Power as the Cause of Motion and a New Foundation of Classical Mechanics

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Laws of motion are derived based on power rather than on force. I show how power extends the law of inertia to include curvilinear motion and I also show that the law of action-reaction can be expressed in terms of the mutual time rate of change of kinetic energies instead of mutual forces. I then compare the laws of motion based on power to Newton's Laws of Motion and I investigate the relation of power to Leibniz's notion of vis viva. I also discuss briefly how the metaphysics of power as the cause of motion can be grounded in a modern version of occasionalism for the purpose of establishing an alternative foundation of mechanics. The laws of motion derived in this paper along with the metaphysical foundation proposed come in defense of the hypotheses that time emerges as an ordered progression of now and that gravitation is the effect of energy transfer between an unobservable substance and all matter in the Universe.

1 Introduction

This paper's central aim is the derivation of laws of motion based on the notion of power rather than on the classical notion of force. Although the derivation of laws of motion is traditionally a subject of mechanics, several references are made herein to the history and philosophy of science. This is necessary because this paper deals primarily with the foundations of mechanics. Specifically, the hypothesis that power is the cause of motion, as contrasted to the Newtonian hypothesis according to which force is the cause of motion, leads to a major revision of the foundations of Classical Mechanics.

Most contemporary philosophers of science focus on the foundational problems of General Relativity and Quantum Mechanics and, unlike their seventeenth-century counterparts, think of Classical Mechanics as unproblematic. Butterfield mentions two errors found in this view that correspond to what he calls the matter-in-motion picture and the particle-in-motion picture [1]. According to the matter-in-motion picture, for example, bodies are collections of particles separated by voids, can move in vacuum and interact with each other, whilst their motion is completely determined by Newton's Second Law. This view has become a part of an "educated layperson's" common sense nowadays but according to Butterfield it is problematic: it does not offer, amongst other things, any explanation of the mechanism(s) of the assumed interactions but resorts to concepts such as forces acting across an intervening void ("action-at-a-distance").

The failure of modern theories to provide solutions to the foundational problems of Classical Mechanics is partly due to the fact that alternative rigid foundations have not been proposed but issues seem to have been further perplexed. Quantum uncertainty and the four-dimensional space-time of relativity have taken the place of the determinism and of the unobservable absolute space and universal time of Classical Mechanics. Mysterious action-at-a-distance still prevails in the quantum world and attempts to quantize gravity and unite Quantum Mechanics with General Relativity have failed to this date. In presenting an alternative system of laws of motion based on power, I aim primarily in the investigation of a new foundation, which offers an alternative approach for solutions to some of the unsolved problems of Classical Mechanics.

In a similar way to the matter-in-motion picture, the notion of force has also become part of an "educated layperson's" common sense, thanks to the empirical support the laws of mechanics have enjoyed over the past 300 years. It is well known, however, that Newton was heavily criticized for his use of the notion of force in an effort to ground his physics on his metaphysics and there is still considerable interest in the metaphysics of his *Principia*. In *Science and Hypothesis*, Poincaré writes [2]:

> When are two forces equal? We are told that it is when they give the same acceleration to the same mass, or when acting in opposite directions they are in equilibrium. This definition is a sham.

In *Principles of Dynamics*, Donald T. Greenwood offers an introduction to the issues raised by Newton's concept of force [3]:

> The concept of force as a fundamental quantity in the study of mechanics has been criticized by various scientists and philosophers of science from shortly after Newton's enunciation of the laws of motion until the present time. Briefly, the idea of a force, and a field force in particular, was considered to be an

intellectual construction, which has no real existence. It is merely another name for the product of mass and acceleration, which occurs in the mathematics of solving a problem. *Furthermore, the idea of force as a cause of motion should be discarded since the assumed cause and effect relationship cannot be proven*. (Italics added)

The questions raised from Newton's definition of force and postulation of absolute space are well known to the philosophers of science and will be further discussed in sections 4, 5 and 6. In the following two sections, 2 and 3, I will show that using the notion of power as *a priori* principle, laws of motion can be derived with remarkably different definitions of inertia and action-reaction. I will then argue in section 4, where I discuss the relation of this alternative system of laws to Newton's, that the existence of a more general principle of motion is even acknowledged by Newton, in his own writings. In section 5, the relation of the notion of power to Leibniz's notion of vis viva is examined. Then, in section 6, I discuss how the metaphysics of power can be grounded in a modern version of occasionalism for the purpose of establishing an alternative foundation of Classical Mechanics. I argue that the alternative foundation proposed, along with an appropriate space-time structure, support a new hypothesis about time and about the nature of gravitation.

2 The axiom of motion

I begin the derivation of the laws of motion by stating the axiom of motion, an expression relating the velocity and the time rate of change of momentum of a particle, to a scalar quantity called the time rate of change of kinetic energy, also known as (instantaneous) power. The status of this axiom is assumed here to be that of *a priori* truth as opposed to a self-evident or empirical principle.

Axiom of Motion: The time rate of change of the kinetic energy of a particle is equal to the scalar product of its velocity and time rate of change of its momentum.

