On the Tidal Evolution of the Earth-Moon System: A Cosmological Model

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We have presented a cosmological model for the tidal evolution of the Earth-Moon system. We have found that the expansion of the universe has immense consequences on our local systems. The model can be compared with the present observational data. The close approach problem inflicting the known tidal theory is averted in this model. We have also shown that the astronomical and geological changes of our local systems are of the order of Hubble constant.

1 Introduction

The study of the Earth-Moon-Sun system is very important and interesting. Newton's laws of motion can be applied to such a system and good results are obtained. However, the correct theory to describe the gravitational interactions is the general theory of relativity. The theory is prominent in describing a compact system, such as neutron stars, black hole, binary pulsars, etc. Einstein theory is applied to study the evolution of the universe. We came up with some great discoveries related to the evolution of the universe. Notice that the Earth-Moon system is a relatively old system (4.5 billion years) and would have been affected by this evolution. Firstly, the model predicts the right abundance of Helium in the universe during the first few minutes after the big bang. Secondly, the model predicts that the universe is expanding and that it is permeated with some relics photons signifying a big bang nature. Despite this great triumphs, the model is infected with some troubles. It is found the age of the universe determined according to this model is shorter than the one obtained from direct observations. To resolve some of these shortcomings, we propose a model in which vacuum decays with time couples to matter. This would require the gravitational and cosmological constant to vary with time too. To our concern, we have found that the gravitational interactions in the Newtonian picture can be applied to the whole universe provided we make the necessary arrangement. First of all, we know beforehand that the temporal behavior is not manifested in the Newton law of gravitation. It is considered that gravity is static. We have found that instead of considering perturbation to the Earth-Moon system, we suggest that these effect can be modeled with having an effective coupling constant (G) in the ordinary Newton's law of gravitation. This effective coupling takes care of the perturbations that arise from the effect of other gravitational objects. At the same time the whole universe is influenced by this setting. We employ a cosmological model that describes the present universe and solves many of the cosmological problems. To our surprise, the present cosmic acceleration can be understood as a counteract due to an increasing gravitational strength. The way how expansion of the universe affects our Earth-Moon system shows up in changing the length of day, month, distance, etc. These changes are found in some biological and geological systems. In the astronomical and geological frames changes are considered in terms of tidal effects induced by the Moon on the Earth. However, tidal theory runs in some serious difficulties when the distance between Earth and Moon is extrapolated backwards. The Moon must have been too close to the Earth a situation that has not been believed to have happened in our past. This will bring the Moon into a region that will make the Moon rather unstable, and the Earth experiencing a big tide that would have melted the whole Earth. We have found that one can account for this by an alternative consideration in which expansion of the universes is the main cause.

2 Tidal theory

We know that the Earth-Moon system is governed by Kepler's laws. The rotation of the Earth in the gravity field of the Moon and Sun imposes periodicities in the gravitational potential at any point on the surface. The most obvious effect is the ocean tide which is greater than the solid Earth tide. The potential arising from the combination of the Moon's gravity and rotation with orbital angular velocity (ω_L) about the axis through the common center of mass is (Stacey, 1977 [1])

$$V = -\frac{Gm}{R'} - \frac{1}{2}\,\omega_L^2 r^2\,, \tag{1}$$

where m is the mass of the Moon, and from the figure below one has

$$\left.\begin{aligned} R'^2 &= R^2 + a^2 - 2aR\cos\psi \\ r^2 &= b^2 + a^2\sin^2\theta - 2ab\cos\psi \end{aligned}\right\}, \tag{2}$$

where $\cos\psi = \sin\theta\cos\lambda$, $b = \frac{m}{M+m}R$, while a is the Earth's radius.

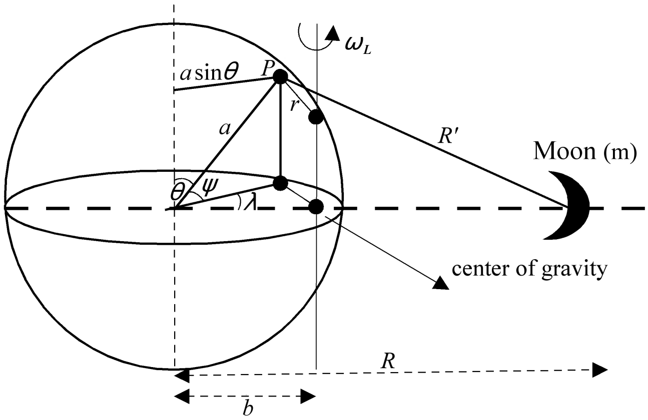

Fig. 1: The geometry of the calculation of the tidal potential of the Moon and a point P on the Earth's surface.

