# Electric Charge as a Form of Imaginary Energy

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Electric charge is considered as a form of imaginary energy. With this consideration, the energy of an electrically charged particle is a complex number. The real part is proportional to the mass, while the imaginary part is proportional to the electric charge. The energy of an antiparticle is given by conjugating the energy of its corresponding particle. Newton's law of gravity and Coulomb's law of electric force are classically unified into a single expression of the interaction between the complex energies of two electrically charged particles. Interaction between real energies (or masses) is the gravitational force. Interaction between imaginary energies (or electric charges) is the electromagnetic force. Since radiation is also a form of real energy, there are another two types of interactions between real energies: the mass-radiation interaction and the radiation-radiation interaction. Calculating the work done by the mass-radiation interaction on a photon, we can derive the Einsteinian gravitational redshift. Calculating the work done by the radiation-radiation interaction on a photon, we can obtain a radiation redshift. This study suggests the electric charge as a form of imaginary energy, so that classically unifies the gravitational and electric forces and derives the Einsteinian gravitational redshift.

## 1 Introduction

It is well known that mass and electric charge are two fundamental properties (inertia and electricity) of matter, which directly determine the gravitational and electromagnetic interactions via Newton's law of gravity [1] and Coulomb's law of electric force [2]. Mass is a quantity of matter [3], and the inertia of motion is solely dependent upon the mass. According to Einstein's energy-mass expression (or Einstein's first law) [4], mass is also understood as a form of real energy. The real energy is always positive. It cannot be destroyed but can be transferred from one form to another. Therefore, the mass is understood not only based on the gravitational interaction but also on the quantity of matter, the inertia of motion, and the energy

Electric charge has two varieties of either positive or negative. It appears always in association with mass to form positive or negative electrically charged particles with different masses. The interaction between electric charges, however, is independent of the mass. Positive and negative charges can annihilate or cancel each other and produce in pair with the total electric charges conserved. So far, the electric charge is understood only based on the electromagnetic interactions. Its own physics meaning of a pure electric charge is still unclear.

In this paper, the pure electric charge is suggested to be a form of imaginary energy. With this suggestion or idea of imaginary energy, we can express an electrically charged particle as a pack of certain amount of complex energy, in which the real part is proportional to the mass and the imaginary part is proportional to the electric charge. We can combine the gravitational and electromagnetic interactions between two electrically charged particles into the interaction between their complex energies. We can also naturally obtain the energy of an antiparticle by conjugating the energy of its corresponding particle and derive the Einsteinian gravitational redshift from the mass-radiation interaction, a type of interaction between real energies.

## 2 Electric charge — a form of imaginary energy

With the idea that the electric charge is a form of imaginary energy, total energy of a particle can be generally expressed as a complex number

$$E = E^M + iE^Q, \tag{1}$$

where $i = \sqrt{-1}$ is the imaginary number. The real energy $\mathrm{Re}(E) = E^M$ is proportional to the particle mass

$$E^M = Mc^2, \tag{2}$$

while the imaginary energy $\mathrm{Im}(E) = E^Q$ is proportional to the particle electric charge defined by

$$E^Q = \frac{Q}{\sqrt{G}}c^2 = \alpha E^M, \tag{3}$$

where $G$ is the gravitational constant, $c$ is the light speed, and $\alpha$ is the charge-mass ratio (or the imaginary-real energy ratio) defined by

$$\alpha \equiv \frac{E^Q}{E^M} = \frac{Q}{\sqrt{G}M}, \tag{4}$$

in the cgs unit system. The imaginary energy has the same sign as the electric charge has. Including the electric charge, we can modify Einstein's first law as

$$E = (1 + i\alpha)Mc^2. \tag{5}$$

The modulus of the complex energy is

$$|E| = \sqrt{1+\alpha^2}Mc^2. \tag{6}$$

For an electrically charged particle, the absolute value of $\alpha$ is a big number. For instance, proton's $\alpha$ is about $10^{18}$ and electron's $\alpha$ is about $-2\times10^{21}$. Therefore, an electrically charged particle holds a large amount of imaginary energy in comparison with its real or rest energy. A neutral particle such as a neutron, photon, or neutrino has only a real energy.

## 3 Unification of Newton's law of gravity and Coulomb's law

Considering two pointlike electrically charged objects with masses $M_1$, $M_2$, electric charges $Q_1$, $Q_2$, and distance $r$, we can unify Newton's law of gravity and Coulomb's law of electric force by the following single expression of the interaction between complex energies

$$\vec{F} = -\,G\,\frac{E_1E_2}{c^4r^3}\,\vec{r}, \tag{7}$$

where $E_1$ is the energy of object one and $E_2$ is the energy of object two. Eq. (7) shows that the interaction between two particles is proportional to the product of their energies and inversely proportional to the square of the distance between them.

