

Physical Consequences of Mathematical Principles

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Physical consequences are derived from the following mathematical structures: the variational principle, Wigner's classifications of the irreducible representations of the Poincaré group and the duality invariance of the homogeneous Maxwell equations. The analysis is carried out within the validity domain of special relativity. Hierarchical relations between physical theories are used. Some new results are pointed out together with their comparison with experimental data. It is also predicted that a genuine Higgs particle will not be detected.

1 Introduction

Physics aims to describe processes which are observed in the real world. For this purpose, mathematical formulations of physical theories are constructed. Mathematical elements of a physical theory can be divided into three sets: elements that play a relative fundamental role and are regarded as cornerstones of the theory's structure, elements used as a derivation tool and final formulas that describe the behavior of a given system. This kind of classification is used here for the convenience of the presentation. In particular, what is regarded here as a fundamental element may, in principle, be derived from more profound mathematical elements.

This work regards the following mathematical structures as cornerstones of the discussion. The variational principle and its relevant Lagrangian density; Wigner's analysis of the irreducible representations of the Poincaré group; the duality invariance of the homogeneous Maxwell equations. Some well known results of these elements are pointed out alongside others that are not very well known. Boldface numbers are used for marking the latter kind of results. It is shown that some of these results fit experimental data whereas others are used as a prediction of yet unknown experimental data.

The discussion is carried out within a framework that is based on the following theoretical elements. First, Special Relativity is regarded as a covering theory and all expressions must be consistent with relativistic covariance. The De Broglie relation between the particle's wave properties and its energy-momentum is used. Another issue is related to the hierarchical relations between physical theories. (A good discussion of this issue can be found in [1], pp. 1–6.) The following lines explain this issue in brief.

Every physical theory applies to a limited set of processes. For example, let us take the problem of moving bodies. It is well known that physical theories yield very good predictions for the motion of planets around the sun. On the other hand, nobody expects that a physical theory be able to predict the specific motion of an eagle flying in the sky. This simple example proves that the validity of a physical theory should be

evaluated only with respect to a limited set of experiments. The set of experiments which can be explained by a physical theory is called its domain of validity. The relations between domains of validity define hierarchical relations between the corresponding theories. For example, given theories A , B and A 's domain of validity is a subset of B 's domain of validity then B 's rank is higher than that of A .

An examination of Newtonian mechanics and relativistic mechanics illustrates the notion of hierarchical relations between theories. Newtonian mechanics is good for low velocity experiments (because its predictions are consistent with the error range of measurements). On the other hand, relativistic mechanics is good even for velocities that approach the speed of light. Two conclusions can be derived from these properties of the theories: First, relativistic mechanics has a more profound basis because it is valid for all experiments where Newtonian mechanics holds *and* for many other experiments where Newtonian mechanics fails. Another aspect of the relations between Newtonian mechanics and relativistic mechanics is that Newtonian mechanics imposes constraints on the form of the low velocity limit of relativistic mechanics. Indeed, the low velocity limit of relativistic mechanics is (and must be) consistent with Newtonian formulas. Below, this kind of constraint is called *constraint imposed by a lower rank theory*. Some of the theoretical derivations included below rely on this principle.

The Lorentz metric used is diagonal and its entries are $(1, -1, -1, -1)$. Greek indices run from 0 to 3. Expressions are written in units where $\hbar = c = 1$. In this system of units there is just one dimension. Here it is taken to be that of length. Therefore, the dimension of a physical quantity is a power of length and is denoted by $[L^n]$. In particular, energy and momentum take the dimension $[L^{-1}]$. The symbol $Q_{,\mu}$ denotes the partial derivative of the quantity Q with respect to x^μ . An upper dot denotes a differentiation with respect to time.

The second section discusses quantum mechanical consequences of the variational principle. The Dirac equation is examined in the third section. The fourth section shows incon-

sistencies of the Klein-Gordon (KG) and the Higgs equations. The fifth section examines results obtained from Wigner’s classification of the irreducible representations of the Poincaré group. Consequences of the duality invariance of the homogeneous Maxwell equations together a regular charge-monopole theory are discussed in the sixth section. The seventh section contains concluding remarks.

