Steady Particle States of Revised Electromagnetics

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A revised Lorentz invariant electromagnetic theory leading beyond Maxwell's equations, and to a form of extended quantum electrodynamics, has been elaborated on the basis of a nonzero electric charge density and a nonzero electric field divergence in the vacuum state. Among the applications of this theory, there are steady electromagnetic states having no counterpart in conventional theory and resulting in models of electrically charged and neutral leptons, such as the electron and the neutrino. The analysis of the electron model debouches into a point-charge-like geometry with a very small characteristic radius but having finite self-energy. This provides an alternative to the conventional renormalization procedure. In contrast to conventional theory, an integrated radial force balance can further be established in which the electron is prevented from "exploding" under the action of its net self-charge. Through a combination of variational analysis and an investigation of the radial force balance, a value of the electronic charge has been deduced which deviates by only one percent from that obtained in experiments. This deviation requires further investigation. A model of the neutrino finally reproduces some of the basic features, such as a small but nonzero rest mass, an angular momentum but no magnetic moment, and long mean free paths in solid matter.

1 Introduction

Maxwell's equations in a vacuum state with a vanishing electric field divergence have served as a basis for quantum electrodynamics (QED) in its conventional form [1]. This theory has been very successful in many applications, but as stated by Feynman [2], there still exist areas within which it does not provide fully adequate descriptions of physical reality. When applying conventional theory to attempted models of the electron, there thus appear a number of incomprehensible and unwieldy problems. These include the existence of a steady particle state, the unexplained point-charge-like geometry, the question of infinite self-energy and the associated physical concept of renormalization with extra added counter terms [3], the lack of radial force balance of the electron under the action of its self-charge [4], and its unexplained quantized charge. Also the models of an electrically neutral state of the neutrino include a number of questions, such as those of a nonzero but small rest mass, a nonzero angular momentum and a vanishing magnetic moment, and excessively long mean free paths for interaction with solid matter.

The limitations of conventional theory have caused a number of authors to elaborate modified electromagnetic approaches aiming beyond Maxwell's equations. Among these there is a theory [5–12] to be described in this paper. It is based on a vacuum state that can give rise to local space charges and an associated nonzero electric field divergence, leading to a current in addition to the displacement current. The field equations are then changed in a substantial manner, to result in a form of extended quantum electrodynamics ("EQED").

In applications of the present theory to photon physics, the nonzero electric field divergence appears as a small quantity, but it still comes out to have an essential effect on the end results [11, 12]. For the steady particle states to be treated here, the field equations contain electric field divergence terms which appear as large contributions already at the outset.

2 Basic field equations

The basic physical concept of the present theory is the appearance of a local electric charge density in the vacuum state in which there are quantum mechanical electromagnetic fluctuations. This charge density is associated with a nonzero electric field divergence. When imposing the condition of Lorentz invariance on the system, there arises a local "space-charge current density" in addition to the displacement current. The detailed deductions are described in earlier reports by the author [5–12]. The revised field equations *in the vacuum* are given by

$$\operatorname{curl}\mathbf{B}/\mu_0 = \varepsilon_0(\operatorname{div}\mathbf{E})\,\mathbf{C} + \varepsilon_0\partial\mathbf{E}/\partial t\,, \qquad (1)$$

$$\operatorname{curl}\mathbf{E} = -\partial\mathbf{B}/\partial t\,, \qquad (2)$$

$$\mathbf{B} = \operatorname{curl}\mathbf{A}\,, \qquad \operatorname{div}\mathbf{B} = 0\,, \qquad (3)$$

$$\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t\,, \qquad \operatorname{div}\mathbf{E} = \bar{\rho}/\varepsilon_0 \qquad (4)$$

for the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, the electric charge density $\bar{\rho}$, the magnetic vector potential $\mathbf{A}$, the electrostatic potential ϕ, and the velocity vector $\mathbf{C}$, where $\mathbf{C}^2 = c^2$. In analogy with the direction to be specified for the current density in conventional theory, the unit vector $\mathbf{C}/c$ depends on the geometry of the particular configuration to be studied.