Replacing $E_1$ and $E_2$ by using the complex energy expression (1), we obtain

$$\begin{aligned}\vec{F} &= -\,G\,\frac{M_1M_2}{r^3}\,\vec{r} + \frac{Q_1Q_2}{r^3}\,\vec{r} - i\,\sqrt{G}\,\frac{M_1Q_2+M_2Q_1}{r^3}\,\vec{r} = \\ &= \vec{F}_{MM} + \vec{F}_{QQ} + i\vec{F}_{MQ}\,. \end{aligned} \tag{8}$$

The first term of Eq. (8) represents Newton's law for the gravitational interaction between two masses $\vec{F}_{MM}$. The second term represents Coulomb's law for the electromagnetic interaction between two electric charges $\vec{F}_{QQ}$. The third term is an imaginary force between the mass of one object and the electric charge of the other object $i\vec{F}_{MQ}$. This imaginary force is interesting and may play an essential role in adhering an electric charge on a mass or in combining an imaginary energy with a real energy. A negative imaginary force adheres a positive electric charge on a mass, while a positive imaginary force adheres a negative electric charge on a mass. Figure 1 sketches all of the interactions between two electrically charged particles as included in Eq. (8).

Electric charges have two varieties and thus three types of interactions: (1) repelling between positive electric charges $\vec{F}_{++}$, (2) repelling between negative electric charges $\vec{F}_{--}$, and (3) attracting between positive and negative electric charges $\vec{F}_{+-}$. Figure 2 shows the three types of the Coulomb interactions between two electric charges.

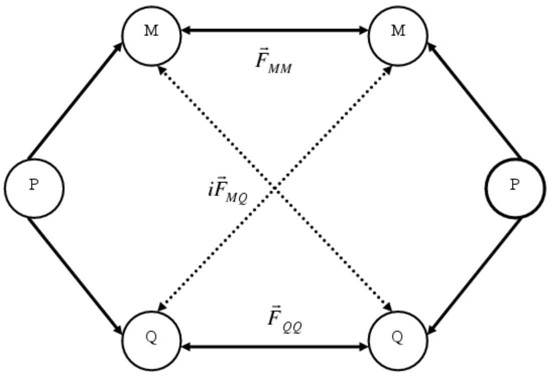

Fig. 1: Interactions between two electrically charged particles. They iclude (1) the gravitational force between masses, (2) the electric force between charges, and (3) the imaginary force between mass and charge.

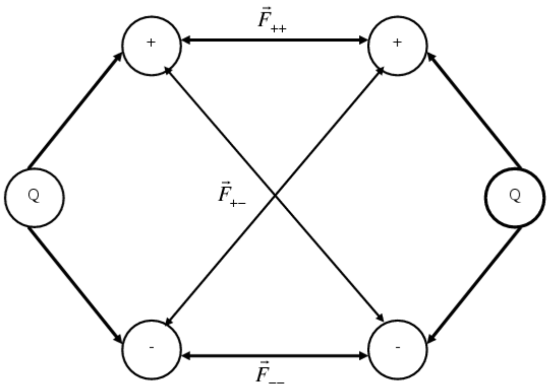

Fig. 2: Interactions between two electric charges. They include (1) repelling between two positive charges, (2) repelling between two negative charges, and (3) attraction between positive and negative charges.

## 4 Energy of antiparticles

The energy of an antiparticle [5, 6] is naturally obtained by conjugating the energy of the corresponding particle

$$E^* = \left(E^M + iE^Q\right)^* = E^M - iE^Q. \tag{9}$$

The only difference between a particle and its corresponding antiparticle is that their imaginary energies (thus their electric charges) have opposite signs. A particle and its antiparticle have the same real energy but have the sign-opposite imaginary energy.

In a particle-antiparticle annihilation process, their real energies completely transfer into radiation photon energies and their imaginary energies annihilate or cancel each other. Since there are no masses to adhere, the electric charges come together due to the electric attraction and cancel each other (or form a positive-negative electric charge pair (+,–)). In a particle-antiparticle pair production process, the radiation photon energies transfer to rest energies with a pair of imaginary energies, which combine with the rest energies to form a particle and an antiparticle.

To describe the energies of all particles and antiparticles, we can introduce a two-dimensional energy space. It is a complex space with two axes denoted by the real energy $\mathrm{Re}(E)$ and the imaginary energy $\mathrm{Im}(E)$. There are two phases in the energy space. In phase I, both real and imaginary energies are positive, while, in phase II, the imaginary energy is negative. Neutral particles including massless radiation photons are located on the real energy axis. Electrically charged particles are distributed between the real and imaginary energy axes. A particle and its antiparticle cannot be located in the same phase of the energy space.