Denoting the kinetic energy by E_k and the momentum by $\mathbf{p}$, the axiom of motion can be expressed as follows:

$$\frac{dE_k}{dt} = \frac{d\mathbf{p}}{dt}\cdot\frac{d\mathbf{r}}{dt}\,, \tag{1}$$

where $\mathbf{r}$ is the position vector of the particle. The momentum $\mathbf{p}$ is defined as

$$\mathbf{p} = m\frac{d\mathbf{r}}{dt}\,. \tag{2}$$

If the mass m of the particle is independent of time t and position $\mathbf{r}$, then by combining equations (1) and (2), the time rate of change of the kinetic energy E_k can be written as follows:

$$\frac{dE_k}{dt} = m\frac{d^2\mathbf{r}}{dt^2}\cdot\frac{d\mathbf{r}}{dt}\,. \tag{3}$$

Corollary I: The kinetic energy of a particle with a constant mass m is given by

$$E_k = \frac{1}{2}\,m\mathbf{v}\cdot\mathbf{v}\,, \tag{4}$$

where $\mathbf{v}$ is defined as

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}\,. \tag{5}$$

Proof: From equation (3) we obtain

$$\frac{dE_k}{dt} = m\frac{d^2\mathbf{r}}{dt^2}\cdot\frac{d\mathbf{r}}{dt} = m\frac{d\mathbf{r}}{dt}\cdot\frac{d}{dt}\left(\frac{d\mathbf{r}}{dt}\right) = m\frac{d}{dt}\left(\frac{1}{2}\frac{d\mathbf{r}}{dt}\cdot\frac{d\mathbf{r}}{dt}\right),$$

which yields

$$E_k = \frac{1}{2}\,m\frac{d\mathbf{r}}{dt}\cdot\frac{d\mathbf{r}}{dt} = \frac{1}{2}\,m\mathbf{v}\cdot\mathbf{v}\,. \tag{6}$$

The axiom of motion is the only principle required for deriving the laws of motion, as it will be shown in the next section.

3 The laws of motion

Law of Inertia: If the time rate of change of the kinetic energy of a particle is zero, the particle will continue in its state of motion.

Proof: If the time rate of change of the kinetic energy of a particle is zero, then from equation (3) we obtain

$$m\frac{d^2\mathbf{r}}{dt^2}\cdot\frac{d\mathbf{r}}{dt} = 0\,. \tag{7}$$

Assuming m remains constant, the following satisfy equation (7)

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}_0\,, \tag{8}$$

$$\frac{d\mathbf{r}}{dt} = 0\,, \tag{9}$$

$$\frac{d^2\mathbf{r}}{dt^2}\cdot\mathbf{v} = 0\,, \tag{10}$$

where $\mathbf{v}_0$ is a constant. Thus, solutions to equation (7) include motion with a constant velocity $\mathbf{v}_0$, given by equation (8), or a state of rest, given by equation (9) and in both these cases the time rate of change of kinetic energy is zero. These are trivial solutions to equation (7) arising when either the velocity or the acceleration of the particle, are null vectors. Yet, these two trivial solutions result in the simplest kinematic states possible and the only two states allowed when there are no forces acting on a particle according to Newton's First Law. However, if power is postulated as the cause of motion there is another trivial solution, that of uniform circular motion, as it will be shown below.

General solutions to equation (10) include all curvilinear paths with a constant kinetic energy E_k. The requirement of a constant kinetic energy could have been included in the statement of the law of inertia but this is obviously redundant since, if the time rate of change of the kinetic energy is zero then kinetic energy is constant. Clearly, the states of motion resulting from (8) and (9) are trivial solutions to (10) with zero velocity and zero acceleration, respectively. From equations (5), (6) and (10) we obtain:

$$\frac{d^2\mathbf{r}}{dt^2}\cdot\frac{d\mathbf{r}}{dt}=0 \Leftrightarrow \frac{d\mathbf{r}}{dt}\cdot\frac{d\mathbf{r}}{dt}=\mathbf{v}\cdot\mathbf{v}=\frac{2E_k}{m}=k\,, \tag{11}$$

where k is a constant equal to twice the kinetic energy per unit mass. Thus, all motion paths that satisfy equation (10) also satisfy the following equation

$$\frac{d\mathbf{r}}{dt}\cdot\frac{d\mathbf{r}}{dt}=k\,, \tag{12}$$

which is equivalent to the statement that the magnitude of velocity, or the speed, must be constant. In the case of motion in a plane, $\mathbf{v}$ can be expressed in polar coordinates as follows:

$$\mathbf{v}=\frac{dr}{dt}\hat{\mathbf{r}}\,+\,r\frac{d\theta}{dt}\hat{\theta}\,. \tag{13}$$

From equations (12) and (13) we obtain:

$$\left(\frac{dr}{dt}\right)^2+\left(r\frac{d\theta}{dt}\right)^2=k^2. \tag{14}$$

A trivial solution to equation (14) is uniform circular motion given by

$$\mathbf{r}(t)=r\,\hat{\mathbf{r}}(t)\,, \tag{15}$$

where r is a constant radius and the unit radial vector $\hat{\mathbf{r}}$ rotates at a constant rate $d\theta/dt$. In the context of this law of inertia, if a particle is in uniform circular motion and the time rate of change of its kinetic energy remains zero, the state of uniform circular motion will be maintained. Notice that no claim of any sort is made herein that zero power is the cause of uniform circular motion. Obviously, a zero of something cannot be the real cause of anything. The only claim made is that if a particle is in uniform circular motion -or in any other curvilinear path that satisfies equation (12) — and power, the postulated cause of motion, remains zero then the particle will continue in its state of motion. I would like to stretch this point because, as it will be discussed further in chapter 4, the laws of motion presented in this paper can be considered as an alternative to Newton's Laws of Motion. Thus, one should refrain from evaluating these laws in the context of Newtonian mechanics, since the two systems of laws are grounded in different metaphysics. The question then of how a particle is set on a uniform circular motion in the first place is a metaphysical one and it will be placed in its proper context in chapter 6.

