \left(E_xB_y - B_xE_y\right)^2\Big]. \tag{133}$$

Making use of the definition of the Poynting vector from (113), we obtain

$$\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \epsilon_{\rm em}^2\,\left(E^2 + c^2B^2\right)^2 -$$
$$- 4\epsilon_{\rm em}^2 c^2\left[(E\times B)_x^2 + (E\times B)_y^2 + (E\times B)_z^2\right] \tag{134}$$

and finally

$$\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \epsilon_{\rm em}^2\,\left(E^2 + c^2B^2\right)^2 - \frac{4}{c^2}\,\left(S_x^2 + S_y^2 + S_z^2\right). \tag{135}$$

Substituting in (123), the transverse term becomes

$$\mathcal{E}_\perp = \frac{1}{4\mu_0}\,\left[\epsilon_{\rm em}^2\,\left(E^2 + c^2B^2\right)^2 - \frac{4}{c^2}\,S^2\right] \tag{136}$$

or

$$\mathcal{E}_\perp = \frac{1}{\mu_0}\,\left[U_{\rm em}^2 - \frac{1}{c^2}\,S^2\right], \tag{137}$$

where $U_{\rm em} = \frac{1}{2}\,\epsilon_{\rm em}(E^2 + c^2B^2)$ is the electromagnetic field energy density.

§11.3. Electromagnetic field strain energy density and the photon. $\boldsymbol{S}$ is the electromagnetic energy flux along the direction of propagation [34, p. 62]. As noted by Feynman [38, pp. 27-1–27-2], local conservation of the electromagnetic field energy can be written as

$$-\frac{\partial U_{\mathrm{em}}}{\partial t} = \nabla \cdot \boldsymbol{S}, \tag{138}$$

where the term $\boldsymbol{E} \cdot \boldsymbol{j}$ representing the work done on the matter inside the volume is 0 in the absence of charges (due to the absence of mass). By analogy with the current density four-vector $j^{\nu} = (c\varrho, \boldsymbol{j})$, where ϱ is the charge density, and $\boldsymbol{j}$ is the current density vector, which obeys a similar conservation relation, we define the Poynting four-vector

$$S^{\nu} = (c\,U_{\mathrm{em}},\, \boldsymbol{S}), \tag{139}$$

where U_{em} is the electromagnetic field energy density, and $\boldsymbol{S}$ is the Poynting vector. Furthermore, as per (138), S^{ν} satisfies

$$\partial_{\nu} S^{\nu} = 0. \tag{140}$$

Using definition (139) in (137), that equation becomes

$$\mathcal{E}_{\perp} = \frac{1}{\mu_0 c^2}\, S_{\nu} S^{\nu}. \tag{141}$$

The indefiniteness of the location of the field energy referred to by Feynman [38, see p. 27-1] is thus resolved: the electromagnetic field energy resides in the distortions (transverse displacements) of the spacetime continuum.

Hence the invariant electromagnetic strain energy density is given by

$$\mathcal{E} = \frac{1}{\mu_0 c^2}\, S_{\nu} S^{\nu} \tag{142}$$

where we have used $\rho = 0$ as per (121). This confirms that S^{ν} as defined in (139) is a four-vector.

It is surprising that a longitudinal energy flow term is part of the transverse strain energy density i.e. $S^2/\mu_0 c^2$ in (137). We note that this term arises from the time-space components of (114) and (115) and can be seen to correspond to the transverse displacements along the *time-space* planes which are folded along the direction of propagation in 3-space as the Poynting vector. The electromagnetic field energy density term U_{em}^2/μ_0 and the electromagnetic field energy flux term $S^2/\mu_0 c^2$ are thus combined into the transverse strain energy density.

The negative sign arises from the signature $(+---)$ of the metric tensor $\eta^{\mu\nu}$.

This longitudinal electromagnetic energy flux is massless as it is due to distortion, not dilatation, of the spacetime continuum. However, because this energy flux is along the direction of propagation (i.e. longitudinal), it gives rise to the particle aspect of the electromagnetic field, the photon. As shown in [39, see pp. 174–175] [40, see p. 58], in the quantum theory of electromagnetic radiation, an intensity operator derived from the Poynting vector has, as expectation value, photons in the direction of propagation.

This implies that the $(pc)^2$ term of the energy relation of Special Relativity needs to be separated into transverse and longitudinal massless terms as follows:

$$\hat{E}^2 = \underbrace{\rho^2 c^4}_{\mathcal{E}_\parallel} + \underbrace{\hat{p}_\parallel^2 c^2 + \hat{p}_\perp^2 c^2}_{massless\ \mathcal{E}_\perp} \tag{143}$$

where $\hat{p}_\parallel$ is the massless longitudinal momentum density. (137) shows that the electromagnetic field energy density term U_{em}^2/μ_0 is reduced by the electromagnetic field energy flux term $S^2/\mu_0 c^2$ in the transverse strain energy density, due to photons propagating in the longitudinal direction. Hence we can write [40, see p. 58]

$$\int_V \frac{1}{\mu_0 c^2}\, S^2\, dV = \sum_k n_k\, h\, \nu_k\,. \tag{144}$$

where h is Planck's constant and n_k is the number of photons of frequency ν_k. Thus the kinetic energy is carried by the distortion part of the deformation, while the dilatation part carries only the rest-mass energy, which in this case is 0.

As shown in (75), (76) and (77), the constant of proportionality to transform energy density squared ($\hat{E}^2$) into strain energy density ($\mathcal{E}$) is $1/(32\,\kappa_0)$:

$$\mathcal{E}_\parallel = \frac{1}{32\,\kappa_0} \left(\rho c^2\right)^2, \tag{145}$$

$$\mathcal{E} = \frac{1}{32\,\kappa_0}\, \hat{E}^2, \tag{146}$$

$$\mathcal{E}_\perp = \frac{1}{32\,\kappa_0} \left(\hat{p}_\parallel^2 c^2 + \hat{p}_\perp^2 c^2\right) = \frac{1}{4\,\mu_0}\, t^{\alpha\beta} t_{\alpha\beta}. \tag{147}$$

Substituting (137) into (147), we obtain

$$\mathcal{E}_\perp = \frac{1}{32\,\kappa_0} \left(\hat{p}_\parallel^2 c^2 + \hat{p}_\perp^2 c^2\right) = \frac{1}{\mu_0} \left(U_{\mathrm{em}}^2 - \frac{1}{c^2}\, S^2\right) \tag{148}$$

and

$$\hat{p}_{\|}^{2}c^{2}+\hat{p}_{\perp}^{2}c^{2}=\frac{32\,\kappa_0}{\mu_0}\left(U_{\rm em}^{2}-\frac{1}{c^{2}}\,S^{2}\right). \tag{149}$$

This suggests that

$$\mu_0 = 32\,\kappa_0\,, \tag{150}$$

to obtain the relation

$$\hat{p}_{\|}^{2}c^{2}+\hat{p}_{\perp}^{2}c^{2}=U_{\rm em}^{2}-\frac{1}{c^{2}}\,S^{2}. \tag{151}$$

§12. Linear elastic volume force. The volume (or body) force X^ν has been introduced in the equilibrium dynamic equation of the *STC* in (46) on Page 239 viz.

$$T^{\mu\nu}{}_{;\mu} = -X^\nu. \tag{152}$$

Comparison with the corresponding general relativistic expression showed that the volume force is equal to zero at the macroscopic local level. Indeed, as pointed out by Wald [14, see p. 286], in General Relativity the local energy density of matter as measured by a given observer is well-defined, and the relation

$$T^{\mu\nu}{}_{;\mu} = 0 \tag{153}$$

can be taken as expressing local conservation of the energy-momentum of matter.