## 5 Quantization of imaginary energy

The imaginary energy is quantized. Each electric charge quantum $e$ (the electric charge of proton) has the following imaginary energy

$$E^e = \frac{e}{\sqrt{G}}\,c^2 \sim 1.67\times 10^{15}\ \text{ergs} \sim 10^{27}\,\text{eV}, \qquad (10)$$

which is about $10^{18}$ times greater than proton's real energy (or the energy of proton's mass). Dividing the size of proton ($10^{-15}$ cm) by proton's imaginary-real energy ratio ($10^{18}$), we obtain a scale length $l_Q = 10^{33}$ cm.

On the other hand, Kaluza-Klein theory geometrically unified the four-dimensional Einsteinian general theory of relativity and Maxwellian electromagnetic theory into a five-dimensional unification theory ([7–9] for the original studies, [10] for an extensive review, and [11, 12] for the field solutions). In this unification theory, the fifth dimension is a compact (one-dimensional circle) space with radius $10^{33}$ cm [13], which is about the order of $l_Q$ obtained above. The reason why the fifth dimensional space is small and compact might be due to that the imaginary energy of an electrically charged particle is many orders of magnitude higher than its real energy. The charge is from the extra (or fifth) dimension [14], a small compact space. A pure electric charge is not measureable and is thus reasonably represented by an imaginary energy.

The imaginary energy of the electric charge quantum is about the thermal energy of the particle at a temperature $T_Q = 2E^e/k_B \sim 2.4\times 10^{31}$ °K. At this extremely high temperature, an electrically charged particle (e.g. proton) has a real energy in the same order of its imaginary energy. According to the standard big bang cosmology, the temperature at the grand unification era and earlier can be higher than about $T_Q$ [15]. To have a possible explanation for the origin of the universe (or the origin of all the matter and energy), we suggest that a large electric charge such as $10^{46}$ Coulombs ($\sim 10^{76}$ ergs) was burned out, so that a huge amount of imaginary energies transferred into real energies at the temperature $T_Q$ and above during the big bang of the universe. This suggestion gives a possible explanation for the origin of the universe from nothing to the real world in a process of transferring a large amount of imaginary energy (or electric charge) to real energy.

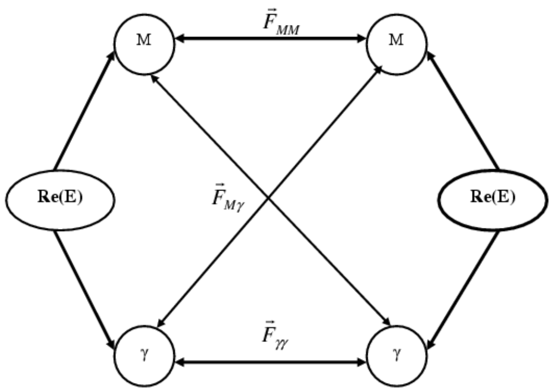

Fig. 3: Three types of gravitational interactions between real energies: (1) the mass-mass interaction, (2) the mass-radiation interaction, and (3) the radiation-radiation interaction.

## 6 Gravitational and radiation redshifts

Real energies actually have two components: matter with mass and matter without mass (i.e. radiation). The interactions between real energies may be referred as the gravitation in general. In this sense, we have three types of gravitations: (1) mass-mass interaction $\vec{F}_{MM}$, (2) mass-radiation interaction $\vec{F}_{M\gamma}$, and (3) radiation-radiation interaction $\vec{F}_{\gamma\gamma}$. Figure 3 sketches all these interactions between real energies.

The energy of a radiation photon is given by $h\nu$, where $h$ is the Planck's constant and $\nu$ is the frequency of the radiation. According to Eq. (7), the mass-radiation interaction between a mass $M$ and a photon $\gamma$ is given by

$$\vec{F} = -\,G\,\frac{Mh\nu}{c^2 r^3}\,\vec{r}, \qquad (11)$$

and the radiation-radiation interaction between two photons $\gamma_1$ and $\gamma_2$ is given by

$$\vec{F}_{\gamma\gamma} = -\,G\,\frac{(h\nu_1)(h\nu_2)}{c^4 r^3}\,\vec{r}. \qquad (12)$$

Newton's law of gravity describes the gravitational force between two masses $\vec{F}_{MM}$. The Einsteinian general theory of relativity has successfully described the effect of matter (or mass) on the space-time and thus the interaction of matter on both matter and radiation (or photon). If we appropriately introduce a radiation energy-momentum tensor into the Einstein field equation, the Einsteinian general theory of relativity can also describe the effect of radiation on the space-time and thus the interaction of radiation on both matter and radiation.