From Kepler's third law one finds

$$\omega_L^2 R^3 = G(M+m)\,, \tag{3}$$

where M is the Earth's mass, so that one gets for $a \ll R$

$$V = -\frac{Gm}{R}\left(1+\frac{1}{2}\frac{m}{M+m}\right) - \\ -\frac{Gma^2}{R^3}\left(\frac{3}{2}\cos\psi-\frac{1}{2}\right)-\frac{1}{2}\,\omega_L^2 a^2\sin^2\theta\,. \tag{4}$$

The first term is a constant that is due to the gravitational potential due to the Moon at the center of the Earth, with small correction arising from the mutual rotation. The second term is the second order zonal harmonics and represents a deformation of the equipotential surface to a prolate ellipsoid aligned with the Earth-Moon axis. Rotation of the Earth is responsible for the tides. We call the latter term tidal potential and define it as

$$V_2 = -\frac{Gma^2}{R^3}\left(\frac{3}{2}\cos\psi-\frac{1}{2}\right). \tag{5}$$

The third term is the rotational potential of the point P about an axis through the center of the Earth normal to the orbital plane. This does not have a tidal effect because it is associated with axial rotation and merely becomes part of the equatorial bulge of rotation. Due to the deformation an additional potential k_2V_2 (k_2 is the Love number) results, so that at the distance (R) of the Moon the form of the potential due to the tidal deformation of the Earth is

$$V_T = k_2V_2 = k_2\left(\frac{a}{R}\right)^3 = -\frac{Gma^5}{R^6}\left(\frac{3}{2}\cos\psi-\frac{1}{2}\right). \tag{6}$$

We can now identify ψ with ϕ_2: the angle between the Earth-Moon line and the axis of the tidal bulge, to obtain the tidal torque (τ) on the Moon:

$$\tau = m\left(\frac{\partial V_T}{\partial\psi}\right)_{\psi=\phi_2} = \frac{3}{2}\left(\frac{Gm^2a^5k_2}{R^6}\right)\sin 2\phi_2\,. \tag{7}$$

The torque causes an orbital acceleration of the Earth and Moon about their common center of mass; an equal and opposite torque exerted by the Moon on the tidal bulge slows the Earth's rotation. This torque must be equated with the rate of change of the orbital angular momentum (L), which is (for circular orbit)

$$L = \left(\frac{M}{M+m}\right)R^2\omega_L\,, \tag{8}$$

upon using (3) one gets

$$L = \frac{Mm}{M+m}(GR)^{\frac{1}{2}}\,, \qquad L = \frac{MmG^{\frac{2}{3}}}{(M+m)^{\frac{1}{3}}}\,\omega_L^{-\frac{1}{3}}\,. \tag{9}$$

The conservation of the total angular momentum of the Earth-Moon system (J) is a very integral part in this study. This can be described as a contribution of two terms: the first one due to Earth axial rotation ($S = C\omega$) and the second term due to the Moon orbital rotation (L). Hence, one writes

$$J = S + L = C\,\omega + \left(\frac{Mm}{M+m}\right)R^2\omega_L\,. \tag{10}$$

We remark here to the fact that of all planets in the solar system, except the Earth, the orbital angular momentum of the satellite is a small fraction of the rotational angular momentum of the planet. Differentiating the above equation with respect to time t one gets

$$\tau = \frac{dL}{dt} = \frac{L}{2R}\frac{dR}{dt} = -\frac{dS}{dt}\,. \tag{11}$$

The corresponding retardation of the axial rotation of the Earth, assuming conservation of the total angular momentum of the Earth-Moon system, is

$$\frac{d\omega}{dt} = -\frac{\tau}{C}\,, \tag{12}$$

assuming C to be constant, where C is the axial moment of inertia of the Earth and its present value is ($C_0 = 8.043\times 10^{37}$ kg m^{-3}). It is of great interest to calculate the rotational energy dissipation in the Earth-Moon system. The total energy (E) of the Earth-Moon system is the sum of three terms: the first one due to axial rotation of the Earth, the second is due to rotation of the Earth and Moon about their center of mass, and the third one is due to the mutual potential energy. Accordingly, one has