2 The Variational Principle

This section is dedicated to the form of a quantum theory of a *massive particle*. Let us examine the pattern obtained in a two slit interference experiment. Here one finds bright and dark strips. A completely dark interference point indicates that a full anti-phase destruction takes place there. Obviously, this property should be obtained in every Lorentz frame of reference. It follows that the phase must depend on a Lorentz scalar.

The quantity which is suitable for this purpose is the action of the system. Thus, let us examine a Lagrangian density of the system and its action

$$S = \int \mathcal{L}(\psi, \psi_{,\mu}) d^4 x^\mu. \tag{1}$$

Now, if the Lagrangian density is a Lorentz scalar then also the action is a Lorentz scalar. Therefore, it is concluded that

1. A relativistically consistent quantum theory may be derived from a Lagrangian density which is a Lorentz scalar.

Another issue is related to the dimension of the quantities. The phase is an argument of an exponent. Therefore, it must be dimensionless. Thus, in the system of units used here the action is dimensionless and satisfies this requirement. It follows that

2. An acceptable Lagrangian density must have the dimension $[L^{-4}]$.

This conclusion means that the wave function ψ acquires a well defined dimension.

Remark:

The foregoing arguments indicate that if one wishes to take an alternative way for constructing a relativistically self-consistent quantum theory, then one must find another physically meaningful quantity that is a dimensionless Lorentz scalar and is suitable for taking the role of the particle’s phase. Apparently, such a quantity does not exist. If this claim is correct then the variational principle is also a necessary condition for constructing a self-consistent relativistic quantum theory.

Another point is related to the independent variables x^μ of the wave function

$$\psi(x^\mu) \tag{2}$$

which is a *single set* of four space-time coordinates. Therefore (2) cannot describe a composite particle, because such a particle requires, besides a description of the space-time location of its center of energy, additional coordinates for describing its internal structure. Therefore,

3. The wave function $\psi(x^\mu)$ describes an elementary structureless pointlike particle.

This result is consistent with the nature of an elementary classical particle (see [2], pp. 46, 47). Below it is applied as a useful criterion for evaluating experimental data.

The Lagrangian density is used here as the cornerstone of the theory. Hence, the particle’s equations of motion are the corresponding Euler-Lagrange equations (see [3], p. 14; [4], p. 16)

$$\frac{\partial}{\partial x^\mu} \frac{\partial \mathcal{L}}{\partial \frac{\partial \psi}{\partial x^\mu}} - \frac{\partial \mathcal{L}}{\partial \psi} = 0. \tag{3}$$

On this basis it is concluded that

4. The particle’s equations of motion are the Euler-Lagrange equations derived from the Lagrangian density.

Obviously, different kinds of Lagrangian density yield different equations of motion. This point is discussed later.

Another issue is the consistency of a quantum theory of a massive particle with the classical theory, where the latter provides an example of constraints imposed by a lower rank theory. The classical limit of quantum mechanics is discussed in the literature (see [5], pp. 19–21 and elsewhere; [6], pp. 25–27, 137–138).

In order to do that, the quantum theory should provide expressions for the energy and the momentum of the particle. As a matter of fact, having an appropriate expression for the energy at the system’s rest frame is enough. Indeed, a Lorentz boost guarantees that the theory provides appropriate expressions for the energy and momentum in any reference frame. Therefore, the following lines examine the construction of an expression for the energy of a massive quantum mechanical particle *in its rest frame*. For this end, let us take the Lagrangian density and construct the following second rank tensor (see [4], p. 19)

$$\mathcal{T}_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \frac{\partial \psi}{\partial x^\mu}} \frac{\partial \psi}{\partial x^\nu} - \mathcal{L} g_{\mu\nu}. \tag{4}$$

Now, density is a 0-component of a 4-vector and the same is true for energy. Hence, energy density is a (0,0) component of a second rank tensor. Moreover, like the dimension of the Lagrangian density, the dimension of $\mathcal{T}_{\mu\nu}$ of (4) is $[L^{-4}]$. This is also the dimension of energy density. Now, in quantum mechanics, the Hamiltonian is regarded as the energy operator. Thus, the entry \mathcal{T}_{00} of (4) is regarded as an expression for the Hamiltonian density

$$\mathcal{H} = \psi \frac{\partial \mathcal{L}}{\partial \psi} - \mathcal{L}. \tag{5}$$