Non-trivial solutions to equation (14) include motion in a plane where the magnitude of the velocity $\mathbf{v}$ remains constant up to sign changes. Such motion possibilities are virtually unlimited, including for instance motion in eight-shaped figures and cycloid paths. However, some of these paths may represent physical possibilities and others may not. Uniform circular motion is a physical possibility in both micro and macro scales and this has been confirmed empirically. The choice of specific curvilinear motions over others as an effect of inertia, if power is postulated to be the cause of motion, is the subject of metaphysics discussed in section 6. The law of inertia presented in this section is a statement that the state of such motions is maintained in the absence of a cause, if power is postulated to be the cause of motion. However, the law does not provide a justification for the existence or preference of certain states of motions over others in the absence of a cause of motion.

General solutions to equation (12) in three-dimensional Euclidean space include motion along any curve. It is known from differential geometry that if a curve is regular, then there exists a reparametrization such that the curve has unit speed [4]. Thus, a particle can be made to move with constant speed along any curve in space using proper arc-length reparametrization resulting in constant kinetic energy and as a consequence, zero power.

The law of inertia is a statement about the tendency of particles to maintain their state of motion when the time rate of change in their kinetic energy is zero and this tendency is called *inertia*. Again, the law of inertia was derived based on the metaphysical hypothesis that power is the cause of motion. A consequence from such hypothesis is that the set of "cause-free" paths now includes all paths where the kinetic energy remains constant, instead of just uniform rectilinear motion and the state of rest defined in Newtonian mechanics. As it will be discussed in section 4.1, from an empirical viewpoint it is irrelevant whether one considers just rectilinear or curvilinear motion as an effect of inertia, since no experiment can be devised to prove that in the case of a freely moving particle. This is because, there is always a cause present affecting the motion of all particles. In the case of Newtonian mechanics, this cause is a gravity force and in the case of the laws of motion discussed in this paper there is always a power cause acting and giving rise to gravitational effects as it will be discussed in chapter 6.

Corollary II: If the time rate of change of the kinetic energy of a particle is zero, linear momentum is conserved.

Proof: As a direct consequence of the law of inertia, if the time rate of change of kinetic energy is zero and the velocity is denoted by $\mathbf{v}$, then from equations (1) and (5) we obtain

$$\frac{d\mathbf{p}}{dt}\cdot\mathbf{v}=0\,. \tag{16}$$

By using equation (2) and since $\mathbf{v}$ is not the null vector in general, we obtain from equation (16) the result:

$$\frac{d(m\mathbf{v})}{dt} = 0 \Rightarrow (m\mathbf{v})_2 - (m\mathbf{v})_1 = 0 \Rightarrow \\ \Rightarrow (m\mathbf{v})_2 = (m\mathbf{v})_1 = m\mathbf{v} = \text{const}\,. \tag{17}$$

Equation (17) is the mathematical statement of the theorem of the conservation of linear momentum [5].

Law of Interaction: To every action there is an equal and opposite reaction; that is, in an isolated system of two particles acting upon each other, the mutual time rate of change of kinetic energies are equal in magnitude and opposite in sign.

Proof: We denote the two interacting particles as m_1 and m_2. Furthermore, we denote m_1 as the agent causing the action in the system. The total kinetic energy of the interacting system of particles is the sum of the kinetic energies of the two particles:

$$E_k = E_{k_1} + E_{k_2}\,. \tag{18}$$

From equations (1), (5) and (18) we obtain

$$\frac{dE_k}{dt} = \frac{d\mathbf{p_1}}{dt}\cdot\mathbf{v_1} + \frac{d\mathbf{p_2}}{dt}\cdot\mathbf{v_2}\,, \tag{19}$$

where $\mathbf{v_1}$ and $\mathbf{v_2}$ are the velocities of the two particles with momentum $\mathbf{p_1}$ and $\mathbf{p_2}$, respectively.

Next, we consider the mutual time rate of change of kinetic energy imposed by the particles upon each other. The time rate of change of kinetic energy of particle m_2, denoted as E_{k_2}, is equal to the action imposed on it by particle m_1, denoted as $E_{k_{12}}$ and given by

$$\frac{dE_{k_2}}{dt} = \frac{d\mathbf{p_2}}{dt}\cdot\mathbf{v_2} = \frac{dE_{k_{12}}}{dt}\,. \tag{20}$$

The time rate of change of the kinetic energy of particle m_1 is equal to the sum of the time rate of change of the kinetic energy of the system due to its action as an agent and that imposed on it by particle m_2 in the form of a reaction and denoted as $E_{k_{21}}$

$$\frac{dE_{k_1}}{dt} = \frac{dE_k}{dt} + \frac{dE_{k_{21}}}{dt} = \frac{d\mathbf{p_1}}{dt}\cdot\mathbf{v_1}\,. \tag{21}$$

By combining equations (19), (20) and (21), we obtain the result:

$$\frac{dE_{k_{12}}}{dt} = -\frac{dE_{k_{21}}}{dt}\,. \tag{22}$$

Equation (22) is the mathematical statement of the law of interaction. According to the law, the reaction on a horse pulling on a cart, — to use Newton's example in the Principia — is equal to the action applied by the horse on the cart. In general, part of the action produced by the horse is used to change its own state of motion and the remaining to change that of the cart. In the case where the total action of the horse is reacted by the cart, from equation (21) it may be seen that dE_k/dt is equal to zero and the state of motion does not change. Then, in this special case, action is equal to reaction *by definition*. This can serve the purpose of clearing any confusion that may arise when the action by the horse on the cart is thought to be equal to the total action produced by the horse, a statement that is not true in the most general case.