It was also pointed out in that section that at the microscopic level, energy is known to be conserved only within the limits of the Heisenberg Uncertainty Principle, suggesting that the volume force may be very small, but not exactly zero. This is analogous to quantum theory where Planck's constant h must be taken into consideration at the microscopic level while at the macroscopic level, the limit $h \to 0$ holds.

In this section, we investigate the volume force and its impact on the equations of the Elastodynamics of the Spacetime Continuum. First we consider a linear elastic volume force. Based on the results obtained, we will then consider a variation of that linear elastic volume force based on the Klein-Gordon quantum mechanical current density.

We investigate a volume force that consists of an elastic linear force in a direction opposite to the displacements. This is the well-known elastic "spring" force

$$X^\nu = k_0\, u^\nu, \tag{154}$$

where k_0 is the postulated elastic force constant of the spacetime continuum volume force. (154) is positive as the volume force X^ν is defined positive in the direction opposite to the displacement [18]. Introduction of this volume force into our previous analysis on Page 239 yields the following relations.

§12.1. Displacement wave equation. Substituting (154) into (51), viz.

$$\mu_0 \nabla^2 u^\nu + (\mu_0 + \lambda_0)\, \varepsilon^{;\nu} = -X^\nu, \tag{155}$$

the dynamic equation in terms of displacements becomes

$$\mu_0 \nabla^2 u^\nu + (\mu_0 + \lambda_0)\, \varepsilon^{;\nu} = -\,k_0\, u^\nu. \tag{156}$$

This equation can be rewritten as

$$\nabla^2 u^\nu + \frac{k_0}{\mu_0}\, u^\nu = -\frac{\mu_0 + \lambda_0}{\mu_0}\, \varepsilon^{;\nu}. \tag{157}$$

This displacement equation is similar to a nonhomogeneous Klein-Gordon equation for a vector field, with a source term.

§12.2. Wave equations. Additional wave equations as shown on Page 240 can be derived from this volume force.

§12.2.1. Dilatational (longitudinal) wave equation. Substituting (154) into (57), viz.

$$(2\mu_0 + \lambda_0)\, \nabla^2 \varepsilon = -X^\nu{}_{;\nu}\,, \tag{158}$$

the longitudinal (dilatational) wave equation becomes

$$(2\mu_0 + \lambda_0)\, \nabla^2 \varepsilon = -k_0\, u^\nu{}_{;\nu}\,. \tag{159}$$

Using $u^\mu{}_{;\mu} = \varepsilon$ from (44) and rearranging, this equation can be rewritten as

$$\nabla^2 \varepsilon + \frac{k_0}{2\mu_0 + \lambda_0}\, \varepsilon = 0\,. \tag{160}$$

This wave equation applies to the volume dilatation ε. This equation is similar to the homogeneous Klein-Gordon equation for a scalar field, a field whose quanta are spinless particles [36].

§12.2.2. Rotational (transverse) wave equation. Substituting (154) into (61), viz.

$$\mu_0 \nabla^2 \omega^{\mu\nu} = -X^{[\mu;\nu]}, \tag{161}$$

the transverse (rotational) wave equation becomes

$$\mu_0 \nabla^2 \omega^{\mu\nu} = -\frac{k_0}{2}\left(u^{\mu;\nu} - u^{\nu;\mu}\right). \tag{162}$$

Using the definition of $\omega^{\mu\nu}$ from (42) and rearranging, this equation can be rewritten as

$$\nabla^2 \omega^{\mu\nu} + \frac{k_0}{\mu_0}\,\omega^{\mu\nu} = 0\,. \tag{163}$$

This antisymmetric equation is also similar to an homogeneous Klein-Gordon equation for an antisymmetrical tensor field.

§12.2.3. Strain (symmetric) wave equation. Substituting (154) into (64), viz.

$$\mu_0 \nabla^2 \varepsilon^{\mu\nu} + (\mu_0 + \lambda_0)\,\varepsilon^{;\mu\nu} = -X^{(\mu;\nu)}, \tag{164}$$

the symmetric (strain) wave equation becomes

$$\mu_0 \nabla^2 \varepsilon^{\mu\nu} + (\mu_0 + \lambda_0)\,\varepsilon^{;\mu\nu} = -\frac{k_0}{2}\left(u^{\mu;\nu} + u^{\nu;\mu}\right). \tag{165}$$

Using the definition of $\varepsilon^{\mu\nu}$ from (41) and rearranging, this equation can be rewritten as

$$\nabla^2 \varepsilon^{\mu\nu} + \frac{k_0}{\mu_0}\,\varepsilon^{\mu\nu} = -\frac{\mu_0 + \lambda_0}{\mu_0}\,\varepsilon^{;\mu\nu}. \tag{166}$$

This symmetric equation is also similar to a nonhomogeneous Klein-Gordon equation for a symmetrical tensor field with a source term and has the same structure as the displacement equation.

§12.3. Electromagnetism. We consider the impact of this volume force on the equations of electromagnetism derived previously. Substituting (154) into (95), viz.

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \frac{1}{2\mu_0}\,X^{\nu}, \tag{167}$$

Maxwell's equations in terms of the rotation tensor become

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \frac{k_0}{2\mu_0}\,u^{\nu}. \tag{168}$$

Separating u^{ν} into its longitudinal (irrotational) component $u^{\nu}_{\parallel}$ and its transverse (solenoidal) component $u^{\nu}_{\perp}$ using the Helmholtz theorem in four dimensions [42] according to

$$u^{\nu} = u^{\nu}_{\parallel} + u^{\nu}_{\perp}\,, \tag{169}$$

substituting for $\omega^{\mu\nu}$ from $F^{\mu\nu} = \varphi_0 \omega^{\mu\nu}$ and for $u_\perp^\nu$ from $A^\mu = -\frac{1}{2}\varphi_0 u_\perp^\mu$, this equation becomes

$$F^{\mu\nu}{}_{;\mu} = \varphi_0 \frac{2\mu_0 + \lambda_0}{2\mu_0} \varepsilon^{;\nu} + \frac{\varphi_0 k_0}{2\mu_0} u_\parallel^\nu - \frac{k_0}{\mu_0} A^\nu. \tag{170}$$

Proper treatment of this case requires that the current density four-vector be proportional to the RHS of (170) as follows:

$$\mu_{\rm em}\, j^\nu = \frac{\varphi_0}{2\mu_0} \left[(2\mu_0 + \lambda_0)\varepsilon^{;\nu} + k_0 u_\parallel^\nu \right] - \frac{k_0}{\mu_0} A^\nu. \tag{171}$$

This thus yields the following microscopic form of the current density four-vector:

$$j^\nu = \frac{\varphi_0}{2\mu_0 \mu_{\rm em}} \left[(2\mu_0 + \lambda_0)\varepsilon^{;\nu} + k_0 u_\parallel^\nu \right] - \frac{k_0}{\mu_0 \mu_{\rm em}} A^\nu. \tag{172}$$

We thus find that the second term is proportional to A^ν as is the second term of the current density obtained from the quantum mechanical Klein-Gordon equation with an electromagnetic field [36, see p. 35].

§12.4. Discussion of linear elastic volume force results. This section has been useful in that consideration of a simple linear elastic volume force leads to equations which are of the Klein-Gordon type. The wave equations that are obtained for the scalar ε, the four-vector u^ν, and the symmetric and antisymmetric tensors $\varepsilon^{\mu\nu}$ and $\omega^{\mu\nu}$ respectively, are all equations that are similar to homogeneous or nonhomogeneous Klein-Gordon equations. The solutions of these equations are well understood [41, see pp. 414–433].