It is explained below why an expression for density is required. Here, density properties can be readily taken from electrodynamics (see [2], pp. 73–75). Density must have the dimension $[L^{-3}]$ and be a 0-component of a 4-vector satisfying the continuity equation

$$j_{,\mu}^{\mu} = 0. \quad (6)$$

At this point, one may take either of the following alternatives:

- A. Use the Hamiltonian density \mathcal{H} together with the density expression and extract the Hamiltonian differential operator H , operating on ψ . The energy is an eigenvalue of this operator:

$$H\psi = E\psi, \quad (7)$$

Now the De Broglie relation

$$i \frac{\partial \psi}{\partial t} = E\psi, \quad (8)$$

yields the differential equation

$$i \frac{\partial \psi}{\partial t} = H\psi. \quad (9)$$

At this point one can construct a Hilbert space that includes all eigenfunctions of the Hamiltonian H .

- B. Use the expression for density as an inner product for ψ and construct an orthonormal basis for the corresponding Hilbert space. Next construct the Hamiltonian matrix. For the i, j functions of the Hilbert space basis, the Hamiltonian matrix element is

$$H_{ij} = \int \mathcal{H}(\psi_i, \psi_{i,\mu}, \psi_j, \psi_{j,\nu}) d^3x. \quad (10)$$

At this point, the Hamiltonian matrix is diagonalized and its energy eigenfunctions and eigenvalues are obtained.

Obviously, the mathematical structures of A and B are relevant to the same data. Therefore, both methods construct one and the same Hilbert space.

Equation (9) makes the following problem. As stated above, the Euler-Lagrange equation (3) is the system's equation of motion. On the other hand, (9) is *another* differential equation. Hence, the following requirement should be satisfied.

5. Requirement 1: The first order differential equation (9) should be consistent with the Euler-Lagrange equation of the theory (3).

The next two sections are devoted to two specific kinds of Lagrangian density of massive particles.

3 The Dirac field

It is shown here that the Dirac field satisfies the requirements derived above and that experimental data support the theory. The formulas are written in the standard notation [3,7].

The Dirac Lagrangian density is

$$\mathcal{L} = \bar{\psi} [\gamma^{\mu} (i\partial_{\mu} - eA_{\mu}) - m] \psi. \quad (11)$$

A variation with respect to $\bar{\psi}$ yields the corresponding Euler-Lagrange equation

$$\gamma^{\mu} (i\partial_{\mu} - eA_{\mu})\psi = m\psi. \quad (12)$$

As stated in section 2, the dimension of a Lagrangian density is $[L^{-4}]$. Therefore, the dimension of ψ is $[L^{-3/2}]$ and the Dirac 4-current

$$j^{\mu} = \bar{\psi} \gamma^{\mu} \psi \quad (13)$$

satisfies the required dimension and the continuity equation (6) (see [7], p. 9). Thus, the density is the 0-component of (13)

$$\rho_{Dirac} = \psi^{\dagger} \psi. \quad (14)$$

Substituting the Dirac Lagrangian density (11) into the general formula (5), one obtains the Dirac Hamiltonian density

$$\mathcal{H} = \psi^{\dagger} [\boldsymbol{\alpha} \cdot (-i\nabla - e\mathbf{A}) + \beta m + eV] \psi. \quad (15)$$

The density $\psi^{\dagger}\psi$ can be factored out from (15) and the expression enclosed within the square brackets is the Dirac Hamiltonian written as a differential operator. Its substitution into (9) yields the well known Dirac quantum mechanical equation

$$i \frac{\partial \psi}{\partial t} = [\boldsymbol{\alpha} \cdot (-i\nabla - e\mathbf{A}) + \beta m + eV] \psi. \quad (16)$$

It is also interesting to note that due to the linearity of the Dirac Lagrangian density (11) with respect to ψ , the Dirac Hamiltonian density (15) as well as the Dirac Hamiltonian *do not contain a derivative of ψ with respect to time*. Hence, (16) is an explicit first order differential equation. It is easily seen that (16) agrees completely with the Euler-Lagrange equation (12) of the Dirac field. It follows that Requirement 1 which is written near the end of section 2 is satisfied.