Using well-known vector identities, equations (1) and (2) can be recast into the local momentum equation

$$\operatorname{div}{}^2\mathbf{S} = \bar{\rho}\,(\mathbf{E} + \mathbf{C} \times \mathbf{B}) + \varepsilon_0 \frac{\partial}{\partial t}\,\mathbf{g} \tag{5}$$

and the local energy equation

$$-\operatorname{div}\mathbf{S} = \bar{\rho}\,\mathbf{E}\cdot\mathbf{C} + \frac{1}{2}\,\varepsilon_0 \frac{\partial}{\partial t}\,w_f\,. \tag{6}$$

Here ${}^2\mathbf{S}$ is the electromagnetic stress tensor,

$$\mathbf{g} = \varepsilon_0\,\mathbf{E} \times \mathbf{B} = \frac{1}{c^2}\,\mathbf{S} \tag{7}$$

can be interpreted as an electromagnetic momentum density with $\mathbf{S}$ denoting the Poynting vector, and

$$w_f = \frac{1}{2}\left(\varepsilon_0\,\mathbf{E}^2 + \mathbf{B}^2/\mu_0\right) \tag{8}$$

representing the electromagnetic field energy density. An electromagnetic source energy density

$$w_s = \frac{1}{2}\,\bar{\rho}\,(\phi + \mathbf{C}\cdot\mathbf{A}) \tag{9}$$

can also be deduced and related to the density (8) as shown earlier [12].

As distinguished from Maxwell's equations, the present theory includes steady electromagnetic states in which all explicit time derivatives vanish in equations (1)–(6). The volume integrals of w_f and w_s then become equal for certain configurations which are limited in space.

3 Steady axisymmetric states

Among the steady axisymmetric states the analysis is here restricted to particle-shaped ones where the configuration is bounded both in the axial and radial directions. There are also string-shaped states being uniform in the axial directions, as described elsewhere [7, 12].

3.1 General features of particle-shaped states

In particle-shaped geometry a frame (r, θ, φ) of spherical coordinates is introduced, where all relevant quantities are independent of the angle φ. The analysis is further limited to a current density $\mathbf{j} = (0, 0, C\bar{\rho})$ and a vector potential $\mathbf{A} = (0, 0, A)$. Here $C = \pm\,c$ represents the two possible spin directions. The basic equations (1)–(4) then take the form

$$\frac{(r_0\,\rho)^2\,\bar{\rho}}{\varepsilon_0} = D\phi = \left[D + (\sin\theta)^{-2}\right](CA)\,, \tag{10}$$

where the dimensionless radial variable $\rho = r/r_0$ has been introduced with r_0 as a characteristic radial dimension, and where the operator $D = D_\rho + D_\theta$ is defined by

$$D_\rho = -\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial}{\partial\rho}\right), \qquad D_\theta = -\frac{\partial^2}{\partial\theta^2} - \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta}\,. \tag{11}$$

The general solution of equations (10) is obtained in terms of a *generating function*

$$F(r, \theta) = CA - \phi = G_0 \cdot G(\rho, \theta)\,, \tag{12}$$

where G_0 stands for a characteristic amplitude and G for a normalized dimensionless part. The solutions become

$$CA = -(\sin^2\theta)\,DF\,, \tag{13}$$

$$\phi = -\left[1 + (\sin^2\theta)\,D\right]F\,, \tag{14}$$

$$\bar{\rho} = -\left(\frac{\varepsilon_0}{r_0^2\,\rho^2}\right)D\left[1 + (\sin^2\theta)\,D\right]F\,. \tag{15}$$

The extra degree of freedom introduced by the nonzero electric field divergence and the inhomogeneity of equations (10) are underlying this general result.

Using expressions (13)–(15), (9), and the functions

$$f(\rho, \theta) = -(\sin\theta)\,D\left[1 + (\sin^2\theta)\,D\right]G\,, \tag{16}$$

$$g(\rho, \theta) = -\left[1 + 2(\sin^2\theta)\,D\right]G \tag{17}$$

integrated field quantities can be obtained which represent a net electric charge q_0, magnetic moment M_0, mass m_0, and angular momentum s_0. The magnetic moment is obtained from the local contributions of the current density, and the mass and angular momentum from those of w_s/c^2 and the energy relation by Einstein. The current density behaves as a common convection current. The mass flow originates from the velocity vector, having the same direction for positive and negative charge elements. Thus the integrated quantities become

$$q_0 = 2\pi\varepsilon_0\,r_0\,G_0\,J_q\,, \qquad I_q = f\,, \tag{18}$$

$$M_0 = \pi\varepsilon_0\,C\,r_0^2\,G_0\,J_M\,, \qquad I_M = \rho\,(\sin\theta)\,f\,, \tag{19}$$

$$m_0 = \pi(\varepsilon_0/c^2)\,r_0\,G_0^2\,J_m\,, \qquad I_m = fg\,, \tag{20}$$

$$s_0 = \pi(\varepsilon_0 C/c^2)\,r_0^2\,G_0^2\,J_s\,, \qquad I_s = \rho\,(\sin\theta)\,fg \tag{21}$$

with the normalized integrals

$$J_k = \int_{\rho_k}^{\infty}\int_0^{\pi} I_k\,d\rho\,d\theta\,, \qquad k = q, M, m, s\,. \tag{22}$$

Here ρ_k are small radii of circles centered around the origin $\rho = 0$ when G is divergent there, and $\rho_k = 0$ when G is convergent at $\rho = 0$.