It should be noted that we cannot simply put

$$\frac{m^2c^2}{\hbar^2} = \frac{k_0}{2\mu_0 + \lambda_0} \tag{173}$$

or

$$\frac{m^2c^2}{\hbar^2} = \frac{k_0}{\mu_0} \tag{174}$$

from the Klein-Gordon equation, as the expression to use depends on the wave equation considered. This ambiguity in the equivalency of the constant $m^2c^2/\hbar^2$ to *STCED* constants indicates that the postulated elastic linear volume force proposed in (154) is not quite correct, even if it is a step in the right direction. It has provided insight into the impact of the volume force on this analysis, but the volume force is not quite the simple linear elastic expression considered in (154).

In the following section, we derive a volume force from the general current density four-vector expression of (175) below. We find that the volume force (154) and consequently the current density four-vector (172) need to be modified.

§13. Derivation of a quantum mechanical volume force. One identification of the volume force based on quantum mechanical considerations is possible by comparing (102), viz.

$$j^{\nu} = \frac{1}{2}\,\frac{\varphi_0}{\mu_{\rm em}\,\mu_0}\left[(2\mu_0+\lambda_0)\,\varepsilon^{;\nu} + X^{\nu}\right], \tag{175}$$

with the quantum mechanical expression of the current density four-vector j^{ν} obtained from the Klein-Gordon equation for a spin-0 particle. The Klein-Gordon equation can also describe the interaction of a spin-0 particle with an electromagnetic field. The current density four-vector j^{ν} in that case is written as [36, see p. 35]

$$j^{\nu} = \frac{i\hbar e}{2m}\left(\psi^{*}\partial^{\nu}\psi - \psi\,\partial^{\nu}\psi^{*}\right) - \frac{e^2}{m}\,A^{\nu}(\psi\psi^{*}), \tag{176}$$

where the superscript $*$ denotes complex conjugation.

The first term of (176) includes the following derivative-like expression:

$$i\left(\psi^{*}\partial^{\nu}\psi - \psi\,\partial^{\nu}\psi^{*}\right). \tag{177}$$

It is generated by multiplying the Klein-Gordon equation for ψ by ψ^{*} and subtracting the complex conjugate [36]. The general form of the expression can be generated by writing

$$\psi \sim \exp(i\phi), \tag{178}$$

which is a qualitative representation of the wave function. One can then see that with (178), the expression

$$\partial^{\nu}(\psi\psi^{*}) \tag{179}$$

has the qualitative structure of (177) although it is not strictly equivalent. However, given that the steps followed to generate (177) are not repeated in this derivation, strict equivalence is not expected. Replacing (177) with (179), the first term of (176) becomes

$$\frac{\hbar e}{2m}\,\partial^{\nu}(\psi\psi^{*}). \tag{180}$$

We see that this term is similar to the first term of (175) and setting them to be equal, we obtain

$$\frac{\varphi_0}{2\,\mu_{\rm em}\,\mu_0}\,(2\mu_0+\lambda_0)\,\varepsilon^{;\nu} = \frac{\hbar e}{2m}\,\partial^{\nu}(\psi\psi^{*}). \tag{181}$$

Similarly, the second terms of (175) and (176) are also similar and setting them to be equal, we obtain

$$\frac{\varphi_0}{2\mu_{\rm em}\,\mu_0}\,X^\nu = -\frac{e^2}{m}\,A^\nu(\psi\psi^*)\,. \tag{182}$$

The equalities (181) and (182) thus result from the comparison of (175) and (176).

The first identification that can be derived from (181) is

$$\varepsilon\,(x^\mu) = \psi\psi^* \tag{183}$$

to a proportionality constant which has been set to 1, given that the norm of the wavefunction itself is arbitrarily normalized to 1 as part of its probabilistic interpretation. Both are dimensionless quantities. ε is the change in volume per original volume as a function of position x^μ, which is stated explicitly in (183), while $\psi\psi^*$ is the probability density as a function of position, and hence is also a proportion of an overall quantity normalized to 1. There are thus many similarities between ε and ψ. This equation leads to the conclusion that the quantum mechanical wavefunction describes longitudinal wave propagations in the *STC* corresponding to the volume dilatation associated with the particle property of an object.

Using (81) viz.

$$A^\nu = -\frac{1}{2}\,\varphi_0\,u^\nu_\perp \tag{184}$$

and (183) in (182), the quantum mechanical volume force is given by

$$X^\nu = \mu_0\mu_{\rm em}\,\frac{e^2}{m}\,\varepsilon\,(x^\mu)\;u^\nu_\perp\,. \tag{185}$$

Using the definition for the dimensionless fine structure constant $\alpha = \,= \mu_{\rm em} c e^2/2h$, (185) becomes

$$X^\nu = 2\,\mu_0\,\alpha\,\frac{h}{m\,c}\,\varepsilon\,(x^\mu)\;u^\nu_\perp \tag{186}$$

or

$$X^\nu = 2\,\mu_0\,\alpha\,\lambda_{\rm c}\,\varepsilon\,(x^\mu)\;u^\nu_\perp\,, \tag{187}$$

where $\lambda_{\rm c} = h/m\,c$ is the electron's Compton wavelength.

Thus the *STCED* elastic force constant of (154) is given by

$$k_0 = \mu_0\mu_{\rm em}\,\frac{e^2}{m} = 2\,\mu_0\,\alpha\,\frac{h}{m\,c} = 2\,\mu_0\,\alpha\,\lambda_{\rm c}\,. \tag{188}$$

The units are [N][m^{-1}] as expected. The volume force is proportional to $\varepsilon\,(x^\mu)\,u^\nu_\perp$ as opposed to just u^ν as in (154):

$$X^\nu = k_0\,\varepsilon\,(x^\mu)\;u^\nu_\perp. \tag{189}$$

The volume force X^ν is proportional to the Planck constant as suspected previously. This explains why the volume force tends to zero in the macroscopic case. The volume force is also proportional to the *STC* volume dilatation $\varepsilon\,(x^\mu)$ in addition to the displacements $u^\nu_\perp$. This makes sense as all deformations, both distortions and dilatations, should be subject to the *STC* elastic spring force. This is similar to an elastic spring law as X^ν is defined positive in the direction opposite to the displacement [18]. The volume force also describes the interaction with an electromagnetic field given that (176) from which it is derived includes electromagnetic interactions.

Starting from (181), and making use of (183), the *STC* electromagnetic shearing potential constant φ_0 of (81) can be identified:

$$\varphi_0 = \frac{2\mu_0}{2\mu_0+\lambda_0}\,\mu_{\rm em}\,\frac{e\hbar}{2m} = \frac{2\mu_0}{2\mu_0+\lambda_0}\,\mu_{\rm em}\,\mu_{\rm B} \tag{190}$$

where the Bohr magneton $\mu_{\rm B} = e\hbar/2m$ has been used. Using (179) and (183) in (176), we obtain

$$j^\nu = \frac{e\hbar}{2m}\,\varepsilon^{;\nu} - \frac{e^2}{m}\,A^\nu\varepsilon\,(x^\mu) \tag{191}$$

or

$$j^\nu = \mu_{\rm B}\,\varepsilon^{;\nu} - \frac{e^2}{m}\,A^\nu\varepsilon\,(x^\mu) \tag{192}$$

with the Bohr magneton.