A Hilbert space can be constructed from the eigenfunctions obtained as solutions of the Dirac equation (16). Here the inner product of the Hilbert space is based on the density of the Dirac function (14). The eigenfunctions of the Hamiltonian are used for building an orthonormal basis

$$\delta_{ij} = \int \psi_i^{\dagger} \psi_j d^3x. \quad (17)$$

Now, the form of an energy eigenfunction is

$$\psi(\mathbf{x}, t) = e^{-iEt} \chi(\mathbf{x}). \quad (18)$$

This form enables a construction of a Hilbert space based on $e^{-iEt}\chi(\mathbf{x})$ (the Schrödinger picture) or on $\chi(\mathbf{x})$ (the Heisenberg picture). Here, in the Heisenberg picture, wave functions of the Hilbert space are time independent.

As is well known, the non-relativistic limit of the Dirac equation agrees with the Pauli equation of a spinning electron (see [7], pp. 10–13). Hence, in accordance with the discussion presented in the first section, the Dirac relativistic quantum mechanical equation is consistent with the constraint imposed by the lower rank theory of the non-relativistic quantum mechanical equations. A related aspect of this constraint is the density represented by the Dirac wave function (14). Indeed, in the non-relativistic limit of Dirac's density, (14) reduces to the product of the "large" components of Dirac's ψ (see [7], pp. 10–13). Hence, (14) agrees with the density of the Pauli-Schrödinger equations $\Psi^\dagger\Psi$. This agreement also proves the compatibility of the Hilbert space of the Pauli-Schrödinger equations with that of the non-relativistic limit of the Dirac equation.

Beside the satisfactory status of Dirac's theory, his equation has an extraordinary success in describing experimental results of electrons and muons in general and in atomic spectroscopy in particular. Moreover, experiments of very high energy prove that quarks are spin-1/2 particles. In particular, high energy experimental data are consistent with the *point-like* nature of electrons, muons and quarks (see [8], pp. 271, 272; [9], p. 149). Hence, the Dirac equation satisfies item 3 of section 2.

4 Lagrangian density of second order equations

This section discusses second order quantum equations of motion (denoted here by SOE) which are derived from a Lagrangian density. The presentation is analogous to that of the previous section where the Dirac equation is discussed. The analysis concentrates on terms containing the highest order derivatives. Thus, the specific form of terms containing lower order derivatives is not written explicitly and all kinds of these terms are denoted by the acronym for Low Order Terms *LOT*. Second order quantum differential equations are derived from Lagrangian densities of the following form:

$$\mathcal{L} = \phi_{,\mu}^* \phi_{,\nu} g^{\mu\nu} + LOT. \quad (19)$$

This form of the Lagrangian density is used for the KG (see [3], p. 38) and the Higgs (see [4], p. 715) fields.

Applying the Euler-Lagrange variational principle to the Lagrangian density (19) one obtains a second order differential equation that takes the following form

$$g^{\mu\nu} \partial_\mu \partial_\nu \phi = LOT. \quad (20)$$

Here, unlike the case of the Dirac field, the dimension of ϕ is L^{-1} . Hence, in order to satisfy dimensional requirements,

the expression for density must contain a derivative with respect to a coordinate. Thus, the 4-current takes the following form (see [3], p. 40; [10], p. 199)

$$j_\mu = i(\phi^* \phi_{,\mu} - \phi_{,\mu}^* \phi) + LOT \quad (21)$$

and the density is

$$\rho = i(\phi^* \dot{\phi} - \dot{\phi}^* \phi) + LOT. \quad (22)$$

The left hand side of (21) is a 4-vector. Therefore, ϕ of SOE is a Lorentz scalar.