At this point a further step is taken by restricting the analysis to a separable generating function

$$G(\rho, \theta) = R(\rho) \cdot T(\theta)\,. \tag{23}$$

The integrands of the normalized forms then become

$$I_q = \tau_0 R + \tau_1 (D_\rho R) + \tau_2 D_\rho (D_\rho R)\,, \tag{24}$$

$$I_M = \rho\,(\sin\theta) I_q\,, \tag{25}$$

$$I_m = \tau_0\tau_3 R^2 + (\tau_0\tau_4 + \tau_1\tau_3) R (D_\rho R) + \tau_1\tau_4 (D_\rho R)^2 + \\ + \tau_2\tau_3\, R D_\rho (D_\rho R) + \tau_2\tau_4 (D_\rho R)\,[D_\rho (D_\rho R)]\,, \tag{26}$$

$$I_s = \rho\,(\sin\theta) I_m\,, \tag{27}$$

where

$$\tau_0 = -(\sin\theta)(D_\theta T) - (\sin\theta) D_\theta \left[(\sin^2\theta)(D_\theta T)\right], \tag{28}$$

$$\tau_1 = -(\sin\theta) T - (\sin\theta) D_\theta \left[(\sin^2\theta) T\right] - \sin^3\theta\,(D_\theta T)\,, \tag{29}$$

$$\tau_2 = -(\sin^3\theta) T\,, \tag{30}$$

$$\tau_3 = -T - 2(\sin^2\theta)(D_\theta T)\,, \tag{31}$$

$$\tau_4 = -2(\sin^2\theta) T\,. \tag{32}$$

The restriction (23) of separability becomes useful here for configurations having sources $\bar{\rho}$ and $\mathbf{j}$ that are mainly localized to a region near the origin, such as for a particle of limited extent. The analysis further concerns a radial function R which can become convergent or divergent at the origin, and a finite polar function T with finite derivatives which can be symmetric or antisymmetric in respect to the "equatorial plane" (midplane) defined by $\theta = \pi/2$. Repeated partial integration of expressions (22) for J_q and J_M leads to the following results as described in detail elsewhere [7, 8, 12]:

- The integrated charge q_0 and magnetic moment M_0 vanish in all cases where R is convergent at the origin and T has top-bottom symmetry as well as antisymmetry in respect to the equatorial plane. These cases lead to models of electrically neutral particles, such as the neutrino;
- The charge q_0 and magnetic moment M_0 are both nonzero provided that R is divergent at the origin and T has top-bottom symmetry. This case leads to models of charged particles, such as the electron. As will be seen from the analysis to follow, the *divergence* of R can still becomes reconcilable with *finite* values of q_0, M_0, m_0, and s_0 provided that the characteristic radius r_0 is made to shrink to the very small values of a point-charge-like state, as also being supported by experimental observations.

3.2 Quantum conditions of steady states

In this analysis a simplified road is chosen by imposing relevant quantum conditions afterwards on the obtained general solutions of the field equations. This is expected to be a rather good approximation to a rigorous approach where the extended field equations are quantized from the outset. The quantized equations namely become equivalent to the original ones in which the field quantities are replaced by their expectation values according to Heitler [13].

The angular momentum (spin) condition to be imposed on a model of the electron in the capacity of a fermion particle, as well as of the neutrino, is combined with equation (21) to result in

$$s_0 = \pi\left(\varepsilon_0 C/c^2\right) r_0^2 G_0^2 J_s = \pm h/4\pi\,. \tag{33}$$

In particular, for a charged particle such as the electron, muon, tauon or their antiparticles, equations (18) and (33) combine to

$$q^* \equiv |q_0/e| = \sqrt{f_0 J_q^2/2J_s}\,, \qquad f_0 = 2\varepsilon_0 ch/e^2. \tag{34}$$

Here q^* is a dimensionless charge which is normalized with respect to the experimentally determined elementary charge "e", and $f_0 \cong 137.036$ is the inverted value of the fine-structure constant.

According to Dirac, Schwinger, and Feynman [14] the quantum condition of the magnetic moment of a charged particle such as the electron becomes

$$M_0 m_0/q_0 s_0 = 1 + \delta_M\,, \qquad \delta_M = 1/2\pi f_0\,, \tag{35}$$

which shows excellent agreement with experiments. Here the unity term of the right hand member is due to Dirac who obtained the correct Landé factor, and δ_M is a small quantum mechanical correction due to Schwinger and Feynman. Conditions (33) and (35) can also be made plausible by elementary physical arguments based on the present picture of a particle-shaped state of "self-confined" radiation [7, 12].