§13.1. Microscopic dynamics of the STC

§13.1.1. Dynamic equations. Substituting (189) into (155), the dynamic equation in terms of displacements becomes

$$\mu_0\nabla^2 u^\nu + (\mu_0+\lambda_0)\,\varepsilon^{;\nu} = -k_0\,\varepsilon\,(x^\mu)\,u^\nu_\perp\,. \tag{193}$$

This equation can be rewritten as

$$\nabla^2 u^\nu + \frac{k_0}{\mu_0}\,\varepsilon\,(x^\mu)\;u^\nu_\perp = -\frac{\mu_0+\lambda_0}{\mu_0}\,\varepsilon^{;\nu}. \tag{194}$$

We note that $\varepsilon(x^\mu)$ is a scalar function of 4-position only, and plays a role similar to the potential $V(\boldsymbol{r}\,)$ in the Schrödinger equation. Indeed, $\varepsilon(x^\mu)$ represents the mass energy structure (similar to an energy potential) impacting the solutions of this equation.

Separating u^ν into its longitudinal (irrotational) component $u^\nu_\parallel$ and its transverse (solenoidal) component $u^\nu_\perp$ using the Helmholtz theorem in four dimensions [42] according to

$$u^\nu = u^\nu_\parallel + u^\nu_\perp\,, \tag{195}$$

we obtain the separated equations

$$\left.\begin{aligned} \nabla^2 u^\nu_\parallel &= -\frac{\mu_0+\lambda_0}{\mu_0}\,\varepsilon^{;\nu} \\ \nabla^2 u^\nu_\perp + \frac{k_0}{\mu_0}\,\varepsilon\left(x^\mu\right)\,u^\nu_\perp &= 0 \end{aligned}\right\}. \tag{196}$$

The wave equation for $u^\nu_\parallel$ describes the propagation of longitudinal displacements, while the wave equation for $u^\nu_\perp$ describes the propagation of transverse displacements.

§13.1.2. Longitudinal displacements equation. Substituting for $\varepsilon^{;\nu}$ from (191) in the first equation of (196), we obtain

$$\nabla^2 u^\nu_\parallel = -\frac{2k_{\rm L}}{\hbar}\left[\frac{e^2}{m}\,j^\nu + eA^\nu\varepsilon\left(x^\mu\right)\right], \tag{197}$$

where the dimensionless ratio

$$k_{\rm L} = \frac{\mu_0+\lambda_0}{\mu_0} \tag{198}$$

has been introduced. Hence the source term on the RHS of this equation includes the mass resulting from the dilatation displacements, the current density four-vector, and the vector potential resulting from the distortion displacements. It provides a full description of the gravitational and electromagnetic interactions at the microscopic level.

§13.1.3. Transverse displacements equation. Substituting for $u^\nu_\perp$ from (81) in the second equation of (196), we obtain

$$\nabla^2 A^\nu + \frac{k_0}{\mu_0}\,\varepsilon\left(x^\mu\right)\,A^\nu = 0\,. \tag{199}$$

Substituting for k_0 from (188), this equation becomes

$$\nabla^2 A^\nu + \mu_{\rm em}\,\frac{e^2}{m}\,\varepsilon\left(x^\mu\right)\,A^\nu = 0 \tag{200}$$

or

$$\nabla^2 A^\nu + 2\alpha\,\frac{h}{mc}\,\varepsilon\left(x^\mu\right)\,A^\nu = 0 \tag{201}$$

and finally

$$\nabla^2 A^\nu + 2\alpha\,\lambda_c\,\varepsilon\left(x^\mu\right)\,A^\nu = 0\,. \tag{202}$$

This equation is similar to a Proca equation except that the coefficient of A^ν is not the familiar $m^2c^2/\hbar^2$. Given that transverse displacements are massless, the Proca equation coefficient is not expected given its usual interpretation that it represents the mass of the particle described by the equation. This is discussed in more details in the next section.

§13.2. Wave equations

§13.2.1. Longitudinal wave equation. Substituting (189) into (158), the longitudinal (dilatational) wave equation becomes

$$\left(2\mu_0 + \lambda_0\right)\nabla^2\varepsilon = -\nabla_\nu\big[k_0\,\varepsilon\left(x^\mu\right)\,u_\perp^\nu\big]\,. \tag{203}$$

Taking the divergence on the RHS, using $u^\nu_{\perp;\nu} = 0$ from (45) and rearranging, this equation can be rewritten as

$$\nabla^2\varepsilon = -\frac{k_0}{2\mu_0 + \lambda_0}\,u_\perp^\nu\varepsilon_{;\nu}\,. \tag{204}$$

Substituting for $u_\perp^\nu$ from (81), for k_0 from (188) and for $\varepsilon_{;\nu}$ from (191), we obtain

$$\nabla^2\varepsilon - 4\,\frac{e^2}{\hbar^2}\,A^\nu A_\nu\,\varepsilon = 4\,\frac{m}{\hbar^2}\,A^\nu j_\nu\,. \tag{205}$$

Recognizing that

$$e^2\,A^\nu A_\nu = P^\nu P_\nu = -m^2c^2, \tag{206}$$

and substituting in (205), the equation becomes

$$\nabla^2\varepsilon + 4\,\frac{m^2c^2}{\hbar^2}\,\varepsilon = 4\,\frac{m}{\hbar^2}\,A^\nu j_\nu\,. \tag{207}$$

This is the Klein-Gordon equation except for the factor of 4 multiplying the ε coefficient and the source term. The term on the RHS of this equation is an interaction term of the form $\boldsymbol{A}\cdot\boldsymbol{j}$.

As identified from (183) and confirmed by this equation, the quantum mechanical wavefunction describes longitudinal wave propagations in the *STC* corresponding to the volume dilatation associated with the particle property of an object. The RHS of the equation indicates an interaction between the longitudinal current density j_ν and the transverse vector potential A^ν. This is interpreted in Electromagnetism as energy in the static magnetic induction field to establish the steady current distribution [43, see p. 150]. It is also the form of the interaction term introduced in the vacuum Lagrangian for classical electrodynamics [44, see p. 428].

Although (207) with the $m^2c^2/\hbar^2$ coefficient is how the Klein-Gordon equation is typically written, (205) is a more physically accurate way of writing that equation, i.e.

$$\frac{\hbar^2}{4}\nabla^2\varepsilon - e^2 A^\nu A_\nu\,\varepsilon = m\,A^\nu j_\nu\,, \tag{208}$$

as the massive nature of the equation resides in its solutions $\varepsilon\,(x^\mu)$. The constant m needs to be interpreted in the same way as the constant e. The constant e in the Klein-Gordon equation is the elementary unit of electrical charge (notwithstanding the quark fractional charges), not the electrical charge of the particle represented by the equation. Similarly, the constant m in the Klein-Gordon equation needs to be interpreted as the elementary unit of mass (the electron's mass), not the mass of the particle represented by the equation. That is obtained from the solutions $\varepsilon\,(x^\mu)$ of the equation.

§13.2.2. Transverse wave equation. Substituting (189) into (161), the transverse (rotational) wave equation becomes

$$\mu_0\nabla^2\omega^{\mu\nu} = -\frac{k_0}{2}\left[(\varepsilon\,u_\perp^\mu)^{;\nu} - (\varepsilon\,u_\perp^\nu)^{;\mu}\right]. \tag{209}$$

Using (42) and rearranging, this equation can be rewritten as

$$\nabla^2\omega^{\mu\nu} + \frac{k_0}{\mu_0}\,\varepsilon\,(x^\mu)\;\omega^{\mu\nu} = \frac{1}{2}\frac{k_0}{\mu_0}\,(\varepsilon^{;\mu}u_\perp^\nu - \varepsilon^{;\nu}u_\perp^\mu)\,. \tag{210}$$

Substituting for $\omega^{\mu\nu}$ using $F^{\mu\nu} = \varphi_0\,\omega^{\mu\nu}$ from (80), for $u_\perp^\nu$ from (81), for k_0 from (188) and for $\varepsilon^{;\nu}$ from (191), we obtain

$$\nabla^2 F^{\mu\nu} + \mu_{\mathrm{em}}\,\frac{e^2}{m}\,\varepsilon\,(x^\mu)\;F^{\mu\nu} = \mu_{\mathrm{em}}\,\frac{e}{\hbar}\;(A^\mu j^\nu - A^\nu j^\mu)\,. \tag{211}$$

This equation can also be written as

$$\nabla^2 F^{\mu\nu} + 2\,\alpha\,\lambda_{\mathrm{c}}\,\varepsilon\,(x^\mu)\;F^{\mu\nu} = \mu_{\mathrm{em}}\,\frac{e}{\hbar}\;(A^\mu j^\nu - A^\nu j^\mu)\,. \tag{212}$$

This is a new equation of the electromagnetic field strength $F^{\mu\nu}$. The term on the RHS of this equation is an interaction term of the form $\boldsymbol{A}\times\boldsymbol{j}$.