The philosophical issues arising from the law of interaction will be discussed in more detail in section 4.

Corollary III: In an isolated system of two particles acting upon each other and both having velocity $\mathbf{v}$, the mutual time rate of change of momentum vectors are equal in magnitude and opposite in direction.

Proof: By denoting the mutual momentum vectors by $\mathbf{p_{12}}$ and $\mathbf{p_{21}}$, from equations (1), (5) and (22) we obtain

$$\frac{d\mathbf{p_{12}}}{dt}\cdot\mathbf{v} = -\frac{d\mathbf{p_{21}}}{dt}\cdot\mathbf{v} \Leftrightarrow \left(\frac{d\mathbf{p_{12}}}{dt} + \frac{d\mathbf{p_{21}}}{dt}\right)\cdot\mathbf{v} = 0\,. \tag{23}$$

Since $\mathbf{v}$ is not in general a null vector, we obtain the result:

$$\frac{d\mathbf{p_{12}}}{dt} = -\frac{d\mathbf{p_{21}}}{dt}\,. \tag{24}$$

In the case where $\mathbf{v}$ is orthogonal to the sum of the mutual time rate of change of the momentum vectors of the two particles, then equation (23) will still hold. However, in this case, the mutual time rate of change of momentum vectors will not in general be equal in magnitude and opposite in direction.

The axiom of motion of section 2, together with the law of inertia and the law of interaction, combined further with the axiom of conservation of energy of isolated systems, provide a framework for deriving the differential equations of motion of particles and by extension of rigid bodies in dynamical motion. Next, I will examine the relation of the laws of motion presented in this section to Newton's Laws of Motion.

4 Power versus force

Newton stated his laws of motion in *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy), first published in 1686 [6]. The Principia was revised by Newton in 1713 and 1726. Using modern terminology, the laws can be stated as follows [3]:

First Law: Every body continuous in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces acting upon it.

Second Law: The time rate of change of linear momentum of a body is proportional to the force acting upon it and occurs in the direction in which the force acts.

Third Law: To every action there is an equal and opposite reaction and thus, the mutual forces of two bodies acting upon each other are equal in magnitude and opposite in direction.

4.1 Newton's First Law: A priori truth or an experimental fact?

Newton's First Law can be deduced from the law of inertia stated in section 3 and specifically from equations (8) and (9), or from corollary II. According to the law of inertia, when the time rate of change of the momentum of a particle is zero, then that particle will either remain at rest or move in a straight line with constant velocity $\mathbf{v}_0$.

It is interesting to recall Newton's comments in Principia that follow the First Law [6]:

> Projectiles continue in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are continuously drawn aside from rectilinear motion, do not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in freer spaces, preserve their motions both progressive and circular for a much longer time.

The first part of Newton's comments regarding the projectile motion is problematic from an empirical perspective. No experiment can be devised where a projectile will move in the absence of gravity. Thus, there can be no cause free motion experiments in the context of Newtonian mechanics in order to observe what the resulting motion would be if the cause were to be removed. Therefore, it seems that Newton was referring to a thought experiment than to a well-established empirical fact. Furthermore, in the remaining part of Newton's comment regarding the First Law, things become even more interesting as he attempts to draw conclusions regarding the validity of the First Law from the motion of rotating bodies, such as spinning disks and planets. This is obviously a peculiar attempt for a connection between the rectilinear motion the First Law deals exclusively with, and rotational motion in the absence of a resisting medium. It appears that Newton's attempt to provide conclusive empirical support of the First Law is fraught with difficulties simply because no experiments can be devised from which the First Law can be inferred from the phenomena and rendered general by induction. This fact turns out to conflict with Newton's statement in the general scholium in book III of the Principia [6]:

> In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive forces of bodies, and the laws of motion and gravitation, were discovered.

The First Law and specifically the statement that bodies remain at rest or move uniformly in a straight line unless a force acts upon them, does not comply with the rules of the (experimental) philosophy Newton claims to abide with. The First Law does not deal with circular orbits, even if such orbits were employed by Newton as an example in his attempt to justify it. The First Law is actually an axiom, which must be accepted without proof, and not a statement derived via the use of inductive methodology. This is again due to the fact that no experiment can be devised on our planet for the purpose of observing what the motion of a projectile would be when there is no force acting upon it. According to Newton's Law of Universal Gravitation, gravity forces act upon a body unless it is set in motion in a region of space sufficiently far away from the influence of other bodies. Is then Newton alluding to the possibility of the existence of a more general First Law similar to the law of inertia of section 3? Let us recall what Poincaré said [2]:

> *The Principle of Inertia*. — A body under the action of no force can only move uniformly in a straight line. Is this a truth imposed on the mind *à priori*? If this be so, how is it that the Greeks have ignored it? How could they have believed that motion ceases with the cause of motion? Or, again, that every body, if there is nothing to prevent it, will move in a circle, the noblest of all forms of motion? If it be said that the velocity of a body cannot change, or there is no reason for it to change, may we not just as legitimately maintain that the position of a body cannot change, or that the curvature of its path cannot change, without the agency of an external cause? Is, then, the principle of inertia, which is not an *à priori* truth, an experimental fact? Have there ever been experiments on bodies acted on by no forces? And, if so, how did we know that no forces were acting?