When a photon of light travels relative to an object (e.g. the Sun) from $\vec{r}$ to $\vec{r}+d\vec{r}$, it changes its energy or frequency from $\nu$ to $\nu + d\nu$. The work done on the photon by the mass-radiation interaction ($\vec{F}_{M\gamma}\cdot d\vec{r}$) is equal to the photon energy change ($hd\nu$), i.e.,

$$-G\,\frac{Mh\nu}{c^2r^2}\,dr = hd\nu\,. \tag{13}$$

Eq. (13) can be rewritten as

$$\frac{d\nu}{\nu} = -\,\frac{GM}{c^2r^2}\,dr\,. \tag{14}$$

Integrating Eq. (14) with respect to $r$ from $R$ to $\infty$ and $\nu$ from $\nu_e$ to $\nu_o$, we have

$$\ln\frac{\nu_o}{\nu_e} = -\,\frac{GM}{c^2R}\,, \tag{15}$$

where $R$ is the radius of the object, $\nu_e$ is the frequency of the light when it is emitted from the surface of the object, $\nu_o$ is the frequency of the light when it is observed by the observer at an infinite distance from the object. Then, the redshift of the light is

$$Z_G = \frac{\lambda_o-\lambda_e}{\lambda_e} = \frac{\nu_e-\nu_o}{\nu_o} = \exp\left(\frac{GM}{c^2R}\right) - 1\,. \tag{16}$$

In the weak field approximation, it reduces

$$Z_G \simeq \frac{GM}{c^2R}\,. \tag{17}$$

Therefore, calculating the work done by the mass-radiation interaction on a photon, we can derive the Einsteinian gravitational redshift in the weak field approximation.

Similarly, calculating the work done on a photon from an object by the radiation-radiation gravitation $\vec{F}_{\gamma\gamma}$, we obtain a radiation redshift,

$$Z_\gamma = \frac{4GM}{15c^5}\,\sigma A T_c^4 + \frac{G}{c^5}\,\sigma A T_s^4\,, \tag{18}$$

where $\sigma$ is the Stephan-Boltzmann constant, $A$ is the surface area, $T_c$ is the temperature at the center, $T_s$ is the temperature on the surface. Here we have assumed that the inside temperature linearly decreases from the center to the surface. The radiation redshift contains two parts. The first term is contributed by the inside radiation. The other is contributed by the outside radiation. The redshift contributed by the outside radiation is negligible because $T_s \ll T_c$.

The radiation redshift derived here is significantly small in comparison with the empirical expression of radiation redshift proposed by Finlay-Freundlich [16]. For the Sun with $T_c = 1.5{\times}10^7$ °K and $T_s = 6{\times}10^3$ °K, the radiation redshift is only about $Z_\gamma = 1.3{\times}10^{-13}$, which is much smaller than the gravitational redshift $Z_G = 2.1{\times}10^{-6}$.

## 7 Discussions and conclusions

A quark has not only the electric charge but also the color charge [17, 18]. The electric charge has two varieties (positive and negative), while the color charge has three values (red, green, and blue). Describing both electric and color charges as imaginary energies, we may unify all of the four fundamental interactions into a single expression of the interaction between complex energies. Details of the study including the color charge will be given in the next paper.

Eq. (1) does not include the self-energy — the contribution to the energy of a particle that arises from the interaction between different parts of the particle. In the nuclear physics, the self-energy of a particle has an imaginary part [19, 20]. The mass-mass, mass-charge, and charge-charge interactions between different parts of an electrically charged particle will be studied in future.

As a summary, a pure electric charge (not observable and from the extra dimension) has been suggested as a form of imaginary energy. Total energy of an electrically charged particle is a complex number. The real part is proportional to the mass, while the imaginary part is proportional to the electric charge. The energy of an antiparticle is obtained by conjugating the energy of its corresponding particle. The gravitational and electromagnetic interactions have been classically unified into a single expression of the interaction between complex energies.

The interactions between real energies are gravitational forces, categorized by the mass-mass, mass-radiation, and radiation-radiation interactions. The work done by the mass-radiation interaction on a photon derives the Einsteinian gravitational redshift, and the work done by the radiation-radiation interaction on a photon gives the radiation redshift, which is significantly small in comparison with the gravitational redshift.

The interaction between imaginary energies is electromagnetic force. Since an electrically charged particle contains many order more imaginary energy than real energy, the interaction between imaginary energies are much stronger than that between real energies.

Overall, this study develops a new physics concept for electric charges, so that classically unifies the gravitational and electric forces and derives the Einsteinian gravitational redshift.

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