Using the standard method (5), one finds that the Hamiltonian density takes the following form (see [3], p. 38; [10], p. 198)

$$\mathcal{H} = \dot{\phi}^* \dot{\phi} + (\nabla\phi^*) \cdot (\nabla\phi) + LOT. \quad (23)$$

An analysis of these expressions shows that, unlike the case of the Dirac equation, SOE theories encounter problems. Some of these problems are listed below.

- One cannot obtain a differential operator representing the Hamiltonian. Indeed, the highest order time derivative of the SOE density (22) is *anti-symmetric* with respect to $\dot{\phi}^*, \dot{\phi}$ whereas the corresponding term of the Hamiltonian density (23) is *symmetric* with respect to these functions (see [11], section 3, which discusses the KG equation). Hence, in the case of SOE theories, one cannot use method *A* of section 2 for constructing a Hilbert space for the system.
- The density associated with the wave function ϕ is an indispensable element of the Hilbert space. The dependence of the SOE density (22) on time-derivatives proves that a SOE Hilbert space *is built on functions of the four space-time coordinates* x^μ . Hence, SOE cannot use the Heisenberg picture where the functions of the Hilbert space are time independent $\psi_H = \psi_S(t_0)$ (see [3], p. 7).
- In the Schrödinger theory $\Psi^*\Psi$ represents density. It follows that like the case of the Dirac field, the dimension of this Ψ is $[L^{-3/2}]$. On the other hand, the dimension of the SOE function ϕ is $[L^{-1}]$. Therefore, the nonrelativistic limit of SOE theories is inconsistent with the Schrödinger theoretical structure.
- Unlike the Dirac Hamiltonian, which is independent of time-derivatives of ψ , the SOE Hamiltonian density has a term containing the *bilinear product* $\dot{\phi}^* \dot{\phi}$. Hence, it is not clear how a SOE analogue of the fundamental quantum mechanical equation (9) can be created. Moreover, it should be proved that this *first order implicit nonlinear differential equation* is consistent with the corresponding *second order explicit differential equation* (20) of SOE, as stated by requirement 1 which is formulated near the end of section 2. Without substantiating the validity of the Hamiltonian, SOE

- theories violate a constraint imposed by a lower rank theory which is explained in the lines that precede (4).
- e. Some SOE theories apply to *real* fields (see [3], p. 26; [4], p. 19 etc.). New problems arise for these kinds of physical objects. Indeed, density cannot be defined for these particles (see [12], pp. 41–43). Moreover, a massive particle may be at rest. In this case its amplitude should be independent of time. But a *real* wave function has no phase. Therefore, in the case of a motionless real particle, *the time-derivative of its wave function vanishes identically*. For this reason, its physical behavior cannot be described by a differential equation with respect to time. Thus, a real SOE particle *cannot* be described by the SOE equation of motion (20) and it cannot have a Hamiltonian.
 - f. Another problem arises for a charged SOE particle. As stated in item *a* above, this particle cannot have a differential operator representing the Hamiltonian. Hence, method *A*, discussed near (7)–(9), cannot be used for a Hilbert space construction. Moreover, the inner product of a time-dependent Hilbert space is destroyed in the case of an external charge that approaches a charged SOE particle (see [13], pp. 59–61). Hence, method *B* does not hold either. It follows that a charged SOE particle has no Hamiltonian. Therefore, a charged SOE particle does not satisfy a constraint imposed by a lower rank theory.

This discussion points out theoretical difficulties of SOE fields. The experimental side responds accordingly. Point 3 of section 2 is useful for evaluating the data. Thus, a field $\psi(x^\mu)$ used in a Lagrangian density describes *an elementary point-like particle*. It turns out that as of today, no scalar pointlike particle has been detected.

In the history of physics, the three π -mesons have been regarded as KG particles and the electrically neutral π^0 member of this triplet was regarded as a Yukawa particle, namely, a real (pseudo) scalar KG particle. However, it has already been established that π -mesons are not elementary pointlike particles but composite particles made of $q\bar{q}$ and they occupy a nonvanishing spatial volume. Thus, as of today, there is no experimental support for an SOE particle. The theoretical and experimental SOE problems mentioned above are regarded seriously here. On the basis of the foregoing analysis, it is predicted here that no genuine elementary SOE particle will be detected. A special case is the following statement: a genuine Higgs particle will not be detected.