In a charged particle-shaped state the electric current distribution generates a total magnetic flux Γ_{tot}. Here we consider the electron to be a system having both quantized angular momentum s_0 and a quantized charge q_0. The magnetic flux should then be quantized as well, and be given by the specific values of the two quantized concepts s_0 and q_0. This leads to the relation

$$\Gamma_{tot} = |s_0/q_0|\,. \tag{36}$$

4 A model of the electron

The analysis in this section will show that finite and nonzero integrated field quantities can be obtained in terms of the shrinking characteristic radius of a point-charge-like state. This does not imply that r_0 has to become strictly equal to zero, which would end up into the unphysical situation of a structureless point.

4.1 The integrated field quantities

The generating function to be considered has the parts

$$R = \rho^{-\gamma} e^{-\rho}, \qquad \gamma > 0\,, \tag{37}$$

$$T = 1 + \sum_{\nu=1}^{n}\left\{a_{2\nu-1}\sin[(2\nu-1)\theta] + a_{2\nu}\cos(2\nu\theta)\right\}. \tag{38}$$

The radial part (37) appears at first glance to be somewhat special. Generally one could have introduced a negative power series of ρ. However, for a limited number of terms, that with the largest negative power will in any case dominate at the origin. Due to the analysis which follows the same series has further to contain one term only, with a locked special value of γ. Moreover, the exponential factor in the form (37) secures the convergence of any moment with R, but will not appear in the end result.

The radial form (37) is now inserted into the integrands (24)–(27). Then the integrals (22) take a form $J_k = J_{k\rho} J_{k\theta}$. Here $J_{k\rho}$ is a part resulting from the integration with respect to ρ, and which is dominated by terms of the strongest negative power. The part $J_{k\theta}$ further results from the integration with respect to θ. In the integrals $J_{k\rho}$ divergences appear when the lower limits ρ_k approach zero. To outbalance this, we introduce a shrinking characteristic radius

$$r_0 = c_0\,\varepsilon\,, \qquad c_0 > 0\,, \qquad 0 < \varepsilon \ll 1\,, \tag{39}$$

where ε is a dimensionless smallness parameter. The integrated field quantities (18)–(21) then become

$$q_0 = 2\pi\varepsilon_0 c_0 G_0 \left[J_{q\theta}/(\gamma-1)\right]\left(\varepsilon/\rho_q^{\gamma-1}\right), \tag{40}$$

$$M_0 m_0 = \pi^2(\varepsilon_0^2 C/c^2) c_0^3 G_0^3 \cdot \\ \cdot \left[J_{M\theta} J_{m\theta}/(\gamma-2)(2\gamma-1)\right]\left(\varepsilon^3/\rho_M^{\gamma-2}\,\rho_m^{2\gamma-1}\right), \tag{41}$$

$$s_0 = \pi(\varepsilon_0 C/c^2) c_0^2 G_0^2 \left[J_{s\theta}/2(\gamma-1)\right]\left(\varepsilon/\rho_s^{\gamma-1}\right)^2. \tag{42}$$

The reason for introducing the compound quantity $M_0 m_0$ in expression (41) is that this quantity appears as a single entity in all finally obtained relations of the present analysis. The configuration with its integrated quantities is now required to scale in such a way that the geometry is preserved by becoming independent of ρ_k and ε. Such a uniform scaling implies that

$$\rho_q = \rho_M = \rho_m = \rho_s = \varepsilon \tag{43}$$

and that the parameter γ has to approach the value 2 from above, as specified by

$$\gamma(\gamma-1) = 2 + \tilde{\delta}\,, \quad 0 \leqslant \tilde{\delta} \ll 1\,, \quad \gamma \approx 2 + \tilde{\delta}/3\,. \tag{44}$$

As a result of this

$$J_{k\theta} = \int_0^\pi I_{k\theta}\, d\theta\,, \tag{45}$$

where

$$I_{q\theta} = -2\tau_1 + 4\tau_2\,, \tag{46}$$

$$I_{M\theta}/\tilde{\delta} = (\sin\theta)(-\tau_1 + 4\tau_2)\,, \tag{47}$$

$$I_{m\theta} = \tau_0\tau_3 - 2(\tau_0\tau_4 + \tau_1\tau_3) + \\ + 4(\tau_1\tau_4 + \tau_2\tau_3) - 8\tau_2\tau_4\,, \tag{48}$$

$$I_{s\theta} = (\sin\theta)\, I_{m\theta}\,. \tag{49}$$

Then

$$q_0 = 2\pi\,\varepsilon_0\, c_0\, G_0\, A_q\,, \tag{50}$$

$$M_0\, m_0 = \pi^2(\varepsilon_0^2 C/c^2)\, c_0^3 G_0^3 A_M A_m\,, \tag{51}$$

$$s_0 = (1/2)\pi(\varepsilon_0 C/c^2)\, c_0^2 G_0^2 A_s \tag{52}$$

with $A_q \equiv J_{q\theta}$, $A_M \equiv J_{M\theta}/\tilde{\delta}$, $A_m \equiv J_{m\theta}$, and $A_s \equiv J_{s\theta}$.