Poincaré continues with his discussion of the principle of inertia by stating that

> Newton's First Law could be the consequence of a more general principle, of which the principle of inertia is only a particular case.

In turn, I argue that the axiom of motion, equation (1), can serve the role of this more general principle and Newton's First Law is indeed a special case of a more general law of inertia, such as the one derived in section 3.

Thus, I essentially argue that Newton's First Law makes reference to phenomena that are just two possibilities within a broader range of possibilities mandated by a more general principle of inertia, such as the law of inertia of section 3. As I will demonstrate in the proceedings, the same holds true with Newton's Third Law. There, matters are even clearer regarding my argument that Newton's laws are just a special case of the laws presented in section 3.

4.2 Newton's Second Law: The metaphysical cause of motion

The mathematical expression of Newton's Second Law, after a suitable choice of units is made is the following [3]:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v})\,. \tag{25}$$

With the Second Law, Newton defines force as the cause of motion and equates it to the time rate of change of momentum. The laws of motion presented in section 3, based on the axiom of motion, challenge the notion that the Newtonian force is the cause of motion and the metaphysical foundation of mechanics. However, in these laws of motion, the metaphysics of force are replaced by those of the time rate of change of kinetic energy, also known as *power*. In a way analogous to Newton's Second Law, the axiom of motion stated in section 2 can be expressed as follows

$$P = \frac{d(E_k)}{dt}\,, \tag{26}$$

where P is the (instantaneous) power and E_k the kinetic energy of a particle.

> When we say force is the cause of motion, we are talking metaphysics. . .

writes Poincaré in *Science and Hypothesis* [2]. This statement made by Poincaré also applies when the time rate of change of kinetic energy, or power, is defined as the cause of motion. Whether using force or power, the physics of the associated laws of motion must be grounded in some metaphysics and this is done in section 6. It is important to understand that the particular choice of a quantity to assume the role of the cause of motion becomes the link between the empirical world of physics and the metaphysics of what exists and is real. Thus, although one can choose either force or power as the basis of stating laws of motion, the metaphysical foundations of such laws will turn out to be profoundly different. Newton used his notion of force to ground his physics in the metaphysics of absolute space and time. In section 6, I will discuss how the notion of power grounds the physics of the laws of motion of section 3 in the metaphysics of a modern version of Cartesian occasionalism and a dual space-time account. It turns out that the view of the world implied by such metaphysics is very different from the Newtonian or Leibnizian ones.

Besides the difference in metaphysics, the alternative to Newton's second law given by equation (26) offers an advantage in resolving some philosophical issues regarding the foundations of Classical Mechanics and in particular the need to consider fictitious forces when applying Newton's Second Law in non-inertial reference frames. In the case of observers at rest in accelerated reference frames in either rectilinear or uniform circular motion, the time rate of change of kinetic energy is zero and thus no additional fictitious power cause is needed to explain the state of motion. Again, this is only true if power is defined as the cause of motion. If force is defined as the cause of motion then in both non-inertial reference frames mentioned fictitious causes must be considered. Specifically, in the case of rectilinear motion, observers at rest in an accelerated frame must assume inertial fictitious forces acting and in the case of observers at rest in a uniformly rotating reference frame, centrifugal forces acting must be assumed.

The same conclusion holds in the case of fictitious Coriolis forces acting on freely moving particles in rotating reference frames. Since such fictitious forces are always orthogonal to the velocity of a particle in motion, for rotating observers it turns out that the time rate of change of kinetic energy of the particle is equal to zero, as obtained by equation (1). The same result is true for observers at rest since in that case the time rate of change of momentum of a freely moving particle is zero. Fictitious forces need to be considered regardless of whether force or power is defined as the cause of motion when a force analysis is carried out. However, when power is defined as the cause of motion, there are no philosophical issues arising from the need to consider fictitious causes of motion in non-inertial reference frames and this is the point just made. Thus, the transition from force to power as the cause of motion leads to a compatibility with the epistemological principle which states that every phenomenon is to receive the same interpretation from any given moving coordinate system. This epistemological principle also plays an important role in the axiomatic foundation of the theory of relativity [7].