In Electromagnetism, this term is the volume density of the magnetic torque (magnetic torque density), and is interpreted as the "longitudinal tension" between two successive current elements (Helmholtz's longitudinal tension), observed experimentally by Ampère (hairpin experiment) [45].

§13.2.3. Strain wave equation. Substituting (189) into (164), the strain (symmetric) wave equation becomes

$$\mu_0\nabla^2\varepsilon^{\mu\nu} + (\mu_0+\lambda_0)\,\varepsilon^{;\mu\nu} = -\frac{k_0}{2}\left[(\varepsilon\, u_\perp^{\mu})^{;\nu} + (\varepsilon\, u_\perp^{\nu})^{;\mu}\right], \tag{213}$$

which can be rewritten as

$$\nabla^2\varepsilon^{\mu\nu} + \frac{\mu_0+\lambda_0}{\mu_0}\,\varepsilon^{;\mu\nu} = \\ = \frac{1}{2}\,\frac{k_0}{\mu_0}\left[\varepsilon\,(u_\perp^{\mu;\nu} + u_\perp^{\nu;\mu}) + (\varepsilon^{;\mu}u_\perp^{\nu} + \varepsilon^{;\nu}u_\perp^{\mu})\right]. \tag{214}$$

Substituting for $u_\perp^{\nu}$ from (81), for k_0 from (188) and for $\varepsilon^{;\nu}$ from (191), we obtain

$$\nabla^2\varepsilon^{\mu\nu} + k_{\rm L}\,\varepsilon^{;\mu\nu} = k_{\rm T}\,\frac{2m}{\hbar^2}\,(A^{\mu}j^{\nu} + A^{\nu}j^{\mu}) + \\ + k_{\rm T}\,\varepsilon\left[\frac{e}{\hbar}\,(A^{\mu;\nu} + A^{\nu;\mu}) + \frac{2e^2}{\hbar^2}\,(A^{\mu}A^{\nu} + A^{\nu}A^{\mu})\right], \tag{215}$$

where the dimensionless ratio

$$k_{\rm T} = \frac{2\mu_0+\lambda_0}{\mu_0} \tag{216}$$

has been introduced and ratio $k_{\rm L}$ has been used from (198). The last term can be summed to $2A^{\mu}A^{\nu}$. This new equation for the symmetrical strain tensor field includes on the RHS symmetrical interaction terms between the current density four-vector and the vector potential resulting from the distortion displacements and between the vector potential and the mass resulting from the dilatation displacements.

§13.3. Simplified wave equations. Inspection of the wave equations derived previously shows that common factors are associated with A^{ν} and j^{ν} in all the equations. We thus introduce the reduced physical variables $\bar{A}^{\nu}$ and $\bar{j}^{\nu}$ defined according to

$$\bar{A}^{\nu} = e\,A^{\nu}, \qquad \bar{j}^{\nu} = \frac{m}{e}\,j^{\nu}, \tag{217}$$

and $\bar{\bar{A}}^\nu$ and $\bar{\bar{j}}^\nu$ defined according to

$$\bar{\bar{A}}^\nu = \frac{2e}{\hbar}\, A^\nu, \qquad \bar{\bar{j}}^\nu = \frac{2m}{e\hbar}\, j^\nu. \tag{218}$$

The various wave equations then simplify to the following.

Longitudinal displacements equation

$$\nabla^2 u_\parallel^\nu = -\frac{2k_\mathrm{L}}{\hbar}\left(\bar{j}^\nu + \varepsilon \bar{A}^\nu\right) \tag{219}$$

$$\nabla^2 u_\parallel^\nu = -k_\mathrm{L}\left(\bar{\bar{j}}^\nu + \varepsilon \bar{\bar{A}}^\nu\right) \tag{220}$$

Transverse displacements equation

$$\nabla^2 \bar{A}^\nu + 2\alpha\,\lambda_\mathrm{c}\,\varepsilon \bar{A}^\nu = 0 \tag{221}$$

$$\nabla^2 \bar{\bar{A}}^\nu + 2\alpha\,\lambda_\mathrm{c}\,\varepsilon \bar{\bar{A}}^\nu = 0 \tag{222}$$

Longitudinal wave equation

$$\frac{\hbar^2}{4}\,\nabla^2\varepsilon - \bar{A}^\nu \bar{A}_\nu\,\varepsilon = \bar{A}^\nu \bar{j}_\nu \tag{223}$$

$$\nabla^2\varepsilon - \bar{\bar{A}}^\nu \bar{\bar{A}}_\nu\,\varepsilon = \bar{\bar{A}}^\nu \bar{\bar{j}}_\nu \tag{224}$$

Transverse wave equation

$$\nabla^2 F^{\mu\nu} + 2\alpha\,\lambda_\mathrm{c}\,\varepsilon\left(x^\mu\right)\, F^{\mu\nu} = \mu_\mathrm{em}\,\frac{e}{\hbar m}\,\left(\bar{A}^\mu \bar{j}^\nu - \bar{A}^\nu \bar{j}^\mu\right) \tag{225}$$

$$\nabla^2 F^{\mu\nu} + 2\alpha\,\lambda_\mathrm{c}\,\varepsilon\left(x^\mu\right)\, F^{\mu\nu} = \frac{1}{2}\,\mu_\mathrm{em}\,\mu_\mathrm{B}\left(\bar{\bar{A}}^\mu \bar{\bar{j}}^\nu - \bar{\bar{A}}^\nu \bar{\bar{j}}^\mu\right) \tag{226}$$

Strain wave equation

$$\begin{aligned}\nabla^2\varepsilon^{\mu\nu} + k_\mathrm{L}\,\varepsilon^{;\mu\nu} &= k_\mathrm{T}\,\frac{2}{\hbar^2}\left(\bar{A}^\mu \bar{j}^\nu + \bar{A}^\nu \bar{j}^\mu\right) + \\ &+ k_\mathrm{T}\,\varepsilon\left[\frac{1}{\hbar}\left(\bar{A}^{\mu;\nu} + \bar{A}^{\nu;\mu}\right) + \frac{2}{\hbar^2}\left(\bar{A}^\mu \bar{A}^\nu + \bar{A}^\nu \bar{A}^\mu\right)\right]\end{aligned} \tag{227}$$

$$\begin{aligned}\nabla^2\varepsilon^{\mu\nu} + k_\mathrm{L}\,\varepsilon^{;\mu\nu} &= \frac{1}{2}\,k_\mathrm{T}\left(\bar{\bar{A}}^\mu \bar{\bar{j}}^\nu + \bar{\bar{A}}^\nu \bar{\bar{j}}^\mu\right) + \\ &+ \frac{1}{2}\,k_\mathrm{T}\,\varepsilon\left[\left(\bar{\bar{A}}^{\mu;\nu} + \bar{\bar{A}}^{\nu;\mu}\right) + \left(\bar{\bar{A}}^\mu \bar{\bar{A}}^\nu + \bar{\bar{A}}^\nu \bar{\bar{A}}^\mu\right)\right]\end{aligned} \tag{228}$$