The uniform scaling due to relations (39) and (43) in the range of small ε requires the characteristic radius r_0 to be very small, but does not specify its absolute value. One possibility of estimating this radius is by a crude modification of the field equations by an effect of General Relativity originating from the circulatory spin motion [7, 12]. This yields an upper limit of r_0 of about 10^{-19} meters for which this modification can be neglected.

As expressed by equations (39) and (43), the present results also have an impact on the question of Lorentz invariance of the electron radius. In the limit $\varepsilon \to 0$ the deductions will thus in a formal way satisfy such an invariance, in terms of a vanishing radius. At the same time the range of small ε becomes applicable to the physically relevant case of a very small but nonzero radius of a configuration having an internal structure.

4.2 The magnetic flux

According to equation (13) the magnetic flux function becomes

$$\Gamma = 2\pi r(\sin\theta)A = -2\pi r_0\,(G_0/c)\,\rho\,(\sin^3\theta)\,DG\,. \tag{53}$$

Making use of equations (37) and (39), it takes the form

$$\Gamma = 2\pi(c_0 G_0/C)\sin^3\theta\left\{\left[\gamma(\gamma-1)+2(\gamma-1)\rho+\rho^2\right]T - \\ - D_\theta T\right\}\left(\varepsilon/\rho^{\gamma-1}\right)e^{-\rho}\,. \tag{54}$$

To obtain a nonzero and finite magnetic flux function at the spherical surface $\rho = \varepsilon$ when γ approaches the value 2 from above, one has then to choose a corresponding dimensionless lower radius limit $\rho_\Gamma = \varepsilon$, in analogy with the condition (43).

In the further analysis a normalized flux function

$$\Psi \equiv \Gamma_{(\rho=\varepsilon,\theta)}/2\pi(c_0 G_0/C) = \sin^3\theta\,(D_\theta T - 2T) \tag{55}$$

is introduced at $\rho = \varepsilon$. A detailed study [8, 9, 12] of this function shows that there is a main magnetic flux

$$\Psi_0 = \Psi(\pi/2) \equiv A_\Gamma\,, \tag{56}$$

which intersects the equatorial plane, and that the total flux of equation (36) also includes that of two separate magnetic "islands" situated above and below the equatorial plane. As a consequence, the derivative $d\Psi/d\theta$ has two zero points at θ_1 and $\theta_2 > \theta_1$ in the range $0 \leqslant \theta \leqslant \pi/2$. These define the particular fluxes Ψ_1 in the range $0 < \theta \leqslant \theta_1$ and Ψ_2 in the range $\theta_2 < \theta < \pi/2$. The total normalized magnetic flux thus becomes

$$\Psi_{tot} = f_{\Gamma f}\,\Psi_0\,, \quad f_{\Gamma f} = [2(\Psi_1 + \Psi_2) - \Psi_0]/\Psi_0\,, \quad (57)$$

where $f_{\Gamma f} > 1$ is the *obtained* flux factor including the additional contributions from the magnetic islands.

4.3 Quantum conditions

For the angular momentum and its associated charge relation (34) the quantum condition becomes

$$q^* = \sqrt{f_0 A_q^2/A_s} \quad (58)$$

according to equations (50) and (52). The magnetic moment condition (35) further reduces to

$$A_M A_m/A_q A_s = 1 + \delta_M\,. \quad (59)$$

Combination of equations (36), (50), (52), and (56) finally yields

$$8\pi f_{\Gamma q} A_\Gamma A_q = A_s\,, \quad (60)$$

where $f_{\Gamma q}$ is the flux factor being *required* by the quantum condition. For a self-consistent solution the two flux factors of equations (57) and (60) have to become equal to a common factor $f_\Gamma = f_{\Gamma f} = f_{\Gamma q}$.

4.4 Variational analysis of the integrated charge

Since the elementary electronic charge appears to represent the smallest quantum of free charge, the question may be raised whether there is a more profound reason for such a charge to exist, possibly in terms of variational analysis. In a first attempt efforts have therefore been made to search for an extremum of the normalized charge (58), under the two subsidiary quantum conditions (59) and (60) and including Lagrange multipliers. The available variables are then the amplitudes $(a_1, a_2, a_3, \ldots)$ of the polar function (38). However, such a conventional procedure is found to be upset by difficulties. It namely applies when there are well-defined and localized points of extremum, but not when such single points are replaced by a flat plateau in parameter space.