$$E = \frac{1}{2}\,C\omega^2 + \frac{1}{2}\,R^2\omega_L^2\left(\frac{Mm}{M+m}\right) - \frac{GMm}{R}\,, \tag{13}$$

and upon using (3) become

$$E = \frac{1}{2}C\omega^2 - \frac{1}{2}\frac{GMm}{R}\,. \tag{14}$$

Thus

$$\frac{dE}{dt} = C\,\omega\frac{d\omega}{dt} - \frac{1}{2}\frac{GMm}{R^2}\frac{dR}{dt}\,, \tag{15}$$

using (8), (11) and (12) one gets

$$\frac{dE}{dt} = -\,\tau(\omega-\omega_L)\,. \tag{16}$$

3 Our cosmological model

Instead of using the tidal theory described above, we rather use the ordinary Kepler's and Newton law of gravitational. We have found that the gravitation constant G can be written as (Arbab, 1997 [2])

$$G_{\rm eff} = G_0 f(t)\,, \tag{17}$$

where $f(t)$ is some time dependent function that takes care of the expansion of the universe. At the present time we have $f(t_0) = 1$. It seems as if Newton's constant changes with time. In fact, we have effects that act as if gravity changes with time. These effects could arise from any possible source (internal or external to Earth). This variation is a modeled effect due to perturbations received from distant matter. This reflects the idea of Mach who argued that distant matter affects inertia. We note here the exact function $f(t)$ is not known exactly, but we have its functional form. It is of the form $f(t) \propto t^n$, where $n > 0$ is an undetermined constant which has to be obtained from experiment (observations related to the Earth-Moon system). Unlike Dirac hypothesis in which G is a decreasing function of time, our model here suggests that G increases with time. With this prescription in hand, the forms of Kepler's and Newton's laws preserve their form and one does not require any additional potential (like those appearing in (5) and (6)) to be considered. The total effect of such a potential is incorporated in $G_{\rm eff}$. We have found recently that (Arbab, 1997 [2])

$$f(t) = \left(\frac{t}{t_0}\right)^{1.3}, \tag{18}$$

where t_0 is the present age of the universe, in order to satisfy Wells and Runcorn data (Arbab, 2004 [3]).

3.1 The Earth-Sun system

The orbital angular momentum of the Earth is given by

$$L_S = \left(\frac{M}{M+M_\odot}\right) R_E^2\,\Omega\,, \tag{19}$$

or equivalently,

$$\left.\begin{aligned} L_S &= \left(\frac{MM_\odot}{M+M_\odot}\right)(G_{\rm eff}R_E)^{\frac{1}{2}} \\ L_S &= \left(\frac{MM_\odot}{M+M_\odot}\right)^{\frac{1}{3}}\left(\frac{G_{\rm eff}^2}{\Omega}\right)^{\frac{1}{3}} \end{aligned}\right\}, \tag{20}$$

where we have replace G by $G_{\rm eff}$, and Ω is the orbital angular velocity of the Earth about the Sun. The length of the year (Y) is given by Kepler's third law as

$$Y^2 = \left(\frac{4\pi^2}{G_{\rm eff}(M_\odot+M)}\right) R_E^3\,, \tag{21}$$

where R_E is the Earth-Sun distance. We normally measure the year not in a fixed time but in terms of number of days. If the length of the day changes, the number of days in a year also changes. This induces an apparent change in the length of year. From (20) and (21) one obtains the relation

$$L_S^3 = N_1 G_{\rm eff} Y^2\,, \tag{22}$$

and

$$L_S^2 = N_2 G_{\rm eff} R_E\,, \tag{23}$$

where N_1, N_2 are some constants involving (m, M, $M_\odot$). Since the angular momentum of the Earth-Sun remains constant, one gets the relation (Arbab, 2009 [4])

$$Y = Y_0\left(\frac{G_0}{G_{\rm eff}}\right)^2, \tag{24}$$

where Y is measured in terms of days, $Y_0 = 365.24$ days. Equation (23) gives

$$R_E = R_E^0\left(\frac{G_0}{G_{\rm eff}}\right), \tag{25}$$

where $R_E^0 = 1.496\times10^{11}$ m. To preserve the length of year (in terms of seconds) we must have the relation