§13.4. Microscopic theory of Electromagnetism. We consider the impact of this volume force on the equations of electromagnetism derived previously. Substituting (186) into (95), Maxwell's equations in terms of the rotation tensor become

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \alpha\,\lambda_{\rm c}\,\varepsilon\,(x^\mu)\,u_\perp^\nu\,. \tag{229}$$

Substituting for $\omega^{\mu\nu}$ using $F^{\mu\nu} = \varphi_0\,\omega^{\mu\nu}$ from (80) and using (81) for $u_\perp^\nu$, this equation becomes

$$F^{\mu\nu}{}_{;\mu} = \varphi_0\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} - 2\,\alpha\,\lambda_{\rm c}\,\varepsilon\,(x^\mu)\,A^\nu. \tag{230}$$

Proper treatment of this case requires that the current density four-vector be proportional to the RHS of (230) as follows:

$$\mu_{\rm em} j^\nu = \varphi_0\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} - 2\,\alpha\,\lambda_{\rm c}\,\varepsilon\,(x^\mu)\,A^\nu. \tag{231}$$

As seen previously, the equations of electrodynamics, in the general case, are identical to the covariant form of Maxwell's equations and are not modified by the volume force (see Page 246). This yields the following microscopic form of the current density four-vector:

$$j^\nu = \frac{\varphi_0}{\mu_{\rm em}}\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} - \frac{2\alpha\lambda_{\rm c}}{\mu_{\rm em}}\,\varepsilon\,(x^\mu)\,A^\nu. \tag{232}$$

Substituting for φ_0 from (190) and for α as in (186) into (232), we obtain

$$j^\nu = \frac{e\hbar}{2m}\,\varepsilon^{;\nu} - \frac{e^2}{m}\,A^\nu \varepsilon\,(x^\mu) \tag{233}$$

or

$$j^\nu = \mu_{\rm B}\,\varepsilon^{;\nu} - \frac{e^2}{m}\,A^\nu \varepsilon\,(x^\mu) \tag{234}$$

using the Bohr magneton.

§14. Discussion and Conclusion. In this paper, we have presented the Elastodynamics of the Spacetime Continuum (*STCED*). This theory describes the deformations of the spacetime continuum by modeling and analyzing the displacements of the elements of the *STC* resulting from the spacetime continuum strains arising from the energy-momentum stress tensor, based on the application of continuum mechanical results to the spacetime continuum. *STCED* provides a fundamental description of the microscopic processes underlying the spacetime continuum. The combination of the spacetime continuum deformations results in the geometry of the *STC*.

We have proposed a natural decomposition of the spacetime metric tensor into a background and a dynamical part based on an analysis from first principles, of the impact of introducing a test mass in the spacetime continuum. We have found that the presence of mass results in strains in the spacetime continuum. Those strains correspond to the dynamical part of the spacetime metric tensor. The applicability of the proposed metric to the Einstein field equations remains open to demonstration.

We have proposed a framework for the analysis of strained spacetime based on the Elastodynamics of the Spacetime Continuum. In this model, the emphasis is on the displacements of the spacetime continuum infinitesimal elements from their unstrained configuration as a result of the strains applied on the *STC* by the energy-momentum stress tensor, rather than on the geometry of the *STC* due to the energy-momentum stress tensor.

We postulate that this description based on the deformation of the continuum is a description complementary to that of General Relativity which is concerned with modeling the resulting geometry of the spacetime continuum. Interestingly, the structure of the resulting stress-strain relation is similar to that of the field equations of General Relativity. This strengthens our conjecture that the geometry of the spacetime continuum can be seen as a representation of the deformation of the spacetime continuum resulting from the strains generated by the energy-momentum stress tensor. The equivalency of the deformation description and of the geometrical description still remains to be demonstrated. It should be noted that these could be considered to be local effects in the particular reference frame of the observer.

We have applied the stress-strain relation of Continuum Mechanics to the spacetime continuum to show that rest-mass energy density arises from the volume dilatation of the spacetime continuum. This is a significant result as it demonstrates that mass is not independent of the spacetime continuum, but rather results from how energy-momentum propagates in the spacetime continuum. Matter does not warp spacetime, but rather matter *is* warped spacetime. The universe consists of the spacetime continuum and energy-momentum that propagates in it by deformation of its (*STC*) structure.

We have proposed a natural decomposition of tensor fields in strained spacetime, in terms of dilatations and distortions. We have shown that dilatations correspond to rest-mass energy density, while distortions correspond to massless shear transverse waves. We have noted that this decomposition of spacetime continuum deformations into a massive dilatation and a massless transverse wave distortion is somewhat reminiscent of wave-particle duality. This could explain why dilatation-measuring apparatus measure the massive "particle" properties of the deformation, while distortion-measuring apparatus measure the massless transverse "wave" properties of the deformation.

The equilibrium dynamic equation of the spacetime continuum is described by $T^{\mu\nu}{}_{;\mu} = -X^\nu$. In General Relativity, the relation $T^{\mu\nu}{}_{;\mu} = 0$ is taken as expressing local conservation of the energy-momentum of matter. The value $X^\nu = 0$ is thus taken to represent the macroscopic local case, while in the general case, the volume force X^ν is retained in the equation. This dynamic equation leads to a series of wave equations as derived in this paper: the displacement (u^ν), dilatational (ε), rotational ($\omega^{\mu\nu}$) and strain ($\varepsilon^{\mu\nu}$) wave equations. The nature of the spacetime continuum volume force and the resulting inhomogeneous wave equations are areas of further investigation.

Hence energy is seen to propagate in the spacetime continuum as deformations of the *STC* that satisfy wave equations of propagation. Deformations can be decomposed into dilatations and distortions. *Dilatations* involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. *Distortions* correspond to a change of shape of the spacetime continuum without a change in volume and are thus massless. Dilatations correspond to longitudinal displacements and distortions correspond to transverse displacements of the spacetime continuum.

Hence, every excitation of the spacetime continuum can be decomposed into a transverse and a longitudinal mode of propagation. We have noted again that this decomposition into a dilatation with rest-mass energy density and a massless transverse wave distortion, is somewhat reminiscent of wave-particle duality, with the transverse mode corresponding to the wave aspects and the longitudinal mode corresponding to the particle aspects.

A continuity equation for deformations of the spacetime continuum has been derived; we have found that the divergence of the strain and rotation tensors equals the gradient of the massive volume dilatation, which acts as a source term.

We have analyzed the strain energy density of the spacetime continuum in *STCED*. We have found that the strain energy density is separated into two terms: the first one expresses the dilatation energy density (the "mass" longitudinal term) while the second one expresses the distortion energy density (the "massless" transverse term). We have found that the quadratic structure of the energy relation of Special Relativity is present in the strain energy density of the Elastodynamics of the Spacetime Continuum. We have also found that the kinetic energy pc is carried by the distortion part of the deformation, while the dilatation part carries only the rest mass energy.

We have derived Electromagnetism from the Elastodynamics of the Spacetime Continuum based on the identification of the theory's antisymmetric rotation tensor $\omega^{\mu\nu}$ with the electromagnetic field-strength tensor $F^{\mu\nu}$.

The theory provides a physical explanation of the electromagnetic potential: it arises from transverse (shearing) displacements of the spacetime continuum, in contrast to mass which arises from longitudinal (dilatational) displacements of the spacetime continuum. Hence sheared spacetime is manifested as electromagnetic potentials and fields.