The plateau behaviour is in fact what occurs here, and an alternative analysis is then applied in terms of an increasing number of amplitudes that are "swept" (scanned) across their entire range of variation [9, 12]. One illustration of this is presented in Fig. 1 for the first four amplitudes, and with a flux factor $f_\Gamma = 1.82$. The figure shows the behaviour of the normalized charge q^* when scanning the ranges of the remaining amplitudes a_3 and a_4. There is a steep barrier in the upper part of Fig. 1, from which q^* drops down to a flat plateau being quite close to the level $q^* = 1$ which represents the experimental value:

- A detailed analysis of the four-amplitude case clearly demonstrates the asymptotic flat plateau behaviour at large amplitudes a_3 and a_4. The self-consistent minimum values of q^* obtained along the perimeter of the plateau have been found to vary from $q^* = 0.969$ for $f_\Gamma = 1.81$ to $q^* = 1.03$ for $f_\Gamma = 1.69$. Consequently, the plateau is found to be slightly "warped", being partly below and partly above the level $q^* = 1$;

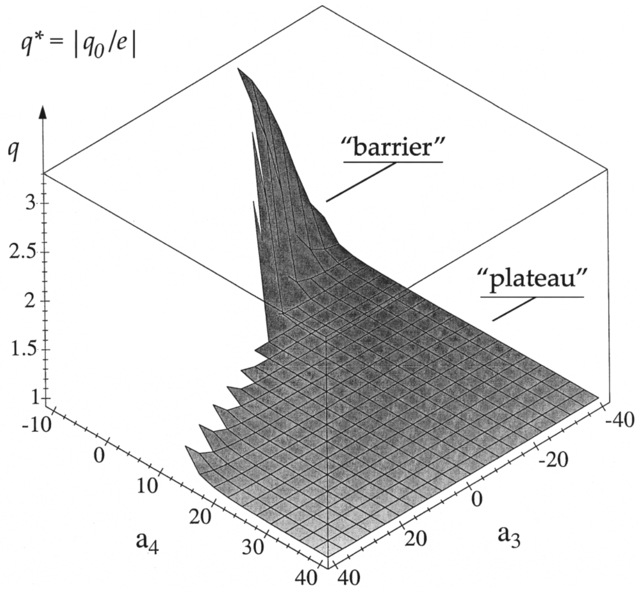

Fig. 1: The normalized electron charge $q^* \equiv |q_0/e|$ as a function of the two amplitudes a_3 and a_4 in the four amplitude case.

- For an increasing number of amplitudes beyond four there is a similar plateau behaviour, with only a slight increase in the level. This is not in conflict with the principle of the variational analysis. Any function q^* can thus have minima in the hyperspace of amplitudes at points where some of these amplitudes vanishes;
- The preserved plateau behaviour at an increasing number of amplitudes can be understood from the fact that the ratio A_q^2/A_s in equation (58) becomes a slow function of the higher "multipole" terms of the expansion (38);
- With these plateau solutions the normalized charge q^* is still left with some additional degrees of freedom. These are eliminated by the analysis of the force balance in the following subsection. There it will be shown that the lowest value of q^* obtained from the variational analysis solely does not become reconcilable with the radial force balance.

4.5 The radial force balance

The fundamental description of a charged particle in conventional theory is deficient also in respect to its radial force balance. Thus, an equilibrium cannot be maintained by the classical electrostatic force $\bar{\rho}\mathbf{E}$ in equation (5) only, but is then assumed to require forces of a nonelectromagnetic character to be present as described by Jackson [4]. In other words, the electron would otherwise "explode" under the action of its self-charge.

Turning to the present revised theory, however, there is an additional magnetic term $\bar{\rho}\,\mathbf{C}\times\mathbf{B}$ in equation (5) which under certain conditions provides the radial force balance of an equilibrium. With the already obtained results based on equations (10)–(15), the integrated radial force of the right-hand member in equation (5) becomes

$$F_r = -2\pi\varepsilon_0 G_0^2 \iint \left[DG + D\left(s^2 DG\right)\right] \cdot \left[\frac{\partial G}{\partial \rho} - \frac{1}{\rho}\, s^2 DG\right] \rho^2 s\, d\rho\, d\theta\,, \qquad (61)$$

where $s \equiv \sin\theta$. For the point-charge-like model of Sections 4.1–4.4 this force is represented by the form

$$F_r = I_+ - I_-\,, \qquad (62)$$

where I_+ and I_- are the positive and negative contributions to F_r. The results are as follows [10]:

- The ratio I_+/I_- in the plateau region of the four-amplitude case decreases from 1.27 at $q^* = 0.98$ to 0.37 at $q^* = 1.01$, thereby passing a sharply defined equilibrium point $I_+/I_- = 1$ at $q^* \cong 0.988$. The remaining degrees of freedom of this case have then been used up;
- With more than four amplitudes slightly higher values of q^* have been obtained in a corresponding plateau region. Even when there exists a force balance at higher values of q^* than that of the four-amplitude solution, the latter still corresponds to the lowest q^* for an integrated radial force balance;
- The obtained small deviation of $q^* \cong 0.988$ from the experimental value $q^* = 1$ is a remaining problem. One possible explanation could be provided by a small quantum mechanical correction of the magnetic flux condition (60), in analogy with the correction δ_M of the magnetic moment condition (59). Another possibility to be further examined is simply due to some uncertainty in the numerical calculations of a rather complex system of relations, being subject to iterations in several consecutive steps;
- The present analysis of the integrated (total) forces, performed instead of a treatment of their local parts, is in full analogy with the earlier deductions of the integrated charge, magnetic moment, mass, and angular momentum.