$$D = D_0\left(\frac{G_{\rm eff}}{G_0}\right)^2, \tag{26}$$

so that

$$Y_0 D_0 = Y D = 3.155\times10^7 \text{ s}\,. \tag{27}$$

This fact is supported by data obtained from paleontology. We know further that the length of the day is related to ω by the relation $D = \frac{2\pi}{\omega}$. This gives a relation of the angular velocity of the Earth about its self of the form

$$\omega = \omega_0\left(\frac{G_0}{G_{\rm eff}}\right)^2. \tag{28}$$

3.2 The Earth-Moon system

The orbital angular momentum of the Moon is given by

$$L = \left(\frac{M}{M+m}\right) R^2\omega_L \tag{29}$$

or,

$$\left.\begin{aligned} L &= \left(\frac{Mm}{M+m}\right)(G_{\rm eff}R)^{\frac{1}{2}} \\ L &= \left(\frac{Mm}{M+m}\right)^{\frac{1}{3}}\left(\frac{G_{\rm eff}^2}{\omega_L}\right)^{\frac{1}{3}} \end{aligned}\right\}, \tag{30}$$

where we have replace G by $G_{\rm eff}$, and ω_L is the orbital angular velocity of the Moon about the Earth. However, the length of month is not invariant as the angular momentum of the Moon has not been constant over time. It has been found found by Runcorn that the angular momentum of the Moon 370 million years ago (the Devonian era) in comparison to the present one (L_0) to be

$$\frac{L_0}{L} = 1.016 \pm 0.003\,. \tag{31}$$

The ratio of the present angular momentum of the Moon (L) to that of the Earth (S) is given by

$$\frac{L_0}{S_0} = 4.83\,, \tag{32}$$

so that the total angular momentum of the Earth-Moon system is

$$J = L + S = L_0 + S_0 = 3.4738\times10^{34}\ \text{Js}\,. \tag{33}$$

Hence, using (17) and (18), (28), (30) and (31) yield

$$\left.\begin{aligned} L &= L_0\left(\frac{t}{t_0}\right)^{0.44} \\ \omega &= \omega_0\left(\frac{t_0}{t}\right)^{2.6}, \quad \omega_L = \omega_{0L}\left(\frac{t}{t_0}\right)^{1.3} \end{aligned}\right\}, \tag{34}$$

where $t = t_0 - t_b$, t_b is the time measured from the present backward. The length of the sidereal month is given by

$$T = \frac{2\pi}{\omega_L} = T_0\left(\frac{t_0}{t}\right)^{1.3}, \tag{35}$$

where $T_0 = 27.32$ days, and the synodic month is given by the relation

$$T_{sy} = \left(\frac{T}{1-\frac{T}{Y}}\right). \tag{36}$$

We notice that, at the present time, the Earth declaration is -5.46×10^{-22} rad/s^2, or equivalently a lengthening of the day at a rate of 2 milliseconds per century. The increase in Moon mean motion is 9.968×10^{-24} rad/s^2. Hence, we found that $\dot{\omega} = -54.8\,\dot{n}$, where $n = \frac{2\pi}{\omega_L}$. The month is found to increase by 0.02788/cy. This variation can be compared with the present observational data.

From (34) one finds

$$\omega\,\omega_L^2 = \omega_0\,\omega_{0L}^2\,. \tag{37}$$

If the Earth and Moon were once in resonance then $\omega = \omega_L \equiv \omega_c$. This would mean that

$$\left.\begin{aligned} \omega_c^3 &= \omega_0\,\omega_{0L}^2 = 516.6\times10^{-18}\ (\text{rad/s})^3 \\ \omega_c &= 8.023\times10^{-6}\ \text{rad/s} \end{aligned}\right\}. \tag{38}$$

This would mean that both the length of day and month were equal. They were both equal to a value of about 9 present days. Such a period has not been possible since when the Earth was formed the month was about 14 present days and the day was 6 hours! Therefore, the Earth and Moon had never been in resonance in the past.

Using the (11) and (34) the torque on the Earth by the Moon is (Arbab, 2005 [4, 5])

$$\tau = -\frac{dL}{dt} = -\frac{dS}{dt}\,, \qquad \tau = -\,\tau_0\left(\frac{t}{t_0}\right)^{0.56}, \tag{39}$$

where $\tau_0 = 3.65\times10^{15}\,\text{N m}$. The energy dissipation in the Earth is given by

$$P = \frac{dE}{dt}\,, \quad \frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}\,C\,\omega^2 - \frac{1}{2}\,\frac{G_{\text{eff}}Mm}{R}\right), \tag{40}$$

where R, ω is given by (30) and (34).