4.3 Newton's Third Law: a special case of a more general action-reaction law?

Newton's Third Law may be deduced from the law of interaction of section 3 and in particular from equation (24) of corollary III. In the scholium following the Laws of Motion, Newton attempts to provide additional support for the Third Law through a host of observations related to various modes of mechanical interaction between bodies. From the closing comments in the scholium, some interesting conclusions can be drawn [6]:

> . . .But to treat of mechanics is not my present business. I was aiming to show by those examples the greater extent and certainty of the third Law of Motion. *For if we estimate the action of the agent from the product of its force and velocity* and likewise the reaction of the impediment from the product of the velocities of its several parts, and the forces of resistance arising from friction, cohesion, weight, and acceleration of those parts, the action and reaction in the use of all sorts of machines will be found always equal to one another. And so far as the action is propagated by the intervening instruments, and at last impressed upon the resisting body, the ultimate action will be always contrary to the reaction. (Italics added)

It is clear that Newton was well aware of the product of velocity and force being a measure of action and of reaction, as defined in the law of interaction of section 3. Newton actually made use of the law of interaction in his scholium above to justify some particular situations where his Third

Law of action-reaction does not apply directly. But why is it the case that Newton stated his Third Law in terms of forces and not in terms of the product of force and velocity he mentions in his scholium quoted above? Why does it appear that a more general law was used to justify some particular situations Newton's Third Law does not directly apply to, but the latter was stated as a law of mechanics? The answer can be found in the attempt to model gravity in Newtonian mechanics as the effect of mutual attraction caused by central forces acting at a distance. The Third Law had to be stated in terms of the mutual action-reaction forces being equal in magnitude and opposite in direction to justify the particular form of Newton's Law of Universal Gravitation. But again, the Third Law fails the requirement set forth by the rules of the experimental philosophy of Newton, for it being deduced from the phenomena; it is just another axiom that must be accepted without proof. Forces acting on different bodies, and especially celestial ones, cannot be experimentally determined to be equal. Only forces acting on the same body can be determined to be equal by experiment.

I have shown that even Newton himself made both indirect and direct use of the notion of power in an attempt to provide a general justification of his Third Law. Can we simply assume that Newton was unaware that there is a single principle that could serve as the basis of a system of laws of mechanics that are in a certain way more general than his laws? I suspect that he was aware of it. But the consequences from stating laws based on this principle of motion would be devastating on the metaphysics of force. If force were to be just an intellectual construction and not the cause of motion, then Newton's whole system of the world was at stake. Motion then would have to be explained based on some other metaphysics, such as Cartesian occasionalism for example and the notion that all causes are due to God, or Spinoza's doctrine that everything is a mode of God [8], or even Leibniz's notion of a living force.

5 Power versus vis viva

Leibniz rejected the doctrine of Cartesian occasionalism and Newtonian substantivalism but his efforts to ground his relationism on the metaphysics of a living force were also met with difficulties. Leibniz realized that for motion to be real, it must be grounded on something that is not mere relation, something absolute and unobservable that serves as its cause [8]. Leibniz stated his laws of motion in his unpublished during his lifetime work *Dynamica de Potentia et Legibus Naturae Corporeae* in which he attempted to explain the world in terms of the conservation laws of vis viva and momentum of colliding bodies.

The laws of inertia and interaction of section 3 were derived from the axiom of motion of section 2. The latter is related to the living force, or vis viva, defined by Leibniz as being a real metaphysical property of a substance. Leibniz measure of vis viva is the quantity mv^2, in contrast to the Cartesian definition of the *quantity of motion* being equal to size multiplied by speed, and later redefined by Newton as being equal to the product of mass and velocity. In turn, the axiom of motion stated in section 2 is related to the time rate of change of vis viva, the quantity Leibniz argued is conserved and a real metaphysical property of a substance, in an effort to support his relational account of space-time.

Leibniz's definition of vis viva as a real metaphysical property of a substance is fraught with difficulties. Roberts has argued that, in his later communications with Samuel Clarke, who was a defender of Newton's substantivalism, Leibniz seems to commit to a richer space-time structure that can support absolute velocities [9]. Roberts' work has cast light into a little known, or maybe misinterpreted, aspect of Leibniz's metaphysics. Specifically, into Leibniz's efforts to come up with laws of motion based on vis viva being a measure of force, while at the same time his relationism implies a space-time structure that is a well-founded phenomenon. This might be an indication of Leibniz's later realization that relationism fails unless absolute velocities are supported by a richer space-time structure than what is commonly referred to as Leibnizian space-time. In section 6, I define an account of space-time that can support relationism and absolute velocities in an attempt to ground the physics of the axiom and laws of motion in the metaphysics of power.

Along these lines, in a similar way to the link between the Newtonian force and momentum, the former being the time rate of change of the latter, I argue that vis viva is actually a quantity of motion and power, its time rate of change, is the cause of motion. In this way the similarities between the laws of conservation of momentum and vis viva become evident, because they are both defined as quantities of motion. In essence, I argue, the time rate of change of vis viva is the real metaphysical cause of motion. Of course, such a switch in the definitions is not compatible with Leibniz's metaphysics. This is because the time rate of change of the kinetic energy of a body moving with constant linear velocity, or even in uniform circular motion, is zero. A zero of something cannot assume the role of a real metaphysical property of a substance and the cause of motion in a Leibnizian world. Despite these metaphysical difficulties I will deal with in more detail in the next section, on the physics side it is clear that the laws of motion of section 3 were derived from a quantity that is proportional to the time derivative of vis viva. Thus, they have a direct link to Leibniz's Laws of Motion [8]. Specifically, Leibniz's laws of conservation of vis viva and momentum can be derived from the laws of inertia and interaction of section 3, respectively, but the details are left out.