5 Irreducible representations of the Poincaré group

The profound significance of Wigner's analysis of the irreducible representations of the Poincaré group (see [14]; [15], pp. 44–53; [16], pp. 143–150) is described by the following words: "It is difficult to overestimate the importance of

this paper, which will certainly stand as one of the great intellectual achievements of our century" (see [16], p. 149). Wigner's work shows that there are two physically relevant classes of irreducible representations of the Poincaré group. One class is characterized by a mass $m > 0$ and a spin s . The second class consists of cases where the self mass $m = 0$, the energy $E > 0$ and two values of helicity. (Helicity is the projection of the particle's spin in the direction of its momentum.) Two values of helicity $\pm s$ correspond to a spin s . Thus, each massive particle makes a basis for a specific irreducible representation that is characterized by the pair of values (m, s) . A massless particle (like the photon) has a zero self mass, a finite energy and two values of helicity (for a photon, the helicity is ± 1).

A result of this analysis is that a system that is stable for a long enough period of time is a basis for an irreducible representation of the Poincaré group (see [15], pp. 48–50). Let us take a photon. Cosmic photons are detected by measuring devices on earth after traveling in space for a very very long time, compared to the duration of an electromagnetic interaction. Therefore, photons must belong to a unique irreducible representation of the Poincaré group. This conclusion is inconsistent with the idea of Vector Meson Dominance (VMD). VMD regards the photon as a linear combination of a massless real photon *and* a massive vector meson. (For a presentation of VMD see [9], pp. 296–303; [17].)

The VMD idea has been suggested in order to explain experimental results of scattering of energetic photons on nucleons. The main points of the data are:

- i. The overall charge of a proton is $+e$ whereas the overall charge of a neutron vanishes. Therefore, charge constituents of a proton and a neutron are different.
- ii. In spite of the data of the previous item, interaction of a hard photon with a proton is nearly the same as its interaction with a neutron.

The theoretical analysis of Wigner's work shows that VMD is unacceptable. Other inconsistencies of VMD with experimental data have also been published [18]. This state of affairs means that the currently accepted Standard Model has no theoretical explanation for the photon-nucleon interaction. This point is implicitly recognized by the PACS category of VMD which does not belong to a theoretical PACS class. Thus, on July 2009, VMD is included in the class of "Other models for strong interactions". Hence, the Standard Model does not provide a theoretical explanation for the scattering data of hard photons on nucleons.

6 Duality transformations of electromagnetic fields

Electromagnetic fields travel in vacuum at the speed of light. Therefore, the associated particle, namely — the photon, is massless. For this reason, it cannot be examined in a frame where it is motionless. This result means that the argument of

point e of section 4 does not hold for electromagnetic fields. It follows that, unlike the wave function of a massive particle, electromagnetic fields can be described by a Lagrangian density that depends on *real functions*. This well known fact is another aspect of the inherent difference between massive and massless particles, which has been obtained by Wigner and discussed in the previous section.

Thus, the system consists of electromagnetic fields whose equations of motion (Maxwell equations) are derived from a Lagrangian density and charge carrying massive particles whose equation of motion (the Lorentz force) is derived from a classical Lagrangian. Below, this theory is called ordinary electrodynamics. All quantities are described by real functions. The action of the system is (see [2], p. 75)

$$S = - \int m \sqrt{1 - v^2} dt - \int A_\mu j_{(e)}^\mu d^4x - \frac{1}{16\pi} \int F_{\mu\nu} F^{\mu\nu} d^4x, \quad (24)$$

where the subscript (e) indicates that j^μ is a current of electric charges, A_μ denotes the 4-potential of the electromagnetic fields, and $F^{\mu\nu}$ is the corresponding fields tensor

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \quad (25)$$

The explicit form of this tensor is

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}. \quad (26)$$

These expressions enable one to derive Maxwell equations (see [2], pp. 78, 79 and 70, 71)

$$F_{,\nu}^{\mu\nu} = -4\pi j_{(e)}^\mu; \quad F^{*\mu\nu}_{,\nu} = 0. \quad (27)$$

Here $F^{*\mu\nu}$ is the dual tensor of $F^{\mu\nu}$

$$F^{*\mu\nu} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}. \quad (28)$$

These tensors satisfy the following relation

$$F^{*\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}, \quad (29)$$

where $\varepsilon^{\mu\nu\alpha\beta}$ is the completely antisymmetric unit tensor of the fourth rank.