We remark that the change in the Earth-Moon-Sun parameters is directly related to Hubble constant (H). This is evident since in our model (see Arbab, 1997 [2]) the Hubble constant varies as $H = 1.11\,t^{-1}$. Hence, one may attribute these changes to cosmic expansion. For the present epoch $t_0 \sim 10^9$ years, the variation of ω, ω_L and D is of the order of H_0 (Arbab, 2009 [4, 5]). This suggests that the cause of these parameters is the cosmic expansion.

Fossils of coral reefs studied by John Wells (Wells, 1963 [7]) revealed that the number of days in the past geologic time was bigger than now. This entails that the length of day was shorter in the past than now. The rotation of the Earth is gradually slowing down at about 2 milliseconds a century. Another method of dating that is popular with some scientists is tree-ring dating. When a tree is cut, you can study a cross-section of the trunk and determine its age. Each year of growth produces a single ring. Moreover, the width of the ring is related to environmental conditions at the time the ring was formed. It is therefore possible to know the length of day in the past from palaeontological studies of annual and daily growth rings in corals, bivalves, and stromatolite. The creation of the Moon was another factor that would later help the planet to become more habitable. When the day was shorter the Earth's spins faster. Hence, the Moon tidal force reduced the Earth's rotational winds. Thus, the Moon stabilizes the Earth rotation and the Earth became habitable. It is thus plausible to say that the Earth must have recovered very rapidly after the trauma of the Moon's formation. It was found that circadian rhythm in higher animals does not adjust to a period of less than 17–19 hours per day. Our models can give clues to the time these animals first appeared (945–1366 million years ago).

This shortening is attributed to tidal forces raised by the Moon on Earth. This results in slowing down the Earth rotation while increasing the orbital motion of the Moon. According to the tidal theory explained above we see that the tidal frictional torque $\tau \propto R^{-6}$ and the amplitude of tides is $\propto R^{-3}$. Hence, both terms have been very big in the past when R was very small. However, even if we assume the rate $\frac{dR}{dt}$ to have been constant as its value now, some billion years ago the Earth-Moon distance R would be very short. This close approach would have been catastrophic to both the Earth and the Moon. The tidal force would have been enough to melt the Earth's crust. However, there appears to be no evidence for such phenomena according to the geologic findings. This fact places the tidal theory, as it stands, in great jeopardy. This is the most embarrassing situation facing the tidal theory.

4 Velocity-dependent Inertia Model

A velocity — dependent inertial induction model is recently proposed by Ghosh (Gosh, 2000 [8]) in an attempt to surmount this difficulty. It asserts that a spinning body slows down in the vicinity of a massive object. He suggested that part of the secular retardation of the Earth's spin and of the Moon's orbital motion can be due to inertial induction by the Sun. If the Sun's influence can make a braking torque on the spinning Earth, a similar effect should be present in the case of other spinning celestial objects. This theory predicts that the angular momentum of the Earth (L'), the torque (τ'), and distance (R') vary as

$$\left.\begin{aligned} L' &= \frac{mM}{(M+m)^{\frac{1}{3}}}\, G_{\rm eff}^{\frac{2}{3}}\, \omega_L^{-\frac{1}{3}} \\ \tau' &= -\frac{L'}{3\omega_L}\, \dot{\omega}_L \\ \dot{R} &= -\frac{2}{3}\,\frac{R}{\omega_L}\, \dot{\omega}_L \end{aligned}\right\}. \tag{41}$$

The present rate of the secular retardation of the Moon angular speed is found to be $\frac{d\omega_L}{dt} \equiv \dot{\omega}_L \approx 0.27\times10^{-23}$ rad s^{-2} leaving a tidal contribution of $\approx -0.11\times10^{-23}$ rad s^{-2}. This gives a rate of $\frac{dR}{dt} \equiv \dot{R} = -0.15\times10^{-9}$ m s^{-1}. Now the apparent lunar and solar contributions amount to $\approx 2.31\times10^{-23}$ rad s^{-2} and $\approx 1.65\times10^{-23}$ rad s^{-2} respectively. The most significant result is that $\frac{dR}{dt}$ is negative and the magnitude is about one tenth of the value derived using the tidal theory only. Hence, Ghosh concluded that the Moon is actually approaching the Earth with a vary small speed, and hence there is no close-approach problem. Therefore, this will imply that the tidal dissipation must have been much lower in the Earth's early history.

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