In addition, the theory provides a physical explanation of the current density four-vector: it arises from the 4-gradient of the volume dilatation of the spacetime continuum. A corollary of this relation is that massless (transverse) waves cannot carry an electric charge or produce a current.

The transverse mode of propagation involves no volume dilatation and is thus massless. Transverse wave propagation is associated with the distortion of the spacetime continuum. Electromagnetic waves are transverse waves propagating in the *STC* itself, at the speed of light.

The Lorentz condition is obtained directly from the theory. The reason for the value of zero is that transverse displacements are massless because such displacements arise from a change of shape (distortion) of the spacetime continuum, not a change of volume (dilatation).

In addition, we have obtained a generalization of Electromagnetism for the situation where a volume force is present, in the general non-macroscopic case. Maxwell's equations are found to remain unchanged, but the current density has an additional term proportional to the volume force X^ν.

The Elastodynamics of the Spacetime Continuum thus provides a unified description of the spacetime deformation processes underlying general relativistic Gravitation and Electromagnetism, in terms of spacetime continuum displacements resulting from the strains generated by the energy-momentum stress tensor.

We have calculated the strain energy density of the electromagnetic energy-momentum stress tensor. We have found that the dilatation longitudinal (mass) term of the strain energy density and hence the rest-mass energy density of the photon is 0. We have found that the distortion transverse (massless) term of the strain energy density is a combination of the electromagnetic field energy density term $U^2_{\rm em}/\mu_0$ and the electromagnetic field energy flux term $S^2/\mu_0 c^2$, calculated from the Poynting vector. This longitudinal electromagnetic energy flux is massless as it is due to distortion, not dilatation, of the spacetime continuum. However, because this energy flux is along the direction of propagation (i.e. longitudinal), it gives rise to the particle aspect of the electromagnetic field, the photon.

We have investigated the volume force and its impact on the equations of the Elastodynamics of the Spacetime Continuum. We have found that a linear elastic volume force leads to equations which are of the Klein-Gordon type. From a variation of that linear elastic volume force based on the Klein-Gordon quantum mechanical current density, we have found that the quantum mechanical wavefunction describes longitudinal wave propagations in the *STC* corresponding to the volume dilatation associated with the particle property of an object. We have derived the wave equations corresponding to the modeled volume force. The longitudinal wave equation is found to correspond to the Klein-Gordon equation with a source term corresponding to an interaction term of the form $\boldsymbol{A}\cdot\boldsymbol{j}$, further confirming that the quantum mechanical wavefunction describes longitudinal wave propagations in the *STC*. The transverse wave equation is found to be a new equation of the electromagnetic field strength $F^{\mu\nu}$, which includes an interaction term of the form $\boldsymbol{A}\times\boldsymbol{j}$ corresponding to the volume density of the magnetic torque (magnetic torque density). The equations obtained reflect a close integration of gravitational and electromagnetic interactions at the microscopic level.

§14.1 Future directions. This paper has presented a linear elastic theory of the Elastodynamics of the Spacetime Continuum for the analysis of the deformations of the spacetime continuum. It provides a fundamental description of gravitational, electromagnetic and some quantum phenomena. Progress has been achieved towards the goal set initially, that the theory should in principle be able to explain the basic physical theories from which the rest of physical theory can be built, without the introduction of inputs external to the theory. Physical explanations of the following phenomena have been derived from *STCED* in this paper:

— ***Decomposition of the metric tensor.*** A decomposition of the metric tensor into its background and dynamical parts is obtained. The dynamical part corresponds to the strains generated in the spacetime continuum by the energy-momentum stress tensor.

— ***Wave-particle duality.*** Every excitation of the spacetime continuum can be separated into a transverse (distortion) and a longitudinal (dilatation) mode of propagation. This decomposition of spacetime continuum deformations into a massive dilatation ("particle") and a massless transverse distortion ("wave") is similar to wave-particle duality.

— ***Nature of matter.*** The longitudinal mode of propagation involves an invariant change in volume of the spacetime continuum. Rest-mass energy, and hence matter, arises from this invariant volume dilatation of the spacetime continuum.

— ***Maxwell's equations.*** Maxwell's equations are derived from the theory, including a generalization when a volume forme X^ν is present.

— ***Nature of Electromagnetism.*** The theory provides a physical explanation of the electromagnetic potential, which arises from transverse (shearing) displacements of the spacetime continuum, and of the current density four-vector, which is the 4-gradient of the volume dilatation of the spacetime continuum.

— ***Lorentz condition.*** The Lorentz condition is obtained directly from the theory.

— ***Electromagnetic radiation.*** The transverse mode of propagation involves no volume dilatation and is thus massless. Electromagnetic waves are transverse waves propagating in the spacetime continuum itself.

— ***Speed of light.*** Energy propagates through the spacetime continuum as deformations of the continuum. The maximum speed at which the transverse distortions can propagate through the spacetime continuum is c, the speed of light.

— ***Quadratic energy relation of Special Relativity.*** This is derived from the strain energy density which is separated into a dilatation energy density term (the "mass" longitudinal term) and a distortion energy density term (the "massless" transverse term). The kinetic energy pc is carried by the distortion part of the deformation, while the dilatation part carries only the rest mass energy.

— ***Nature of photons.*** The strain energy density of the electromagnetic field includes a longitudinal electromagnetic energy flux which is massless as it is due to distortion, not dilatation, of the spacetime continuum. However, because this energy flux is along the direction of propagation (i.e. longitudinal), it gives rise to the photon, the particle aspect of the electromagnetic field.

— ***Nature of the wavefunction.*** The quantum mechanical wavefunction describes longitudinal wave propagations in the spacetime continuum corresponding to the volume dilatation associated with the particle property of an object.

— ***Klein-Gordon equation.*** The longitudinal wave equation derived from a quantum mechanically derived volume force corresponds to the Klein-Gordon equation with a source term corresponding to an interaction term of the form $\boldsymbol{A} \cdot \boldsymbol{j}$.

— ***Magnetic torque density equation.*** The transverse wave equation is found to be a new equation of the electromagnetic field strength $F^{\mu\nu}$, which includes an interaction term of the form $\boldsymbol{A} \times \boldsymbol{j}$ corresponding to the magnetic torque density.

A solid foundation of the *STCED* theory has been laid, from which further expansion can be achieved. The basic physical theory from which the rest of physical theory can be built is not complete. For example, the basic physical constants such as Planck's constant h, the elementary electrical charge e, the elementary mass of the electron m, should be derivable from the fundamental constants κ_0, μ_0, ρ_0 and others characterizing the spacetime continuum. They should also be physically explained by the theory.

This we believe can be achieved by using a more complete theory of the spacetime continuum and of the Elastodynamics of the Spacetime Continuum.

In this section, we suggest future directions to extend the theory of *STCED*. The following areas of exploration are being suggested as candidates worthy of further study:

— Exploration of alternative Volume forces derived from other identifications of related physical results.

— The incorporation of Torsion in the theory, based on Élie Cartan's differential forms formulation.

— Extension of the theory based on the evolution of Continuum Mechanics in the last one hundred years, including Eshelbian Mechanics [46] and the Mechanics of Generalized Continua [47].

— Extension of the theory to include Defects, such as dislocations and disinclinations. Given that the spacetime continuum behaves as a deformable medium, there is no reason not to expect dislocations and other defects to be present in the *STC*.

A more sophisticated theory of *STCED* is expected to provide additional insight into the fundamental nature of the spacetime continuum and of physical theory.