The Lorentz force, which describes the motion of a charged particle, is obtained from a variation of the particle's coordinates (see [2], pp. 49–51)

$$ma_{(e)}^\mu = e F^{\mu\nu} v_\nu. \quad (30)$$

The foregoing expressions describe the well established theoretical structure of ordinary electrodynamics. Let us see the results of introducing duality transformations. Duality transformations (also called duality rotations by $\pi/2$) of electromagnetic fields take the following form (see [19], pp. 252, 551; [20], p. 1363)

$$\mathbf{E} \rightarrow \mathbf{B}, \quad \mathbf{B} \rightarrow -\mathbf{E}. \quad (31)$$

These transformations can be put into the following tensorial form

$$F^{\mu\nu} \rightarrow F^{*\mu\nu}; \quad F^{*\mu\nu} \rightarrow -F^{\mu\nu}. \quad (32)$$

An examination of the *homogeneous* Maxwell equations

$$F_{,\nu}^{\mu\nu} = 0; \quad F^{*\mu\nu}_{,\nu} = 0, \quad (33)$$

proves that they are invariant under the duality transformations (32). On the other hand, an inequality is obtained for the inhomogeneous Maxwell equation

$$F^{*\mu\nu}_{,\nu} \neq -4\pi j_{(e)}^\mu. \quad (34)$$

This problem can be settled by the introduction of the notion of magnetic monopoles (called briefly monopoles). Thus, duality transformations of the electromagnetic fields (32) are augmented by the following transformation that relates charges and monopoles

$$e \rightarrow g; \quad g \rightarrow -e, \quad (35)$$

where g denotes the monopole strength.

Two things are established at this point:

1. The theoretical foundation of ordinary electrodynamics (24), and its equations of motion (27) and (30).
2. The mathematical form of duality transformations (32) and (35).

Now, a theory for a system of monopoles and electromagnetic fields (called below monopole electrodynamics) is obtained from the application of duality transformations to ordinary electrodynamics. The action principle of this system is

$$S = - \int m \sqrt{1 - v^2} dt - \int A_{(m)\mu} j_{(m)}^\mu d^4x - \frac{1}{16\pi} \int F_{(m)\mu\nu}^* F_{(m)}^{*\mu\nu} d^4x, \quad (36)$$

where the subscript (m) denotes that the quantities pertain to monopole electrodynamics. Here the fields are derived from a 4-potential

$$F_{(m)\mu\nu}^* = A_{(m)\nu,\mu} - A_{(m)\mu,\nu}, \quad (37)$$

which is analogous to (25). Maxwell equations of monopole electrodynamics are

$$F_{(m)\mu\nu}^*{}_{,\nu} = -4\pi j_{(m)}^\mu; \quad F_{(m)}^{*\mu\nu}{}_{,\nu} = 0 \quad (38)$$

and the Lorentz force is

$$ma_{(m)}^\mu = gF_{(m)}^{*\mu\nu}v_\nu. \quad (39)$$

Thus, we have two theories for two distinct systems: ordinary electrodynamics for a system of charges and fields and monopole electrodynamics for a system of monopoles and fields. The first system does not contain monopoles and the second system does not contain charges. The problem is to find the form of a unified theory that describes the motion of charges, monopoles and fields. Below, such a theory is called a charge-monopole theory. The charge-monopole theory is a higher rank theory whose domain of validity includes those of ordinary electrodynamics and of monopole electrodynamics as well. On undertaking this assignment, one may examine two postulates:

1. Electromagnetic fields of ordinary electrodynamics are identical to electromagnetic fields of monopole electrodynamics.
2. The limit of the charge-monopole theory for a system that does not contain monopoles agrees with ordinary electrodynamics and limit of the charge-monopole theory for a system that does not contain charges agrees with monopole electrodynamics.