6 The metaphysics of power

Before I discuss the metaphysics of power and specifically the notion that power is the cause of motion, I will briefly review the philosophical debate about the ontology of space-time. I argue that the space-time debate and the debate about the cause of motion are closely related in the sense that an answer to the former provides an answer to the latter. Thus, I essentially argue that the space-time debate is not a mere philosophical one and its resolution will have a decisive impact on which laws of motion and gravitation are assigned the status of "laws of nature" as opposed to that of mere heuristics.

6.1 The space-time debate

The publication of Newton's Principia in 1686 was the cause of the start of one of the most interesting debates in the history of the philosophy of science, dealing mainly with the ontology of space-time. Leibniz ignited the debate by arguing that Newton's substantival space-time, the notion that space and time exist independently of material things and their spatiotemporal relations, was not a well-founded phenomenon. Leibniz confronted Newtonian substantivalists with his relationism, based on which space is defined as the set of (possible) relations among material things and the only well-defined quantities of motion are relative ones [10]. Newton just grounded his physics in the metaphysics of force and absolute space and time. For Newton, the only well-defined quantities of motion are the absolute ones, like absolute position, velocity and acceleration. Substantivalism and relationism then appear in modern literature as two completely different accounts of space-time.

The key issue regarding the space-time debate, which is still alive by the way, is whether it does really make sense to speak of *either* a substantival *or* a relational account of space-time. Since diametrically opposite views of this kind have only led to sharp conflict and irreconcilable differences, maybe it would make sense to investigate whether both a substantival and relational space-time is a possibility. This two-level approach seems not to have been considered seriously because it implies a superfluous world. However, both Newtonian substantivalism and Leibnizian relationism are fraught with difficulties. On one hand, the metaphysics of Newtonian force require the postulation of unobservables, like absolute space. On the other hand, in Leibniz's relationism, for motion to be real, it must be grounded in something that is not mere relation, something absolute and unobservable that serves as its cause, what Leibniz called a vis viva [9]. The differences seem to reconcile when a two-level, or if I may call it a dual, space-time account is postulated and I will throw in here the term *substantival relationism*.

6.2 From cause-free motion to gravitation

The hypothesis about the duality of space-time just put forward is next examined in the context of gravitation and its observable effects, i. e. the motion of celestial bodies and free-falling particles. This step is of great importance since any laws of motion must account for all observable phenomena including those that are attributed to gravitation. Newton accomplished the step of grounding the physics of the Laws of Motion to his metaphysics of substantival space and universal time, by assuming that the cause of gravitation was also some type of force. Next, in what was a remarkable achievement in the history of science, he derived the famous Law of Universal Gravitation (LUG). In a similar way, I argue that power is the cause of gravitation in order to maintain a compatibility with the axiom and laws of motion of sections 2 and 3, respectively. Thus, the time rate of change of a potential energy function $E_p(r)$ is the cause of gravitation and equation (1), the axiom of motion, becomes

$$\frac{dE_k}{dt} = \frac{d\mathbf{p}}{dt} \cdot \frac{d\mathbf{r}}{dt} = -\frac{dE_p}{dt}\,. \tag{27}$$

The law of conservation of mechanical energy can be derived from equation (27) as follows:

$$\frac{dE_k}{dt} = -\frac{dE_p}{dt} \;\Leftrightarrow\; \frac{d}{dt}(E_k + E_p) = 0 \;\Leftrightarrow\; \Leftrightarrow\; E_k + E_p = \text{const}\,. \tag{28}$$

The Law of Universal Gravitation may be restated as follows:

Law of Universal Gravitation: All particles move in such a way as for the time rate of change of their kinetic energy to be equal the time rate of change of their potential energy.

In fact, I argue that Newton's Law of Universal Gravitation is a statement about the form of the potential function $E_p(r)$ in equation (27) and thus it can assume a variety of interpretations regarding mechanisms giving rise to it. If we postulate that energy transfer affects all particles in motion, in accordance with equation (27), this can support the hypothesis that gravitation is the result of energy transfer between all bodies in motion with some substance. Substantival space-time can serve the role of this substance and can facilitate the energy transfer to and from all bodies in motion and in such a way that all spatiotemporal quantities evolve according to certain rules giving rise to the well-known potential function $E_p(r)$ first discovered by Newton.

Since the above metaphysics are compatible with the concept of a mechanical universe, one could then postulate the existence of some type of mechanism that facilitates the transfer of energy between all bodies in motion and substantival space-time. This mechanism must be part of the substance level, whereas at the phenomenal level its effect is the observed motions. According to this dual scheme, at the phenomenal level the only well-founded quantities of motion are relative ones and space-time is relational, whereas, at the substance level, the only well-defined quantities of motion are the absolute ones and the space-time is substantival.

6.3 A new foundation of mechanics

The hypothesis just made, attributing gravitation to energy transfer between all bodies in motion and substantival space-time requires that at every instance something must accomplish this task and bring about the perceived effects. I will relate this to occasionalism in the following way: according to Nicolas Malebranche and other seventeenth-century Cartesian occasionalists, what we actually call causes are really no more than *occasions* on which, in accordance with his own laws, God acts to bring about the effect [11]. If one were to replace the notion of God by the notion of a mechanism, then a modern (or mechanical) occasionalist could assert that what we actually call causes are no more than occasions on which a mechanism acts to bring about the effect. In this sense we immediately resolve two more issues: first, time emerges as an ordered progression of instances, or nows, on which the mechanism acts to bring about the effect. Then, the matter-in-motion picture [1] is better illuminated by asserting that all motion and interactions of material bodies are facilitated by a mechanism that operates based on its own rules rather than taking place due to forces or based on rules inherent in the bodies themselves.