Submitted on January 31, 2013

1. Einstein A. Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*, 1916, vol. 49. English translation reprinted in Lorentz H. A., Einstein A., Minkowski H, and Weyl H. The Principle of Relativity; A Collection of Original Memoirs on the Special and General Theory of Relativity. Dover Publications, New York, pp.109–164, 1952.
2. Sakharov A.D. Vacuum Quantum Fluctuations in Curved Space and the Theory of Gravitation. *Soviet Physics–Doklady*, 1968, vol. 12, 1040–1041.
3. Tartaglia A. A Strained Space-time to Explain the large Scale Properties of the Universe. *International Journal of Modern Physics: Conference Series*, 2011, vol. 3, 303–311.
4. Tartaglia A., Radicella N., Sereno M. Lensing in an elastically strained spacetime. *Journal of Physics: Conference Series*, 2011, vol. 283, 012037.
5. Millette P.A. On the Decomposition of the Spacetime Metric Tensor and of Tensor Fields in Strained Spacetime. *Progress in Physics*, 2012, vol. 4, 5–8.
6. Millette P.A. The Elastodynamics of the Spacetime Continuum as a Framework for Strained Spacetime. *Progress in Physics*, 2013, vol. 1, 55–59.
7. Millette P.A. Derivation of Electromagnetism from the Elastodynamics of the Spacetime Continuum. *Progress in Physics*, 2013, vol. 2, 12–15.
8. Millette P.A. Strain Energy Density in the Elastodynamics of the Spacetime Continuum and the Electromagnetic Field. *Progress in Physics*, 2013, vol. 2, 82–86.
9. Deser S. Covariant decomposition of symmetric tensors and the gravitational Cauchy problem. *Annales de l'I.H.P., Section A*, 1967, vol. 7 (2), 149–188.
10. Krupka D. The Trace Decomposition Problem. *Contributions to Algebra and Geometry*, 1995, vol. 36 (2), 303–315.
11. Straumann N. Proof of a decomposition theorem for symmetric tensors on spaces with constant curvature. Cornell University arXiv: gr-qc/0805.4500.
12. Chen X.-S., Zhu B.-C. Physical decomposition of the gauge and gravitational fields. Cornell University arXiv: gr-qc/1006.3926.
13. Chen X.-S., Zhu B.-C. Tensor gauge condition and tensor field decomposition. Cornell University arXiv: gr-qc/1101.2809.
14. Wald R. M. General Relativity. The University of Chicago Press, Chicago, 1984.
15. Szabados L.B. Quasi-Local Energy-Momentum and Angular Momentum in GR: A Review Article. *Living Rev. Relativity*, 2004, vol. 7, 4.

16. Jaramillo J.L., Gourgoulhon E. Mass and Angular Momentum in General Relativity. Cornell University arXiv: gr-qc/1001.5429.
17. Segel L.A. Mathematics Applied to Continuum Mechanics. Dover Publications, New York, 1987.
18. Flügge W. Tensor Analysis and Continuum Mechanics. Springer-Verlag, New York, 1972.
19. Padmanabhan T. Gravitation, Foundations and Frontiers. Cambridge University Press, Cambridge, 2010.
20. Eddington A.S. The Mathematical Theory of Relativity. Cambridge University Press, Cambridge, 1957.
21. Kaku M. Quantum Field Theory; A Modern Introduction. Oxford University Press, Oxford, 1993.
22. Lawden D.F. Tensor Calculus and Relativity. Methuen & Co, London, 1971.
23. Horie K. Geometric Interpretation of Electromagnetism in a Gravitational Theory with Space-Time Torsion. Cornell University arXiv: hep-th/9409018.
24. Sidharth B.G. The Unification of Electromagnetism and Gravitation in the Context of Quantized Fractal Space Time. Cornell University arXiv: gen-ph/0007021.
25. Wu N. Unification of Electromagnetic Interactions and Gravitational Interactions. Cornell University arXiv: hep-th/0211155.
26. Rabounski D. A Theory of Gravity Like Electrodynamics. *Progress in Physics*, 2005, vol. 2, 15–29.
27. Wanas M.I. and Ammar S.A. Space-Time Structure and Electromagnetism. Cornell University arXiv: gr-qc/0505092.
28. Shahverdiyev S.S. Unification of Electromagnetism and Gravitation in the Framework of General Geometry. Cornell University arXiv: gen-ph/0507034.
29. Chang Y.-F. Unification of Gravitational and Electromagnetic Fields in Riemannian Geometry. Cornell University arXiv: gen-ph/0901.0201.
30. Borzou A. and Sepangi H.R. Unification of Gravity and Electromagnetism Revisited. Cornell University arXiv: gr-qc/0904.1363.
31. Chernitskii A.A. On Unification of Gravitation and Electromagnetism in the Framework of a General-Relativistic Approach. Cornell University arXiv: gr-qc/0907.2114.
32. Baylis W.E. Electrodynamics, A Modern Geometric Approach. Birkhäuser, Boston, 2002.
33. Jackson J.D. Classical Electrodynamics, 2nd ed. John Wiley & Sons, New York, 1975.
34. Charap J.M. Covariant Electrodynamics, A Concise Guide. The John Hopkins University Press, Baltimore, 2011.
35. Barut A.O. Electrodynamics and Classical Theory of Fields and Particles. Dover Publications, New York, 1980.
36. Greiner W. Relativistic Quantum Mechanics, Wave Equations. Springer-Verlag, New York, 1994.
37. Misner C.W., Thorne K.S., Wheeler J.A. Gravitation. W.H. Freeman and Company, San Francisco, 1973.

38. Feynman R.P., Leighton R.B., Sands M. Lectures on Physics, Volume II, Mainly Electromagnetism and Matter. Addison-Wesley Publishing Company, Reading, Massachusetts, 1975.
39. Loudon R. The Quantum Theory of Light, Third Edition. Oxford University Press, Oxford, 2000.
40. Heitler W. The Quantum Theory of Radiation, Third Edition. Dover Publications Inc., New York, 1984.
41. Polyanin A.D. Handbook of Linear Partial Differential Equations for Engineers and Scientists. Chapman & Hall/CRC, New York, 2002.
42. Woodside D.A. Uniqueness theorems for classical four-vector fields in Euclidean and Minkowski spaces. *Journal of Mathematical Physics*, 1999, vol. 40, 4911–4943.
43. Cook D.M. The Theory of the Electromagnetic Field. Dover Publications, New York, 2002.
44. Müller-Kirsten H.W. Electrodynamics: An Introduction Including Quantum Effects. World Scientific Publishing, Singapore, 2004.
45. Rousseaux G. On the interaction between a current density and a vector potential: Ampère force, Helmholtz tension and Larmor torque. 2006, http://hal.archives-ouvertes.fr/docs/00/08/19/19/DOC/Larmor4.doc
46. Maugin G.A.. Configurational Forces; Thermomechanics, Physics, Mathematics, and Numerics. CRC Press, New York, 2011.
47. Altenbach H., Maugin G.A., Erofeev V., Eds. Mechanics of Generalized Continua. Springer-Verlag, Berlin, 2011.

Vol. 5, 2012 **ISSN 1654-9163**

THE

ABRAHAM ZELMANOV JOURNAL

The journal for General Relativity, gravitation and cosmology

TIDSKRIFTEN

ABRAHAM ZELMANOV

Den tidskrift för allmänna relativitetsteorin, gravitation och kosmologi

The Abraham Zelmanov Journal is a non-commercial, academic journal registered with the *Royal National Library of Sweden.* This journal was typeset using LaTeX typesetting system.

The Abraham Zelmanov Journal är en ickekommersiell, akademisk tidskrift registrerat hos *Kungliga biblioteket.* Denna tidskrift är typsatt med typsättningssystemet LaTeX.