It turns out that these postulates are mutually contradictory.

A charge-monopole theory that relies (implicitly) on the first postulate has been published by Dirac many years ago [21, 22]. (Ramifications of Dirac monopole theory can be found in the literature [20].) This theory shows the need to define physically unfavorable irregularities along strings. Moreover, the form of its limit that applies to a system of monopoles without charges is inconsistent with the theory of monopole electrodynamics, which is derived above from the duality transformations. Therefore, it does not satisfy the constraint imposed by a lower rank theory. The present experimental situation is that in spite of a long search, there is still no confirmation of the existence of a Dirac monopole (see [23], p. 1209).

The second postulate was used for constructing a different charge-monopole electrodynamics [24, 25]. This postulate guarantees that the constraints imposed by the two lower rank theories are satisfied. Moreover, this theory does not introduce new irregularities into electrodynamics. Thus, it is called below regular charge-monopole theory. The following statements describe important results of the regular charge-monopole theory: The theory can be derived from an action principle, whose limits take the form of (24) and (36), respectively. Charges do not interact with bound fields of monopoles; monopoles do not interact with bound fields of charges; radiation fields (namely, photons) of the systems are identical and charges as well as monopoles interact with them. Another result of this theory is that the size of an elementary monopole g is a free parameter. Hence, the theory is

relieved from the huge and unphysical Dirac's monopole size $g^2 = 34.25$.

The regular charge-monopole theory is constructed on the basis of the second postulate. This point means that it is not guided by new experimental data. However, it turns out that it explains the important property of hard photon-nucleon interaction which is mentioned in the previous section. Indeed, just assume that quarks carry a monopole and postulate that the elementary monopole unit g is much larger than the electric charge e (probably $|g| \simeq 1$). This property means that photon-quark interaction depends mainly on monopoles and that the photon interaction with the quarks' electric charge is a small perturbation. Therefore, the very similar results of photon-proton and photon-neutron scattering are explained. (Note also that all baryons have a core which carries three units of magnetic charge that attracts the three valence quarks. The overall magnetic charge of a hadron vanishes.) Other kinds of experimental support for the regular charge-monopole theory have been published elsewhere [26].

7 Concluding remarks

This work is based on the main assumption of theoretical physics which states that results derived from physically relevant mathematical structures are expected to fit experimental data [27]. Three well known mathematical structures are used here: the variational principle, Wigner's analysis of the irreducible representations of the Poincaré group and duality transformations of electromagnetic fields.

The paper explains and uses three points which are either new or at least lack an adequate discussion in textbooks.

1. Constraints are imposed by a lower rank theory on properties of the corresponding limit of a higher rank theory (see a discussion in the Introduction).
2. The need to prove consistency between the Euler-Lagrange equation obtained from a Lagrangian density and the quantum mechanical equation $i\partial\psi/\partial t = H\psi$ which holds for the corresponding Hamiltonian.
3. The field function $\psi(x^\mu)$ describes an elementary pointlike particle (see the discussion near (2)).

Points 1 and 2 are useful for a theoretical evaluation of the acceptability of specific physical ideas. Point 3 is useful for finding an experimental support for these ideas.

The main results of the analysis presented in this work are as follows: Dirac equation is theoretically consistent and has an enormous experimental support. Second order quantum mechanical equations (like the Klein-Gordon and the Higgs equations) suffer from many theoretical problems and have no experimental support. (π -mesons are not pointlike, therefore, they are not genuine Klein-Gordon particles.) Real fields cannot be used for a description of massive particles. The idea of Vector Meson Dominance is inconsistent with Wigner's analysis of the irreducible representations of the Poincaré group.

Therefore, VMD is unacceptable and the Standard Model has no theoretical explanation for the data of a scattering process of an energetic photon on nucleon. Monopole theories that introduce irregularities along strings are inconsistent with point 1 of this section and have no experimental support. The regular charge monopole theory [24–26] is consistent with point 1 and has experimental support.

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