The concept of time as a collection of nows is in fact similar to that found in Barbour [12]. The main difference with the view I express here is that time emerges due to the actions of a mechanism hidden in substantival space-time in an orderly fashion and has a direction, i. e. there is an arrow of time. More importantly, the universal clock of Newton is now part of the mechanism that resides in substantival space-time but at the phenomenal level time and motion cannot be separated because there is no motion without time and no time without motion, i. e. time and motion are inextricably related.

What I argue essentially is that gravitation has an external cause to the phenomenal level and space-time is a substance of some kind that facilitates the energy transfer required for the manifestation of gravitational effects. These ideas may not be completely new. What is new here is the derivation of a system of laws of motion based on the notion of power. Power allows grounding the physics that all phenomena are caused by energy transfer, including those attributed to gravitation, to the metaphysics of substantival space-time being a giant mechanism and a substance. Since the times of the Greeks, Anaximander of Miletus (c. 650 BCE) expressed the view that

> The apeiron, from which the elements are formed, is something that is different (from the elements).

Then, Newton argued that all motion must be referenced to an absolute, unobservable space. Even in general relativity space-time retains its substantival account and it exists independently of the events occurring in it [10]. Baker has argued that the space-time of general relativity must be a substance and attempts to support this claim of his based on the observed expansion of the Universe [13]. Baker's argument about the requirement of a carrier of gravitational energy from its source to a detector, if it is to be compelling, must apply to all forms of energy transfer traditionally assumed to take place in vacuum. But such generalization can be further coupled with the hypothesis that some causes are external to the world of observable phenomena. In Wiithrich there are references made to the hypothesis that gravity forces have an external cause in an attempt to explain the failure in quantizing the field equations of general relativity [14]. Thus, arguments have already been made in favor of the hypothesis that space-time is some kind of a substance and that any causal connections attributed to gravitation are apparent. Usually, arguments leading to such provocative hypotheses are treated at the level of epistemological skepticism but as McCabe argues the hypothesis, for instance, that our universe is part of a computer simulation implementation generates empirical predictions and it is therefore a falsifiable hypothesis [15]. One question that arises from this discussion is the following: does the existence of external causes imply that our world is some type of virtual reality? My own answer to this important question is both yes and no. Yes, because according to the hypothesis there are external causes to the world of perceived phenomena and thus part of another world. No, because a cause being external and unobservable does not preclude it being part of an all-encompassing entity, which we can call Universe. Therefore, the answer to the question seems to depend on how one defines *Universe*. But the presence of external causes to the world of observable phenomena must not be rejected *a priori* on the basis that it leads to the provocative virtual reality hypothesis and experimental physics must pursue seriously its falsification or corroboration. Although such task is highly challenging, the state-of-the-art in precision instrumentation has reached levels that allow the initiation of a program of this nature.

7 Summary

The axiom and laws of motion presented in sections 2 and 3, respectively, are:

Axiom of Motion: The time rate of change of the kinetic energy of a particle is the scalar product of its velocity and time rate of change of its momentum.

Law of Inertia: If the time rate of change of the kinetic energy of a particle is zero, the particle will continue in its state of motion.

Law of Interaction: To every action there is an equal and opposite reaction; that is, in an isolated system of two particles acting upon each other, the mutual time rate of change of kinetic energies are equal in magnitude and opposite in sign.

A restatement of the Law of Universal Gravitation was presented in section 6 as follows:

Law of Universal Gravitation: All particles move in such a way as for the time rate of change of their kinetic energy to be equal to the time rate of change of their potential energy.

In section 4, I argued that the above laws of motion are, in a certain sense, more general than Newton's, and that this claim is even supported by Newton's own writings, especially in the case of the Third Law. Furthermore, in section 5, I discussed the relation of the axiom and laws of motion to Leibniz's laws of the conservation of vis viva and momentum. I argued that kinetic energy can be defined as a quantity of motion and its time derivative as the cause of motion, in a similar way to the Newtonian force being the time derivative of momentum and a postulated cause of motion.

In section 6, I discussed how the axiom and laws of motion of sections 2 and 3, combined further with a modified version of Cartesian occasionalism and a dual space-time account form an alternative foundation of classical mechanics in the context of a mechanical Universe. Specifically, I proposed a substantival-relational account of space-time and a mechanism residing in the substance level whose actions coordinate all motion and interactions. I argued that the proposed foundation supports the hypothesis about gravitation being the effect of energy transfer between all bodies in motion and substantival space-time and I stated a version of the Law of Universal Gravitation which is compatible with the hypothesis that power is the cause of motion. These metaphysics also provide solutions to some foundational problems of Classical Mechanics, such as the matter-in-motion picture and the emergence and direction of time. Finally, I briefly referred to the ramifications on the nature of our physical reality when the cause of gravitation is considered part of an unobservable substance. I argued that the soundness of the virtual reality or computer simulation hypothesis depends on how Universe is defined. The fact that such hypothesis about the nature of our reality is provocative should not be an excuse for rejecting *a priori* external causes of motion and gravitation. Theoretical physicists ought to seriously investigate new models incorporating such assumptions about the nature of our physical reality and experimental physicists should pursue their falsification.

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