With the obtained radial force balance, we finally return to the radial constant c_0 of equation (39). As shown earlier [7], the mass and magnetic moment become $m_0 = K_m/c_0$ and $M_0 = K_M c_0$ where K_m and K_M include the normalized integrals A_m, A_M, and A_s. Introducing the relation $h\nu = m_0 c^2$ by Planck and Einstein and the related Compton wavelength $\lambda_C = c/\nu = h/m_0 c$ combination with $m_0 = K_m/c_0$ then yields $6\pi c_0/\lambda_C = A_m/A_s$. In the radial force balance $A_m/A_s = 1.07$. Choosing the three-fold circumference based on the radius c_0 to be equal to the Compton wavelength then results in masses of the electron, muon, and tauon which deviate by only seven percent from the experimental values. This three-fold circumference requires further investigation.

5 A model of the neutrino

The electrically neutral steady states described in Section 3.1 will now be used as a basis for models of the neutrino. Since the analysis is restricted to a steady particle-shaped configuration, it includes the concept of a nonzero rest mass. This is supported by the observed neutrino oscillations. The present neutrino models are described in detail elsewhere [7, 12, 15], and will only be outlined in this section.

5.1 A convergent generating function

A separable generating function is now adopted, having a convergent radial part R and a polar part T of top-bottom symmetry, as given by

$$R = \rho^\gamma e^{-\rho}\,, \qquad T = \sin^\alpha\theta\,, \qquad (63)$$

where $\gamma \gg \alpha \gg 1$. At increasing values of ρ the part R first reaches a maximum at $\rho = \hat{\rho} = \hat{r}/r_0 = \gamma$, after which it drops steeply to zero at large ρ. Therefore $\hat{r} = \gamma r_0$ can be taken as an effective radius of the configuration. Inserting the forms (63) into equations (24)–(32) and the integrated expressions (20)–(22) for the total mass and angular momentum, we obtain the ratio

$$J_m/J_s = 15/38\,\gamma\,. \qquad (64)$$

Combination of equations (20), (21), (64), and the quantum condition (33) then yields the mass-radius relation

$$m_0\hat{r} = m_0\gamma r_0 = 15h/152\pi c \cong 7{\times}10^{-44}\ [\mathrm{kg}{\times}\mathrm{m}]\,. \qquad (65)$$

For a case with top-bottom antisymmetry of T there is little difference as compared to the result obtained here.

5.2 A divergent generating function

We now turn to a generating function having a divergent radial part of the same form (37) as that for the electron model, and with a polar part of top-bottom antisymmetry. When $\rho = r/r_0$ increases from $\rho = 0$, the radial part decreases from a high level, down to $R = 1/e$ at $\rho = 1$, and further to very small values. Thus $\hat{r} = r_0$ can here be taken as an effective radius of the configuration.

The analysis of the radial integrals is analogous to that of the electron model. To obtain nonzero and finite values of mass m_0 and angular momentum s_0, a shrinking effective radius $\hat{r}$ and a shrinking amplitude factor G_0 are introduced through the relations

$$\hat{r} = r_0 = c_r \cdot \varepsilon \,, \qquad G_0 = c_G \cdot \varepsilon^{\beta}, \tag{66}$$

where c_r, c_G, and β are positive constants and $0 < \varepsilon \ll 1$. Expressions (20) and (21) then take the forms

$$m_0 = \pi(\varepsilon_0/c^2) c_r c_G^2 (2\gamma-1)^{-1} J_{m\theta}\big[\varepsilon^{1+2\beta}/\rho_m^{2\gamma-1}\big]\,, \tag{67}$$

$$s_0 = \pi(\varepsilon_0 C/c^2) c_r^2 c_G^2 [2\,(\gamma-1)]^{-1} J_{s\theta}\Big[\varepsilon^{2(1+\beta)}/\rho_s^{2(\gamma-1)}\Big], \tag{68}$$

where the lower limits ρ_m and ρ_s of the integrals (22) have been introduced. For nonzero and finite values of m_0 and s_0 it is then required that

$$\rho_m = \varepsilon^{(1+2\beta)/(2\gamma-1)}\,, \qquad \rho_s = \varepsilon^{(1+\beta)/(\gamma-1)}\,. \tag{69}$$

With the quantum condition (33) relations (66)–(69) further combine to

$$m_0\,\hat{r} = \frac{h}{2\pi c}\,\frac{\gamma-1}{2\gamma-1}\,(J_{m\theta}/J_{s\theta})\,\varepsilon\,. \tag{70}$$

The ratio $J_{m\theta}/J_{s\theta}$ is here expected to become a slow function of the profile shapes of $T(\theta)$ and $I_{m\theta}$, as obtained for a number of test functions for $I_{m\theta}$. An additional specific example with $\gamma = 3$ and $\beta = 3/2$ yields $\rho_m = \varepsilon^{4/5}$ and $\rho_s = \varepsilon^{5/4}$ making ρ_m and ρ_s almost linear functions of ε. In a first crude approximation relation (70) can therefore be written as

$$m_0\hat{r} \cong 2{\times}10^{-43}\,\varepsilon\;[\mathrm{kg}{\times}\mathrm{m}]\,. \tag{71}$$

5.3 Neutrino penetration into solid matter

The mass m_0 has to be reconcilable with observed data. The upper bounds of the neutrino mass are about 4.7 eV for the electron-neutrino, 170 keV for the muon-neutrino, and 18 MeV for the tauon-neutrino. Neutrinos can travel as easily through the Earth as a bullet through a bank of fog. They pass through solid matter consisting of nucleons, each having a radius $r_N \cong 6{\times}10^{-15}$ meters. Concerning the present neutrino models, there are the following options:

- With the result (65) the ratio $\hat{r}/r_N$ becomes about 10^6, 40, and 0.4 for the electron-neutrino, muon-neutrino, and the tauon-neutrino. The interaction with the electron-neutrino is then expected to take place between the short-range nucleon field as a whole and a very small part of the neutrino field. The latter field could then "heal" itself in terms of a restoring tunneling effect. Then the electron-neutrino would represent the "fog" and the nucleon the "bullet". The mean free paths of the muon- and tauon-neutrinos would on the other hand become short for this option;
- With the result (71) the corresponding values of $\hat{r}/r_N$ become about $4{\times}10^6\,\varepsilon$, $100\,\varepsilon$, and ε, respectively. Here sufficiently small values of ε would make the neutrino play the role of the "bullet" and the nucleon that of the "fog".

6 Conclusions

The present steady electromagnetic equilibria, and their applications to leptons, have no counterparts in conventional theory. The electron model, and that of the muon, tauon and corresponding antiparticles, embrace new aspects and explanations of a number of so far unsolved problems:

- To possess a nonzero electric net charge, the characteristic radius of the particle-shaped states has to shrink to that of a point-charge-like geometry. This agrees with experimental observation;
- Despite the success of the conventional renormalization procedure, physically more satisfactory ways are needed in respect to the infinite self-energy problem of a point-charge, and to the extra added counter terms by which a finite result is obtained from the difference of two "infinities". Such a situation is avoided through the present theory where the "infinity" (divergence) of the generating function is outbalanced by the "zero" of a shrinking characteristic radius;
- In the present approach the Lorentz invariance of the electron radius is formally satisfied at the limit $r_0 \to 0$. At the same time the theory includes a parameter range of small but nonzero radii being reconcilable with an internal structure;
- In contrast to conventional theory, an integrated radial force balance can be provided by the present space-charge current density which prevents the electron from "exploding" under the action of its electric self-charge. Possibly a corresponding situation may arise for the bound quarks in the interior of baryons. Here the strong force provides an equilibrium for their mutual interactions, but this does not fully explain how the individual quarks are kept in equilibrium in respect to their self-charges;
- The variational analysis results in a parameter range of the normalized charge q^* which is close to the experimental value $q^* = 1$. Within this range the remaining degrees of freedom in the analysis become exhausted when imposing the additional condition of an integrated radial force balance. This results in $q^* \cong 0.99$ which deviates by only one percent from the experimental value. The reason for the deviation is not clear at the present stage, but it should on the other hand be small enough to be regarded as an experimental support of the theory. It can also be taken as an indirect confirmation of a correctly applied value

of the Landé factor, because a change of the latter by a factor of two would result in entirely different results. Provided that the value $q^* = 1$ can be obtained after relevant correction, the elementary charge would no longer remain as an independent constant of nature, but is then derived from the velocity of light, Planck's constant, and the permittivity of the vacuum.

The steady states having a vanishing net charge also form possible models for a least some of the basic properties of the neutrino:

- A small but nonzero rest mass is in conformity with the analysis;
- The steady state includes an angular momentum, but no magnetic moment;
- Long mean free paths are predicted in solid matter, but their detailed comparison with observed data is so far an open question.

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